# Current-Driven Magnetization Dynamics and Domain-Wall Motion in Itinerant Ferromagnets 

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## Spin transfer

Spin-polarized electrons impinging on a ferromagnet transfer spin angular momentum inducing a spin torque on the magnetization


Spin torque can have dramatic consequences for magnetic dynamics in nanostructures, leading to various instabilities which contain reach physics and are promising for useful applications

## Current-driven bulk dynamics

Mean-field s-d picture:


Spins drifting through a large position-dependent exchange field nearly adiabatically follow local magnetization direction, exerting a torque on the magnetic moment

Inhomogeneous magnetization thus leads to a coupling between spin and orbital degrees of freedom

Equation of motion for the spin density:

$$
\partial_{t} \mathbf{s}=-\gamma \mathbf{s} \times \mathbf{H}-\partial_{x_{i}} \cdot \mathbf{J}_{i}-\frac{\mathbf{s}-\mathbf{s}_{0}}{\tau_{\sigma}}
$$

## Domain-wall motion





## Also:

S. S. P. Parkin et al.:Transition-metal race-track memory
H. Ohno et al.: Domain walls in GaMnAs wires

## Theories

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$$
\text { PRL 92, } 08660 \text { I (2004) }
$$

In general there are spin-transfer and momentum-transfer terms in the equation of motion
L. Berger, JAP 55, I954 (I984); JAP 7 I, 272 I (I992)

Only the former contributes to the motion of smooth domain walls, while the later is proportional to the wall resistance


FIG. 1. Time-averaged wall velocity as a function of spin current, $j_{\mathrm{s}}$, in the weak pinning case ( $V_{0} \leqslant K_{\perp} / \alpha$ ).

Current-Spin Coupling for Ferromagnetic Domain Walls in Fine Wires

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PRL 95, I 07204 (2005)
No intrinsic pinning!
Pinning comes from an extrinsic potential


## Theories (cont.)

Roles of Nonequilibrium Conduction Electrons on the Magnetization Dynamics of Ferromagnets

Presence of spin-orbit scattering is required for current-driven domain-wall motion

Critical current then vanishes in the absence of pinning disorder

$$
\begin{aligned}
\frac{\partial \mathbf{M}}{\partial t}= & -\gamma \mathbf{M} \times \mathbf{H}_{e f f}+\frac{\alpha}{M_{s}} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}-\frac{b_{J}}{M_{s}^{2}} \mathbf{M} \\
& \times\left(\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x}\right)-\frac{c_{J}}{M_{s}} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x}
\end{aligned}
$$

Current-Induced Magnetization Dynamics in Disordered Itinerant Ferromagnets

## Yaroslav Tserkovnyak, ${ }^{1}$ Arne Brataas, ${ }^{2}$ and Gerrit E. W. Bauer ${ }^{3}$

Treat magnetization dynamics and electron transport self-consistently (time-dependent SDFT)

Include disorder and spin-orbit scattering at the level of kinetic equation, using Keldysh formalism for spin dynamics in the presence of electric field

Derive a general LLG-like equation of motion that can be applied to a variety of problems

## Current-driven spin-wave instabilities

$$
\begin{gathered}
i \hbar \frac{\partial \phi}{\partial t}=-\frac{\hbar^{2}}{2 m} \Delta \phi+J_{H} M \phi-\frac{\hbar^{2}}{2 m}\left(U_{+\beta} \nabla_{i} U_{\beta+}\right) \nabla_{i} \phi \\
E=\int\left(J(\nabla \mathbf{M})^{2}-K(\boldsymbol{\nu} \cdot \mathbf{n})^{2}+\frac{1}{c} j_{i} A_{i}^{\text {eff }}\right) d V \\
\frac{\partial \mathbf{M}}{\partial t}=\frac{g|e|}{2 m c}[\mathbf{f} \times \mathbf{M}] \quad \mathbf{f}=-\frac{\delta E}{\delta \mathbf{M}}=J \Delta \mathbf{M}+\frac{2 K}{M^{2}}(\boldsymbol{\nu} \cdot \mathbf{M}) \boldsymbol{\nu}+\frac{\hbar}{2 M} \frac{j_{i}}{e}\left[\nabla_{i} \mathbf{n} \times \mathbf{n}\right] \\
\omega=\widetilde{J} k^{2}-Q \frac{\mathbf{j} \cdot \mathbf{k}}{e}+\widetilde{K} . \quad \omega_{\min }=\widetilde{K}-\left(Q \frac{j}{e}\right)^{2} \frac{\cos ^{2} \alpha}{4 \widetilde{J}}=\omega_{0}\left(1-\frac{j^{2}}{32 j_{0}^{2}} \cos ^{2} \alpha\right)
\end{gathered}
$$

