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Tunneling between 2DEG electron layers:anomalous sensitivity to spin-orbit coupling

with V. Zyuzin and M. Raikh, *cond-mat/0605019*

Optical conductivity of 2DEG with spin-orbit coupling

with A.-Kh. Farid, *cond-mat/0603058*

KITP Santa Barbara, May 4 2006

Outline

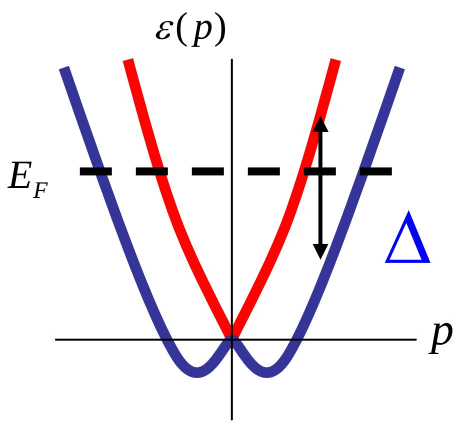
- How to determine spin-orbit coupling when it is obscured by disorder?

- Tunneling in double-layer structures:
  clean layers
  diffusive layers

- Key ingredient: correlated disorder

- I-V curves and their robustness with respect to e-e interactions

Motivation



Spin-orbit splitting of bands

$$H_{SO} = \alpha(s_x p_y - s_y p_x)$$

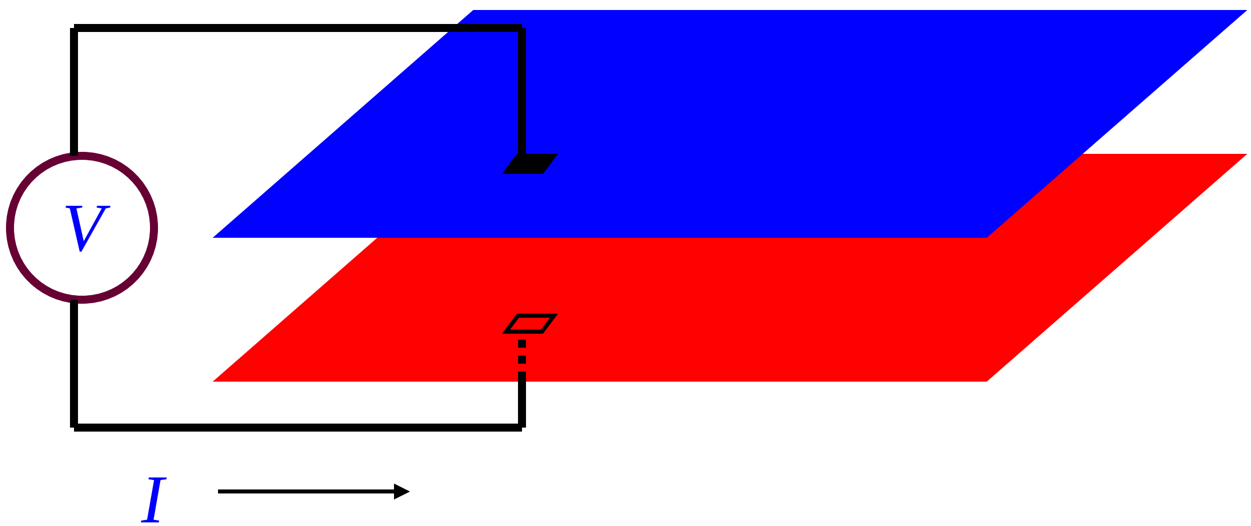
$$\varepsilon(p) = \frac{p^2}{2m} \pm \Delta, \quad \Delta \approx \alpha p_F$$

How to measure SO splitting Δ ?

Large splitting $\Delta \gg 1/\tau$ $\longrightarrow$ SdH oscillations

Small splitting $\Delta \ll 1/\tau$ $\longrightarrow$ *anomalous sensitivity to SO coupling?*

Motivation: search for anomalous sensitivity



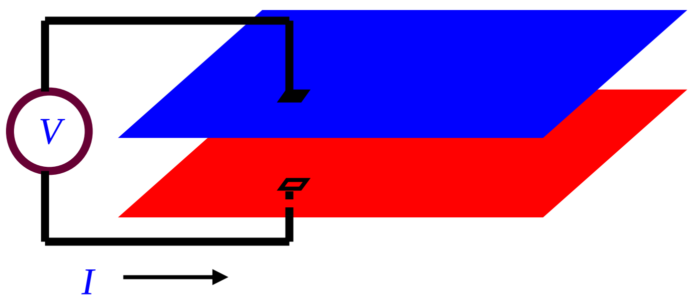
Interlayer tunneling

Weak splittings $\Delta \ll 1/\tau$ ⟶ will be resolved

If the disorder in the two layers is correlated!

Large splitting $\Delta \gg 1/\tau$ ⟶ will not be resolved!

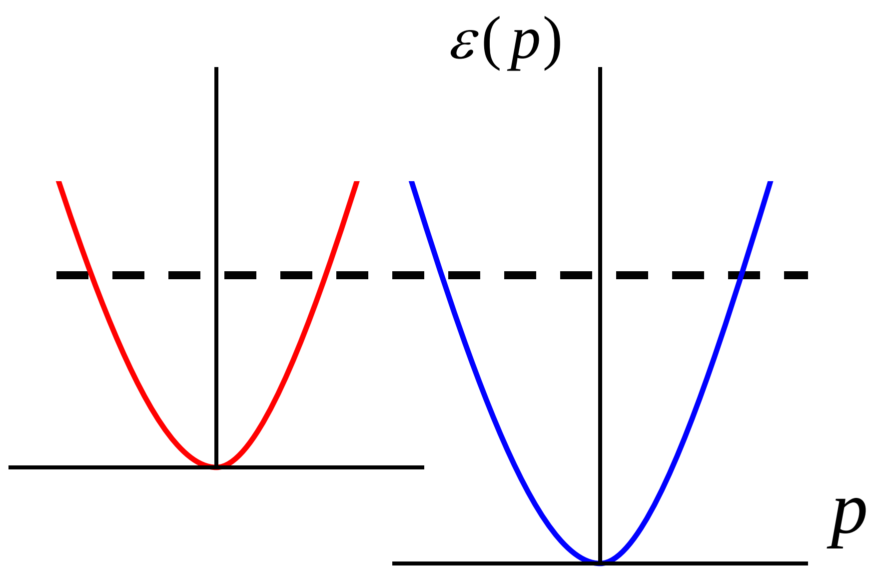
Introduction: tunneling in the absence of SO



Coherent tunneling:

$$H_T = t\int d^2x\, \psi_1^\dagger(\vec{x})\psi_2(\vec{x}) + \text{c.c}$$

$$= t\sum_{\vec{p}} a_1^\dagger(\vec{p})a_2(\vec{p}) + \text{c.c}$$



Momentum ***and*** energy conservation forbidden!

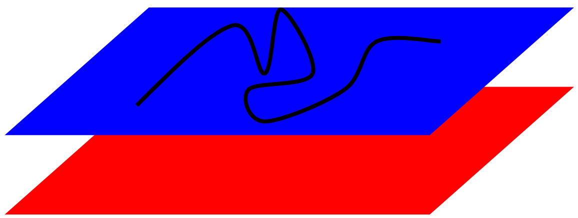
$$I(V) \sim V\delta(V)$$

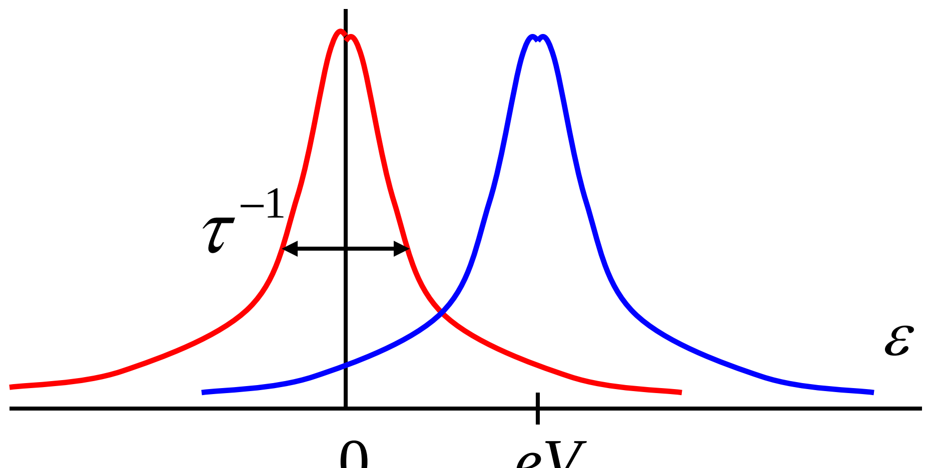
Zheng and MacDonald, '93

Tunneling in the presence of disorder

Plane waves corresponding to different energies are orthogonal

$$I(V) \sim t^2 \int_0^{eV} d\varepsilon \int d^2 p \, A_1(\varepsilon, \vec{p}) A_2(\varepsilon - eV, \vec{p})$$





$$I(V) \sim \frac{V\tau^{-1}}{(eV)^2 + \tau^{-2}}$$

Uncorrelated disorder opens up finite tunneling

Correlated disorder

Correlations of disorder potential in the two layers suppress tunneling and narrows *I-V* curves

