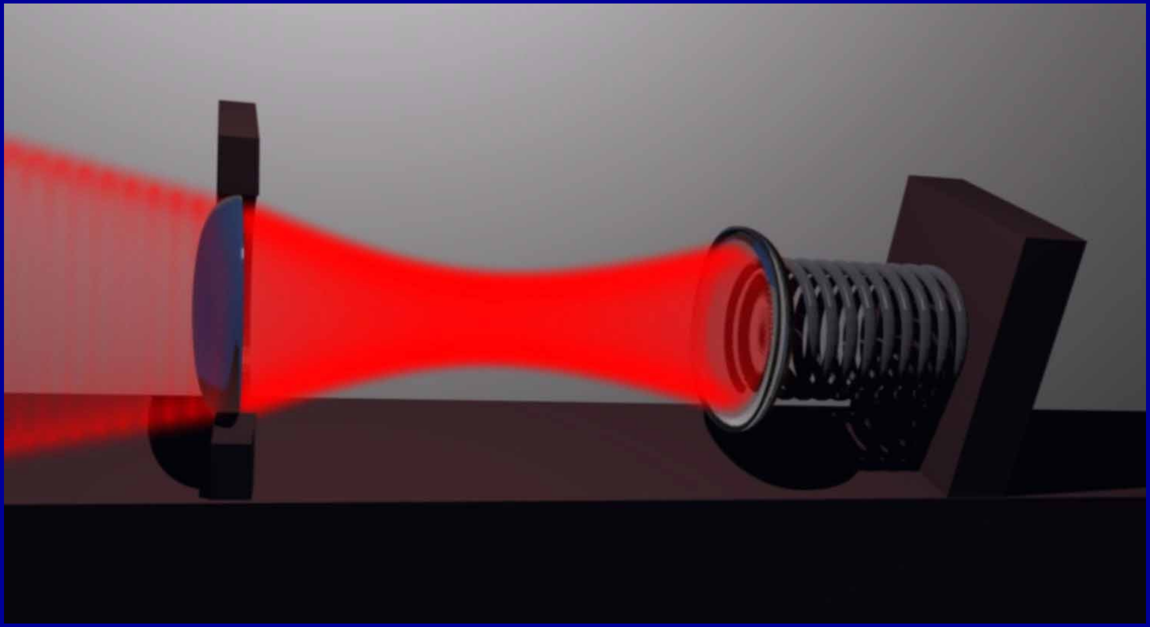
Superconductor-based nanomechanics

Yaroslav M. Blanter

Kavli Institute of Nanoscience, Delft University of Technology

- Introduction to nanomechanics
- Backaction and self-sustained oscillations in a SQUID
- Backaction on a Duffing oscillator
- Phonon blockade

Optomechanics



From T. Kippenberg, EPFL

Typical size: several μm
Frequency: up to 10 MHz
Q-factor: up to 10^4
Coupling: weak

Couples phonons to light via radiation pressure

$$H = \hbar\omega b^\dagger b + M\dot{x}^2/2 + M\omega_0^2 x^2/2 + gxb^\dagger b$$

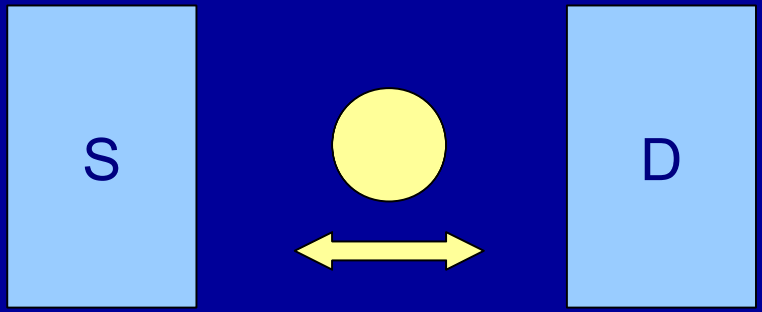
Light

Mechanical resonator

Coupling

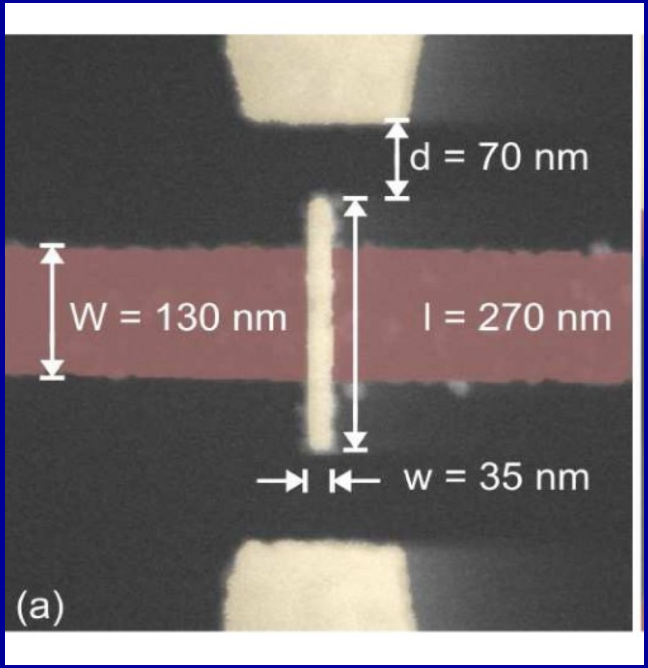
Shuttling

L. Y. Gorelik et al (Chalmers)
PRL **80**, 4526 (1998)



Couples phonons to charge due to the position dependence of tunnel rates and of Coulomb-induced force

D. R. König and E. M. Weig (LMU)
APL **101**, 213111 (2012)

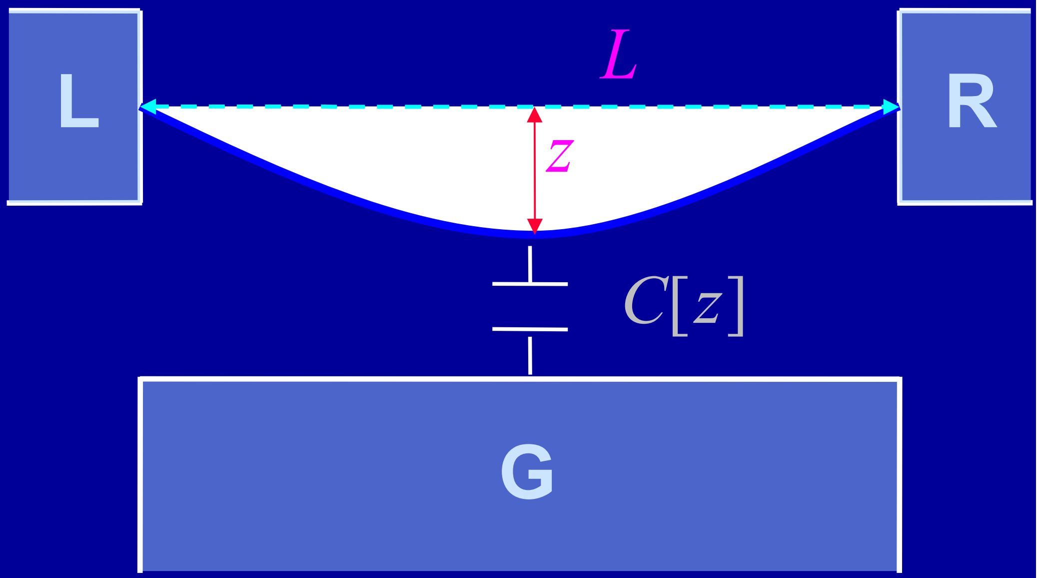


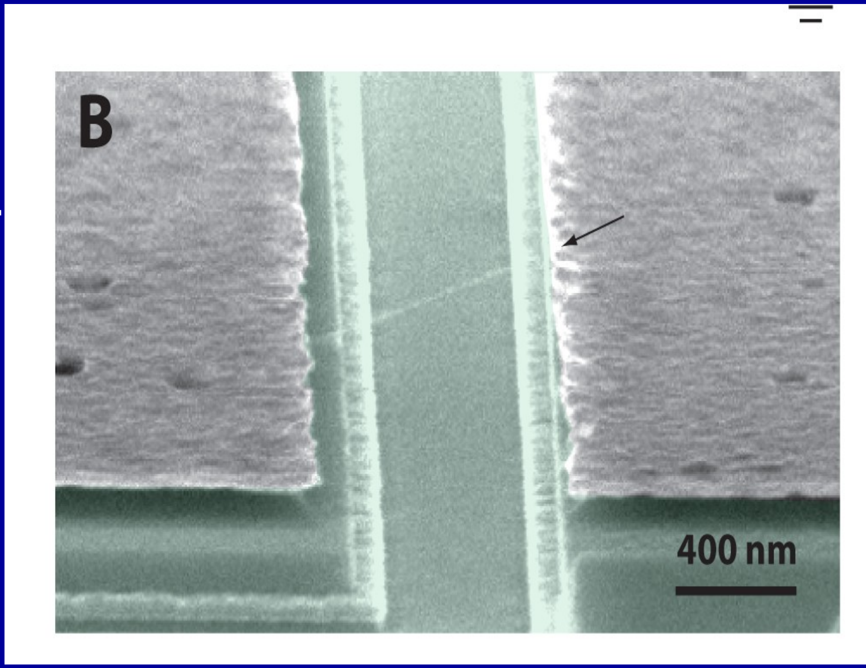
Size: 400 nm
Frequency: 9 MHz
Q-factor: high
Coupling: strong

