KITP Spintronnics Program Seminar April 11, 2006

Restoring lost coherence of a two-level system in a mesoscopic spin bath

Lu J. Sham
University of California, San Diego

Collaborators

Wang Yao, graduate student, graduating this year
Ren-Bao Liu, postdoc, now on faculty at
Chinese University of Hong Kong

Research funded by NSA/ARO and NSF DMR

W. Yao, Ren-Bao Liu, and L. J. Sham, cond-mat/0508441

Issues and Problem

1. Interaction between a micro object with a macro system: decoherence, control, measurement
2. Meso takes the place of macro:
   a. Macro: N→∞
   b. Meso: N large enough to assume the functions of macro but small enough for its functions to be N dep
3. Formulation: a two-level system in contact with a mesoscopic interacting spin bath.
   a. Coherence loss in a two-level state
   b. Restoration of its coherence

One electron in a III-V quantum dot

- Spin relaxation time (or longitudinal)
  - measured T_1~ 50 μs -- 20 ms
- Spin decoherence time (or transverse)
  - $T_2 > T_2^* \sim 10$ ns

GaAs gated dot (Kouwenhoven group, Marcus group), SAQD (Abstreiter group).

GaAs fluctuation dot ensemble or time average of a single dot (Steel et al.), InAs SAQD (Braun et al.), and gated dot (Lukin/Marcus group).

Focus on the decoherence problem ($T_2 << T_1$)

- Solution of the spin dynamics of a single electron in the mesoscopic bath of a dot in time scale much shorter than T_1
- How much is $T_2 > T_2^*$? How to measure T_2?
- Design of measures to restore electron spin coherence

Spin decoherence of a localized electron

Theories antecedent

- Spectral diffusion: Stochastic model of frequency fluctuation
- Born-Markov approx for low energy excitation mediation
  - Electron-nuclear hyperfine interaction
  - Phonon mediated longitudinal spin fluctuations
- Quantum treatment of electron-nuclei

- Anderson et al. RMP 53, PR 59- phonon ex, PR 62 - ind. of interact.
- De Sousa & Das Sarma, PRB 03 - n-n dipolar interaction, T_2 obtained from time dependence of spin echo
- De Sousa, Shenvi & Whaley, PRB 72, 045330 - GaAs n (I>1/2)

Semenov & Kim PRL 04, exp growth of T_2 with lowering of temperature
Golovach, Khaestskii and Loss PRL 04, $T_2 \sim 2$ T

- Merkulov, Efros & Rosen, PRB 02 - inhomogeneous broadening
- Khaetskii, Loss & Glazman, PRL 02 inhomogeneous broadeni

- Witzel, De Sousa & Das Sarma, PRB (2005)
- Shenvi, De Sousa & Whaley, PRB (2005), hyperfine, exact diagonalization of a small cluster

Decoherence process in an open system

Hamiltonian

$$\hat{H} = \sum_{\pm} |\pm\rangle \hat{H}_{\pm} \langle \pm|$$

Terms leading to T_1 removed

Quantum system $\beta|+\rangle + \alpha|-\rangle$ Initial state

spin bath $(\beta|+\rangle + \alpha|-\rangle)|J\rangle \xrightarrow{t} \beta|+\rangle|J^+\rangle + \alpha|-\rangle|J^-\rangle$

Reduced density matrix

Entanglement

$$\hat{\rho} = \begin{bmatrix} \beta\beta^* & \beta\alpha^* \\ \alpha\beta^* & \alpha\alpha^* \end{bmatrix} \rightarrow \begin{bmatrix} |\beta|^2\langle J^+|J^+\rangle & \boxed{\beta\alpha^*\langle J^+|J^-\rangle} \\ \alpha\beta^*\langle J^-|J^+\rangle & |\alpha|^2\langle J^-|J^-\rangle \end{bmatrix}$$

Ensemble average over bath state J

spontaneous decay: from ρ_{++} to ρ_{--} negligible when $T_2 << T_1$

decoherence: Coherence ρ_{+-} decays with time

No a priori stochastic assumption

Spin decoherence of an electron in a III-V quantum dot

Mechanisms of decoherence (transverse dephasing)