Completely correlated disorder

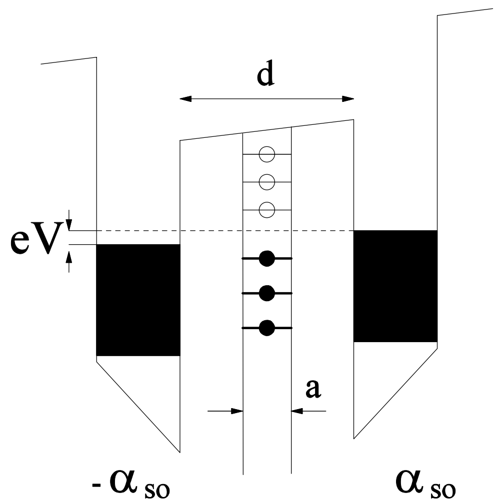
Eigenstates corresponding to different energies are orthogonal

$$I(V) \sim V\delta(V)$$

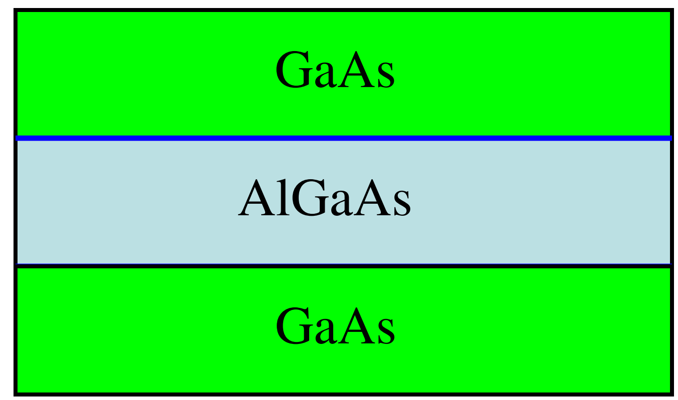
Zheng and MacDonald, ‘93

Role of spin-orbit coupling

SO coupling will break orthogonality, if it is different in two layers



$$H_{SO} \sim \nabla U \cdot \vec{s} \times \vec{p}$$

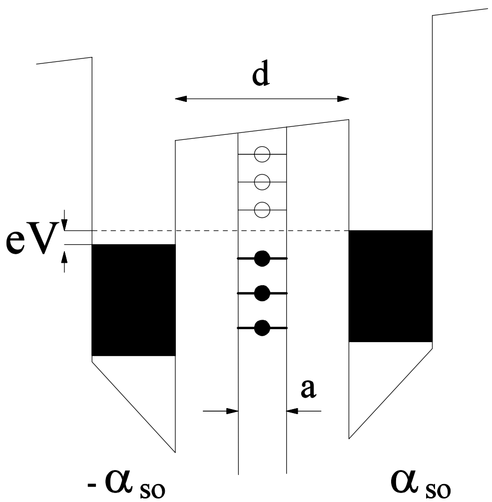


SO couplings are opposite for a symmetric structure

$$H_{\rm so}^{(L)} = \alpha_{\rm so}(\mathrm{p} \times \sigma)_z \qquad H_{\rm so}^{(R)} = -\alpha_{\rm so}(\mathrm{p} \times \sigma)_z$$

The model

Dopants are randomly distributed inside the δ-layer of size a



Correlators of disorder potentials:

$$S_{AB} = \langle V_A(x) V_B(0) \rangle$$

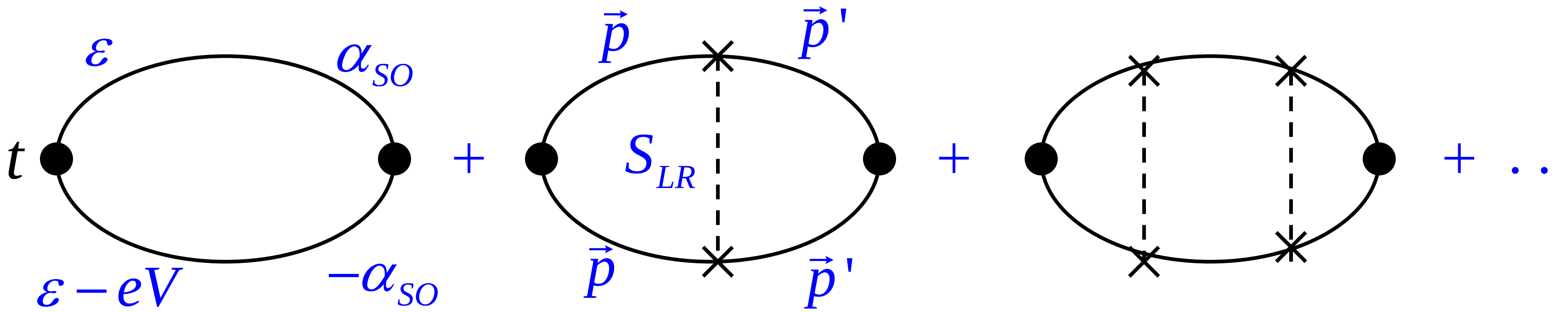
$$S_{LR}(q) = 2\pi\nu N_d \left| U(q) \right|^2 e^{-qd}$$

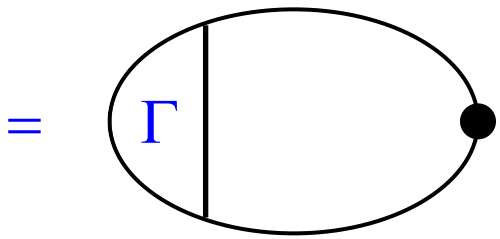
$$S_{LL}(q) = 2\pi\nu N_d \left| U(q) \right|^2 \frac{\sinh(qa)}{qa} e^{-qd}$$

$qa \gg 1$: $S_{LL} \gg S_{LR}$ Completely uncorrelated disorder

$a \ll d$: $S_{LL} \to S_{LR}$ Fully correlated disorder

Calculation of the tunneling current





$$\Gamma(\omega) = t\,\frac{(\omega + i/\tau)^2 - 4\Delta^2}{(\omega + i/\tau)(\omega + i/\tau_0) - 4\Delta^2}$$

Interlayer vertex function

$$\tau_0^{-1} = \int d^2q\ (S_{LL} - S_{LR})$$

Fully correlated disorder: $\tau_0 = \infty$ Uncorrelated disorder: $\tau_0 = \tau$

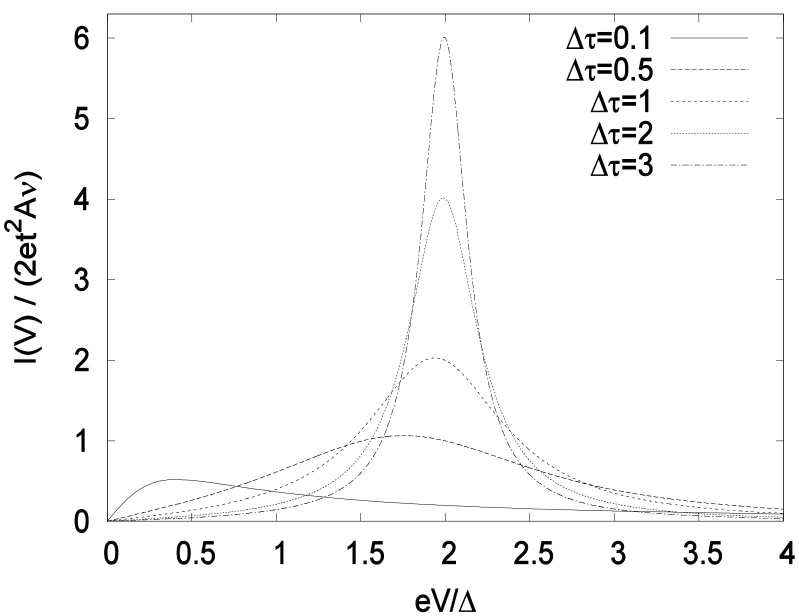
Low-frequency pole in $\Gamma(\omega)$ at $i\omega = 4\Delta^2\tau + \tau_0^{-1}$

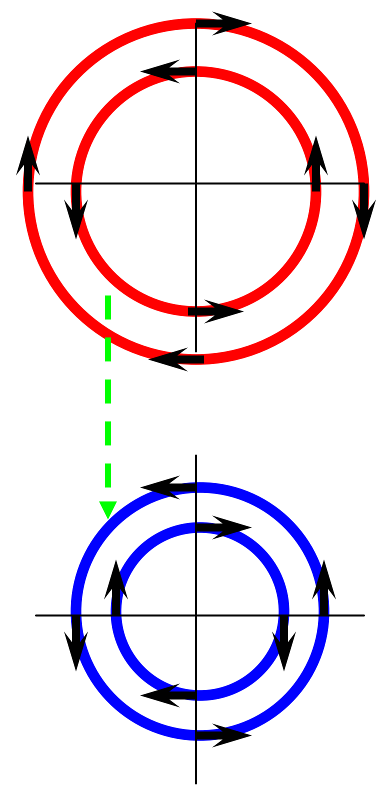
$$\begin{cases} \Delta\tau \ll 1 \\ \tau_0^{-1} \ll \tau^{-1} \end{cases}$$

I-V characteristics: non-interacting electrons

$$I(V) \sim \frac{4\Delta^2\tau^{-1} + (e^2V^2 + \tau^{-2})\tau_0^{-2}}{(e^2V^2 - 4\Delta^2 - \tau^{-1}\tau_0^{-1})^2 + e^2V^2(\tau_0^{-1} + \tau_0^{-1})^2}$$

