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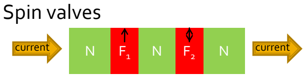
Gilbert Damping, Heat Pumping, and Spin-Pumping

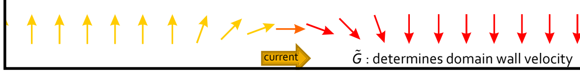
Gilbert Damping

- Dissipation rate
  - $\dot{E} = \mathbf{H}_{\text{eff}} \cdot \frac{d\mathbf{M}}{d\tau} = \frac{1}{\gamma^2} \dot{\mathbf{m}} \left[\tilde{G}(\mathbf{m}) \dot{\mathbf{m}} \right]$
- Switching rate in an applied external field
- Spin transfer torque (MRAM)
  - Spin valves



$\tilde{G}$: determines critical current

  - Domain wall motion



$\tilde{G}$: determines domain wall velocity

Outline

- Magnetization Dynamics
- Gilbert Damping
- Scattering Theory
- Heat Pumping
- Spin Pumping
- Magnetic Insulators
- General Approach – Collective Coordinates
- Kambersky (Kubo)
- Conclusions

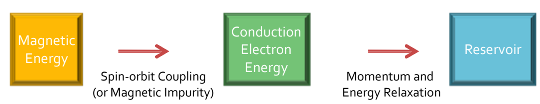
- Collaboration
  - Kjetil Hals (Trondheim, Norway)
  - Andre Kapelrud (Trondheim, Norway)
  - Anh Kiet Nguyen (Trondheim, Norway)
  - Yaroslav Tserkovnyak (UCLA, USA)
  - Gerrit E. W. Bauer (TU Delft, The Netherlands)
  - Paul J. Kelly (Twente, the Netherlands)
  - Anton Starikov (Twente, the Netherlands)
  - Zhe Yuan (Twente, the Netherlands)

Gilbert Damping

- Landau-Lifshitz-Gilbert (LLG) dynamics

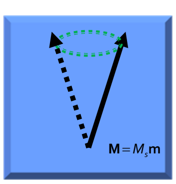
$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \mathbf{m} \times \left[\frac{\tilde{G}(\mathbf{m})}{\gamma^2 M_s} \frac{\partial \mathbf{m}}{\partial t} \right] - \gamma \mathbf{m} \times \mathbf{h}(t)$$

- Phenomenological introduction of dissipation
  - T. L. Gilbert, Phys. Rev. 100, 1243 (1955)
- Gilbert damping qualitively well understood (Kubo)



- *Controlled* quantitative agreement theory and experiments ?
- Damping in nano-scale ferromagnets?

Magnetization Dynamics



Landau-Lifshitz-Gilbert

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \mathbf{m} \times \left[\frac{\tilde{G}(\mathbf{m})}{\gamma^2 M_s} \frac{\partial \mathbf{m}}{\partial t} \right] - \gamma \mathbf{m} \times \mathbf{h}(t)$$

$\mathbf{H}_{\text{eff}}$: effective magnetic field
$\tilde{G}(\mathbf{m})$: Gilbert damping
$\mathbf{h}(t)$: fluctuating magnetic field

Fluctuation-dissipation theorem:

$$\langle h_i(t) h_j(t') \rangle = 2 k_B T \frac{\tilde{G}_{ij}}{\gamma^3 M_s^2} \delta(t - t')$$