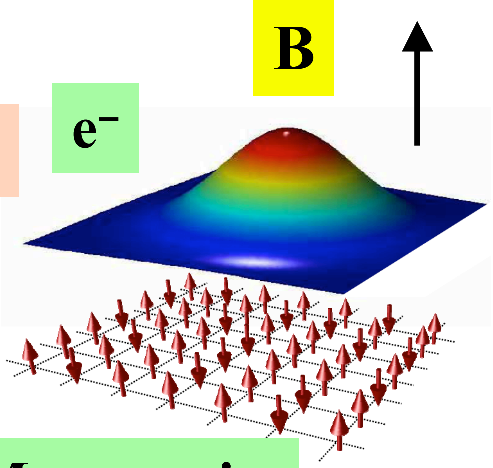
- **Electron-phonon: negligible below 1 K**
- **Decay by spontaneous emission: negligible**
- **Ultimate decoherence source, in general terms of two-level system with a spin bath**
  - **TLS - bath spin interaction**
  - **bath spin-spin interaction**

Single electron spin plus N interacting nuclear spins

Electron-nuclear hyperfine interaction: a

Longitudinal coupling ~(a/N)$S^z J^z{}_n$

cf Leggett



Transverse coupling ~(a/N) $(S^+ J^-{}_n + S^- J^+{}_n)$ contributes to negligible electron spin flips because of energy mismatch:
$\omega_e \gg T \gg \omega_N$

Mesoscopic:
N~ 10^5 - 10^6

second order perturbation →

Nuclear-nuclear spin interaction via single electron spin $A \sim a^2/(N^2\,\omega_e)$

Nuclear-nuclear dipolar interaction B~b

$\omega_e \sim a \gg \omega_N \gg b \sim A$

gen

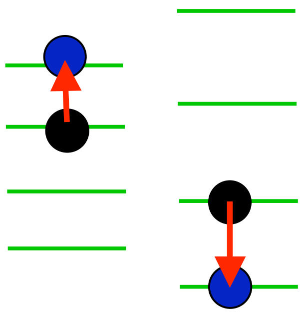
Neglect of electron spin-flip processes →

$$\hat{H} = \sum_{\pm} |\pm\rangle \hat{H}_{\pm} \langle \pm|$$

where $H_\pm$ operate on the nuclear spins

Nuclear pair flip excitations

$|m>_1\ |m'>_2 <==> |m-1>_1\ |m'+1>_2$ mapped to a pseudo-spin 1/2



Reduced Hamiltonian on nuclear pair excitation k for electron spin state $|\pm\rangle$

$$\widehat{\mathcal{H}}_k^{\pm} = \mathbf{h}_k^{\pm}\cdot\hat{\boldsymbol{\sigma}}_k + h_{kz}^{\pm}\hat{\mathsf{I}}_k$$