Clean limit $\Delta \gg \tau^{-1}$



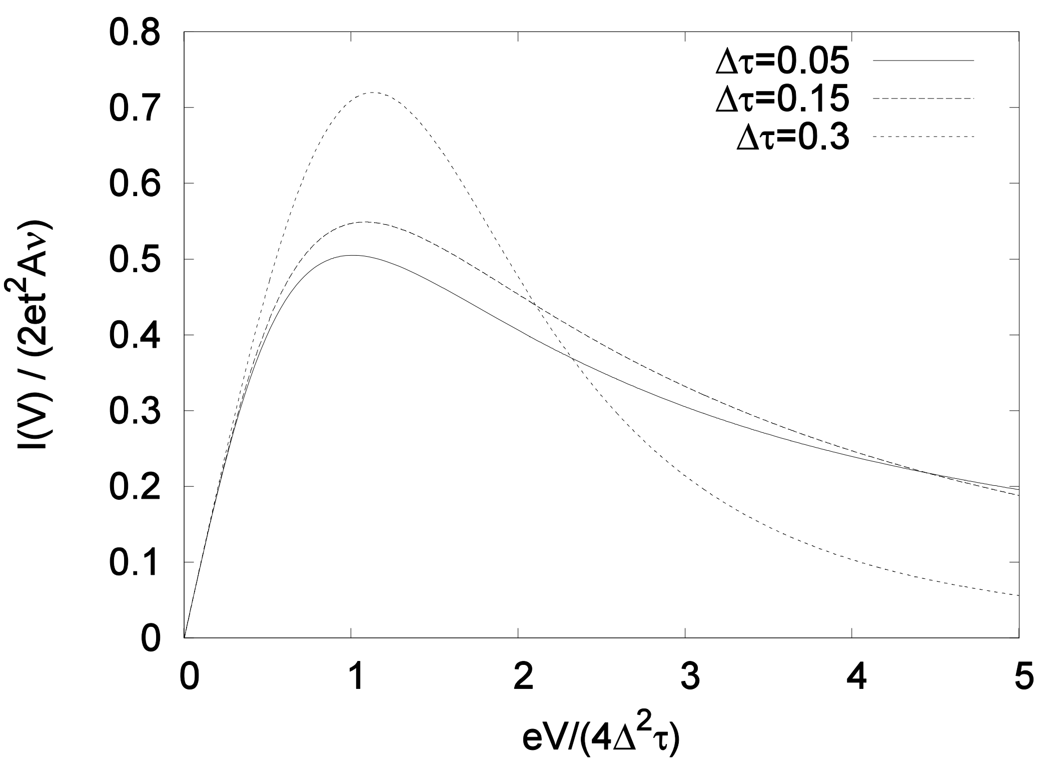


I-V curves: diffusive limit $\Delta \ll \tau^{-1}$

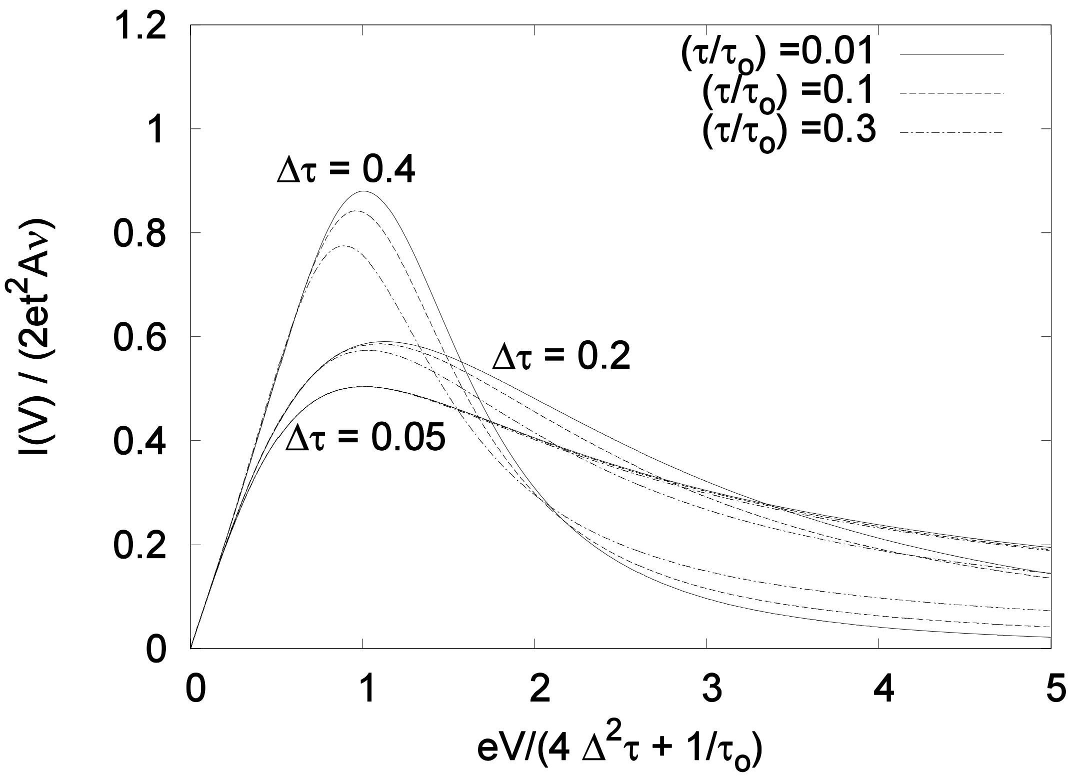
Fully correlated disorder $\tau_0 = \infty$

Peak position shifts towards lower frequencies:

$$eV \sim \Delta \quad \rightarrow \quad eV \sim \Delta^2 \tau$$



Diffusive limit, but $\tau_0^{-1} \neq 0$



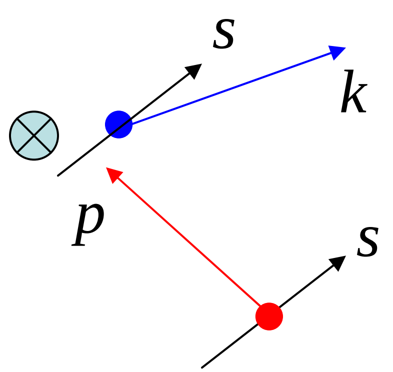
Peak position is rather accurately at

$$eV = 4\Delta^2\tau + \tau_0^{-1}$$

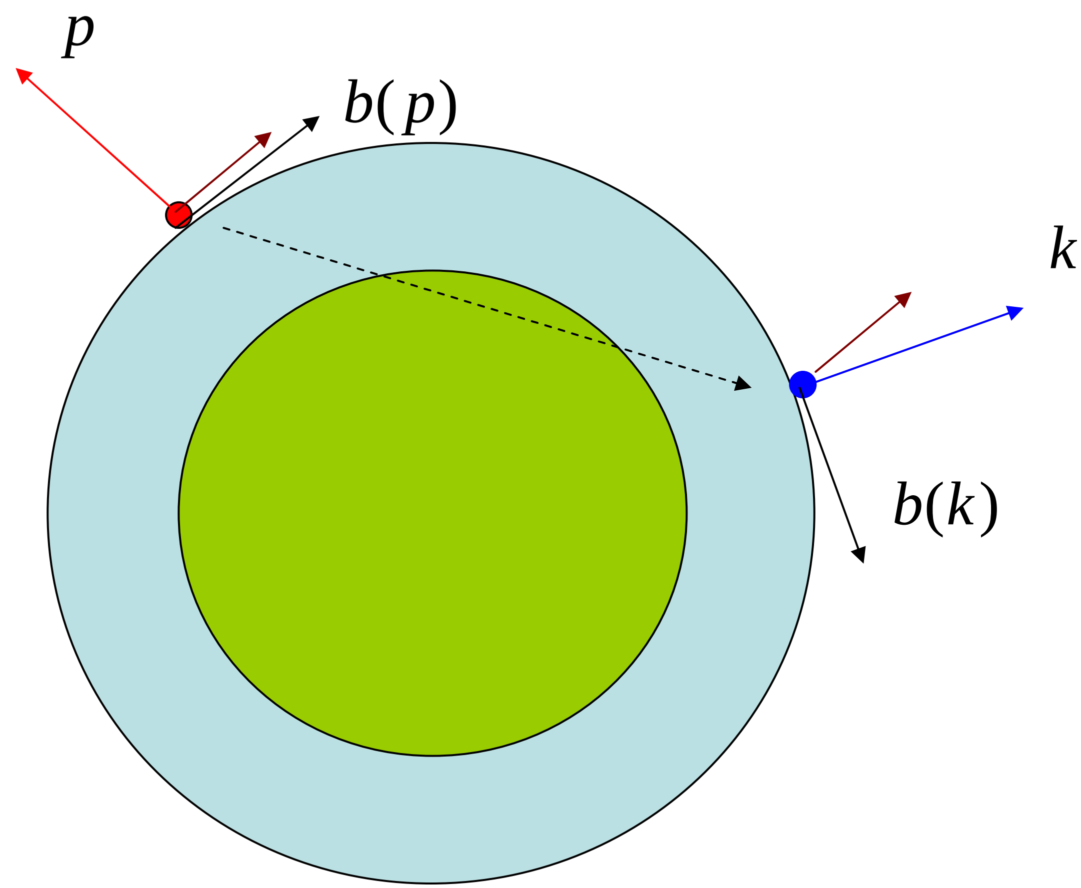
What is the meaning of $4\Delta^2\tau + \tau_0^{-1}$?

Combined decoherence rate

Spin relaxation



Impurity scattering
(spin-conserving!)



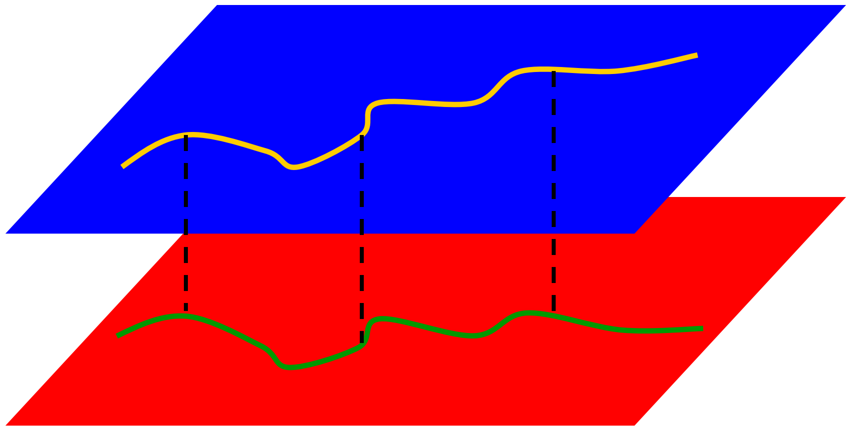
After scattering the electron
is no longer in an eigenstate:
spin starts to precess

$\Delta\tau$ is the angle of precession
between two consecutive collisions

$$1/\tau_s = 2\Delta^2\tau$$

(Dyakonov-Perel spin relaxation; DP ’71)