Theory of Gilbert Damping

- B. Heinrich et al. Phys. Stat. Solidi 23, 501 (1967)
- V. Kambersky, Can. J. Phys. 48, 2906 (1970)
- V. Korenman and R. E. Prange, Phys. Rev. B 6, 2769 (1972)
- V. S. Lutovinov and M. Y. Reizer, Zh. Eksp. Teor. Fiz. 77, 707 (1979)
- V. L. Safanov and H. N. Bertram, Phys. Rev. B 61, R14893 (2000)
- J. Kunes and V. Kambersky, Phys. Rev. B 76, 134416 (2002)
- V. Kambersky, Phys. Rev. B 76, 134416 (2007)
- E. Simanek and B. Heinrich, Phys. Rev. B 67, 144418 (2003)
- K. Gilmore et al., Phys. Rev. Lett. 99, 027204 (2007)
- A. Brataas et al., Phys. Rev. Lett. 101, 037207 (2008)
- I. Garate and A. H. MacDonald, Phys. Rev. B 79, 0664403 (2009); 064404 (2009)
- A. Starikov et al. Phys. Rev. Lett. 105, 236601 (2010).
- A. Brataas et al., Phys. Rev. B 84, 0544416 (2011).
- Y. Li et al. Phys. Rev. B 84, 014412 (2011).
- H. Ebert et al. Phys. Rev. Lett. 107, 066603 (2012).

Detour: Landauer Conductance

- Nano-scale conductor

- Landauer-Buttiker formula

$$I = GV, \quad G = \sum_{ij} |t_{ij}|^2$$

- Sample contact determines the size of the conductance, but irreversible process takes places in reservoirs.
- Fisher-Lee: Landauer-Buttiker is equal to Kubo

Energy Pumping

- Scattering matrix

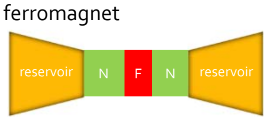
- Reflection (r) and transmission (t) amplitudes
  - Matrices in spin-space and space spanned by incoming transverse wave-guide modes
- Scattering matrix $S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$
- Energy pumping

$$I_E^{(\text{pump,left})} + I_E^{(\text{pump,right})} = \frac{\hbar}{4\pi} \mathrm{Tr}\left[\dot{S}\dot{S}^\dagger\right]$$

J. E. Avron et al. PRL 87, 236601 (2001)
M. Moskalets et al. PRB 66, 035306 (2002); PRB 66, 205320 (2002)

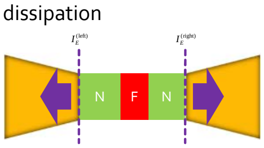
Scattering Theory: Gilbert Damping

- Single ferromagnet



- Energy dissipation

Magnetization precession



- Gilbert damping determined by energy conservation

$$\dot{E} = I_E^{(\text{left})} + I_E^{(\text{right})}$$

Gilbert Damping as Energy Pumping

- Ferromagnets:
  - S-matrix depends on magnetization direction

$$S(\tau) = S(\mathbf{m}(\tau)), \quad \Rightarrow \frac{\partial S}{\partial \tau} = \sum_i \frac{\partial S}{\partial m_i} \dot{m}_i$$

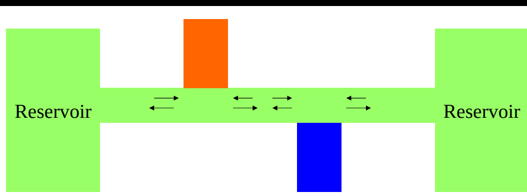
$$I_E^{(\text{pump})} = \frac{\hbar}{4\pi} \mathrm{Tr}\left[\dot{S}\dot{S}^\dagger\right] = \frac{\hbar}{4\pi} \sum_{ij} \dot{m}_i \dot{m}_j \mathrm{Tr}\left[\frac{\partial S}{\partial m_i} \frac{\partial S}{\partial m_j}^\dagger\right] = \dot{E} = \frac{1}{\gamma^2} \dot{\mathbf{m}}\left[\tilde{\mathbf{G}}(\mathbf{m})\dot{\mathbf{m}}\right]$$

- Gilbert damping in ferromagnets (dissipation):
  - Determined from non-dissipative scattering matrix

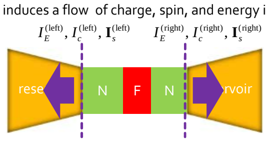
$$\tilde{G}_{ij}(\mathbf{m}) = \frac{\gamma^2 \hbar}{4\pi} \mathrm{Re}\left\{ \mathrm{Tr}\left[\frac{\partial S}{\partial m_i} \frac{\partial S^\dagger}{\partial m_j}\right]\right\}$$