Pseudo-fields

$$\mathbf{h}_k^{\pm} = (\pm A_k + B_k, 0, D_k \pm E_k)$$

H-f mediated

Intrinsic n-n

Diag n-n energy

Hyperfine energy

Independent nuclear spin pair correlation approximation

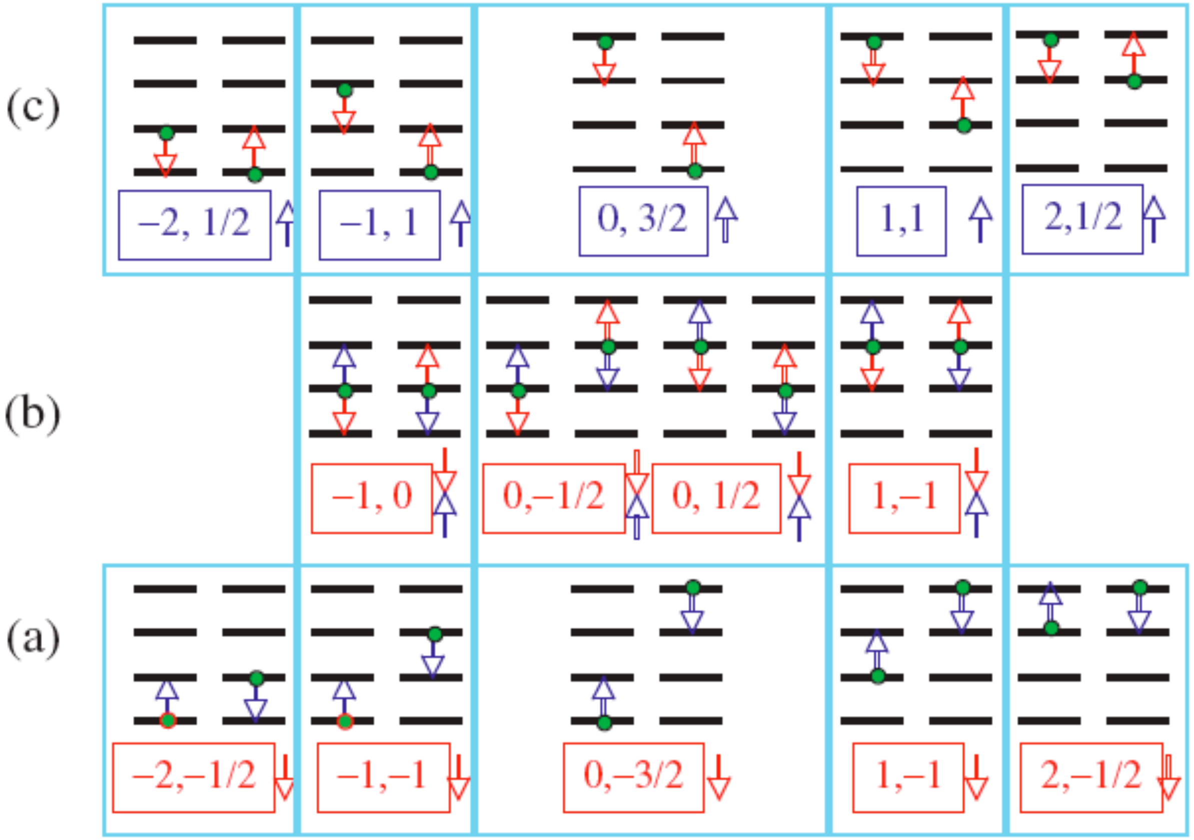
Entanglement overlap

$$\langle J_-(t)|J_+(t)\rangle = \langle J|\prod_k\left[e^{i\mathcal{H}_k^- t}e^{-i\mathcal{H}_k^+ t}\right]|J\rangle$$

Bath spin pair excitation states

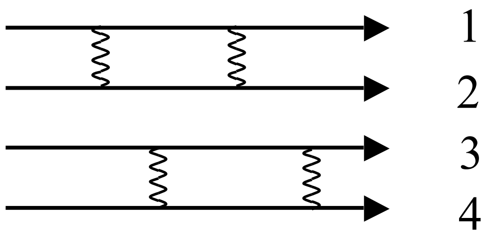
$$k_{nm} = j_n + j_m, \qquad -2I+1 \le k_{nm} \le 2I-1;$$
$$\ell_{nm} = \frac{1}{2}(j_n - j_m), \qquad -I + \frac{1}{2}|k| \le \ell_{nm} \le I - \frac{1}{2}|k| - 1.$$

$k =$	-3	-2	-1	0	1	2	3
				$\frac{3}{2}$			
			1		1		
		$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$	
$\ell =$	0		0		0		0
		$-\frac{1}{2}$		$-\frac{1}{2}$		$-\frac{1}{2}$	
			-1		-1		
				$-\frac{3}{2}$			