## Beyond instability?

## Effect of Spin Current on Uniform Ferromagnetism: Domain Nucleation

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A large spin current applied to a uniform ferromagnet leads to a spin-wave instability as pointed out recently. In this Letter, it is shown that such spin-wave instability is absent in a state containing a domain wall, which indicates that nucleation of magnetic domains occurs above a certain critical spin current. This scenario is supported also by an explicit energy comparison of the two states under spin current.

PRL 94, 07660 (2005)


FIG. 1. Schematic phase diagram under spin current $j_{s}$ in the absence of pinning potential. (a) $K_{\perp}<K$. Above $j_{\mathrm{s}}^{\text {cr }}$, uniform ferromagnetism collapses into multidomain structure in which domain walls are flowing due to spin current. The threshold, $j_{\mathrm{s}}^{\text {depin }}$, for "depinning" from $K_{\perp}$ is below $j_{\mathrm{s}}^{\text {cr }}$. (b) $K_{\perp} \gg 8 K$ Energy of the single-wall state ( $E_{\mathrm{dw}}$ ) is compared with that of the uniformly magnetized state, $E_{\text {uni }}=0$. Multidomain state here remains at rest. In the gray region, $\left(j_{\mathrm{s}}>j_{\mathrm{s}}^{\text {depin }}\right)$, the domain wall starts to flow but is unstable, suggesting a new ground state.

## Magnetization instability driven by spin torques

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(Presented on 11 November 2004; published online 5 May 2005)
Electric currents in a ferromagnet film produce adiabatic and nonadiabatic torques on magnetization. When the current density is sufficiently large, these torques drive the uniform magnetization into spatially and temporally chaotic motion of magnetization. We predict several key characteristics of the magnetization instability by calculating the current-induced domain wall creation, annihilation and dynamics. © 2005 American Institute of Physics. [DOI: 10.1063/1.1849591]

JAP 97, I0C703 (2005)


FIG. 3. Time evolution of the spatially averaged magnetization component along the easy axis $\left\langle M_{x}\right\rangle / M_{s}$. Inset: time evolution of the magnetization of a single mesh in the sample. CS indicates the chaotic states. The parameters are $H_{\text {ext }}=10^{3} \mathrm{Oe}, b_{J}=2.5 \times 10^{3} \mathrm{~m} / \mathrm{s}, \xi=0.0025$ and $\alpha=0.02$.

## Basic theoretical questions

- Critical current? Extrinsic or intrinsic?
- Domain-wall velocity
- Domain-wall deformation
- Current-driven instabilities? Chaotic behavior? Existence of a multidomain "ground state"?
- Basic questions concerning magnetization dynamics in real ferromagnets. Gilbert damping?