Brataas et al. PRL 2008

Pumping



Pumping induces a flow of charge, spin, and energy into the reservoirs



Magnetization precession can pump charge, spin, and energy into the reservoirs

Gilbert Damping and S-matrix

- Gilbert damping in terms of S-matrix

$$\tilde{G}_{ij}(\mathbf{m}) = \frac{\gamma^2 \hbar}{4\pi} \mathrm{Re}\left\{ \mathrm{Tr}\left[\frac{\partial S}{\partial m_i} \frac{\partial S^\dagger}{\partial m_j}\right]\right\}$$

- In general, Gilbert damping is
  - anisotropic and
  - depends on magnetization direction.
- Agrees with Kubo linear response formulation
  - Fluctuation-dissipation theorem
- Suitable to ab-initio first principles band structure calculations
  - S-matrix:
    - Giant magnetoresistance
    - Spin-transfer torque
    - Spin-pumping

Scattering Theory: Gilbert Damping

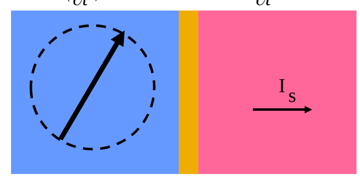
- Gilbert damping

$$\tilde{G}_{ij}(\mathbf{m}) = \frac{\gamma^2 \hbar}{4\pi} \mathrm{Re}\left\{ \mathrm{Tr}\left[\frac{\partial S}{\partial m_i} \frac{\partial S^\dagger}{\partial m_j} \right] \right\}$$

- Two contributions:
  - Spin-orbit induced, damping
    - Spin loss in ferromagnet
    - Proportional to volume of ferromagnet
  - Spin-pumping induced non-local damping
    - Spin loss outside ferromagnet (non-local)
    - Proportional to surface of ferromagnet

Ferromagnetic Resonance

$$\left(\frac{\partial \vec{m}}{\partial t}\right) = -\vec{m} \times H_{\mathrm{eff}} + \alpha \vec{m} \times \frac{\partial \vec{m}}{\partial t}$$



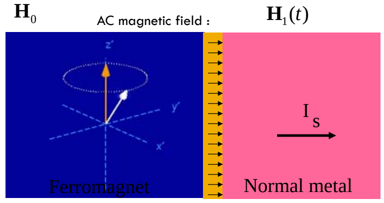
α is *enhanced* in thin films $\alpha = \alpha^{(0)} + \alpha'$

$$\alpha' = \frac{g_L A_r}{4\pi M}, \qquad A_r = \frac{1}{2} \sum_{nm} \left[\left| r^{\uparrow}_{nm} - r^{\downarrow}_{nm} \right|^2 + \left| t'^{\uparrow}_{nm} - t'^{\downarrow}_{nm} \right| \right]$$

Relation to Spin-Pumping

Rotating magnetization direction: $\mathbf{m}(t)$

Static magnetic field: $\mathbf{H}_0$ AC magnetic field : $\mathbf{H}_1(t)$

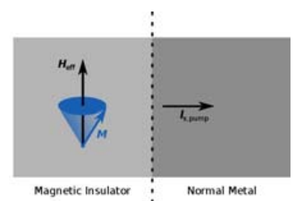


Phenomenological description: Silsbee, Janossy, Monod, Hurdequint, Berger

$$\mathbf{I}_s^{\mathrm{pump}} = \frac{\hbar}{4\pi} g_r^{\uparrow\downarrow} \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}$$

Tserkovnyak, Brataas, Bauer PRL 88, 117601 (2002).
Brataas, Tserkovnyak, Bauer, Halperin, PRB 66, R060404 (2002).
Tserkovnyak, Brataas, Bauer, Halperin, RMP 77, 1375 (2005).
Brataas, Tserkovnyak, Bauer, PRL 101, 037207 (2008).

Spin-pumping - Magnetic Insulators



Pumping enhances damping
- Well-known for macrospins (Tserkovnyak, Brataas, and Bauer)
- What about textures?