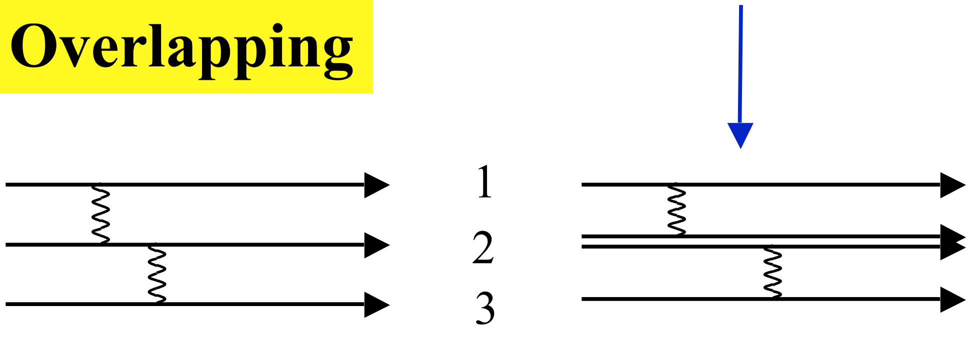


Justification of independent pair correlation approximation

Non-overlapping



Overlapping



Error estimate of higher order correlations negligible if

$$\max\left[\sqrt{N}, a/(b^2\omega_e^4)^{1/6}\right] \ll N \ll a/b$$

meso

statistics

induced n-n interaction

direct n-n interaction

at time t ~ T_2 << 1/b

- **Short time behavior in agreement with perturbation theory of nuclear pair flip interaction.**

Solution for the electron spin coherence

Initial state $\rho^{e} \otimes \rho^{N} \longrightarrow \rho(t)$

Spin coherence at time t $\rho^{e}_{+-} = \langle + | \,\mathrm{Tr}_{N}\,[\rho(t)]\, | - \rangle$

$$\rho^{\mathrm{e}}_{\mu,\nu}(t) = \mathcal{L}_{\mu,\nu}(t)\,\rho^{\mathrm{e}}_{\mu,\nu}(0)$$

NB: the superoperator replaced by a multiplicative factor in t

$$\mathcal{L}_{+,-}(t) = \langle J_{-}(t) | J_{+}(t) \rangle \sum_{J} P_{J} e^{-i(\Omega_{\mathrm{e}}+\mathcal{E}_{J})t}$$

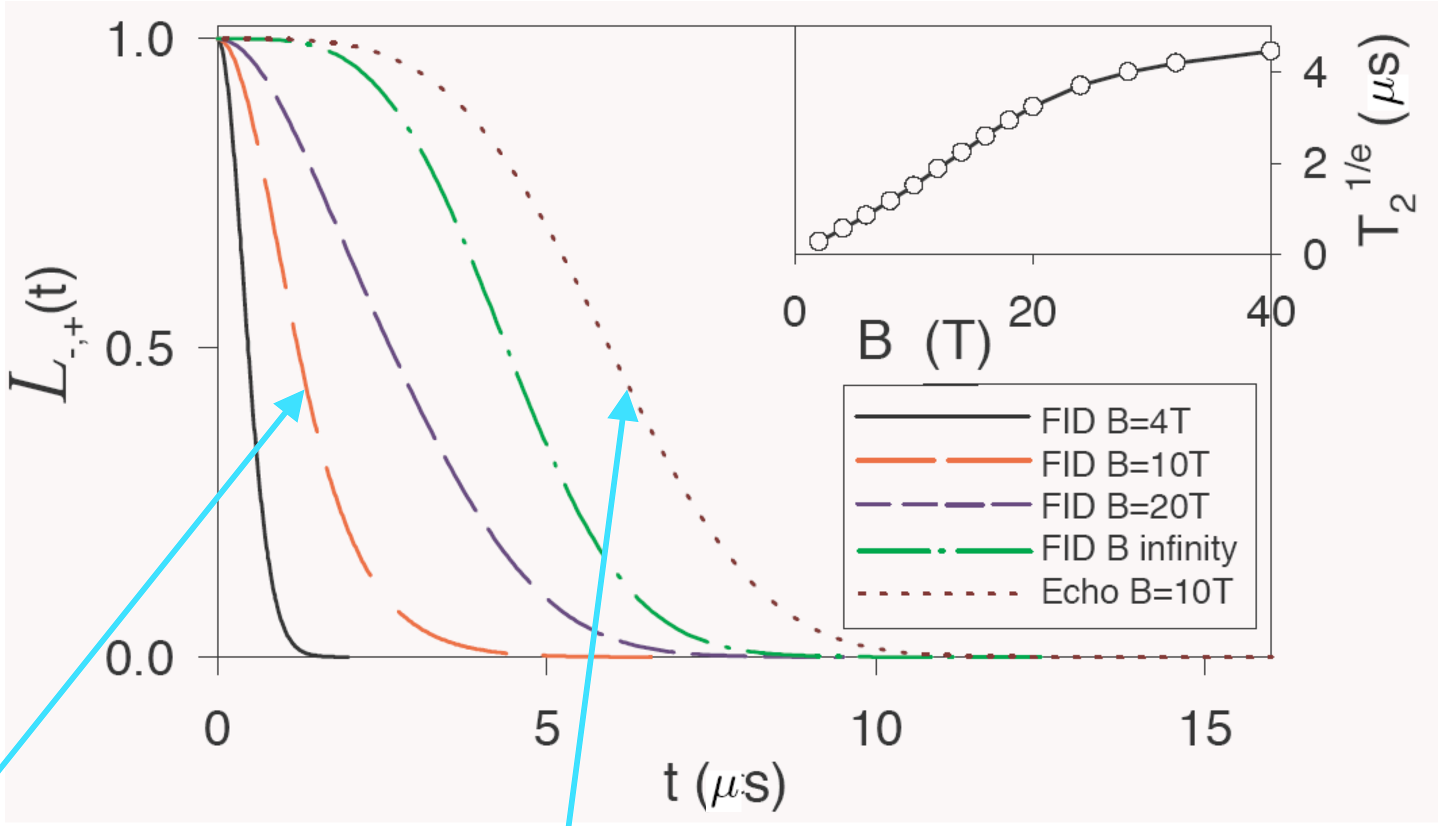
Entanglement overlap of collective nuclear states when el spin is - or +