Double-clamped beam

S. Sapmaz et al (Delft)
PRB **67**, 235414 (2003)

G. Steele et al (Delft)
Science **325**, 1103 (2009)



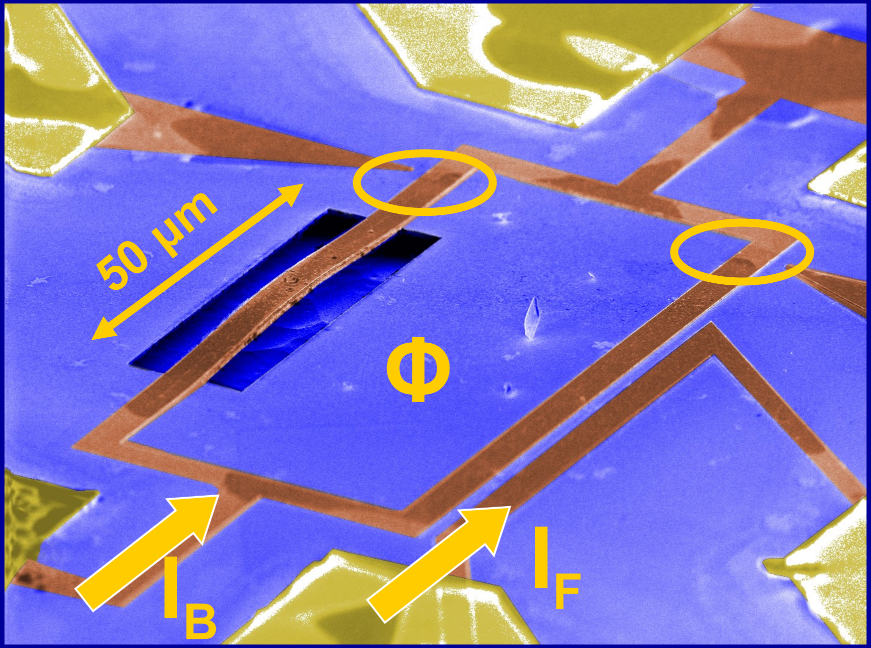


Couples phonons to charge due to the Coulomb-induced force

Size: 500 nm
Frequency: 140 MHz
Q-factor: over 10^5
Coupling: strong

Suspended SQUID

S. Etaki et al (Delft)
Nature Physics **4**, 785 (2008)

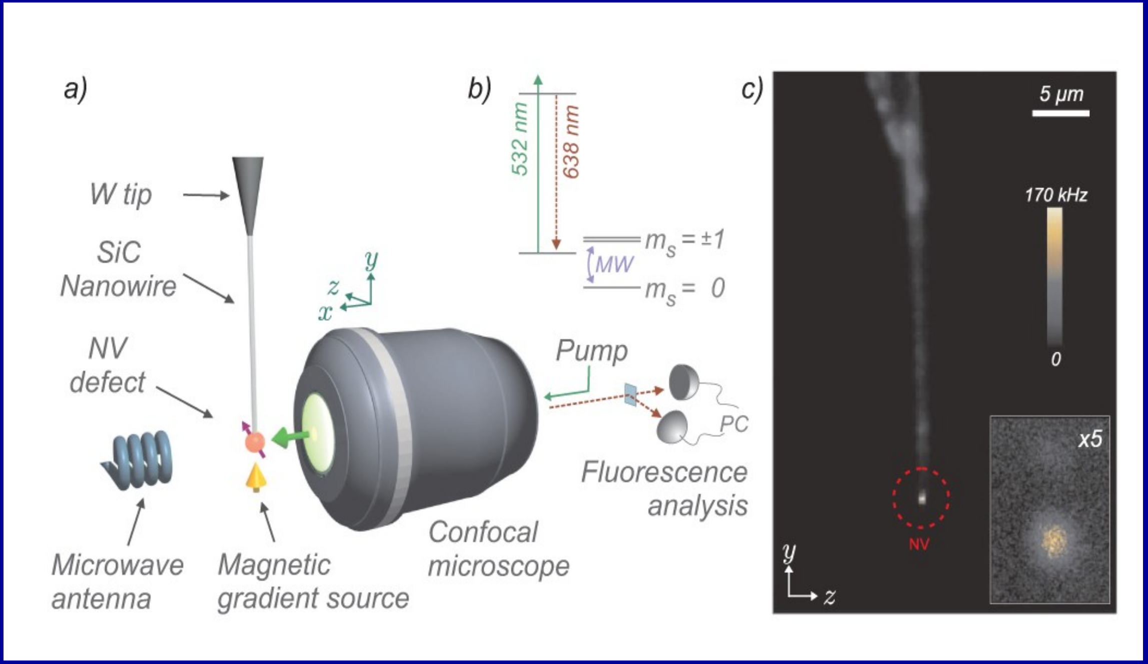


Size: 50 μm
Frequency: 2 MHz
Q-factor: 18000
Coupling: moderate

Couples phonons to superconducting phase due to the area variation

Spin embedded in a resonator

O. Arcizet et al (Inst. NEEL – Grenoble)
Nature Physics **7**, 879 (2011)



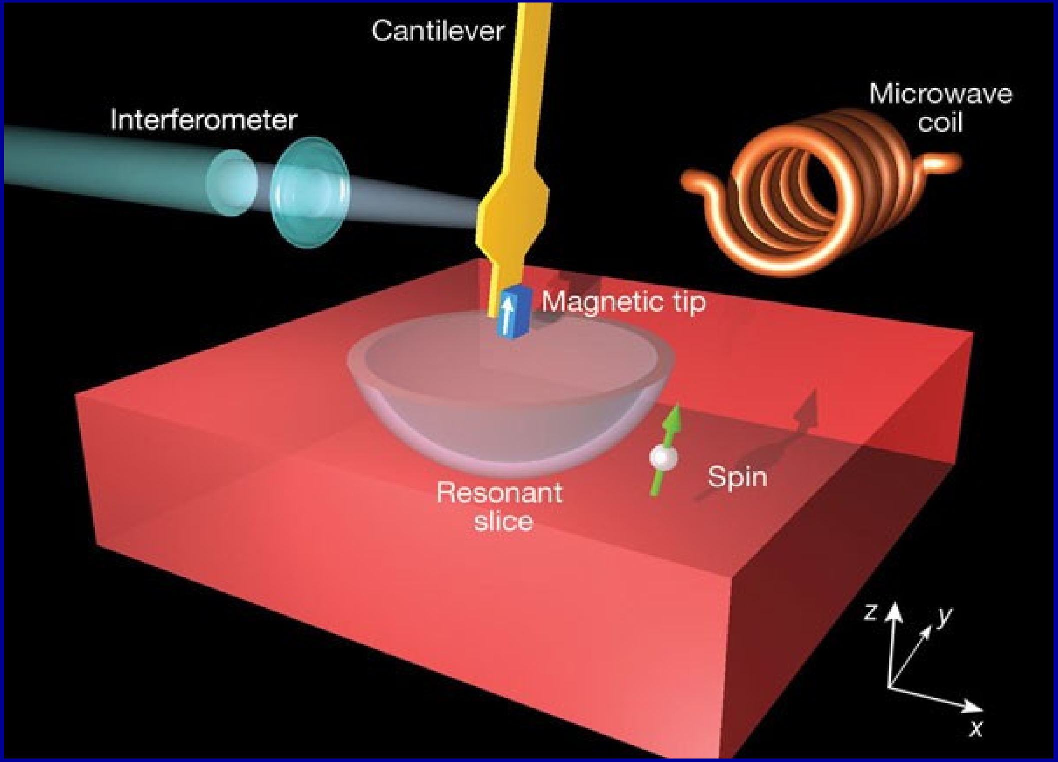
Couples phonons to spin via inhomogeneous magnetic field

Size: 10 µm
Frequency: 1 MHz
Q-factor: 10000
Coupling: weak

Nanomechanics: Functions

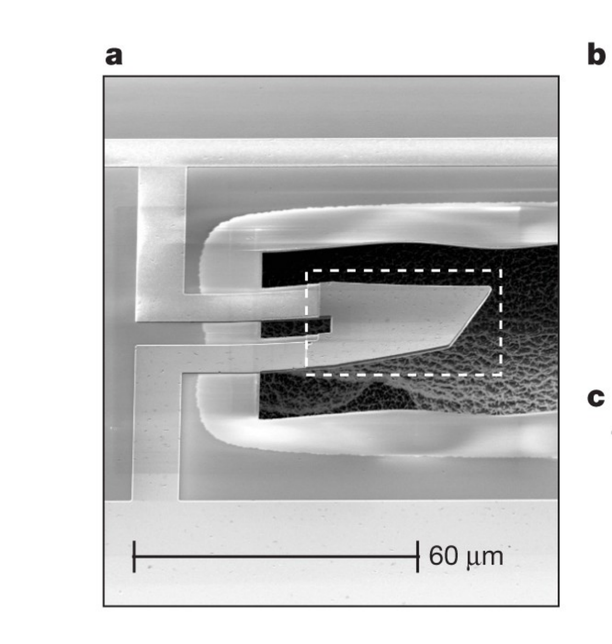
- Detectors
- Shuttles
- Circuit elements
- Quantum state transducers