## Applying circuit-theory ideas

Consider first spin-wave dispersion in multilayer superlattices. Normal interlayers mediate equilibrium supercurrents and nonequilibrium spin pumping


$$
\text { F: } \quad \omega(q)=\frac{\omega_{0}+(b q)^{2} \omega_{x}}{1+i(b q)^{2} \alpha^{\prime}} \quad \text { AF: } \quad \omega(q)=\frac{ \pm(b q) \omega_{x}}{1 \pm i(b q) \alpha^{\prime}}
$$

$$
\omega_{x}=\frac{\gamma J_{x}}{M_{s} d}, \quad \alpha^{\prime}=\frac{\gamma g^{\uparrow \downarrow}}{4 \pi M_{s} d S}, \quad \text { and } \omega_{0}=\gamma H_{\mathrm{eff}}
$$

## Stoner model

In a mean-field view of itinerant ferromagnetism, there is only one species of electrons experiencing an exchange field that has to be determined self-consistently

$$
\gamma \hbar \mathbf{H}_{\mathrm{xc}}[\hat{\rho}](\mathbf{r}, t) \approx \Delta_{\mathrm{xc}} \mathbf{m}(\mathbf{r}, t)
$$

$$
\begin{aligned}
\hat{\mathcal{H}} & =\left[\mathcal{H}_{0}+U(\mathbf{r}, t)+V[\hat{\rho}](\mathbf{r}, t)\right] \hat{1} \\
& +\gamma \hbar \hat{\boldsymbol{\sigma}} \cdot\left(\mathbf{H}+\mathbf{H}_{\mathrm{xc}}[\hat{\rho}]\right)(\mathbf{r}, t)+\hat{\mathcal{H}}_{\sigma}
\end{aligned}
$$

Adiabatic Local Spin-Density Approximation (the "bare" model of itinerant ferromagnetism)

## Bulk dynamics: Keldysh+LDA



Equation of motion for the spin density:

$$
\partial_{t} \mathbf{s}=-\gamma \mathbf{s} \times \mathbf{H}-\partial_{x_{i}} \cdot \mathbf{J}_{i}-\frac{\mathbf{s}-\mathbf{s}_{0}}{\tau_{\sigma}}
$$

## spin dephasing

This leads to Gilbert damping:

$$
\alpha(q)=\frac{1}{\tau_{\sigma} \Delta_{\mathrm{xc}}}+\frac{v_{F \uparrow}^{2}+v_{F \downarrow}^{2}}{6 \tau\left(\nu_{\uparrow}^{-1}+\nu_{\downarrow}^{-1}\right) \Delta_{\mathrm{xc}}^{2} s_{0}} q^{2}
$$

Compare to DFT/Fermi-liquid result in clean limit: $\quad \frac{1}{\tau} \rightarrow \frac{(\gamma H)^{2}+(2 \pi T)^{2}}{E_{F}}$
Mineev, PRB 72, I444I8 (2005); Qian and Vignale, PRL 88, 056404 (2002); Halperin and Hohenberg, PR I88, 898 (I969)

## Kinetic equation

$$
\begin{aligned}
& \hbar \partial_{t} \mathbf{g}_{\mathbf{k s}}+\hbar\left(\mathbf{v}_{\mathbf{k}} \cdot \partial_{\mathbf{r}}\right)\left[\mathbf{g}_{\mathbf{k} s}-\Delta_{\mathrm{x}} \mathbf{u} \delta\left(\varepsilon_{k s}\right)\right]-\Delta_{\mathrm{xc}} \mathbf{Z} \times \mathbf{g}_{\mathbf{k} s} \\
& +s \Delta_{\mathrm{xc}} \mathbf{Z} \times \mathbf{u} \operatorname{sign}\left(\varepsilon_{k s}\right)+\frac{s \hbar e}{\pi \xi \nu_{s}}\left(\mathbf{E} \cdot \mathbf{v}_{\mathbf{k}}\right) \Delta_{\mathrm{xc}} \mathbf{z} \times \mathbf{u} \delta\left(\varepsilon_{k s}\right) \\
& -e\left(\mathbf{E} \cdot \partial_{\mathbf{k}}\right) \mathbf{g}_{\mathbf{k} s}=\pi \xi \sum_{s^{\prime}} \int d k^{\prime} \delta\left(\varepsilon_{k^{\prime} s^{\prime}}-\varepsilon_{k s}\right)\left[\mathbf{g}_{\mathbf{k}^{\prime} s^{\prime}}-\mathbf{g}_{\mathbf{k s}}\right. \\
& \left.+\left(s-s^{\prime}\right) \mathbf{u} \operatorname{sign}\left(\varepsilon_{k s}\right)\right]+\left(\nu_{-s} / \nu_{s}-1\right) \hbar e\left(\mathbf{E} \cdot \mathbf{v}_{\mathbf{k}}\right) \mathbf{u} \delta\left(\varepsilon_{k s}\right) \\
& -\frac{\hbar}{\tau_{\sigma}}\left(\mathbf{g}_{\mathbf{k} s}-s \mathbf{u}\left[\operatorname{sign}\left(\varepsilon_{k s}\right)+\frac{\hbar e}{\pi \xi \nu_{s}} \mathbf{E} \cdot \mathbf{v}_{\mathbf{k}} \delta\left(\varepsilon_{k s}\right)\right]\right)
\end{aligned}
$$

