Proper definition of spin current

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Shi, Zhang, Xiao, and Niu
(PRL, 96, 076604, 2006) , also see (cond-mat 0503505)

Culcer, Sinova, Sintsyn, Jungwirth, MacDonald , and Niu
(PRL,93,046602,2004)

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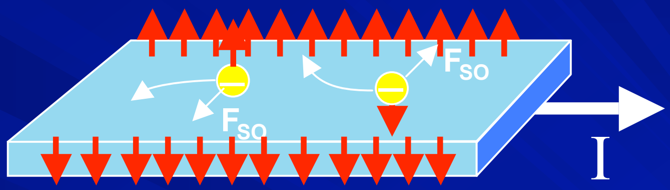
Spin Hall effect: theory

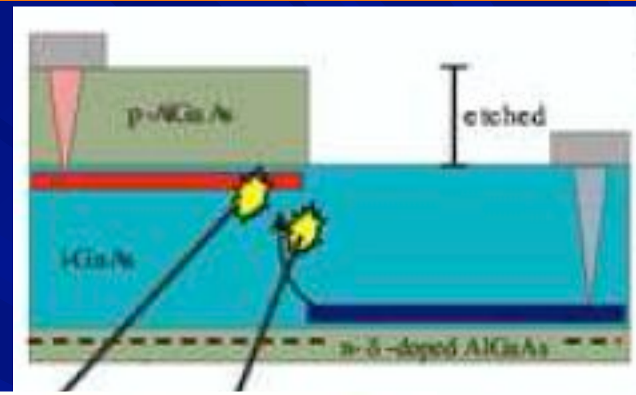
- Extrinsic:
  - Dyakonov and Perel (71),
  - J. E. Hirsch (99),
  - S. Zhang (00)
- Intrinsic:
  - Murakami et al Science (03)
  - Sinova et al PRL (04)
- And many more …

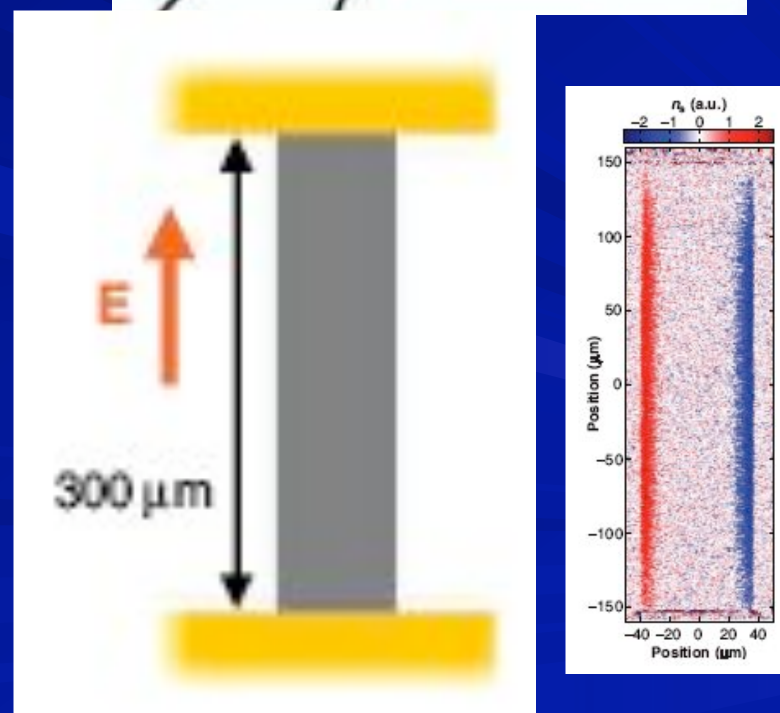


Spin Hall effect: experiments

- Rashba 2D holes
  - Wunderlich et al PRL (05)

- n-type semiconductors
  - Kato et al Science (04)





A logical gap

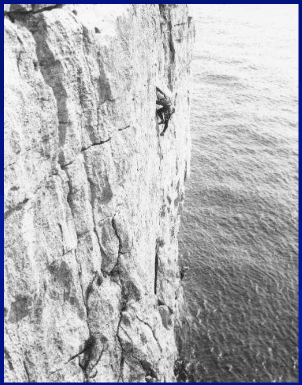
- Spin current calculated for the bulk
  does not connect uniquely to
  spin accumulation at sample boundary.

- Spins can be generated locally at the boundary.

- Spins relax differently at the boundary than in the bulk.