Gilbert Damping and Spin-pumping

- Gilbert damping

$$\tilde{G}_{ij}(\mathbf{m}) = \frac{\gamma^2 \hbar}{4\pi} \mathrm{Re}\left\{ \mathrm{Tr}\left[\frac{\partial S}{\partial m_i} \frac{\partial S^\dagger}{\partial m_j} \right] \right\}$$

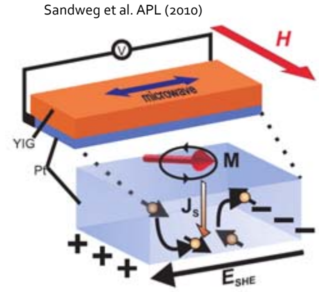
- No spin-orbit interaction

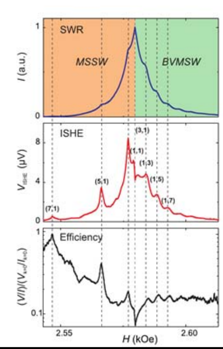
$$S = \frac{1}{2}\left[\left(S_\uparrow + S_\downarrow\right) 1 + \left(S_\uparrow - S_\downarrow\right) \sigma \bullet \mathbf{m} \right] \Rightarrow \frac{\partial S}{\partial m_i} = \frac{1}{2}\left[\left(S_\uparrow - S_\downarrow\right) \sigma_i \right]$$

$$\tilde{G}_{ij}(\mathbf{m}) = \frac{\gamma^2 \hbar}{16\pi} \mathrm{Re}\left\{ \mathrm{Tr}\left[\left(S_\uparrow - S_\downarrow\right)\left(S_\uparrow - S_\downarrow\right)^\dagger \right] \right\} = \frac{\gamma^2 \hbar}{8\pi} \mathrm{Re}\left\{ \mathrm{Tr}\left[1 - S_\downarrow S_\uparrow^* \right] \right\}$$

Experimental Motivation

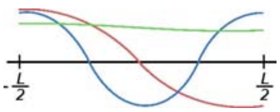
Sandweg et al. APL (2010)



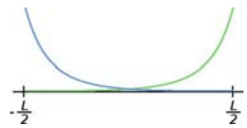


Spin Wave Mode Types

- Volume modes: Standing waves across film



- Surface modes



Analytical Solutions

- Long wavelength limit $(QL \ll 1)$
  - Dipole field is homogenous
  - Transverse standing waves, mode n

$$\omega = \omega_R + i(\alpha + \Delta\alpha)\omega_I$$

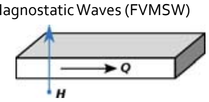
$$\Delta\alpha = \left(2 - \delta_{n,0}\right)\Delta\alpha_{\text{macrospin}},\ \ \Delta\alpha_{\text{macrospin}} = \frac{\gamma\hbar^2 g_\perp}{2e^2 L M_s}$$

- Short wavelength limit $(QL \gg 1)$
  - In-plane exchange energy dominates

$$\Delta\alpha \to 0$$

Dipolar Spin Wave Modes

- Forward Volume Magnostatic Waves (FVMSW)



- Backward Volume Magnetostatic Waves (BVMSW)

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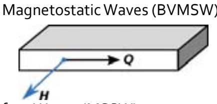
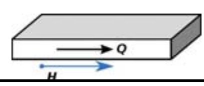


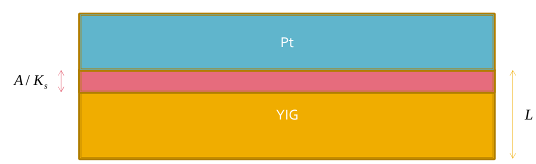
- Magnetostatic Surface Waves (MSSW)



