

# Spin Dynamics of Hole Systems

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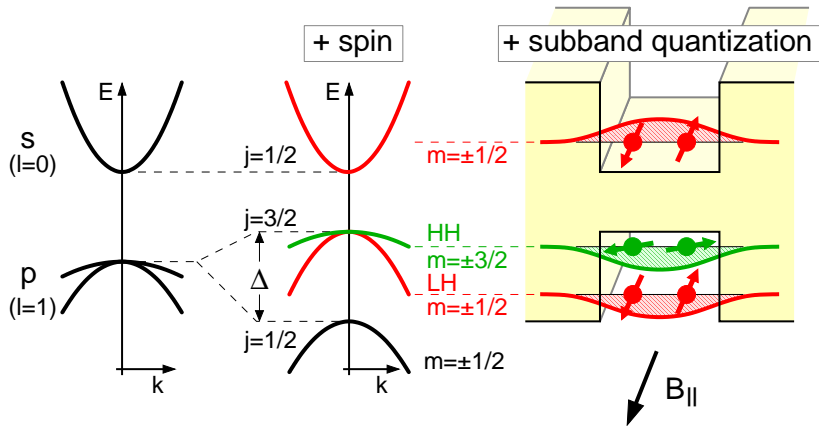
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experiments:

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D. Wasserman, and S. A. Lyon  
(Princeton University)

June 1, 2006

# Electron versus Hole Subbands

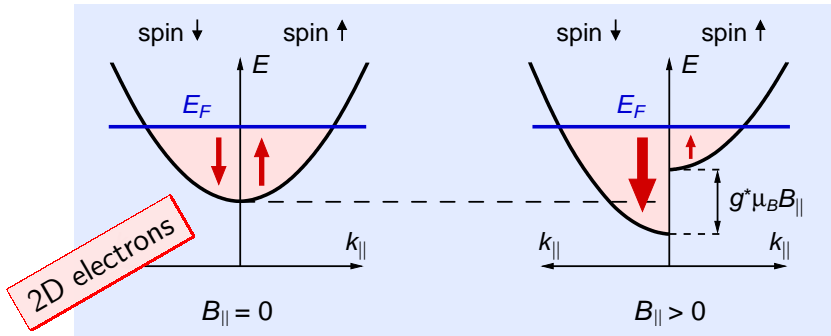


HH states: spin orientation fixed by subband quantization

# Overview

- ▶ Charge carriers in semiconductors:  
spin-1/2 electrons versus spin-3/2 holes
- ▶ Zeeman splitting and spin polarization  
of 2D hole systems in an in-plane magnetic field
- ▶ exchange-correlation enhancement of Zeeman splitting
- ▶ multipole expansion of spin density matrix
- ▶ spin precession of hole systems

# Zeeman Energy Splitting and Spin Polarization



electron systems: fully spin polarized when  $g^* \mu_B B_{||} > E_F$

(measurement: magnetoresistance)

hole systems: qualitatively different from that!

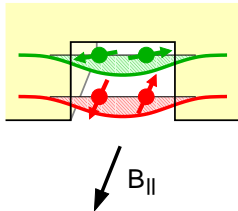
# Zeeman Energy Splitting of HH Systems

- ▶ frozen spin of HH states
- ▶ no Zeeman splitting linear in  $B_{\parallel}$  ( $g_{\parallel}^* \approx 0$ )  
[van Kesteren *et al.*, PRB **41**, 5283 (1990)]
- ▶ Zeeman splitting is a higher-order effect

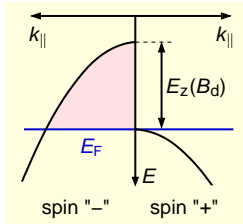
$$\Delta E_{\text{HH}} \propto \frac{B_{\parallel}^3}{E_{\text{HH}} - E_{\text{LH}}}$$

⇒

Zeeman splitting of HH states competes with HH-LH splitting



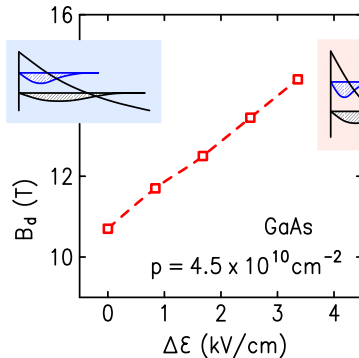
PRB **71**, 113307 (2005)