Inhomogeneous broadening

Origin of stat distribution: ensemble or repeated meas.

Free induction decay for a single bath state

GaAs dot (h=3 nm x r=15 nm)



FID decoherence time ≠ Spin echo decay time

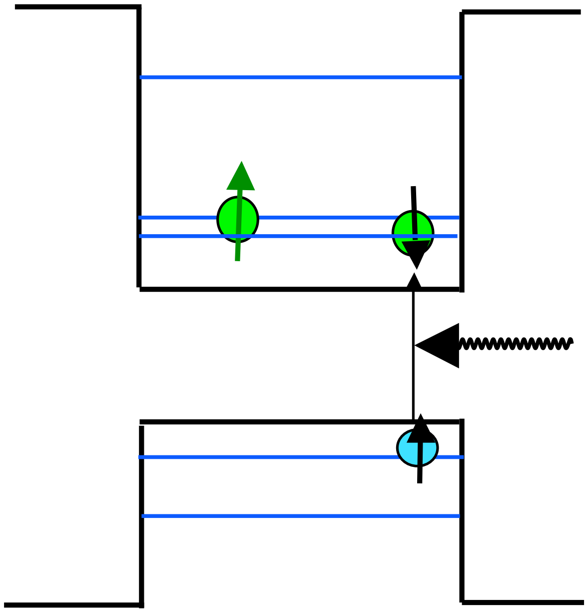
Electron spin qubit and single qubit gate

Single particle levels

Multi-particle states

Spin State to Trion

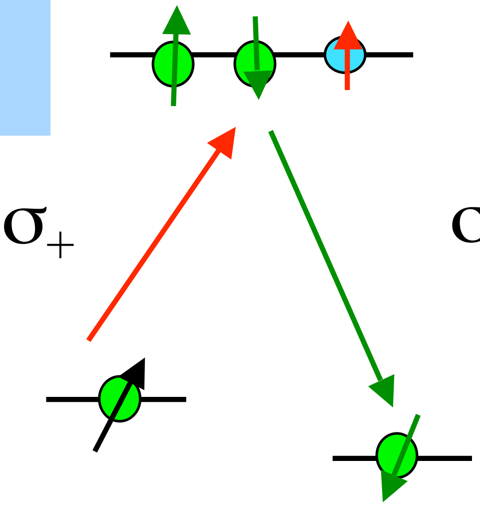
Adiabatic NR Raman Spin-flip Process



Trion

Magnetic field in x

σ_+



σ_+

Theo est. op time ~< 10 ps

Pochung Chen, C. Piermarocchi, L.J. Sham, D. Gammon, and D.G. Steel, PRB **69**, 075320 (2004).