MSSW Geometry

- Analytical solution

$$\Delta\alpha = \frac{\gamma\hbar K_s}{4\pi M_s A}\frac{h}{e^2} g_\perp \frac{\omega_H}{\omega_M}\left[\frac{\omega_H}{\omega_M} + \frac{1}{2} - \frac{K_s^2}{4\pi M_s^2 A}\right]^{-1}$$
$$\sim \Delta\alpha_{\text{macrospin}}(LK_s / A)$$



Linearized LLG equation

Equation of motion, x- and y-components

$$\begin{pmatrix} \frac{\omega_H}{\omega_M} + i\alpha\frac{\omega}{\omega_M} - \frac{A}{2\pi M_s^2}\left(\frac{d^2}{d\xi^2} - Q^2\right) & -i\frac{\omega}{\omega_M} \\ i\frac{\omega}{\omega_M} & \frac{\omega_H}{\omega_M} + i\alpha\frac{\omega}{\omega_M} - \frac{A}{2\pi M_s^2}\left(\frac{d^2}{d\xi^2} - Q^2\right) \end{pmatrix} \mathbf{m}(\xi) = \int_{-L/2}^{L/2} d\xi G_{xy}(\xi - \xi')\mathbf{m}(\xi')$$

$$\omega_H = \gamma H_i \qquad \omega_M = 4\pi\gamma M_s$$

Linearized boundary conditions (top interface)

$$\chi = \frac{L\hbar^2 g_\perp}{4Ae^2}$$

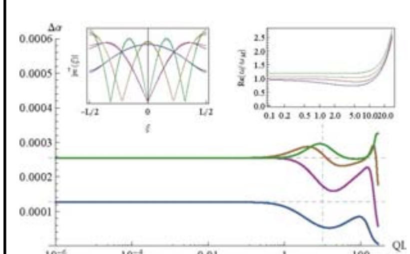
$$\left(L\frac{d}{d\xi} + i\omega\chi + \frac{LK_s}{A}\cos(2\theta)\right)m_x(\xi)\Big|_{\xi=L/2} = O$$

$$\left(L\frac{d}{d\xi} + i\omega\chi + \frac{LK_s}{A}\cos^2(\theta)\right)m_y(\xi)\Big|_{\xi=L/2} = O$$

K_s: surface anisotropy energy

BVMSW Geometry

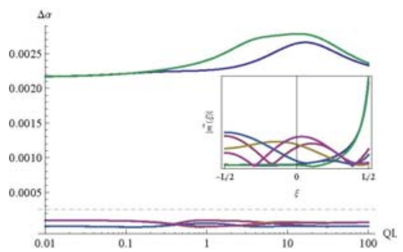
$L = 100\text{nm},\ M_s = 1750/4\pi\ \text{G},\ hg_\perp / e^2 = 1.2\text{•}10^{14}\text{cm}^{-2}$



- 2 modes per transverse wave
- No asymmetry between mode pairs

MSSW Geometry

$L = 100\text{nm}, M_s = 1750/4\pi \text{ G}, hg_\perp / e^2 = 1.2 \cdot 10^{14} \text{cm}^{-2}, K_s = 0.05 \text{ erg cm}^{-2}$



- n=0 mode becomes surface mode

Scattering Theory

- Force

$$\mathbf{F} + \mathbf{f}(t) = -\frac{\partial H(\mathbf{m})}{\partial \mathbf{X}}$$

- Average force

$$\mathbf{F} = -\frac{1}{2\pi i} \int d\varepsilon \mathrm{Tr}\left(S^\dagger \frac{\partial S}{\partial \mathbf{X}} \right)$$

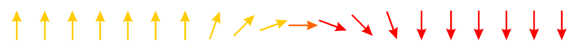
- Fluctuating force

$$\langle f_a(t) f_b(t') \rangle = 2k_B T \Gamma_{ab} \delta(t - t')$$

$$\Gamma_{ab} = \frac{\hbar}{4\pi} \int d\varepsilon \left(-\frac{\partial f}{\partial \varepsilon} \right) \mathrm{Tr}\left[\frac{\partial S}{\partial X_a} \frac{\partial S}{\partial X_b}^\dagger \right]$$