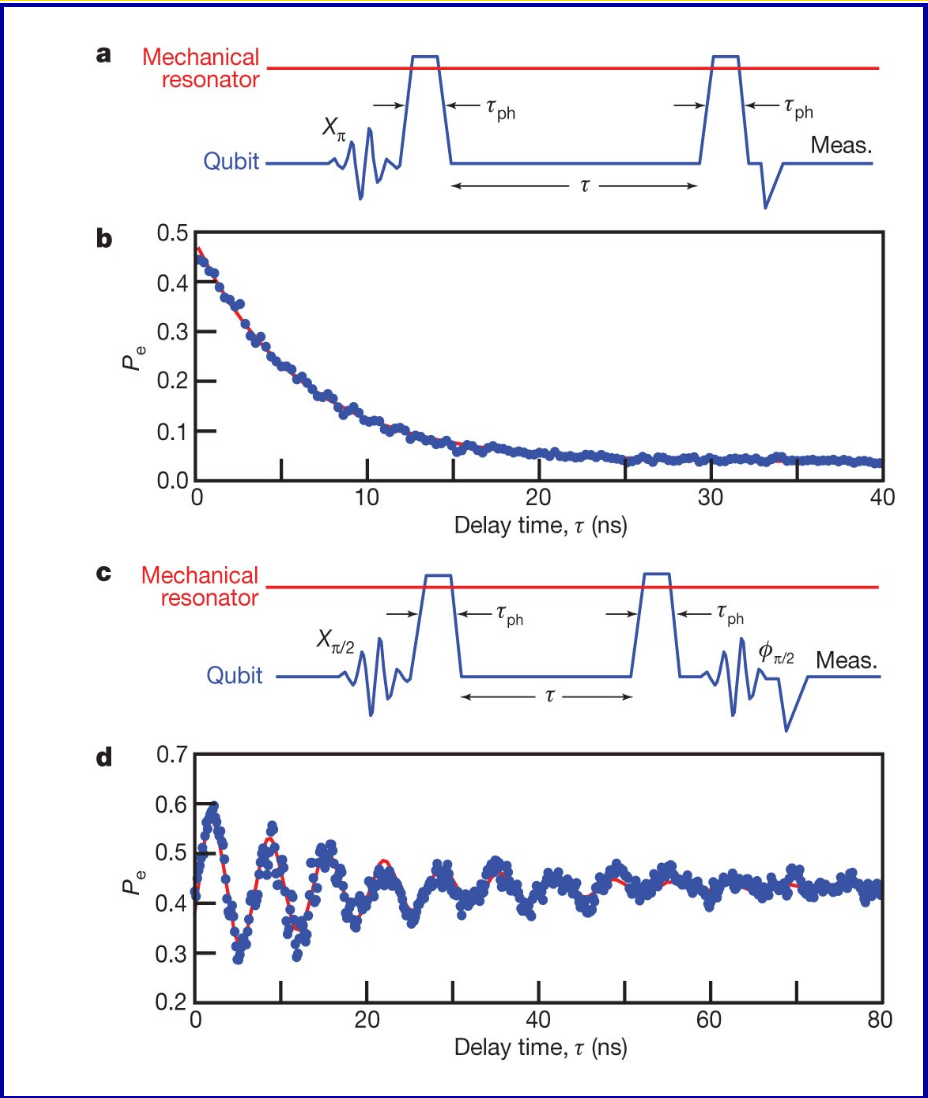
Detection of a single spin



D. Rugar et al (IBM Almaden)
Nature **430**, 329 (2004)

Quantum detection of mechanical oscillations





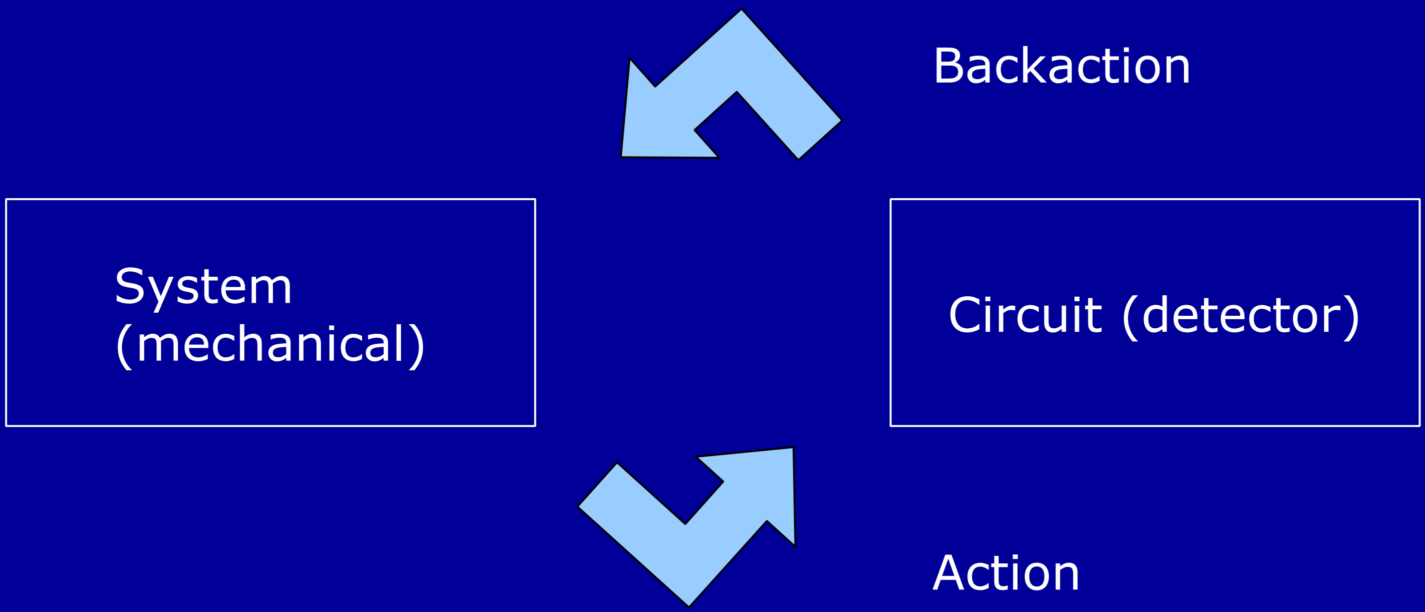
A. D. O'Connell et al (UCSB)
Nature **464**, 697 (2010)

A mechanical resonator coupled to a superconducting qubit

Nanomechanics: Issues

- Coupling: Want stronger coupling b/w a mechanical system and a detector
- Backaction
- Origin of dissipation
- For quantum detection: need to cool down or to have high frequencies

Need to understand coupling and back-action!!

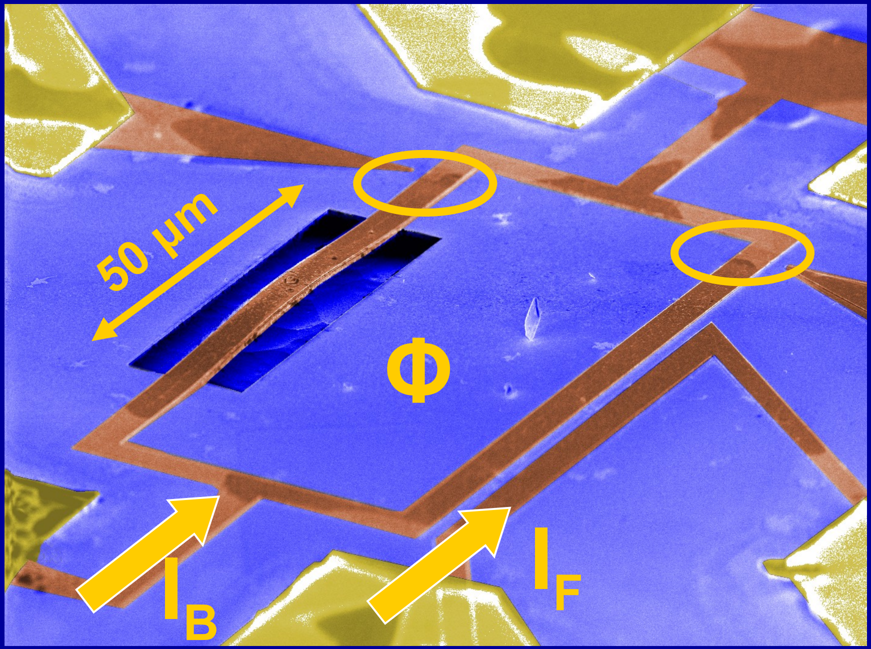


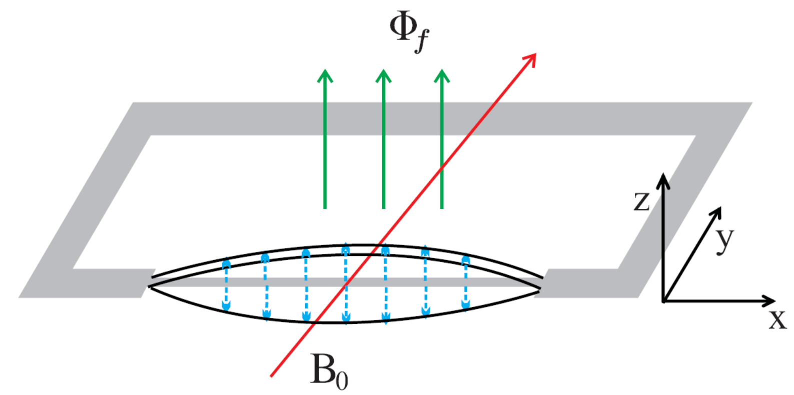
Quantum regime

In a nutshell: To explore interesting quantum physics with mechanical resonators, we need high frequencies and strong coupling

Classical SQUID: experiment

S. Etaki et al, Nature Physics **4**, 785 (2008)

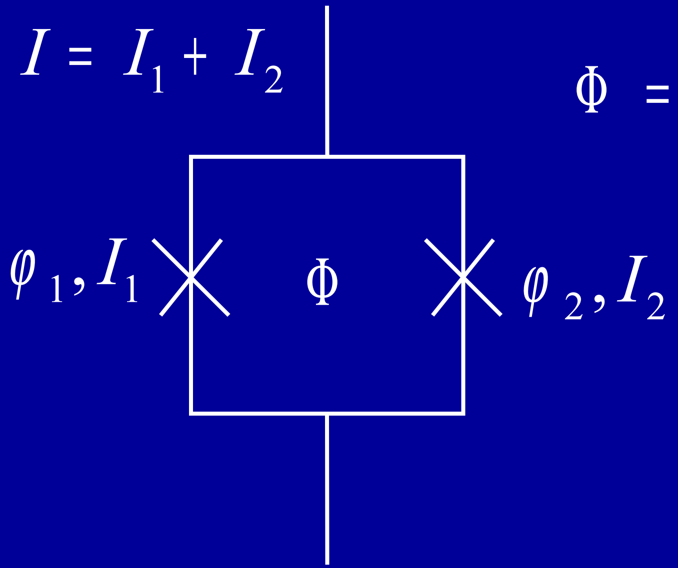




$$\delta\Phi = aBlx, \ a \approx 0.91$$

x - displacement

dc SQUID



$$\Phi = \Phi_0(\varphi_2 - \varphi_1)/(2\pi) \qquad \Phi_0 \equiv \frac{\pi \hbar c}{e}$$

Josephson junctions:

$$I_{1,2} = I_0 \sin\varphi_{1,2}$$

Total current through the loop:

$$I = 2I_0 \cos\left(\frac{\pi\Phi}{\Phi_0}\right)\sin\varphi$$

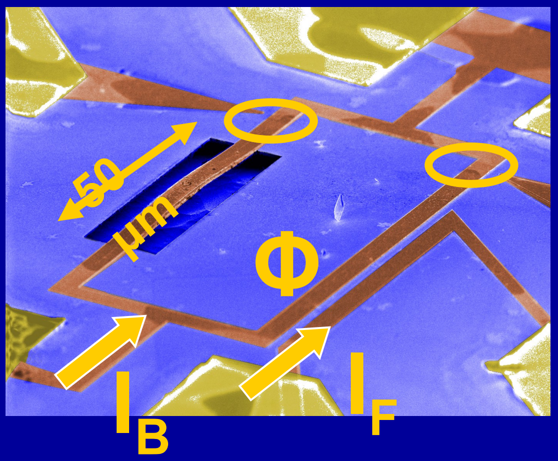
Very sensitive detector of magnetic field