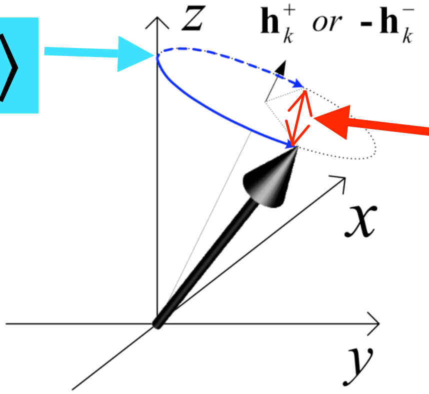
Decoherence sources: optical & spin

Physical picture of causes of decoherence

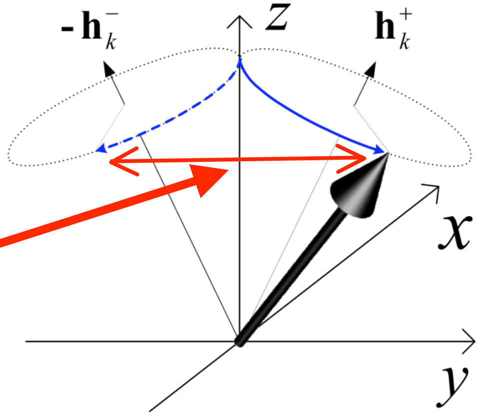
Hyperfine-mediated interaction

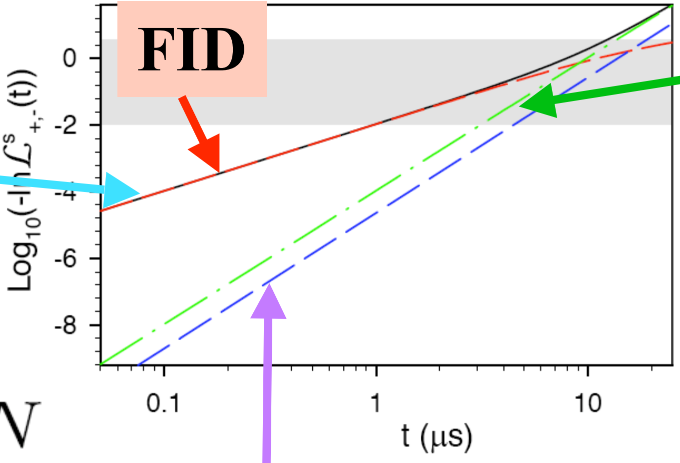
Intrinsic n-n interaction

$|J\rangle$



Spin coherence decreases as e-n entanglement increases





FID

exp[- $(t/T_{2,B})^4$]

exp[- $(t/T_{2,A})^2$]