Return to Gilbert Damping

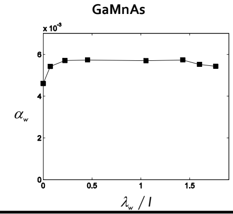
Domain Wall Motion Friction



Walker ansatz:

$$I_E^{(\text{pump})} = \frac{\hbar}{4\pi} \mathrm{Tr}\left[\dot{S} \dot{S}^\dagger \right] = \frac{\hbar (\dot{r}_w)^2}{4\pi} \mathrm{Tr}\left[\frac{\partial S}{\partial r_w} \frac{\partial S}{\partial r_w}^\dagger \right]$$

GaMnAs



Comparison with LLG equation:

$$\alpha_w = \frac{\hbar \lambda_w}{8\pi A S_0} \mathrm{Tr}\left\{ \frac{\partial S}{\partial r_w} \frac{\partial S^\dagger}{\partial r_w} \right\}$$

K. Hals, A. Nguyen and A. Brataas, Phys. Rev. Lett. 102, 256601 (2009).

General Magnetization Textures

- Soft coordinate representation

$$\mathbf{m}(\mathbf{r}t) = \sum_\alpha \mathbf{m}_{st}(\mathbf{r}; X_\alpha(t))$$ Soft coordinate

- Equation of collective coordinate motion

$$\eta \dot{\mathbf{X}} + \mathbf{F} - \Gamma \dot{\mathbf{X}} = 0$$ Thiele PRL 1973, Tretiakov et al. PRL (2008)

  - Gyrotropic tensor: η
  - Force: $\mathbf{F}$
  - Dissipation: Γ
- Additionally, fluctuation-dissipation theorem

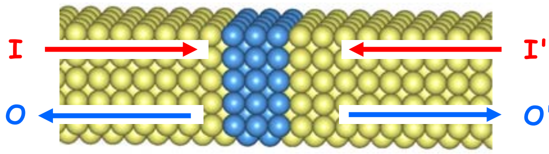
$$\eta \dot{\mathbf{X}} + [\mathbf{F} + \mathbf{f}(t)] - \Gamma \dot{\mathbf{X}} = 0$$

$$\langle f_a(t) f_b(t') \rangle = 2k_B T \Gamma_{ab} \delta(t - t')$$

Ab-initio Implementation

- Paul Kelly (Twente)
- Anton Starikov (Twente)
- Zhe Yuan (Twente)

Scattering Theory of Transport



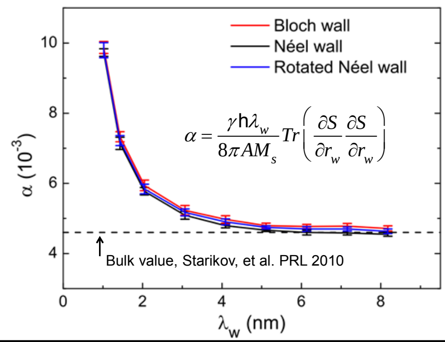
Scattering Matrix

$$\begin{pmatrix} O \\ O' \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} I \\ I' \end{pmatrix}$$

First-principles (DFT) implementation using TB-MTOs

conductance: $\mathcal{G} = \frac{dI}{dV} = \frac{2e^2}{h} \mathrm{Tr}\{\mathbf{t}\,\mathbf{t}^\dagger\}$

Gilbert Damping α for Domain wall

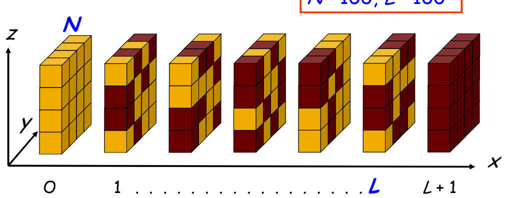


Disorder

- lateral unit cell contains N atoms; periodic in the yz plane
- scattering region contains L layers
- calculation of conductance scales as $N^3 L$
- required Brillouin zone sampling scales as $1 / N$