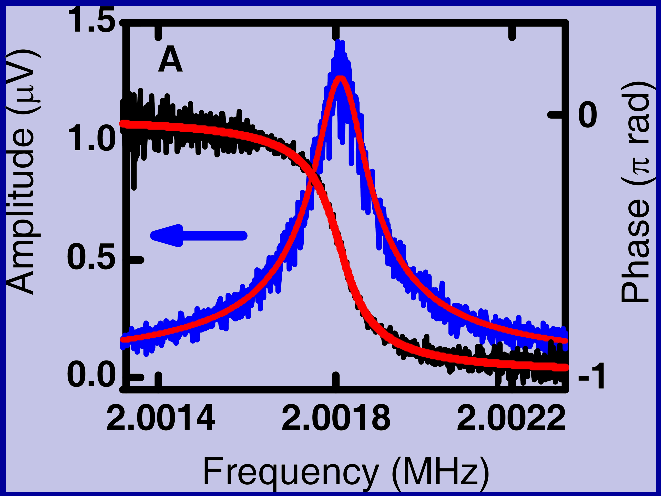
Coupling to mechanical motion:

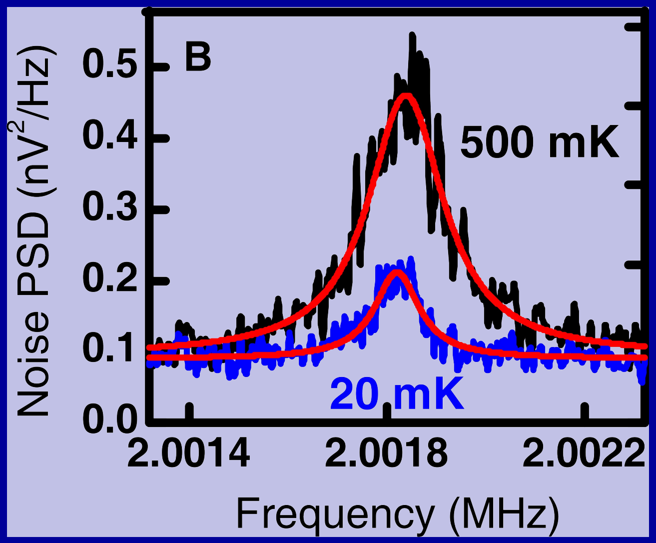
$$E_c = -E_J \cos\left(\frac{\pi\Phi}{\Phi_0} + \alpha x\right) = g_1 x \sin\left(\frac{\pi\Phi}{\Phi_0}\right) + g_2 x^2 \cos\left(\frac{\pi\Phi}{\Phi_0}\right)$$

Classical SQUID: experiment

S. Etaki et al, Nature Physics **4**, 785 (2008)



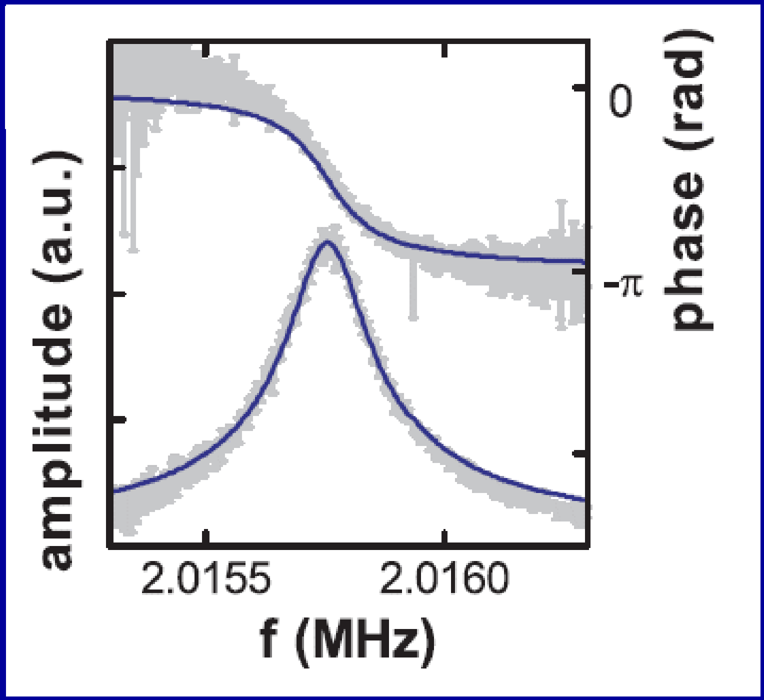




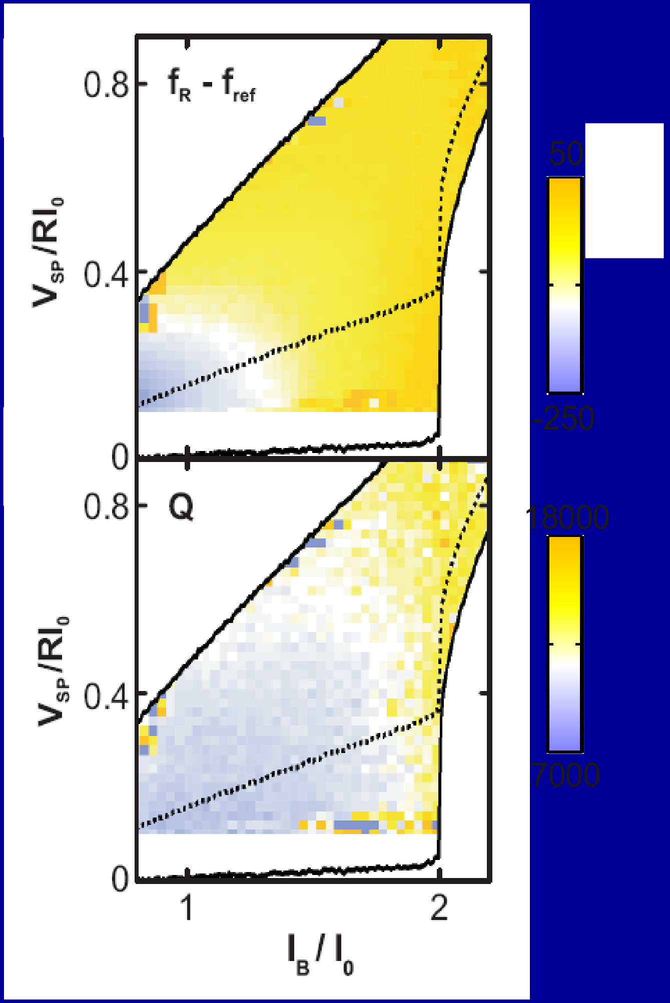
Harmonic oscillator response at
$f = 2$ MHz and $Q = 18\,000$

Backaction - experiments

Driven response:

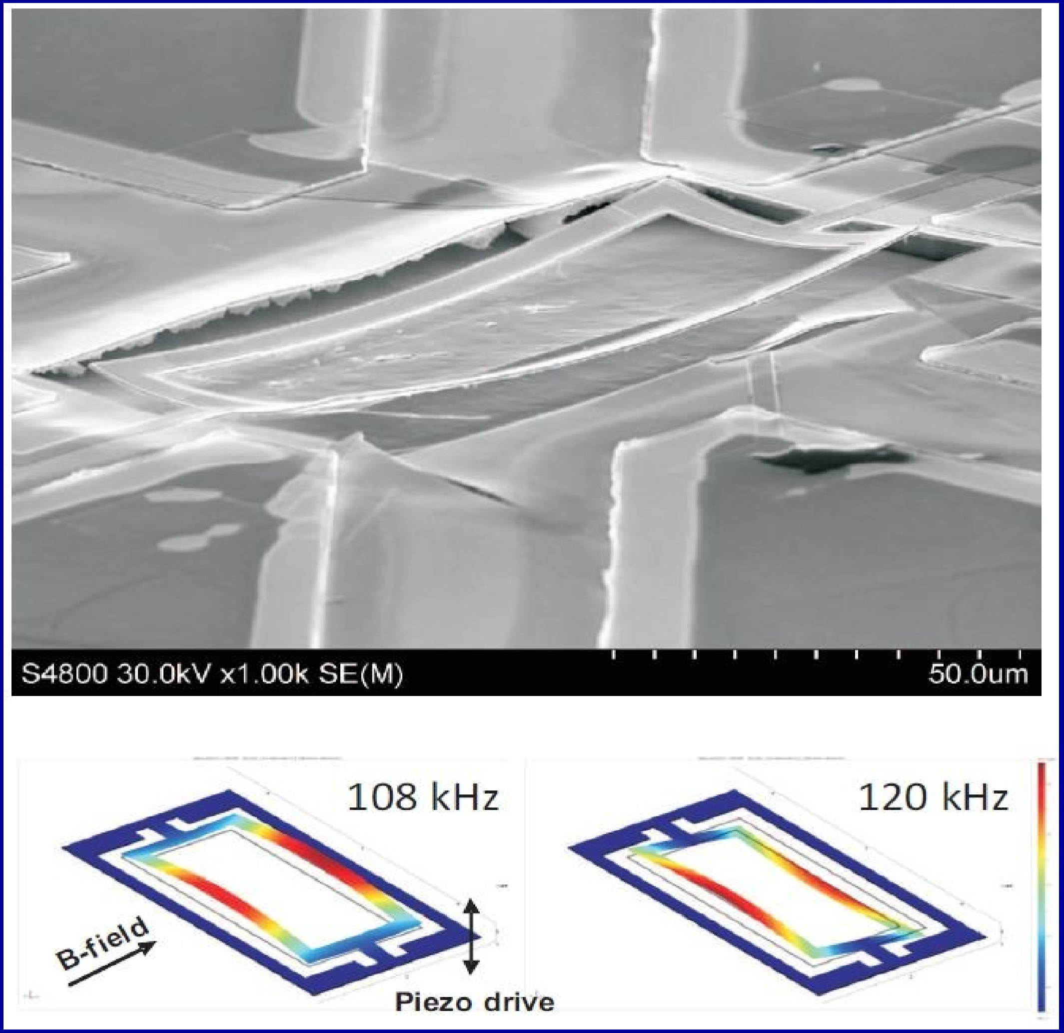


M. Poot et al, PRL **105**, 207203 (2010)



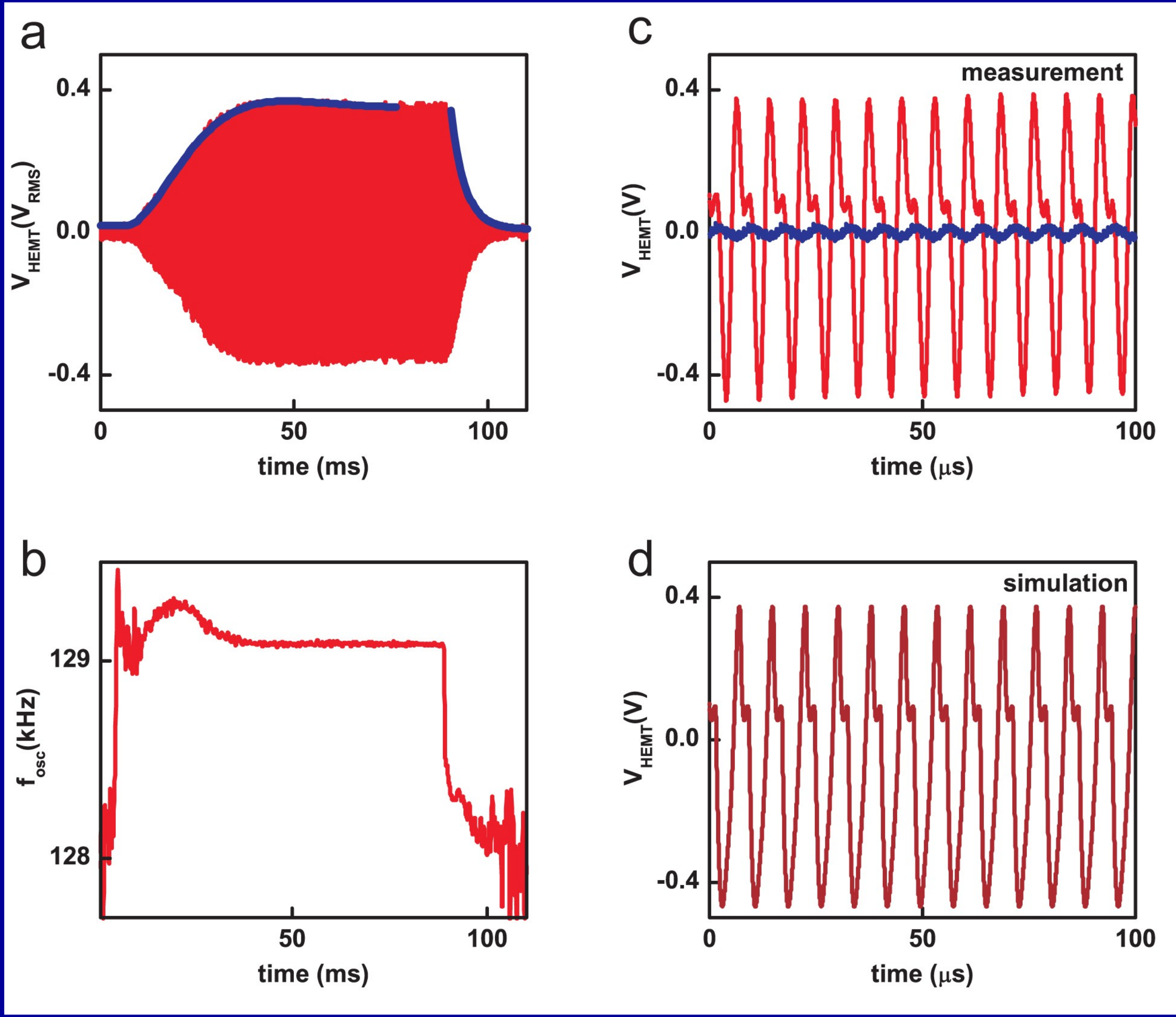
Vary bias conditions of the SQUID, measure the frequency response and extract resonance frequency f_R and quality factor Q

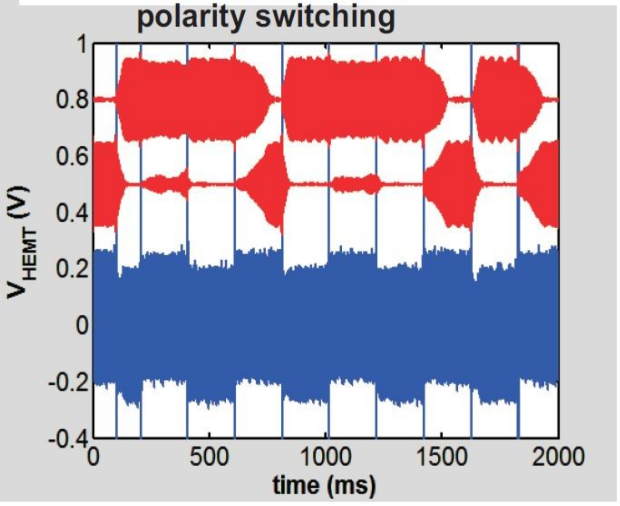
Torsional SQUID



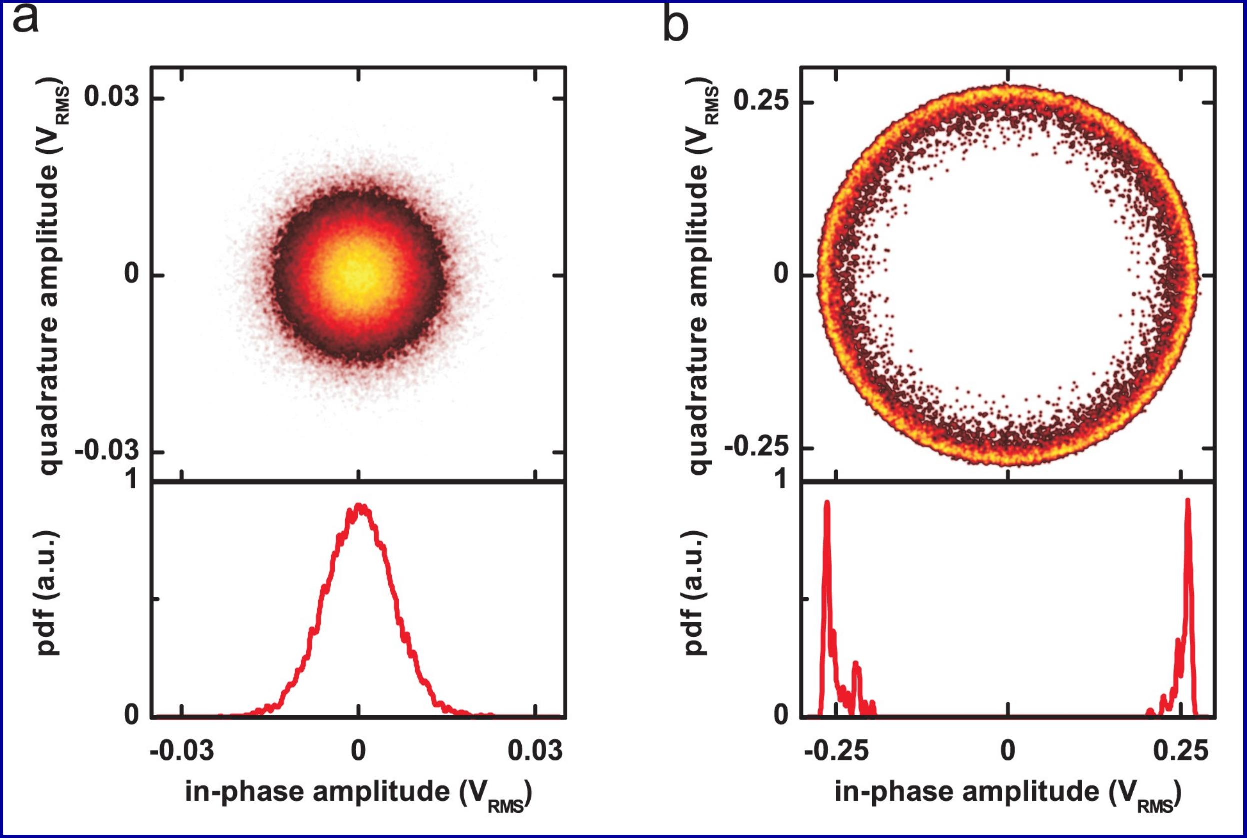
S. Etaki, F. Konschelle, H. Yamaguchi, YMB, H. S. J. van der Zant, Nature Comm. **4**, 1803 (2013)

Time-resolved measurements



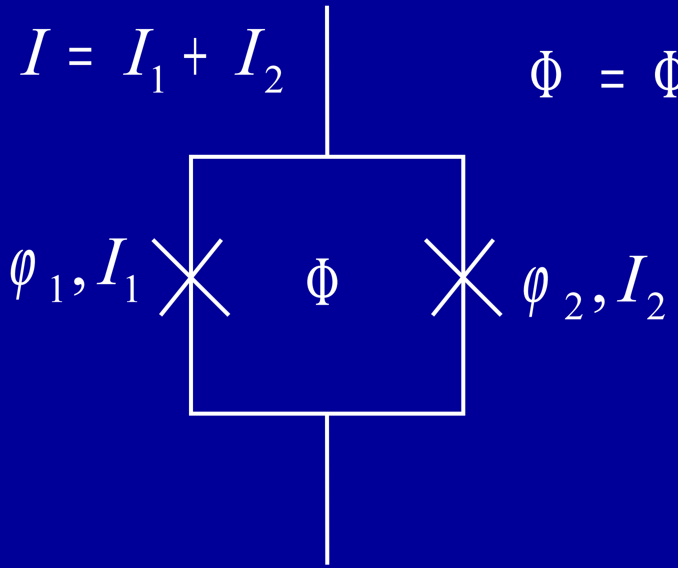


Self-sustained oscillations



Backaction – Theory

M. Poot et al, PRL **105**, 207203 (2010)



$$\Phi = \Phi_a + Blax + L(I_1 - I_2)/2 = \Phi_0(\varphi_2 - \varphi_1)/(2\pi)$$

Motion

Inductive coupling

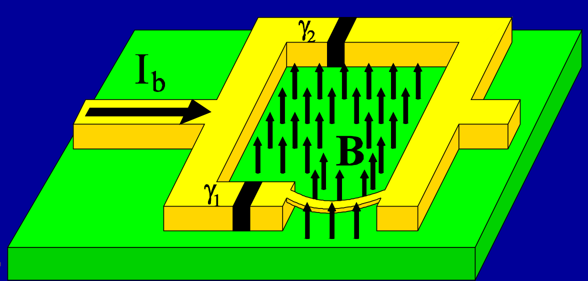
Josephson junctions:

$$I_{1,2} = I_0 \sin\varphi_{1,2} + \frac{V_{1,2}}{R} + C\dot{V}_{1,2},\ V_{1,2} = \frac{\Phi_0}{2\pi}\dot{\varphi}_{1,2}$$