Combined decoherence rate



τ_0 determines typical time for coherent propagation of in the two layers

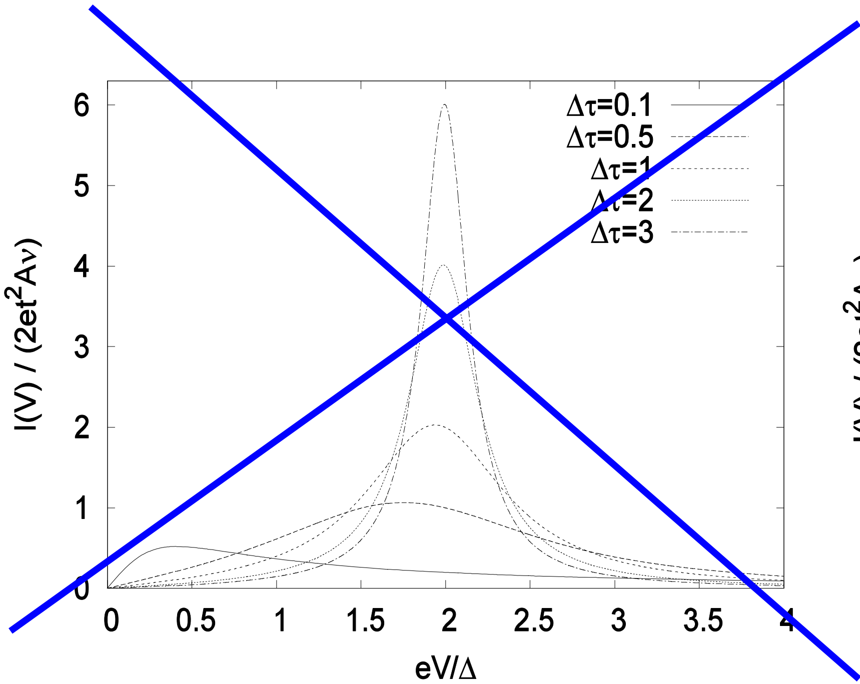
Factor ‘2’ : both layers are affected

$$4\Delta^2\tau + \tau_0^{-1}$$

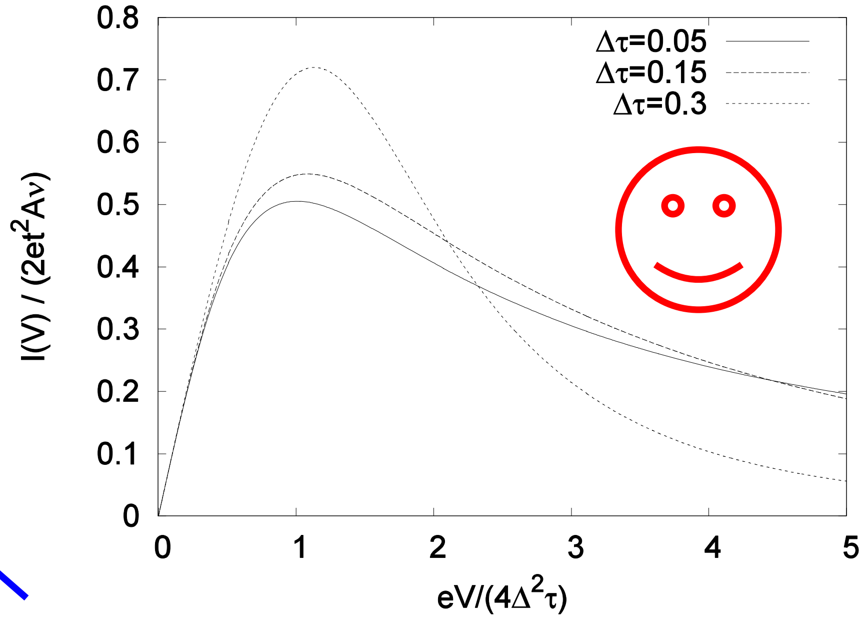
Decoherence of ***spin*** wave function

Decoherence of ***orbital*** wave function

What about electron-electron interactions?



$\Delta \gg \tau^{-1}$



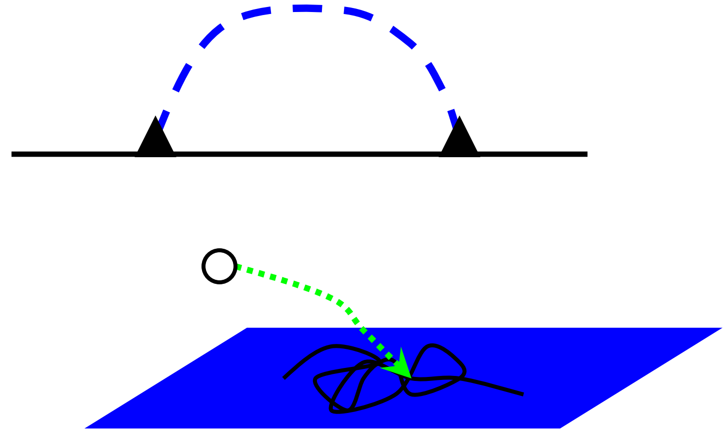
$\Delta \ll \tau^{-1}$

Will these peaks “survive” effects of electron-electron interactions?

Tunneling with e-e interactions: $eV \ll \tau^{-1}$

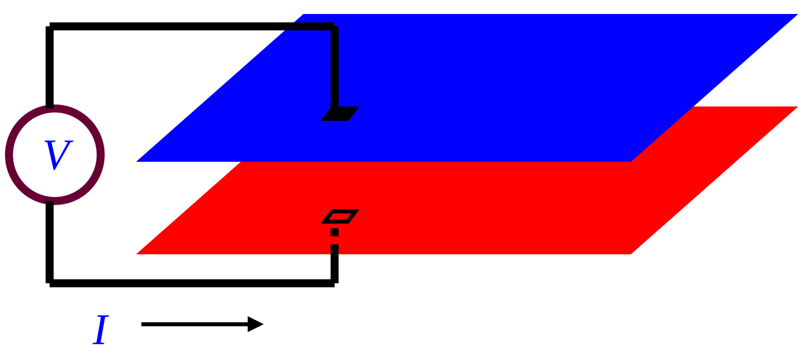
Density of states $\nu(\varepsilon)$

$$\delta\nu(\varepsilon) \sim -\frac{1}{E_F\tau}\ln\left[\frac{1}{\varepsilon\tau}\right]\ln\left[\frac{\tilde{E}}{\varepsilon}\right]$$



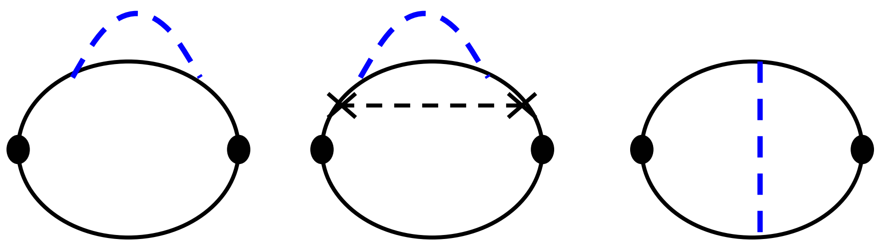
Altshuler, Aronov, and Lee, ‘81

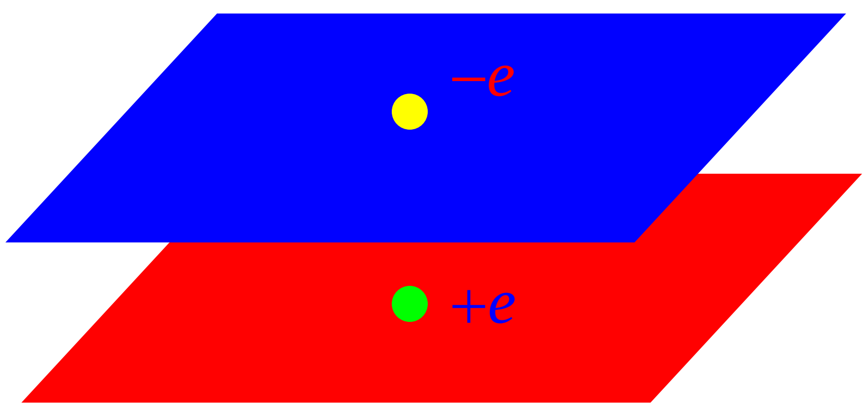
I-V characteristics $\longrightarrow$ Rudin, Aleiner, and Glaman, ‘97



Tunneling with e-e interactions: $eV \ll \tau^{-1}$

$$\delta\nu(\varepsilon) \sim -\frac{1}{E_F\tau}\ln\left[\frac{1}{\varepsilon\tau}\right]\ln\left[\frac{\tilde{E}}{\varepsilon}\right]$$





$$\delta I \sim \frac{eV}{E_F\tau}\ln\left[eV\tau\right]\ln\left[\kappa d\right]$$

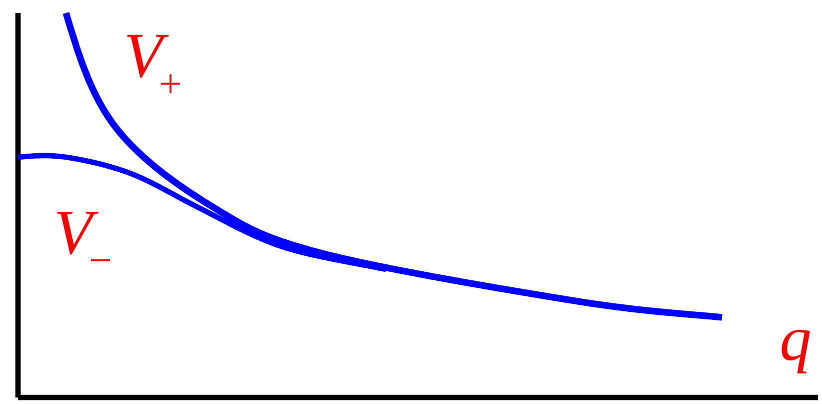
I-V characteristics

Rudin, Aleiner, and Glaman, '97

$$V_{LL} = \frac{2\pi e^2}{q} \qquad V_{LR} = \frac{2\pi e^2}{q}e^{-qd}$$

$$V_+ = V_{LL} + V_{LR} \qquad V_- = V_{LL} - V_{LR}$$

Only anti-symmetric channel contributes



Tunneling with e-e interactions: $\tau^{-1} \to 0$

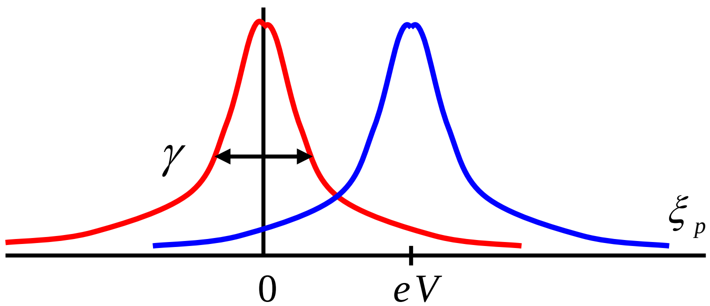
Jungwirth and MacDonald, '96 $$I(V) \sim t^2 \int_0^{eV} d\varepsilon \int d^2p\, A_1(\varepsilon, \vec{p}) A_2(\varepsilon - eV, \vec{p})$$