Note, however, that Xiao, Zangwill, and Stiles [PRB 73, 054428 (2006)] criticized a phenomenological introduction of $\tau_{\sigma}$ arguing that there should be no spin dephasing towards a local equilibrium distribution

Skadsem et al. have recently verified this result for spin-orbit and magnetic disorder in the self-consistent Born approximation

## Magnetic equation of motion


m

$$
\partial_{t} \mathbf{m}=-\gamma \mathbf{m} \times \mathbf{H}_{\text {eff }}+\alpha \mathbf{m} \times \partial_{t} \mathbf{m}+\mathcal{P}\left[1-\mathbf{m} \times\left(\beta+\frac{\partial_{t}}{\Delta_{\mathrm{xc}}}\right)\right]\left(\mathbf{j} \cdot \partial_{\mathbf{r}}\right) \mathbf{m}
$$

$$
\alpha_{\mathrm{LDA}}=\beta \quad \alpha_{s-d}=\eta \beta
$$

The key parameter is the normalized spin-dephasing rate $\beta=1 /\left(\tau_{\sigma} \Delta_{\mathrm{xc}}\right)$
The first current-driven term (which is analogous to the Re $g^{\dagger \downarrow}$ torque in spin valves) leads to spin-wave instability of a uniform magnetization when the current-induced "Doppler shift" equals the natural frequency

The $\beta$ term (analogous to the $\operatorname{Im} g^{\dagger \downarrow}$ torque in spin valves), however, restores uniform magnetization stability in itinerant ferromagnets!

## Domain-wall motion

Current-driven Nèel wall:

Wall width is determined by the interplay of magnetic anisotropies and stiffness:

$$
\mathbf{H}=\left(K m_{z}+H\right) \mathbf{z}-K_{\perp} m_{x} \mathbf{x}+A \nabla^{2} \mathbf{m}
$$



$$
W=\sqrt{A / K}
$$

Li and Zhang, PRB 70, 0244 I7 (2005)

$$
\begin{aligned}
v_{i} & =-\mathcal{P} j \\
v_{f} & =-(\beta / \alpha) \mathcal{P} j
\end{aligned}
$$

Effectively perfect spin conversion into domain-wall motion despite microscopic spin dephasing!

Wall distortion in LDA is determined entirely by the new dynamic term:

$$
1-\frac{W_{f}}{W} \approx \frac{(\mathcal{P} j)^{2}}{2 \gamma A}\left[\frac{1}{\gamma K_{\perp}}\left(1-\frac{\beta}{\alpha}\right)^{2}-\frac{\hbar}{\Delta_{\mathrm{xc}}} \frac{\beta}{\alpha}\right]
$$

## Concluding remarks

- Current-driven magnetization dynamics in "bulk" needs more attention, theoretically and especially experimentally
- We pointed out important qualitative differences between mean-field $s$-d and self-consistent Stoner models
- It is crucial to understand the origin of ferromagnetic damping and its role in current-driven dynamics. Can it be described microscopically in terms of single-electron dephasing due to spin-orbit and/or magnetic disorder?
- What is the correct description of magnetism in transition metals? Current-driven dynamics appears to be a useful tool to address this question