## Depopulation of Minority HH Spin Subband due to $B_{||}$

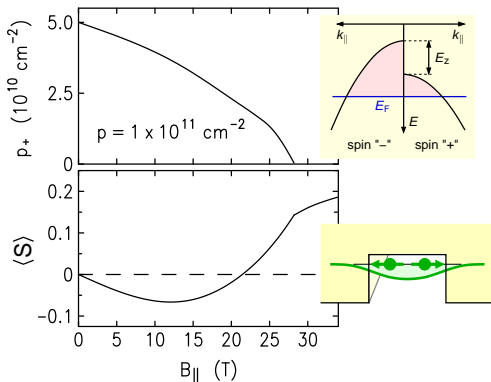
determined experimentally  
via magnetoresistance

small HH-LH splitting  
↕  
large Zeeman splitting  
↕  
small  $B_d$



large HH-LH splitting  
↕  
small Zeeman splitting  
↕  
large  $B_d$

# Zeeman Energy Splitting versus Spin Polarization: Holes



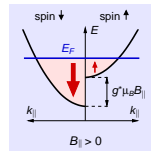
symmetric (100)  
 $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ -  
 $\text{GaAs}$  QW  
 $p = 1 \times 10^{11} \text{ cm}^{-2}$   
 $w = 150 \text{ \AA}$

## Heavy hole systems:

- ▶ Zeeman energy splitting not simply related with spin polarization
- ▶ spin polarization always  $\ll 1$
- ▶ sign reversal of the spin polarization at  $B_{||} > 0$

PRB 71, 113307 (2005)

# Why are these experiments interesting?

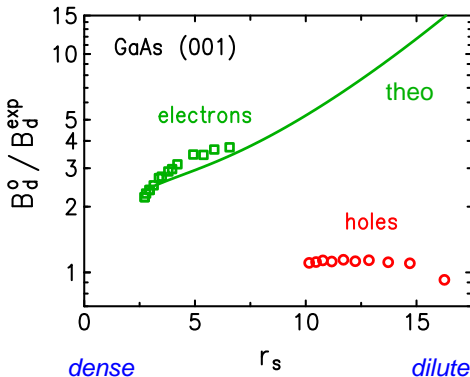


- **Known for dilute electron systems**
  - ▶ Spin susceptibility enhanced because of exchange-correlation
  - ▶ large enhancement for
    - small densities (“dilute systems”)
    - large  $m^*$   $\Rightarrow$  **use holes**
- **2D hole systems in  $B_{\parallel}$  studied experimentally by many groups**
- **Holes mediate ferromagnetism in GaMnAs**
- **Spin Hall effect in hole systems**



# Spin Susceptibility of Dilute 2D Systems

enhancement  
of spin  
susceptibility



density parameter  $r_s = 1/a_B^* \sqrt{\pi n} \propto m^*$

► **electrons:** enhanced spin susceptibility due to exchange-correlation

► **holes:** no enhancement despite large  $r_s$

PRB **72**, 195321 (2005)

# Why is exchange-correlation in hole systems different?

# Spin 1/2 Systems (Electrons):

- $2 \times 2$  spin density matrix
- 4 independent parameters: density + vector of spin polarization
- **Systematic scheme:** multipole expansion of spin density matrix
  - $l = 0$ : monopole  $\equiv$  charge density
  - $l = 1$ : dipole  $\equiv$  spin polarization

- **Compare:**

Multipole expansion of Hamiltonian for spin 1/2 systems


$$\hat{H} = \underbrace{\left[ \frac{p^2}{2m^*} + V(\mathbf{r}) \right]}_{l=0} \mathbb{1}_{2 \times 2} + \underbrace{\frac{g^*}{2} \mu_B \mathbf{B} \cdot \boldsymbol{\sigma}}_{l=1}$$

$l = 0$ :  
dynamics of  
charge density

$l = 1$ :  
spin polarization for  $B > 0$

- Orbital and spin dynamics decoupled because wave functions factorize

# Spin 3/2 Systems (Holes):

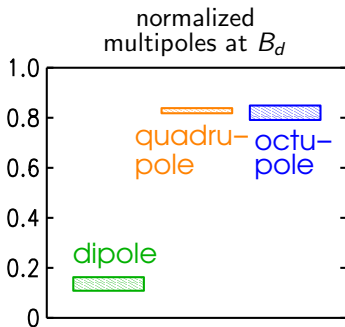
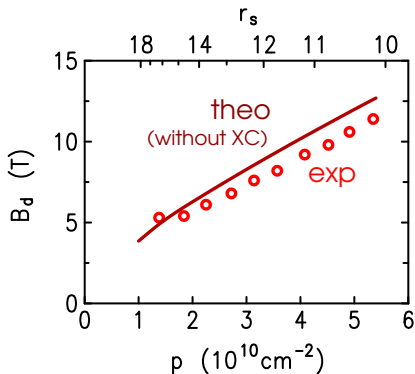
- $4 \times 4$  spin density matrix
- 16 independent parameters:  $\Rightarrow$  very complex 

- Multipole expansion of spin density matrix
  - $l = 0$ : monopole  $\equiv$  charge density
  - $l = 1$ : dipole  $\equiv$  spin polarization ( $B > 0$ )
  - $l = 2$ : quadrupole  $\equiv$  HH-LH splitting
  - $l = 3$ : octupole  $\equiv$  supplement to spin polarization ( $B > 0$ )  
 $\Rightarrow$  gauge invariant!