$$T_{2,B} \approx b^{-1/2}\mathcal{A}^{-1/2}N^{1/4}$$

n-n interaction

$$T_{2,A} \approx \Omega_e \mathcal{A}^{-2} N$$

hyperfine

Spin echo: removes HMI but only reduces n-n i

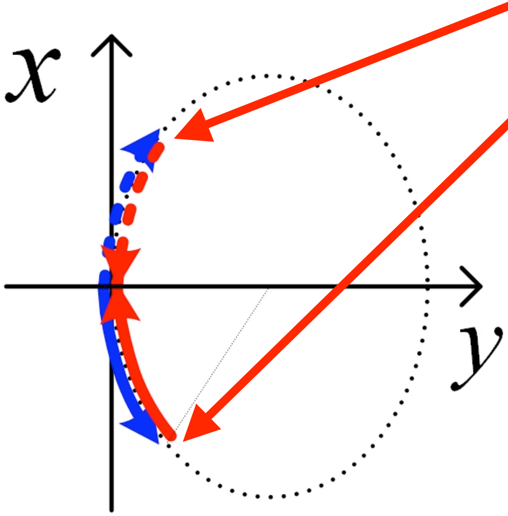
Dot: d=6.2 nm, r=25 nm, B=12 T

2 pulses on electron and effects on nuclear spin dynamics

Hyperfine-mediated interaction

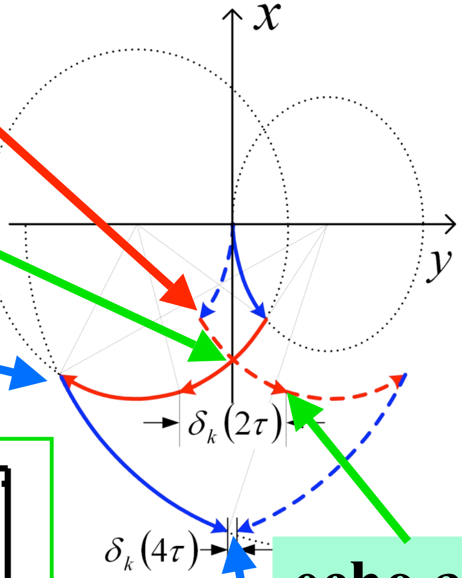
Intrinsic n-n interaction

π pulse at τ



dis-entanglement at (√2)τ

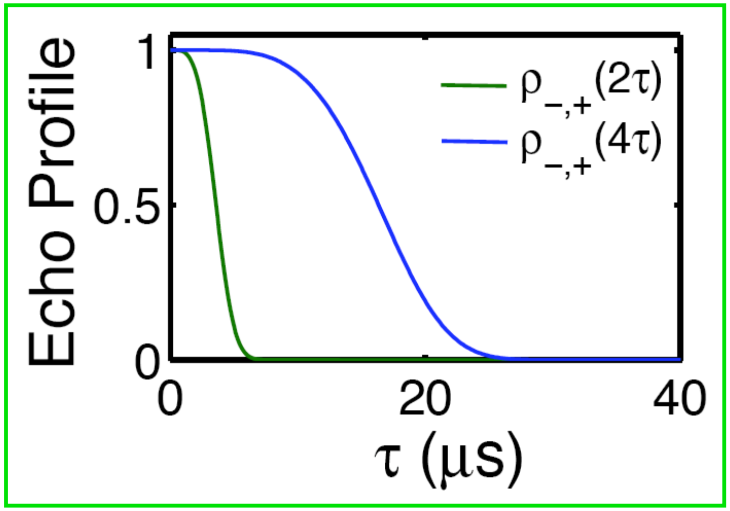
CP: π pulse at 3τ



echo at 2τ

echo at 4τ

No N state prep!

