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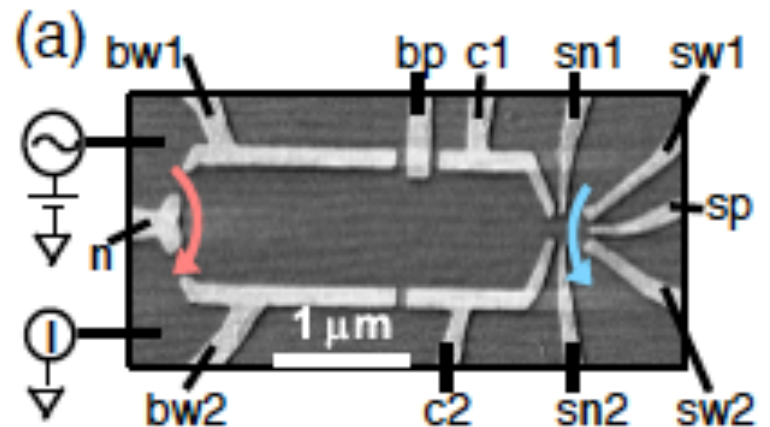
United Nations  
Educational, Scientific and  
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IAEA  
International Atomic Energy Agency

M.N.Kiselev

## Thermoelectric transport through Kondo nano-devices

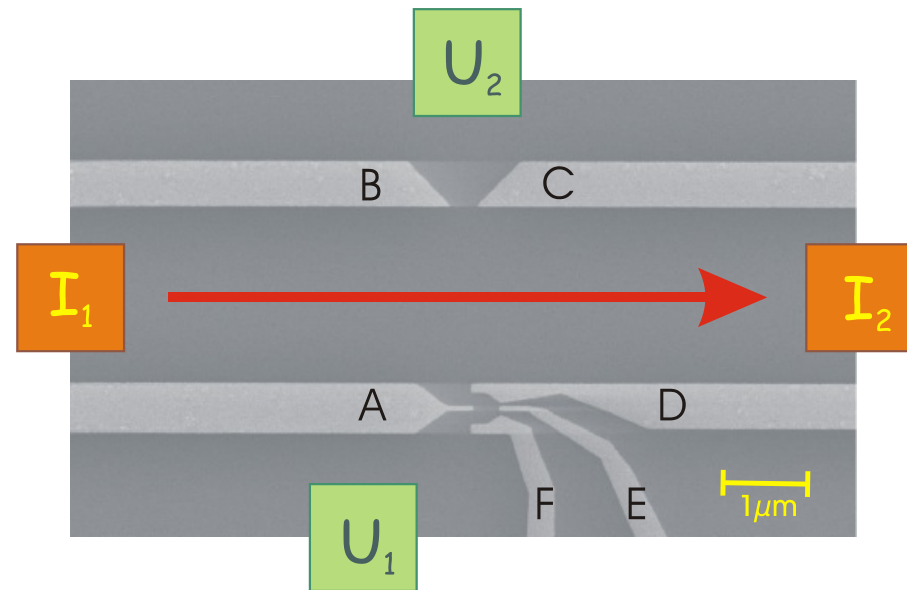
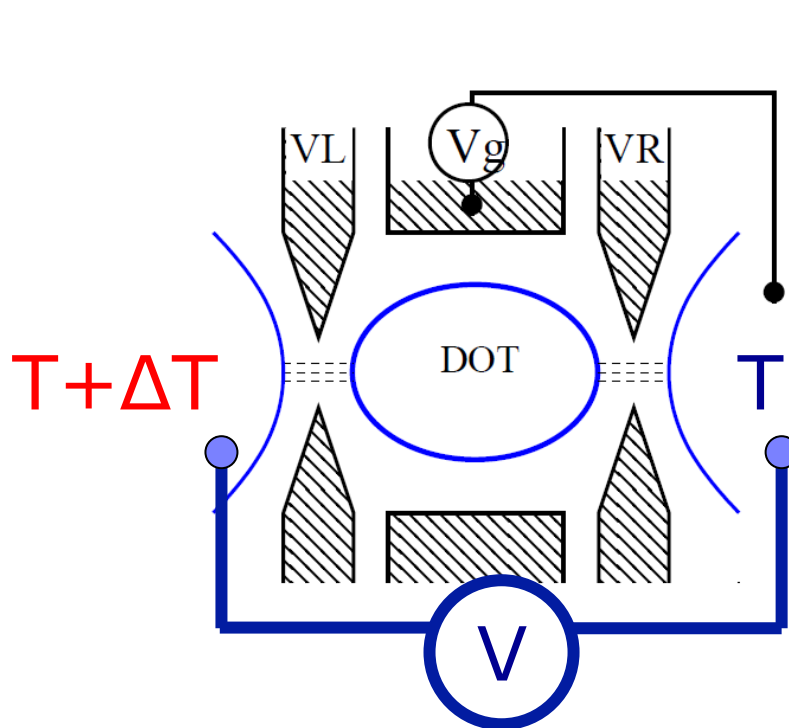