Work needed

- Theorists need to work out boundary spin accumulation for specific boundaries:
  - Steep wall



Shallow wall

- Experimentalists need to find good ways to measure the spin current:
  - go to the bulk

Need a proper definition of spin current

-

- Conventional definition

$$\vec{J}_s = \langle s_z \dot{\vec{r}} \rangle$$

- New definition

$$\hat{\mathcal{J}}_s = \frac{d(\hat{\mathbf{r}}\hat{s}_z)}{dt}$$

Spin continuity equation

- An electric field can generate a torque density if symmetry is low enough
- An electric field can always generate a torque dipole density even if the average torque density is zero
- A new spin current

$$\frac{\partial S_z}{\partial t} + \nabla \cdot \mathbf{J}_s = \mathcal{T}_z$$

$$\mathcal{T}_z(\mathbf{r}) = -\nabla \cdot \mathbf{P}_\tau(\mathbf{r})$$

$$\mathcal{J}_s = \mathbf{J}_s + \mathbf{P}_\tau$$

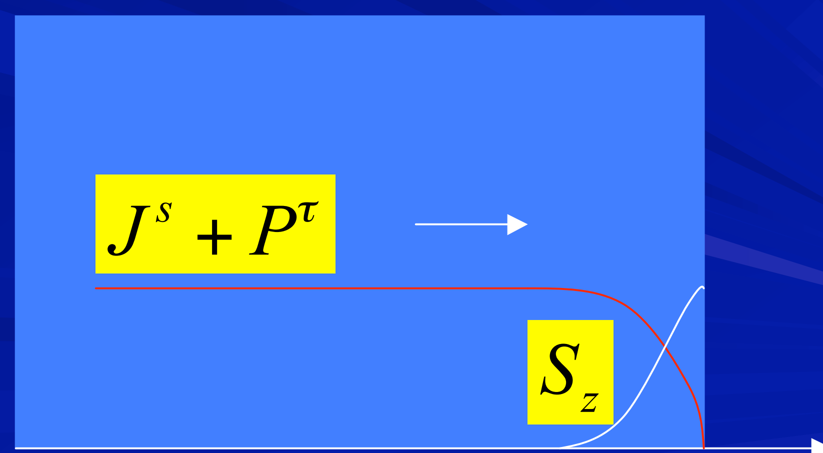
Torque dipole density:
Culcer *et al*
(PRL 93 046602 2004)

Spin accumulation

- A new spin continuity equation
  - Spin generation by inhomogeneous spin current
- Application to a boundary with shallow confinement

$$\frac{\partial S_z}{\partial t} + \nabla \cdot \mathcal{J}_s = -\frac{S_z}{\tau_s}$$

$$\int S_z dx = \tau_s J^s$$



Energy dissipation due to spin current

- Spin displacement: $(\hat{\mathbf{r}}\hat{s}_z)$
- Spin force: $\mathbf{F}_s$

Zeeman field gradient
(g factor gradient + Zeeman field)

- Energy shift: $V = -\mathbf{F}_s \cdot (\hat{\mathbf{r}}\hat{s}_z)$
- Spin current: $\hat{\mathcal{J}}_s = \frac{d(\hat{\mathbf{r}}\hat{s}_z)}{dt}$
- Power dissipation: $dQ/dt = \mathcal{J}_s \cdot \mathbf{F}_s$

Provide a method for
spin current measurement

- Cannot discuss dissipation using conventional current

Onsager relation

Spin-charge conductivity tensor:

$$\begin{pmatrix} \mathbf{J}^s \\ \mathbf{J}^c \end{pmatrix} = \begin{pmatrix} \overleftrightarrow{\sigma}^{ss} & \overleftrightarrow{\sigma}^{sc} \\ \overleftrightarrow{\sigma}^{cs} & \overleftrightarrow{\sigma}^{cc} \end{pmatrix} \begin{pmatrix} \mathbf{F}^s \\ \mathbf{E} \end{pmatrix}$$

Onsager relation:

$$\sigma_{cs}^{yx} = -\sigma_{sc}^{xy}$$

Can be shown easily using linear response

$$H = H_0 - \mathbf{E}\cdot(-e\mathbf{r}) - \mathbf{F}^s\cdot(s_z\mathbf{r})$$

$$\sigma_{mn} = -\sum_{\alpha\neq\beta} \frac{\hbar\,\mathrm{Im}[\langle\alpha|\dot{d}_m|\beta\rangle\langle\beta|\dot{d}_n|\alpha\rangle]}{(\epsilon_\alpha-\epsilon_\beta)^2}(f_\alpha - f_\beta)$$