overall scaling $N^2 L$

$NL \sim 10^4$ atoms
N ~100, L ~100



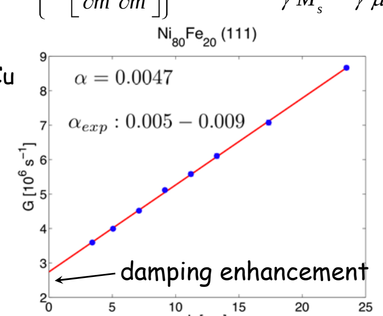
Scattering Theory versus Kubo

- Scattering theory versus Kambersky (Kubo)
- ***Formally Equivalent***
- In practice:
  - Kambersky
    - Evaluated with additional assumptions
    - Band-structure and lifetime broadening not treated equally
    - Relaxation time
  - Scattering approach:
    - No further assumptions
    - Consistent with other transport calculations
      - Giant magnetoresistance
      - Spin-pumping (non-local damping)
    - No explicit phonon contribution

Damping Calculations: Permalloy

$$G = \frac{\gamma^2 \hbar}{4\pi} \mathrm{Re}\left\{ Tr\left[\frac{\partial S}{\partial m}\frac{\partial S^\dagger}{\partial m}\right]\right\} \qquad \alpha = \frac{G}{\gamma M_s} = \frac{G}{\gamma \mu_s L}$$

Cu|Py|Cu



Relation to Kambersky (Kubo)

- Time-dependent (adiabatic) LDA

$$i\hbar \partial_t \psi = \hat{H}\{\mathbf{m}(t)\}\psi, \quad \mathbf{m}(t)\text{: magnetization direction}$$

- Linear response

$$\mathbf{m}(t) = \mathbf{m}_0 + \mathbf{u}(t) \qquad \hat{H}\{\mathbf{m}(t)\} \approx \hat{H}_{st} + \sum_i u_i(t)\partial_i \hat{H}$$

- Energy dissipation

$$\dot{E} = \left\langle \frac{d\hat{H}}{d\tau} \right\rangle = i \sum_{ij} \dot{u}_i \dot{u}_j \lim_{\omega \to 0} \frac{\partial \chi_{ij}(\omega)}{\partial \omega}$$

- Retarded correlation function

$$\chi_{ij}(\tau - \tau') = -\frac{i}{\hbar}\theta(\tau - \tau')\left\langle \left[\partial_i \hat{H}(\tau), \partial_j \hat{H}(\tau')\right]\right\rangle_0$$

Relation to Kambersky (Kubo)

- Exact agreement
  - Kubo (Kambersky, Simanek, Heinrich)
  - Scattering theory (this work)
- Derivation sketch
  - S-matrix: $\hat{S}(\varepsilon,\mathbf{m}) = 1 - 2\pi i \hat{T}(\varepsilon,\mathbf{m})$
  - T-matrix: $\hat{T} = \hat{V}\left[1 + \hat{G}^{(+)}\hat{T}\right]$, $\hat{V}$: scattering off ferromagnet
  - Green's function: $\hat{G}^{(+)}(\varepsilon,\mathbf{m}) = \left[\varepsilon + i\eta - H(\mathbf{m})\right]^{-1}$
  - Linear response:

$$\partial_\tau \hat{S} = -2\pi i \left[1 + \hat{V}_{st}\hat{G}_{st}^{(+)}\right] \sum_i \dot{u}_i \partial_i \hat{H} \left[1 + \hat{V}_{st}\hat{G}_{st}^{(+)}\right]$$

Conclusions

- Magnetization dynamics
  - Slow precession (adiabatic)
  - Landau-Lifshitz-Gilbert phenomenology
- Gilbert damping

$$\tilde{G}_{ij}(\mathbf{m}) = \frac{\gamma^2\hbar}{4\pi}\mathrm{Re}\left\{\mathrm{Tr}\left[\frac{\partial S}{\partial m_i}\frac{\partial S^\dagger}{\partial m_j}\right]\right\}$$

  - Determined by equilibrium (elastic) scattering matrix
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