- compare: Hamiltonian (“Luttinger Hamiltonian”)
  - $l = 0$ : kinetic energy + potential
  - $l = 1$ : Zeeman term
  - $l = 2$ : HH-LH splitting
  - $l = 3$ : negligible
- dynamics of multipole moments coupled,  
because wave functions cannot be factorized PRB **70**, 125301 (2004)

# Spin Multipoles of 2D Hole Systems

GaAs (001) heterojunction



octupole = new degree of freedom of hole spins at  $B_{||} > 0$

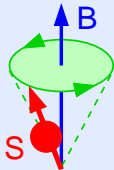
needed: exchange-correlation energy as a function of all four multipoles

# Spin Precession

Electrons ( $S = 1/2$ ):

$$\hat{H} = H_0 + \frac{1}{2} \mathbf{S} \cdot \mathcal{B}$$

$$\frac{d\mathbf{S}}{dt} = \frac{i}{\hbar} [\hat{H}, \mathbf{S}]$$



$$\langle \dot{\mathbf{S}} \rangle = \frac{1}{\hbar} \langle \mathbf{S} \rangle \times \mathcal{B}$$

$$\frac{d|\langle \mathbf{S} \rangle|}{dt} = 0$$

$\langle \mathbf{S} \rangle$  = Bloch vector

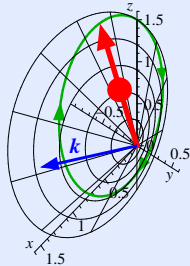
$\mathcal{B}$  = Dresselhaus field  $\mathcal{B}(\mathbf{k})$

$\Rightarrow$  Dyakonov-Perel  
spin relaxation

cond-mat/0605390

Holes ( $S = 3/2$ ):

$$\hat{H} = \hat{H}_0 + \cancel{\hat{H}_1} + \hat{H}_2 + \cancel{\hat{H}_3}$$



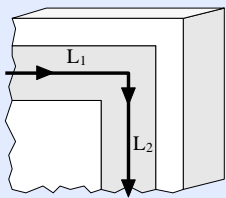
$\mathcal{B} = 0!$

oscillating  
spin  
polarization

$\langle \dot{\mathbf{S}} \rangle \rightarrow$  dipole  $\mathbf{S}$   
+ quadrupole  $\mathbf{Q}$   
+ octupole  $\mathbf{O}$

$$\frac{d|\langle \mathbf{S} \rangle|}{dt} \neq 0$$

# Holes Turning a Corner (B=0!)



[fabricated by Grayson *et al.*,  
APL **86**, 032101 (2005)  
– but as of yet only *n*-type!]

in region  $L_1$ :

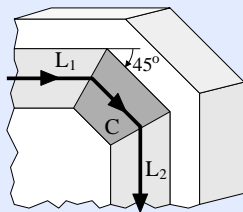
$$|\mathbf{S}(t)|^2 = |\mathbf{O}(t)|^2 = 0$$

in region  $L_2$  (ballistic regime):

$$|\mathbf{S}(t)|^2 = 0$$

$$|\mathbf{Q}(t)|^2 = \frac{1}{16} + \frac{3}{16} \cos^2(\omega t)$$

$$|\mathbf{O}(t)|^2 = \frac{3}{16} \sin^2(\omega t)$$



in region C (ballistic regime):

$$|\mathbf{S}(t)|^2 = \frac{9}{80} \sin^2(\omega t)$$

$$|\mathbf{Q}(t)|^2 = \frac{1}{64} + \frac{15}{64} \cos^2(\omega t)$$

$$|\mathbf{O}(t)|^2 = \frac{39}{320} \sin^2(\omega t)$$

$$\omega = (E_{\text{HH}} - E_{\text{LH}})/\hbar$$

robust against disorder if  $\omega > 1/\tau_s$

cond-mat/0605390

## Summary & Outlook

- ▶ Usually spin polarization of 2D holes at  $B_{\parallel} > 0$  only small  
Sign reversal of spin polarization at  $B_{\parallel} > 0$
  - ▶ Spin density matrix of holes characterized by 4 multipoles  
Spin octupole: new degree of freedom of holes at  $B_{\parallel} > 0$
  - ▶ Usually no exchange-correlation enhancement  
of spin susceptibility of HH states
  - ▶ 2DHSs on (113)GaAs: Spin susceptibility enhanced *only*  
when  $B_{\parallel}$  does give rise to a spin polarization
  - ▶ **required**: XC energy and spin susceptibility as a function  
of all 4 spin multipoles
  - ▶ Spin precession of holes: alternating spin polarization at  $B = 0$
- ⇒ Dyakonov-Perel spin relaxation of holes

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See PRB **70**, 125301 (2004), PRB **71**, 113307 (2005),  
PRB **72**, 195321 (2005), cond-mat/0605390