Oscillator:

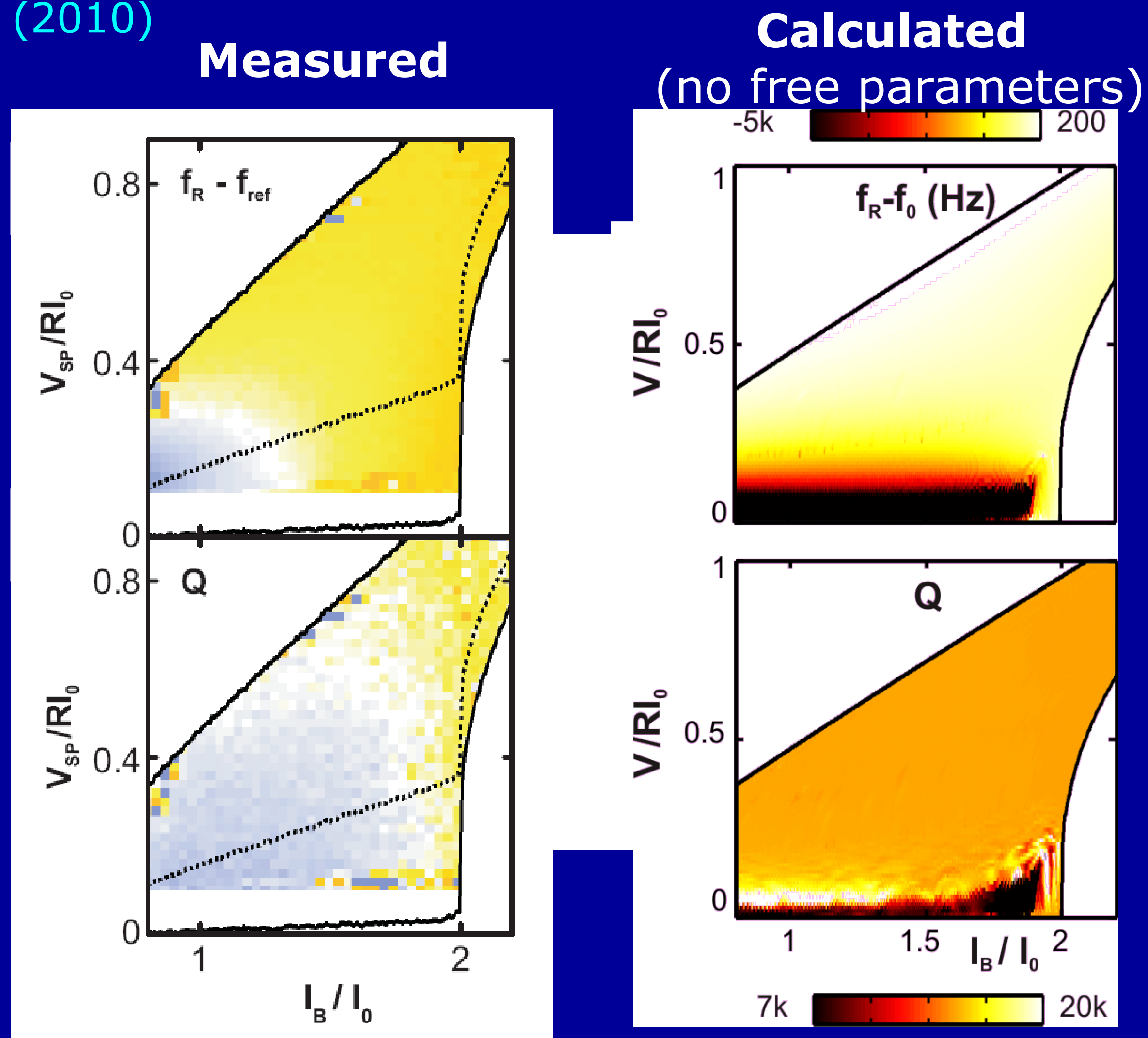
$$M\ddot{x} + \frac{M\omega}{Q}\dot{x} + M\omega^2 x = F\cos\omega t + aBlI_1$$

Lorentz force



Backaction

M. Poot et al, PRL **105**, 207203 (2010)



- Frequency shift and damping of the same order as measured
- Largest backaction occurs at lowest voltages
- No perfect quantitative agreement:
  Flux noise?
  SQUID asymmetry?

Back-action and self-oscillations

$$M\ddot{x} + \frac{M\omega}{Q}\dot{x} + M\omega^2 x = F\cos\omega t + aBlI_1$$

For self-oscillations we need Q < 0

Overdamped:

$$I_1 = V/R \propto \sqrt{\left(\frac{I}{I_c}\right)^2 - 1},\ I_c(x) = 2I_c \cos\frac{\pi\Phi(x)}{\Phi_0}$$

– renormalization of the frequency

Finite capacitance: correction

$$\delta I_1 = C\dot{V} \propto \dot{x}\frac{\partial\Phi}{\partial x}\left(\sqrt{\left(\frac{I}{2I_c\cos\frac{\pi\Phi(x)}{\Phi_0}}\right)^2 - 1}\right)^{-1}$$

Renormalizes the quality factor and may yield self-oscillations

Back-action in a Duffing oscillator

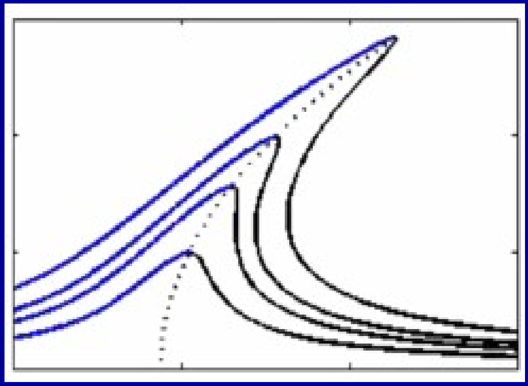
O. Shevchuk, R. Fazio, YMB
in preparation

Physical system: Again SQUID coupled to a mechanical resonator

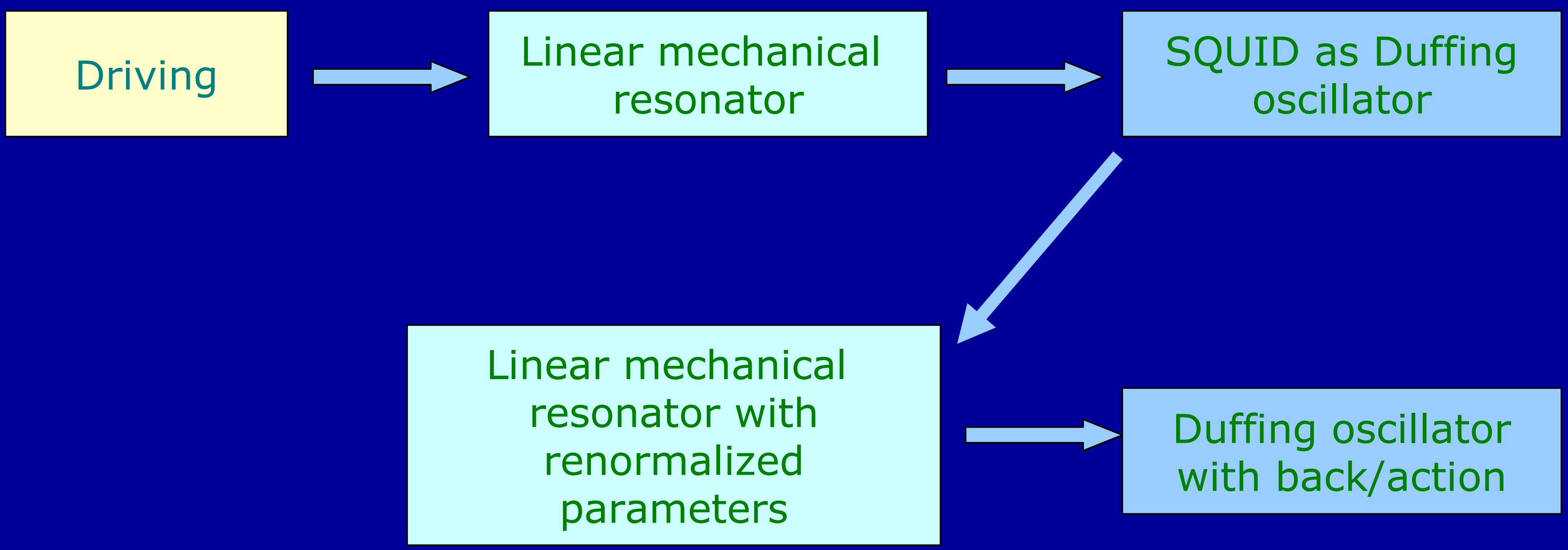
Driving → Linear mechanical resonator → SQUID as Duffing oscillator (Josephson parametric amplifier)

SQUID plasma frequency: $\omega_P = \sqrt{16 E_C E_J \left|\cos(\pi\Phi/\Phi_0)\right|}$

Back-action is efficient: $\omega_P \sim \omega_0$



Perturbation series



Model

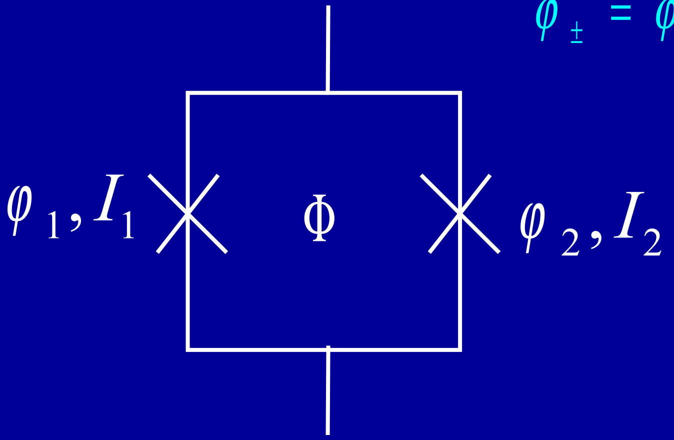
Driving → Linear mechanical resonator → SQUID as Duffing oscillator

$$H = E_J\left(\frac{\dot{\varphi}_+^2}{2\tilde{\omega}_P^2} - 2\cos\varphi_+\cos\varphi_-\right) + \frac{m\dot{u}^2}{2} + \frac{m\omega_0^2u^2}{2} + F_d u \sin\omega_d t$$