I-*V* characteristics: $I(V) \sim V\delta(V)$ when interactions are neglected

$$A(\varepsilon, \vec{p}) = \frac{\gamma(\xi_p)}{(\varepsilon - \xi_p)^2 + \gamma^2(\xi_p)}$$

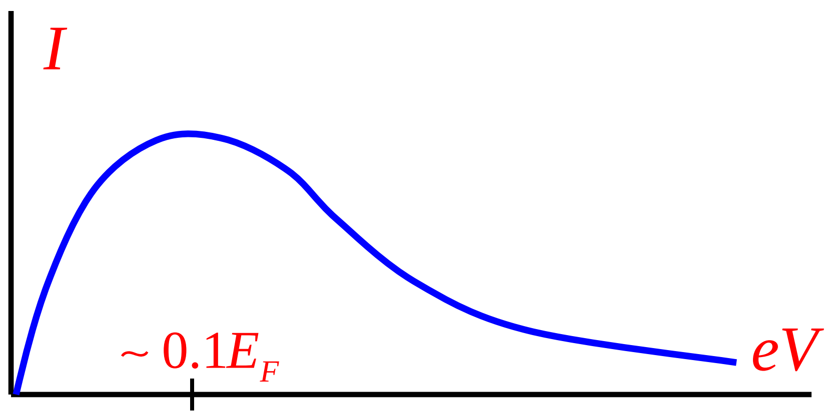
$$\gamma(\xi_p, T) = \frac{\xi_p^2 + T^2}{4\pi E_F} \ln \frac{E_F}{(\xi_p, T)}$$

Chaplik '72, Hodges, Smith and Wilkins,'72
Zheng and Das Sarma, '96



$$I(V) \sim \frac{V\tau^{-1}}{(eV)^2 + \tau^{-2}}$$

1) Fast growth of $\gamma(\xi_p)$ with ξ_p
2) Absence of a scale other than E_F

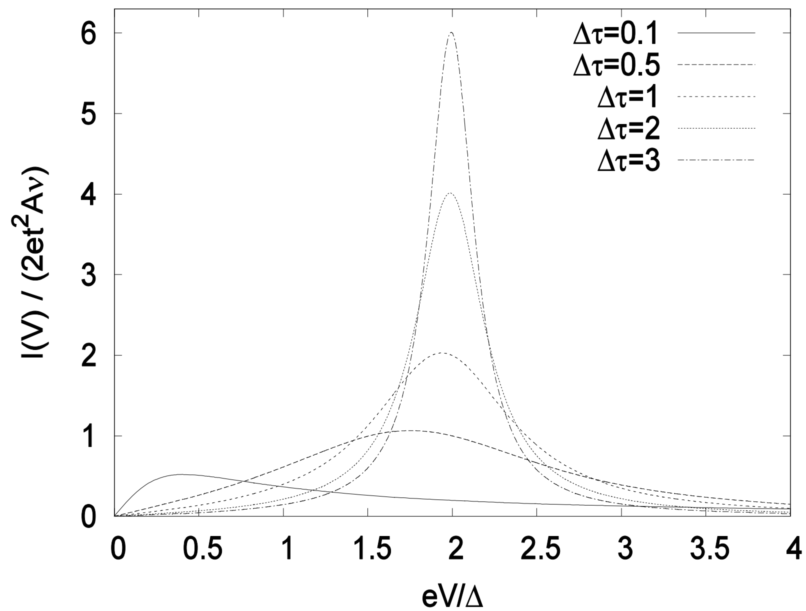


Tunneling with SO and e-e interactions:

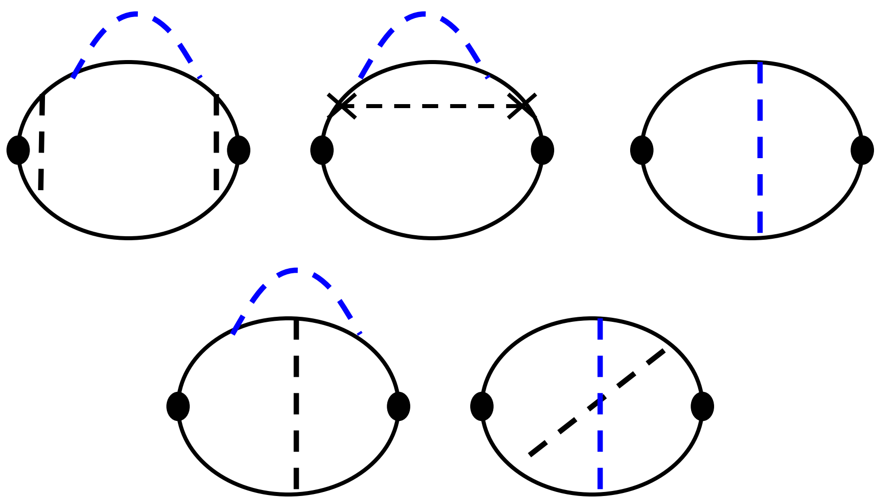
Clean limit $\Delta \gg \tau^{-1}$

$$\Delta \ll 0.1 E_F$$

Peak ***cannot*** be resolved!



Dirty limit $\Delta \ll \tau^{-1}$?



Even first order diagrams are complicated

Eigenstates formalism

$$I(V) \sim t^2 \int_0^{eV} d\varepsilon \int d^2\vec{r}_1 d^2\vec{r}_2 \; A_L(\varepsilon, \vec{r}_1, \vec{r}_2) A_R(\varepsilon - eV, \vec{r}_2, \vec{r}_1)$$

Non-averaged spectral function:

$$A(\varepsilon, \vec{r}_1, \vec{r}_2) = \operatorname{Im} \sum_m \frac{\psi_m(\vec{r}_1)\psi^*{}_m(\vec{r}_2)}{\varepsilon - \varepsilon_m - \Sigma_m(\varepsilon)}$$

$$I(V) \sim t^2 \sum_m \int_0^{eV} d\varepsilon \;\; \operatorname{Im} \frac{1}{\varepsilon - \varepsilon_m - \Sigma_m(\varepsilon)} \;\; \operatorname{Im} \frac{1}{\varepsilon - eV - \varepsilon_m - \Sigma_m(\varepsilon - eV)}$$

To the first order,

$$I(V) \sim \frac{t^2}{V^2} \sum_m \int_0^{eV} d\varepsilon \left[\gamma_\varepsilon(\varepsilon - eV) + \gamma_{\varepsilon - eV}(\varepsilon) \right]$$

$$\gamma_\varepsilon(\omega) = \nu^{-1} \sum_m \langle \delta(\varepsilon - \varepsilon_m) \operatorname{Im} \Sigma_m(\omega) \rangle$$

$$\gamma_\varepsilon(\omega) \sim \frac{\omega}{E_F \tau}$$

Disorder-averaged inverse lifetime
Abrahams, Anderson, Lee,
and Ramakrishnan, ‘81

Peak smearing

Comparing the first-order correction with the peak height → $\frac{\delta I}{I} \sim \frac{1}{E_F \tau} \ll 1$

Physical meaning:

$$\gamma_\varepsilon(\omega) \sim \frac{\omega}{E_F \tau}$$

Peak is "shifted" towards lower voltages:

$$eV \sim 4\Delta^2 \tau \ll \tau^{-1}$$

At these voltages width is small:

$$\gamma(eV) \sim \frac{eV}{E_F \tau} \ll 4\Delta^2 \tau \approx eV$$

→ Peak is **protected** by disorder

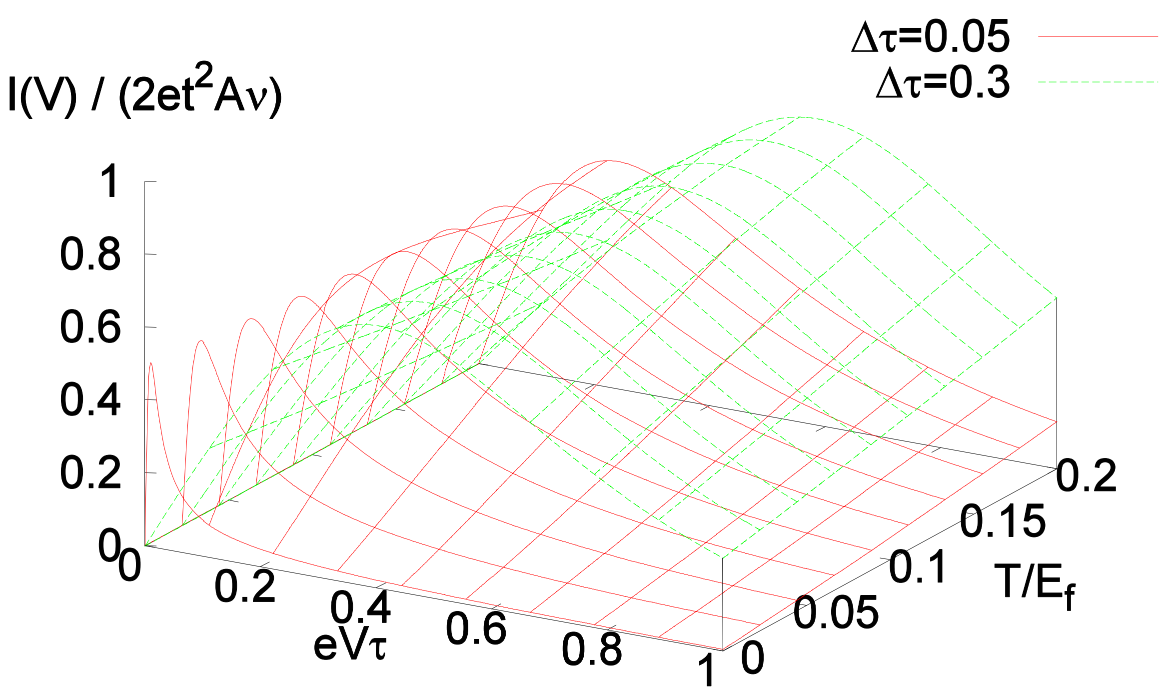
Temperature broadening

$$2\Delta^2\tau \to 2\Delta^2\tau + \gamma_T \qquad \gamma_T = \frac{T}{2E_F\tau}\ln\frac{\tilde{T}}{T}$$