Onsager relation cannot be established using the conventional current

Inverse spin Hall effect

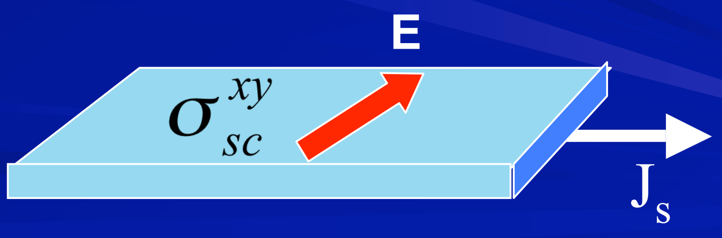
- Transverse charge current induced by spin force:

$$J_c^y = \sigma_{cs}^{yx} F_x^s$$

- Measure spin Hall current based on Onsager relation

$$J_s^y = \sigma_{sc}^{xy} E_y$$

$$\sigma_{sc}^{xy} = -\sigma_{cs}^{yx}$$



Spin Hall conductivity - intrinsic

	Rashba	k^3 Rashba	Luttinger
σ_{xy}^{s0}	$-e/8\pi$	$9e/8\pi$	$\frac{e(\gamma_1+2\gamma_2)}{12\pi^2\gamma_2}(k_H - k_L)$
σ_{xy}^{τ}	$e/4\pi$	$-9e/4\pi$	$-\frac{3e\gamma_1}{12\pi^2\gamma_2}(k_H - k_L) + \frac{e}{6\pi^2}k_H$
σ_{xy}^{s}	$e/8\pi$	$-9e/8\pi$	$\frac{e(\gamma_2-\gamma_1)}{6\pi^2\gamma_2}(k_H - k_L) + \frac{e}{6\pi^2}k_H$

Table I: Intrinsic spin Hall coefficients for a few widely studied semiconductor models, including the Rashba model for 2D electrons [7], k-cubed Rashba model for 2D holes [13] and Luttinger model for 3D holes [6, 10]. For Rashba models, we only list the "universal" values for the case when both bands are occupied.

cond-mat/0503505 version 4 (last version before PRL)

Spin Hall conductivity - disordered

(a) Rashba model

Impurity potential	Born approx.	Definition of spin current	
		$\langle J_s \rangle$	$\mathcal{J}_s$
$\delta(\mathbf{r})$	1st/higher	0[37]	0
$V_{\mathbf{p}-\mathbf{p}'}$	1st	0[38]	0
	higher	0	**Finite**

(b) Cubic Rashba model

Impurity potential	Born approx.	Definition of spin current	
		$\langle J_s \rangle$	$\mathcal{J}_s$
$\delta(\mathbf{r})$	1st/higher	Finite[35,36]	0
$V_{\mathbf{p}-\mathbf{p}'}$	1st	Finite	0
	higher	Finite	**Finite**

TABLE I: Spin Hall effect in the (a) (linear) Rashba model and (b) cubic Rashba model, with various types of the impurity potential for the two different definitions: $\langle J_s \rangle$ is the conventional spin current, $J_s = \frac{1}{2}\{v_y, s_z\}$, and $\mathcal{J}_s$ is the conserved effective spin current, $\mathcal{J}_s = \langle J_s \rangle + P_\tau$. We show the new results in the boldface.

cond-mat/0503475: Spin Hall effect of conserved current
Sugimoto, Onoda, Murakami, Nagaosa

Spin Hall insulators

- Charge insulator with a spin Hall effect

  -- Murakami, Nagaosa, Zhang
- However, all insulators with spin-orbit coupling are spin Hall insulators based conventional spin current, e.g. undoped

GaAs,	Si,	Ge.
0.001,	0.0015,	0.0017 e/a

spin Hall conductivity (Yao)

No spin Hall in simple insulators based on our proper spin current

- Kubo formula:

$$\sigma_{xy}^{sc} = e \sum_{\alpha \neq \beta} \frac{\hbar \, \mathrm{Im}[\langle \alpha | s_z \dot{x} + \dot{s}_z x | \beta \rangle \langle \beta | \dot{y} | \alpha \rangle]}{(\varepsilon_\alpha - \varepsilon_\beta)^2} (f_\alpha - f_\beta)$$

- For localized eigenstates, we can use

$$< \alpha | \dot{x} s_z + x \dot{s}_z | \beta > = < \alpha | x s_z | \beta > (\varepsilon_\alpha - \varepsilon_\beta) / i\hbar$$

$$< \beta | \dot{y} | \alpha > = < \beta | y | \alpha > (\varepsilon_\alpha - \varepsilon_\beta) / i\hbar$$

- Then

$$\sigma_{xy}^{sc} = \frac{e}{\hbar} \sum_\alpha f_\alpha < \alpha | [s_z x, y] | \alpha > = 0$$

- How about band insulators with localized Wannier orbitals?

Conclusions

- New spin current:
  - conventional spin current + torque dipole density
  - Time derivative of spin displacement
- Spin continuity equation:

$$\frac{\partial S_z}{\partial t} + \nabla \cdot \mathcal{J}_s = -\frac{S_z}{\tau_s}$$

  - Basis for study spin accumulation in the bulk
  - -- Spin accumulation at boundaries with shallow confinement
- Can define a spin force
  - Measurement from heat generation
  - Onsager relation is satisfied and can also be used for measuring spin current through the inverse spin Hall
- Altered results on spin Hall conductivity
  - sign reversal on intrinsic values
  - Vanishing values with infinitesimal disorder
  - No spin Hall effect in simple insulators