SQUID

Driven mechanical resonator

$$\varphi_\pm = \varphi_1 \pm \varphi_2$$



$$\Phi = \Phi_0\varphi_-/(2\pi) = \Phi_a + \xi u; \xi u \ll \Phi_a$$

Coupling

Duffing oscillator

Driving → Linear mechanical resonator → SQUID as Duffing oscillator

Van der Pol transformations: $U = \varphi_+ \cos\omega\tau + \dot{\varphi}_+ \omega^{-1} \sin\omega\tau \,;\, U = r\cos\theta$

$$\left(\omega^2 - 8\omega_P^2 \cos\frac{\pi\Phi}{\Phi_0}\left(1 - r^2/8\right)\right)^2 + \omega^2\delta^2 = \zeta^2 \left(\frac{1}{m\lambda\,\omega_P}\right)^2 \left(F_d^2 + \left(E_J \frac{\zeta}{2}\sin\frac{\pi\Phi}{\Phi_0} r\right)^2 - 4E_J\zeta\,\delta\,\sin\frac{2\pi\Phi}{\Phi_0} r\right)$$

Valid for small *r*

Dissipation in the detector

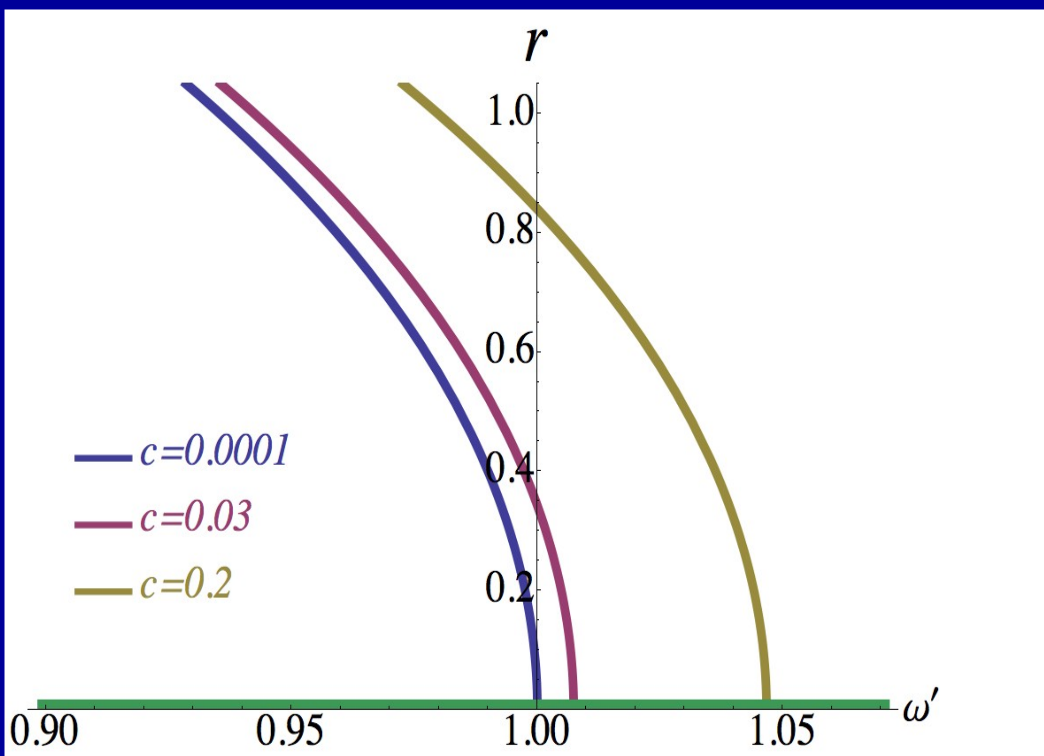
Mechanical dissipation

Back-action

$\lambda \propto \sim Q^{-1}$

Duffing oscillator

$$\left(\omega^2 - 8\omega_P^2 \cos\frac{\pi\Phi}{\Phi_0}\left(1 - r^2/8\right)\right)^2 + \omega^2\delta^2 = \zeta^2\left(\frac{F_d}{m\lambda\omega_P}\right)^2$$



No dissipation, no backaction

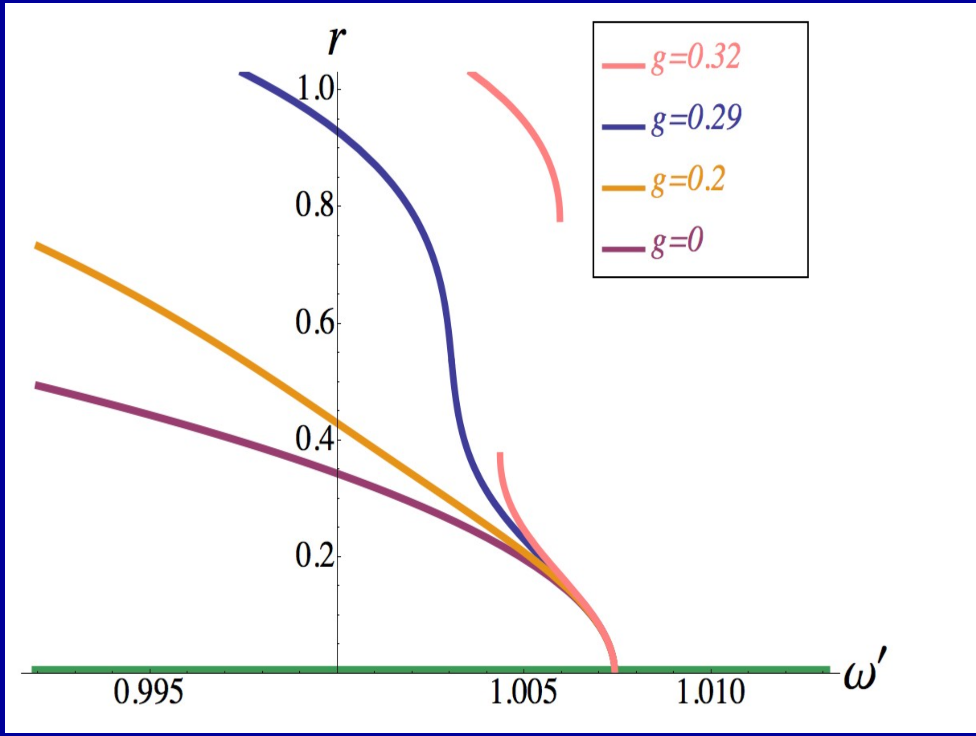
$$\delta = 0; c \propto \zeta F_d$$

$$\omega' = \omega \Big/ \left(2\sqrt{2}\omega_P \cos\frac{\pi\Phi}{\Phi_0}\right)$$

Dissipation kills the non-trivial solution

Duffing oscillator

With backaction, no dissipation



Phonon blockade

N. Didier, S. Pugnetti, YMB, and R. Fazio, PRB **84**, 054503 (2011)

A non-linear mechanical oscillator can not be resonantly driven into a state with more than one phonon

How to detect phonon blockade?

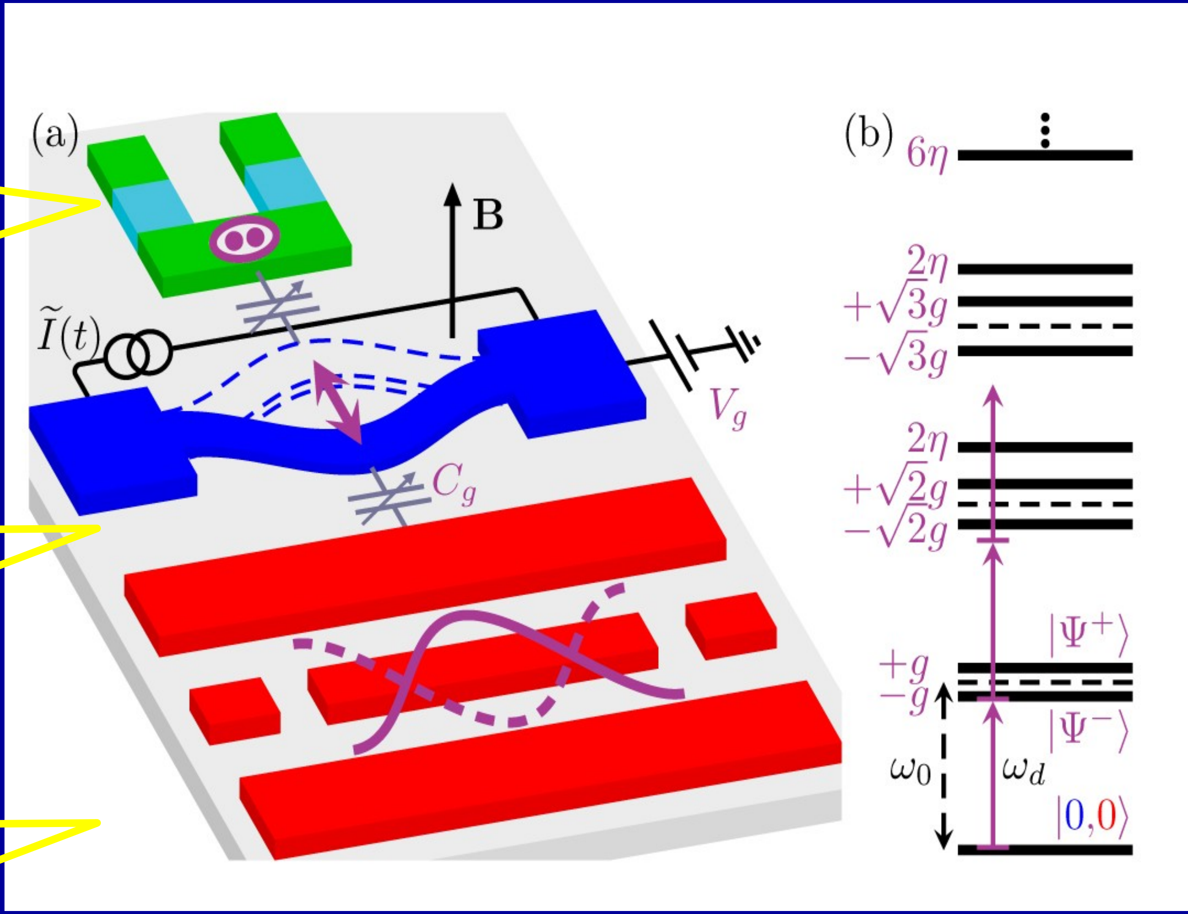
Coupling to a superconducting microwave resonator!