Abrahams, Anderson, Lee, and Ramakrishnan, '81

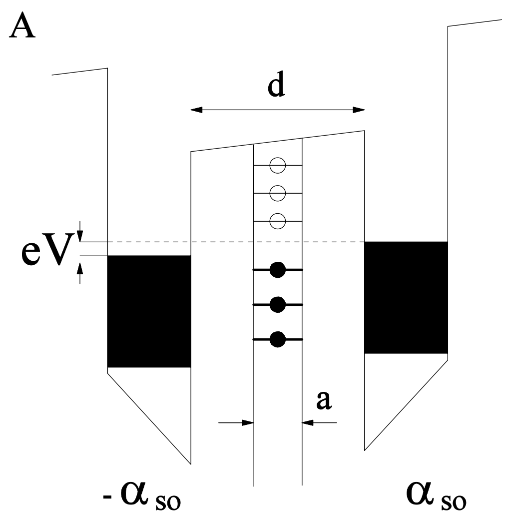
Fully correlated disorder

$$I(V) \sim \frac{V(2\Delta^2\tau^{-1} + \gamma_T)}{e^2V^2 + 4(2\Delta^2\tau^{-1} + \gamma_T)^2}$$



Estimates

1. Uncorrelated part of disorder must be weak: $4\Delta^2\tau > \tau_0^{-1}$



$$\tau_0^{-1} = 2\pi\nu N_d \int d\vec{q}\, |U(q)|^2 \left(\tfrac{\sinh(qa)}{qa} - 1\right) e^{-qd}$$

$$\Delta\tau > a/2d$$

$a \sim 10nm$

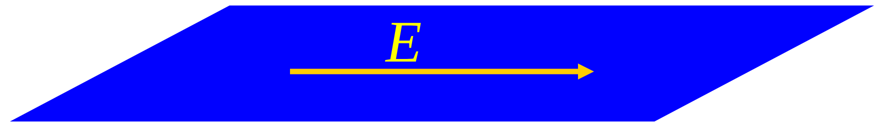
$d \sim 50 - 100nm$

2. Temperature should not be high: $2\Delta^2\tau > \dfrac{T}{2E_F\tau}\ln\dfrac{\tilde{T}}{T}$

$$T < 4(\Delta\tau)^2 E_F$$

Optical conductivity of 2DEG

$\sigma(\omega, q)$



Optical conductivity in not sensitive to interactions in the homogeneous limit $q \to 0$

Galilean invariance → Parabolic spectrum $\varepsilon(p) = \dfrac{p^2}{2m}$

Electric current depends only on the total momentum
and is not modified by electron-electron collisions
in the homogeneous limit

$$\vec{j} = e \sum \frac{\vec{p}}{m}$$

Many-body effects have no effect on $\sigma(\omega, 0)$

Spin-orbit coupling breaks Galilean invariance

Particle moving in electric field: $\vec{E} = -e^{-1}\nabla U$

In the reference frame moving with the electron velocity $\vec{v} = \dfrac{\vec{p}}{m}$
there is a magnetic field

$$\vec{B} = \frac{\vec{v} \times \vec{E}}{c}$$

This magnetic field leads to the Zeeman energy which is momentum-dependent

$$H_{SO} = \frac{g}{2m^2c^2}\nabla U \cdot \vec{s} \times \vec{p}$$

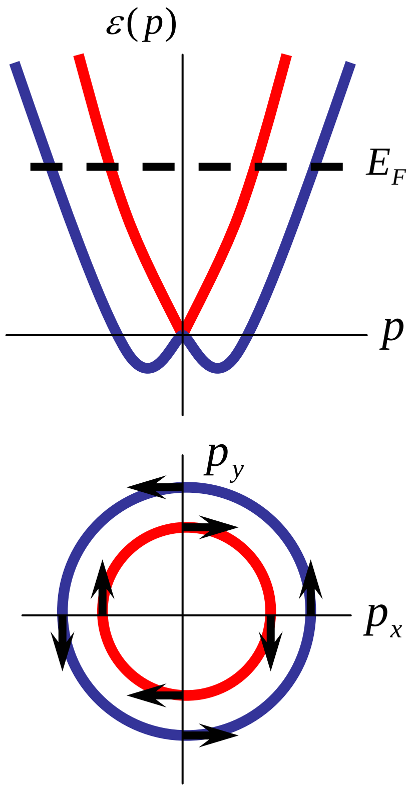
Electric current is spin-dependent
and is not conserved during
electron-electron interactions

$$\vec{j} = e\sum \vec{v} = e\sum\left(\frac{\vec{p}}{m} + \nabla_p H_{SO}\right)$$

$$\sum \frac{\vec{p}}{m} = const \longrightarrow \vec{j} \neq const$$

Homogeneous optical
conductivity **can probe**
many-body effects

Electron eigenstates



Spin degeneracy is lifted by $H_{SO} = \lambda(s_x p_y - s_y p_x)$

Eigenvalues:

$$\varepsilon(p) = \frac{p^2}{2m} + a\lambda p$$

chirality $a = 1, -1$

Eigenstates:

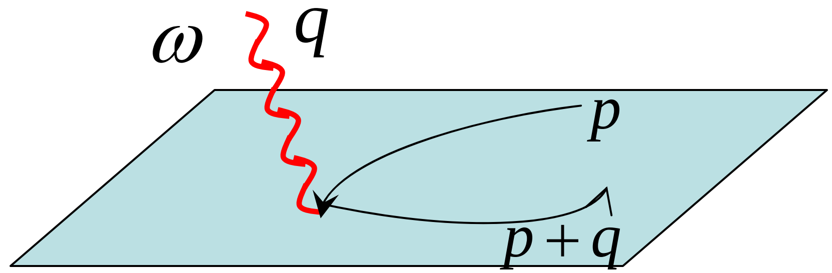
$$\psi_p^a = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\theta/2} \\ a e^{-i\theta/2} \end{pmatrix}$$

Different Fermi momenta:

$$p_F^a = p_F - am\lambda$$

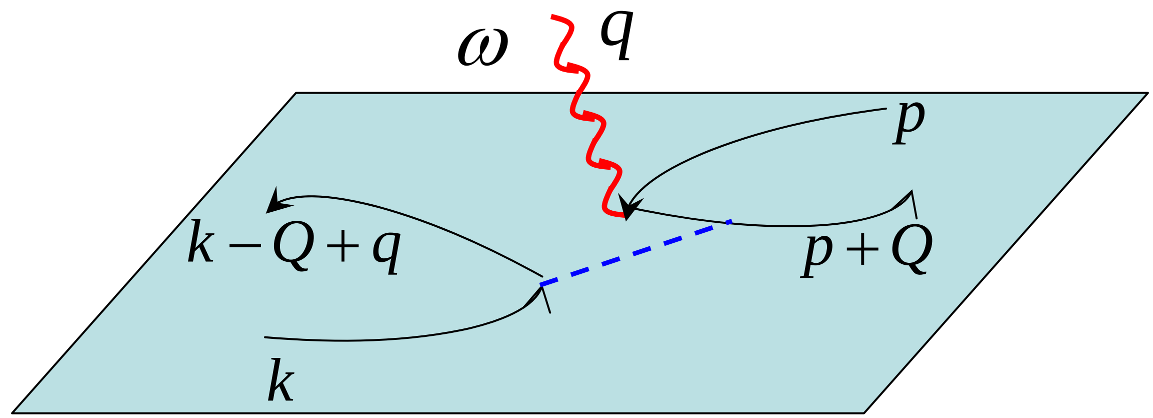
Beyond RPA: two-particle channel

Landau damping:



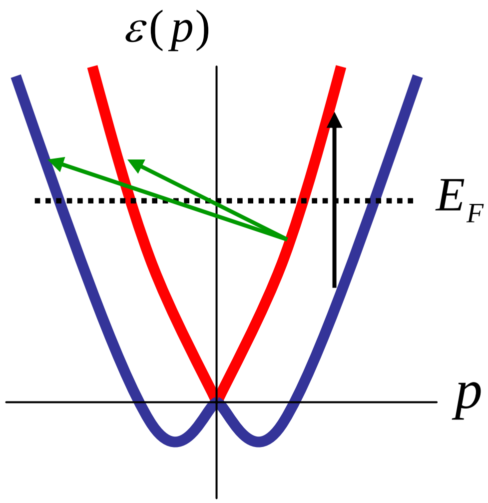
$$\omega = \vec{q} \cdot \vec{v} < q v_F$$

Two-pair channel:



Two pairs moving in opposite direction can have large energy while having negligible total momentum

Modification of Landau damping



Direct transitions between subbands are possible:

1) 'Conventional' Landau damping

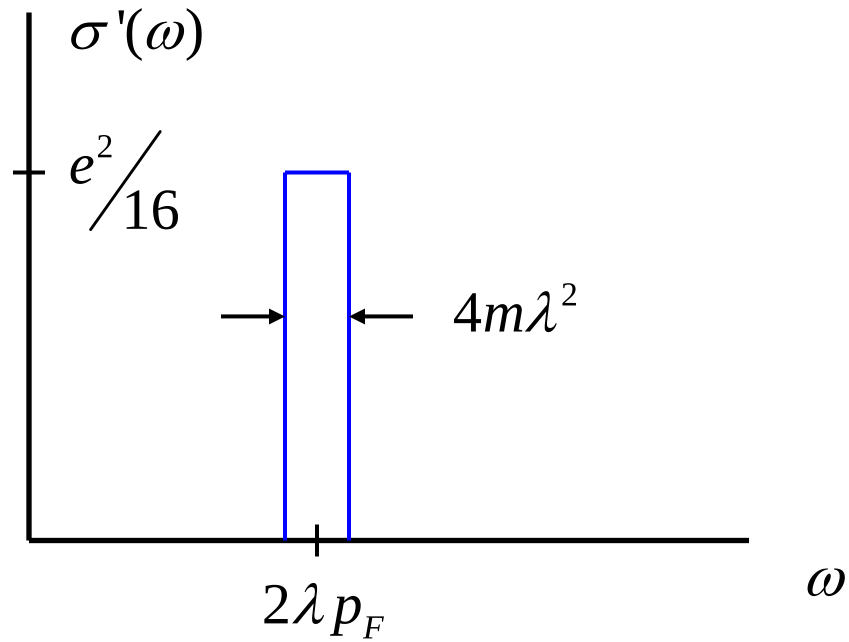
2) 'Combined' or 'chiral' resonance

Energy constraint

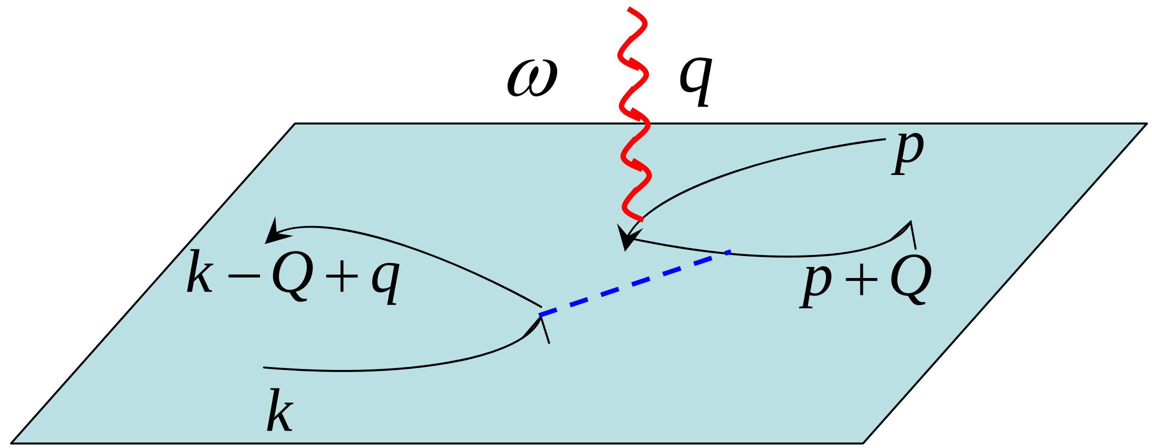
$$\omega = \varepsilon_1(p) - \varepsilon_2(p) = 2\lambda p$$

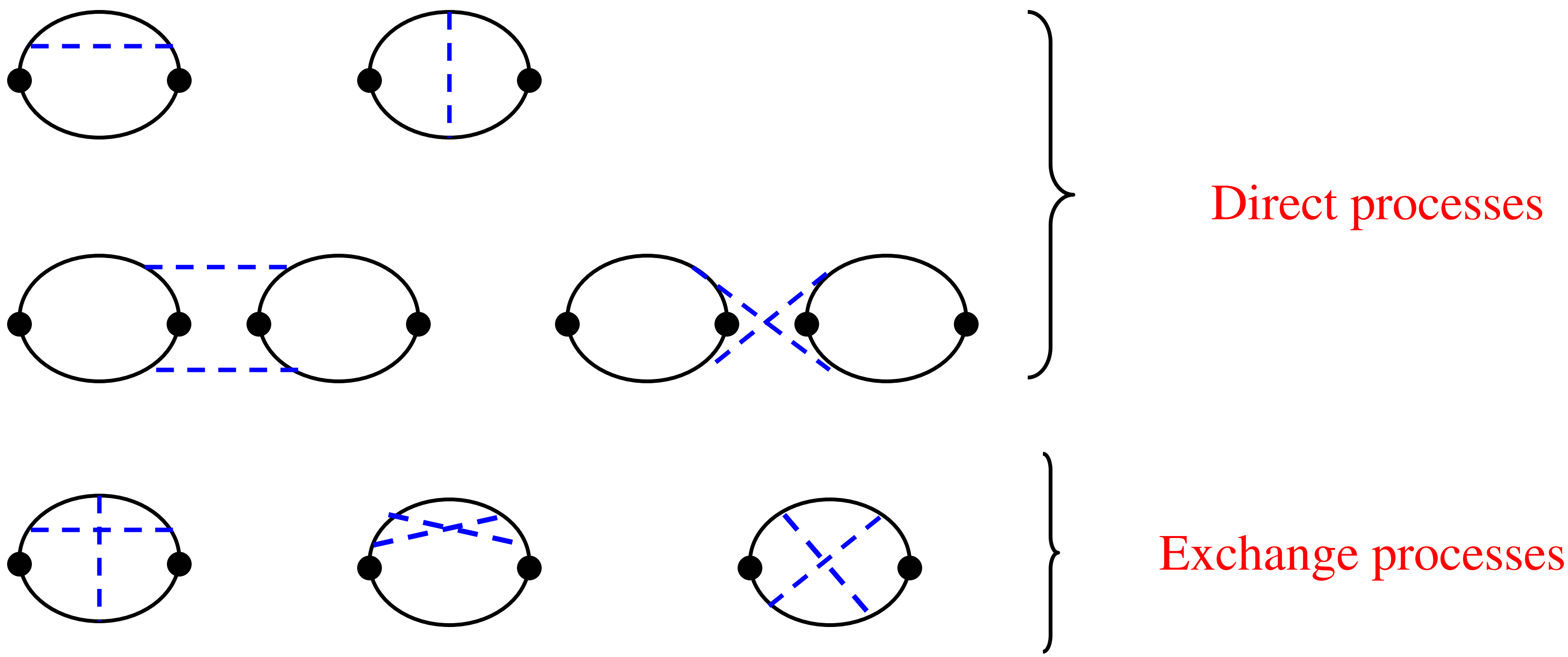
Partition constraint

$$p_F - m\lambda < p < p_F + m\lambda$$



Two-particle channel with spin-orbit





2D diagrammatic calculations, Reizer & Vinokur: 2000 ⟶ wrong

Our method: many-body transitions in the presence of external field

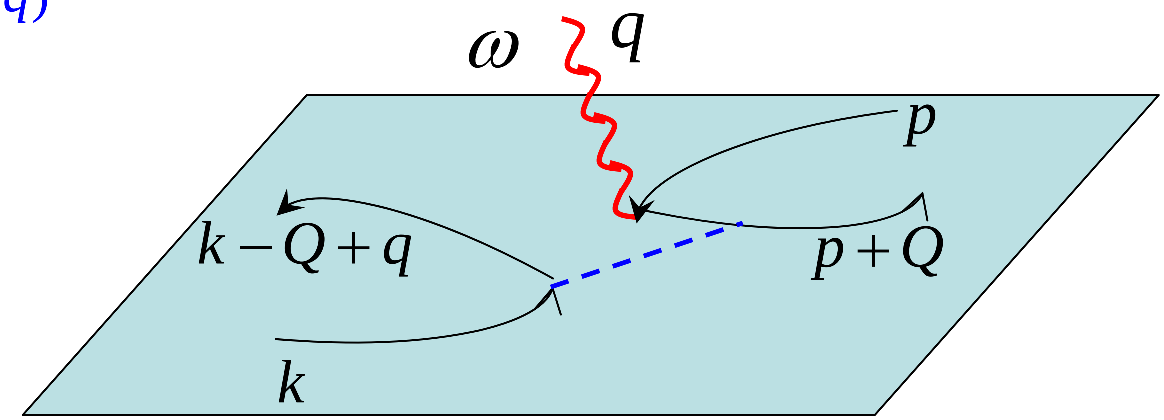
$$\phi(\vec{x},t) = \phi_0 e^{-i\omega t + i\vec{q}\cdot\vec{x}} + \phi_0^* e^{i\omega t - i\vec{q}\cdot\vec{x}}$$

$$-\frac{dW}{dt} = \text{absorption} - \text{emission}$$

all real transitions

$$-\frac{dW}{dt} = 2q^2 \left|\phi_0\right|^2 \sigma'(\omega, q)$$

real part of the optical conductivity



Formalism: Golden Rule

Two-particle wave function

$$\Psi_{pk} = \frac{1}{\sqrt{2}}\left(\psi_p(x_1)\psi_k(x_2) - \psi_p(x_2)\psi_k(x_1)\right)$$

Need transition probabilities:

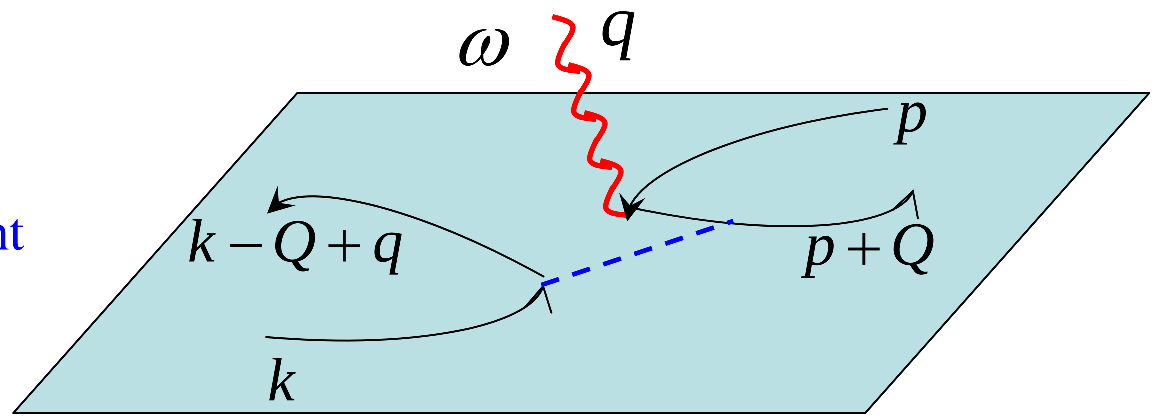
$$\Psi_{pk} \to \Psi_{p'k'}$$

$$i\hbar\frac{\partial\Psi}{\partial t} = H_0\Psi + H_1\Psi$$

$$H_1 = e\phi(\vec{x}_1) + e\phi(\vec{x}_2) + V(\vec{x}_1 - \vec{x}_2)$$

$$H_0 = -\frac{\hbar^2\nabla_1^2}{2m} - \frac{\hbar^2\nabla_2^2}{2m}$$

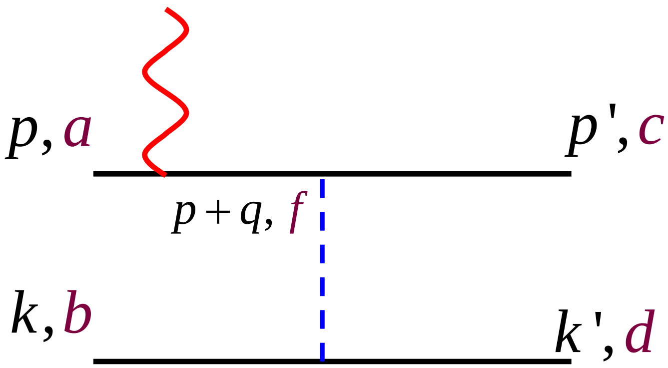
Second-order time-dependent perturbation theory in H_1