T.K.T. Nguyen, MK and V.E. Kravtsov, PRB 82, (2010)  
MK and Z. Ratiani (2013)

KITP, November 21 2013

# Thermoelectric transport through nanostructures

thermopower  $S = -\frac{V}{\Delta T} = \frac{G_{12}}{G}$  thermovoltage



$$I = G \cdot V + G_{12} \cdot \Delta T = 0$$

# Thermoelectric transport: FL description

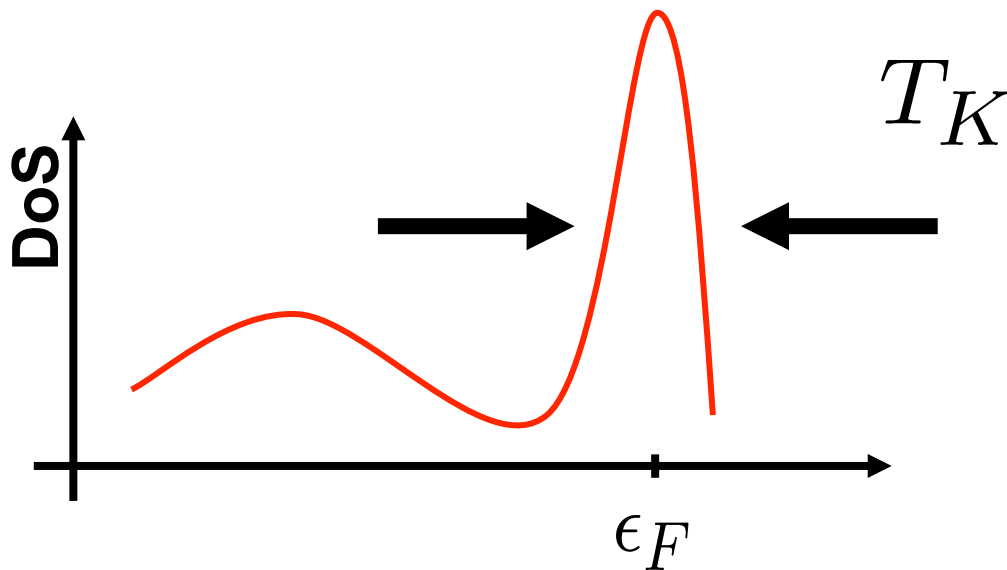
Bulk metals (Fermi Liquid Theory):

$$S \sim T/\epsilon_F \xrightarrow{\text{strongly correlated metals}} S \sim T/T^*$$
$$T^* \ll \epsilon_F$$

strong electron-electron interaction

resonance scattering effects

Example: Kondo effect



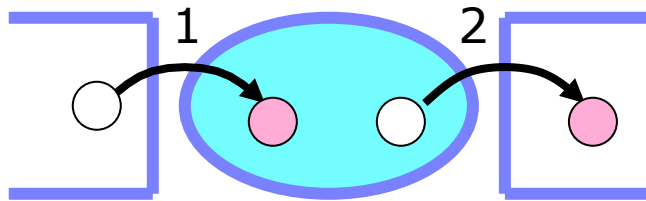
$$S \sim T/T_K$$

Q: How do the effects of strong electron correlations manifest themselves in the thermoelectric transport through the nanostructures?

Q: What are possible mechanisms for enhancement of the thermoelectric power?

Q: Is the thermo-transport through nanostructures always characterized by the Fermi-Liquid concept?