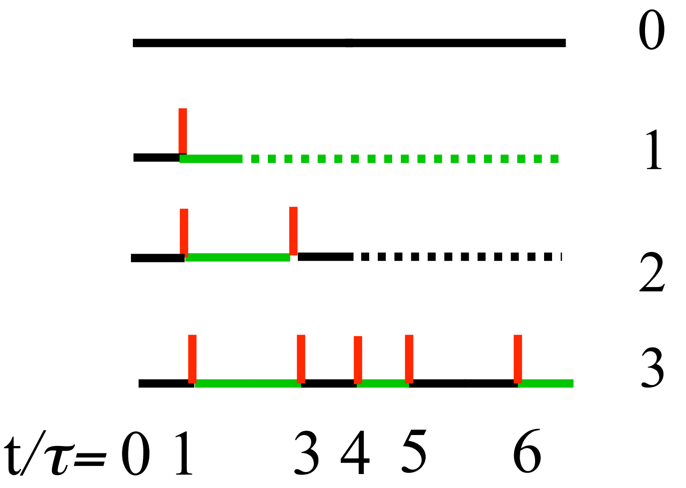


Concatenated pulses for control of entanglement

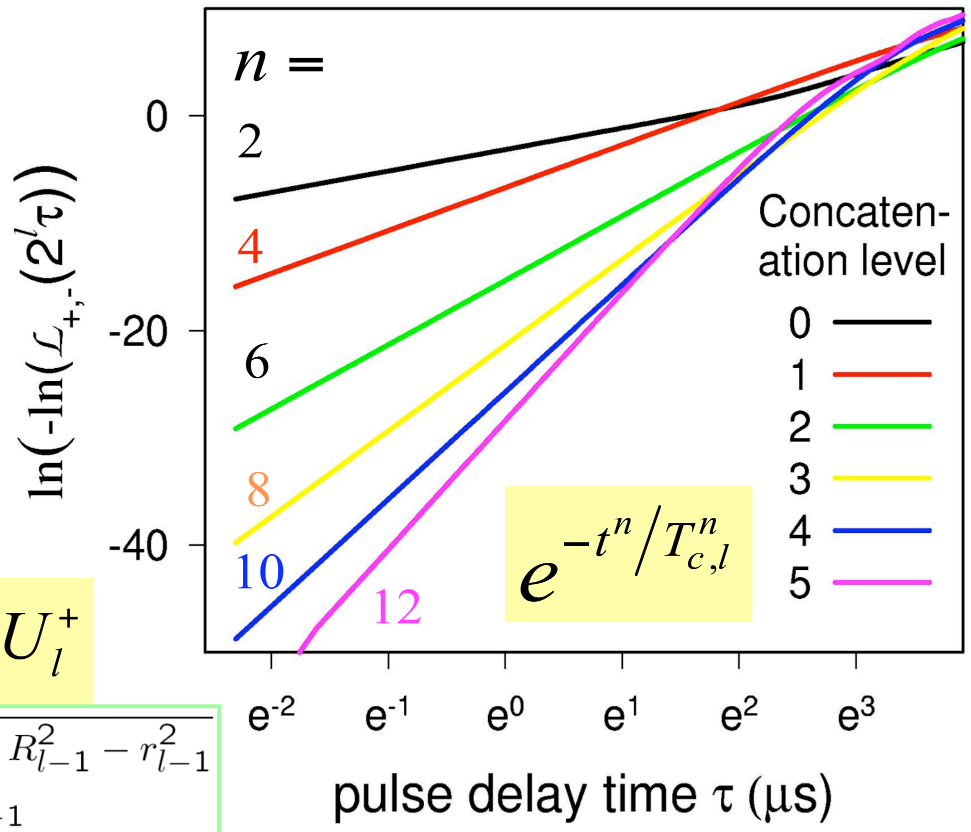
Concatenated dynamical decoupling Khojasteh & Lidar, PRL 95, 180501 (2005)

Adopt concatenation idea but not to decouple the e-n interaction

π-pulse sequences for concatenation level l =



$L_{[001]} = 5.7$ nm, $r_0 = 20$ nm, $B_{ext} = 10$ T



$$U_0^+ \equiv \exp\left(-iH^+\tau\right), \quad U_{l+1}^+ = U_l^- U_l^+$$

Rot $\sin\theta_l^{\pm}\hat{n}_l^{\pm} \equiv \mathbf{R}_l \pm \mathbf{r}_l$

$$\begin{aligned} \mathbf{R}_l &= 2\mathbf{R}_{l-1}\sqrt{1 - R_{l-1}^2 - r_{l-1}^2} \\ \mathbf{r}_l &= 2\mathbf{R}_{l-1} \times \mathbf{r}_{l-1} \end{aligned}$$

Spin qubit decoherence by nuclear spins

- Quantum solution by method of the bath spin correlation → decoherence by entanglement
- Meso bath + bath spin interaction → a swap of evolution paths of bath when the qubit spin is flipped.
- Spin echo, CPMG and concatenated pulses, alt. to dynamical decoupling and complementary to
  - decoherence free space
  - quantum error correction codes
- Mesoscopic spin bath model may be applied to other systems