Our method applied

$$W_{pk\to p'k'} = \frac{2\pi}{\hbar}\left|M\right|^2 \delta(\xi_p^a + \xi_k^b - \xi_{p'}^c - \xi_{k'}^d + \hbar\omega)\delta(\vec{p} + \vec{k} - \vec{p}\,' - \vec{k}\,' + \hbar\vec{q})$$

$M =$

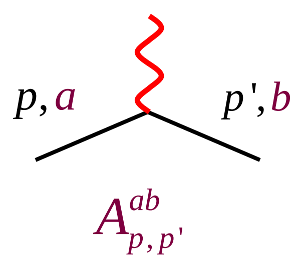


$$= e\phi_0 \frac{A^{af}_{p,p+q} A^{fc}_{p+q,p'} V_{k'-k} A^{fd}_{k,k'}}{\xi_k^a - \xi_{p+q}^f + \omega}$$

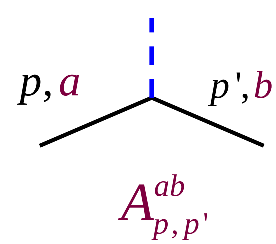
$$A^{ab}_{p,p'} = \left(\chi^b_{p'}\right)^\dagger \cdot \chi^a_p = \frac{1}{2}\left(e^{i(\theta_p - \theta_{p'})/2} + abe^{-i(\theta_p - \theta_{p'})/2}\right)$$

Projection of spin states before and after scattering

How to read graphs:



or



Optical conductivity

$$\sigma'(\omega,q)=\frac{1}{2q^2|\phi_0|^2}\frac{-dQ}{dt}$$

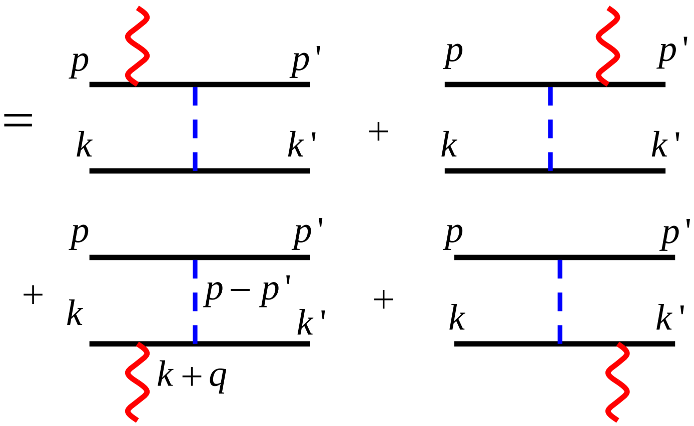
$$\frac{-dW_{abs}}{dt}=\sum_{pkp'k'}W_{pk\to p'k'}n_p n_k(1-n_{p'})(1-n_{k'})$$

$$\frac{-dQ}{dt}=\frac{-dW_{abs}}{dt}+\frac{dW_{em}}{dt}$$

$$\frac{dW_{em}}{dt}=\frac{dW_{abs}}{dt}\exp(-\omega/T)$$

Detailed balance principle

$M =$



Each matrix element is not small in q

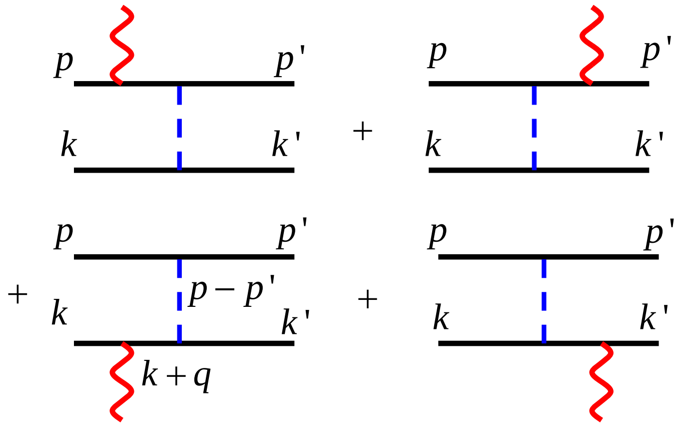
but they ***twice*** interfere ***pairwise*** almost canceling each other leading to $M \sim q^2$

Optical conductivity vanishes in the homogeneous limit $q \to 0$

as $\sigma(\omega,q) \sim q^2$

Optical conductivity with Spin-Orbit coupling

$M =$



The four terms interfere only ***once*** leading to $M \sim q$

For a short-range interaction $V = const$

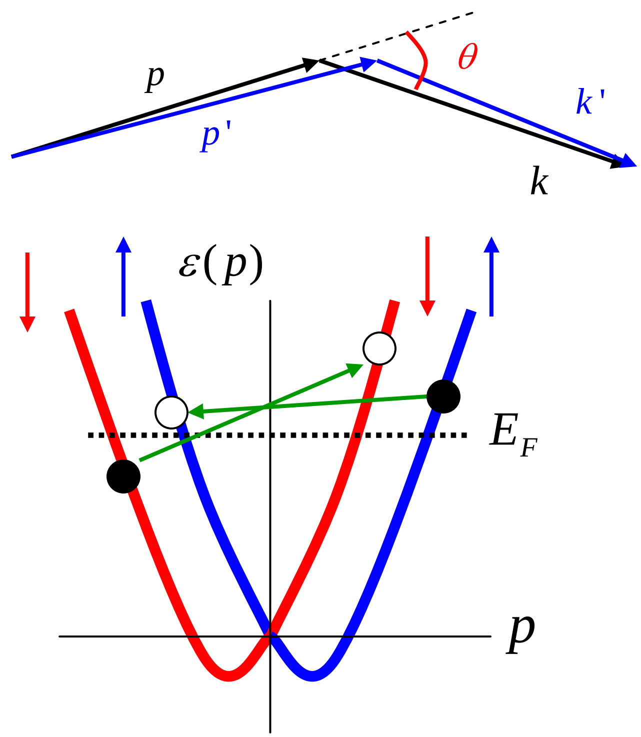
$$\sigma'(\omega) \sim \frac{e^2 \lambda^2 m^2 V^2}{v_F^2 \omega^2} \left[\omega^2 + (2\pi T)^2 \right]$$

This contribution is the result of the interplay of spin-orbit coupling and interaction

Discussion: I

$$\sigma'(\omega) \sim \frac{e^2\lambda^2 m^2 V^2}{v_F^2\omega^2}\left[\omega^2 + (2\pi T)^2\right]$$

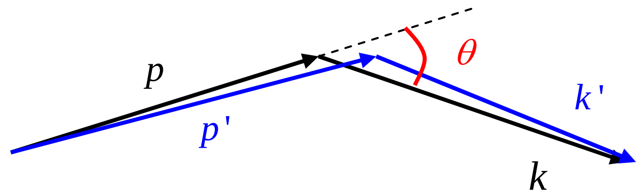
Absence of logarithmic factor – suppression of collinear scattering



No interplay of interactions and spin-orbit coupling in 1D

Chirality of colliding particles is conserved:
No change in the total velocity (thus, current) during a collision in 1D

Discussion: II



Large-angle scattering: $\theta \sim 1$

↓

Exchange processes are equally important as the direct processes

Numerical estimates

$$\sigma'(\omega) \sim \frac{e^2\lambda^2 m^2 V^2}{v_F^2\omega^2}\left[\omega^2 + (2\pi T)^2\right]$$

Two-pair contribution is enhanced for $T \gg \omega \to 0$

Coulomb interaction: $V \sim e^2 / p_F$

$$\frac{\sigma^{(2)}}{\sigma^{(1)}} \sim \frac{\lambda^2}{v_F^2} \quad \frac{T^2}{\omega^2} \quad \frac{\kappa^2}{p_F^2}$$

screening radius: $\kappa \sim p_F \sim 1\times10^6\,\text{cm}^{-1}$

$E_F \sim 5.5$ meV

mean free path: $l \sim 1\mu m$

$\omega_{\min} \sim 1/\tau = v_F / l$

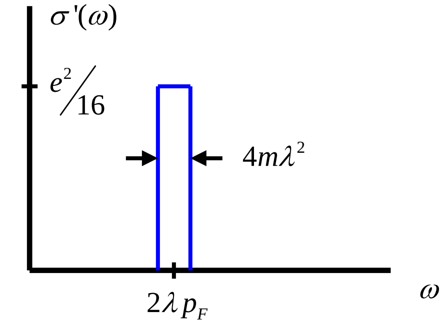
~ 0.01

$\frac{T^2}{1/\tau^2} \sim 10$ for $T \sim 30K$

$$\frac{\sigma^{(2)}}{\sigma^{(1)}} \sim 0.1$$

Conclusions

Spin-orbit coupling results in a single-pair absorption $\sigma_1'(\omega)$ which is narrow in frequency



Combined effects of spin-orbit coupling and electron-electron interactions result in a broader contribution from many-particle excitations

Spin-orbit coupling makes optical conductivity a probe for many-body effects