Phonon blockade

Cooper pair box (makes the oscillator non-linear)

Mechanical oscillator

Superconducting microwave resonator



Phonon blockade

$$\hat{H}_{osc} = \hbar\omega_0 \hat{a}^\dagger \hat{a} + \hbar\eta\, \hat{a}^\dagger \hat{a}^\dagger \hat{a}\hat{a}$$

$$\hat{H}_{SMR} = \hbar\omega_c \hat{b}^\dagger \hat{b}$$

$$\hat{H}_{\text{int}} = \hbar g\left(\hat{a}^\dagger \hat{b} + \hat{a}\hat{b}^\dagger\right)$$ (beam-splitter interaction, disregard radiation pressure)

$$\hat{H}_{driv} = \hbar\varepsilon\, \hat{a}^\dagger \exp\left(-i\omega_d t\right) + h.c.$$

Lindblad equation: Takes into account finite lifetime of phonons and photons

$$\dot{\rho} = \frac{1}{i\hbar}\left[\hat{H}, \hat{\rho}\right] + L\hat{\rho}$$

$$L_{osc}\hat{\rho} = \frac{1}{2}\gamma_{osc}\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger \hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger \hat{a}\right)$$

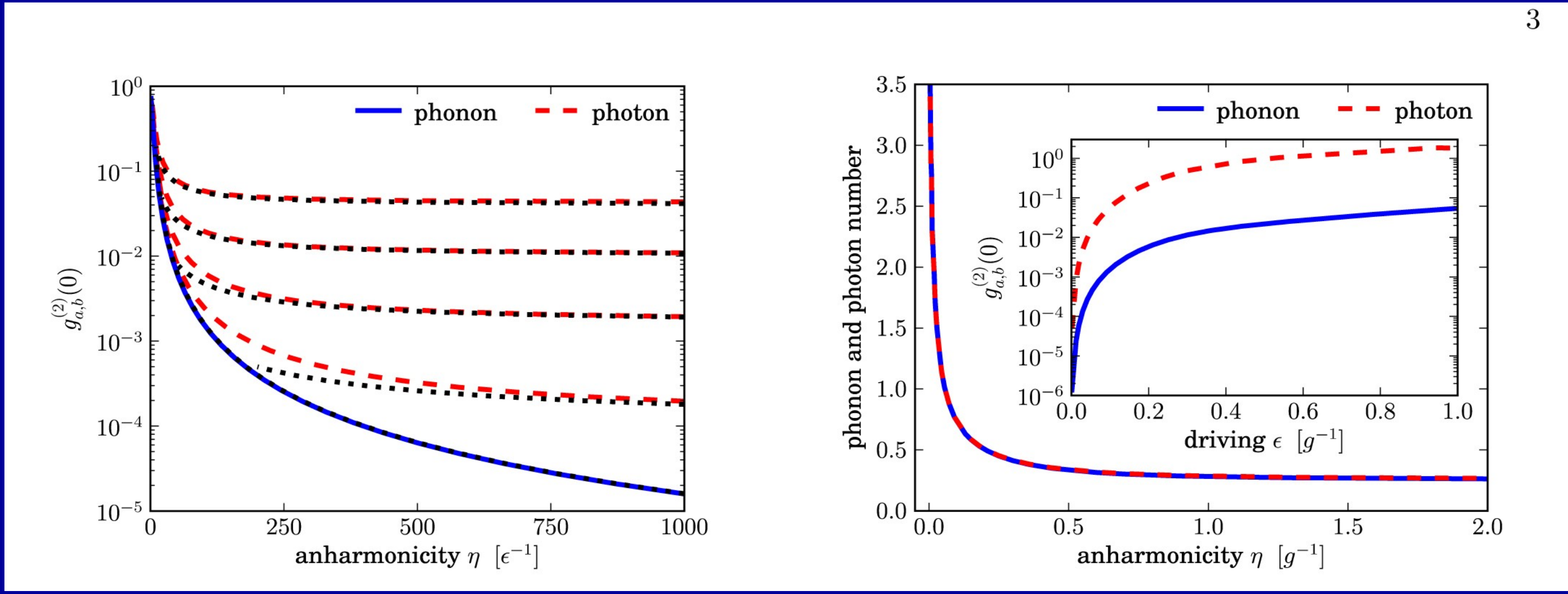
$$L_{SMR}\hat{\rho} = \frac{1}{2}\gamma_{SMR}\left(2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{b}^\dagger \hat{b}\hat{\rho} - \hat{\rho}\hat{b}^\dagger \hat{b}\right)$$

Phonon blockade

Two-point correlation function:

Didier et al, Phys. Rev. B **84**, 054503 (2011)

$$g^{(2)}(\tau) = \lim_{t\to\infty} \frac{\left\langle y^\dagger(t) y^\dagger(t+\tau) y(t+\tau) y(t) \right\rangle}{\left\langle y^\dagger(t) y(t) \right\rangle^2}$$



n=1 states: maximally entangled Bell states

Photons probe phonons

The phonotonic junction

Turn the driving off after the initial state has been prepared:

$$\hat{H}_{osc} = \hbar\omega_0 \hat{a}^\dagger \hat{a} + \hbar\eta\, \hat{a}^\dagger \hat{a}^\dagger \hat{a}\hat{a} + \hbar\omega_c \hat{b}^\dagger \hat{b} + \hbar g\left(\hat{a}^\dagger \hat{b} + \hat{a}\hat{b}^\dagger\right)$$

Two parameters: phase and imbalance

$$\varphi = \arg(\hat{a}^\dagger \hat{b})$$

$$z = \frac{\langle n_a - n_b \rangle}{\langle n_a + n_b \rangle}$$

Dynamics:

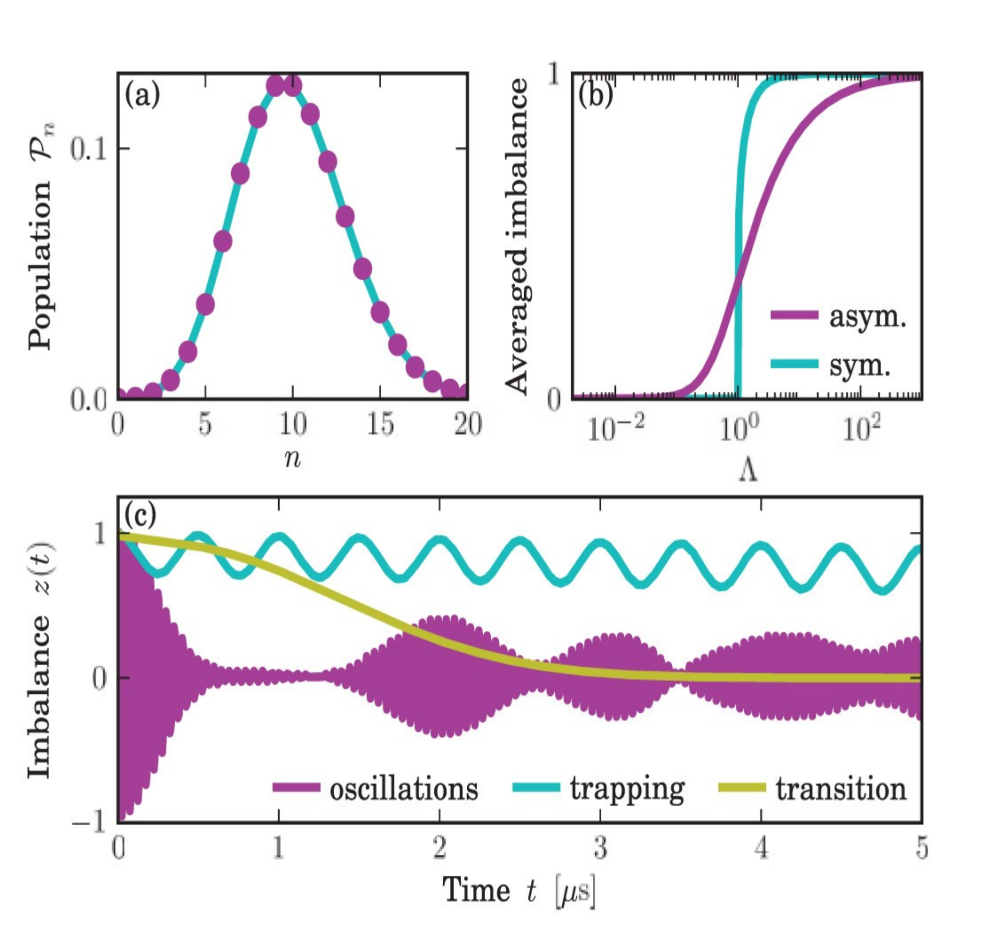
$$\dot{z} = \sqrt{1 - z^2}\sin\varphi\,;$$

$$\dot{\varphi} = \Lambda(1 + z) - \frac{z}{\sqrt{1 - z^2}}\cos\varphi\,;$$

$$\Lambda \equiv \eta\,\langle n_a + n_b \rangle / 2g$$

The phonotonic junction

Turn the driving off after the initial state has been prepared:



Critical coupling:

$$\Lambda_c \equiv \eta \langle n_a + n_b \rangle / 2g_c = 2$$

With dissipation:
Quantum revivals

Conclusions

- Classical mechanical resonator coupled to a SQUID: shift of frequency and suppression of the quality factor; agrees with the experiment. Self-oscillations in a torsional SQUID

- Back-action may considerably modify properties and induce additional instabilities in a Duffing oscillator

- Phonon blockade: phonons are entangled with photons.

Non-linear nanomechanical systems

Delft-Theory

François Konschelle
Olga Shevchuk

Delft-Experiment

Menno Poot
Samir Etaki
Herre van der Zant

SNS Pisa

Stefano Pugnetti
Nicolas Didier
Rosario Fazio

NTT

Imran Mahboob
Koji Onomitsu
Hiroshi Yamaguchi

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