# Sequential tunneling at Coulomb blockade

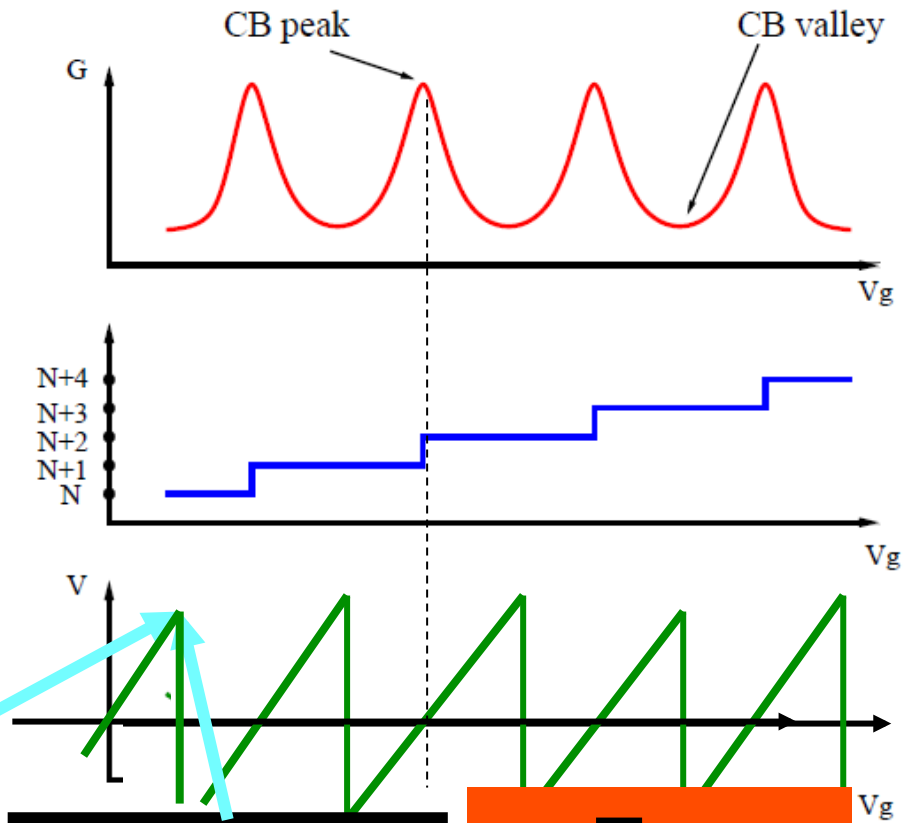


$$S_{\text{Mott}} \equiv -\frac{\pi^2 k}{3 e} kT \frac{d \ln G}{dE_F}$$

For a bulk metal  
 $eS \sim T/E_F \ll 1$

Mott's rule would give  
 for sequential tunneling  
 $eS \sim 1$

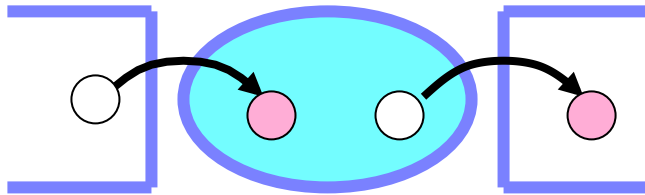
Beenakker & Staring 1992



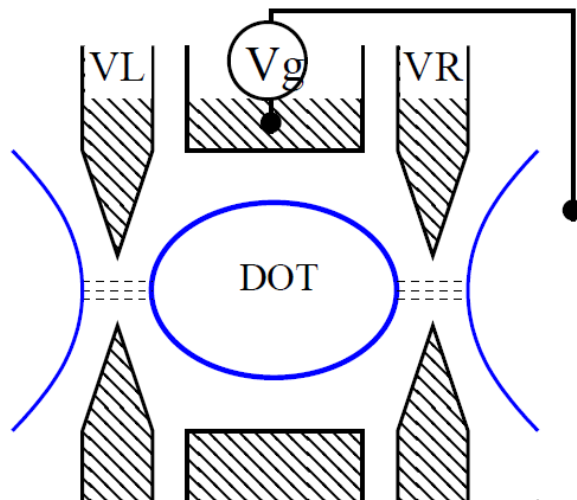
Beenakker & Staring  
 $eS \sim E_C / T \gg 1$

Too large??

# Effect of co-tunneling at weak coupling



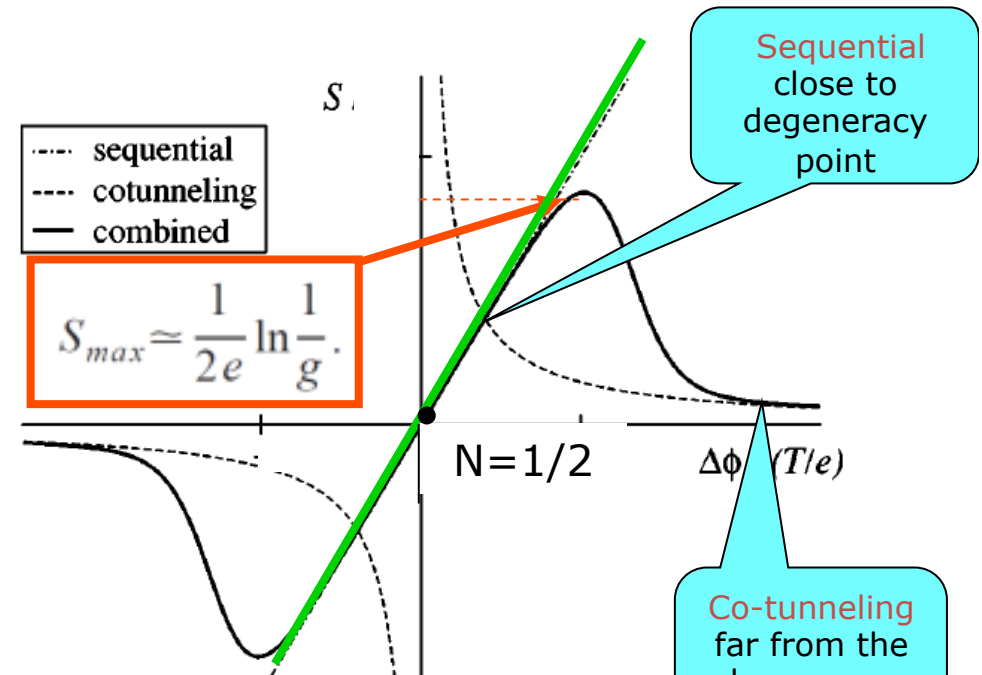
Turek & Matveev, 2002



Weak coupling

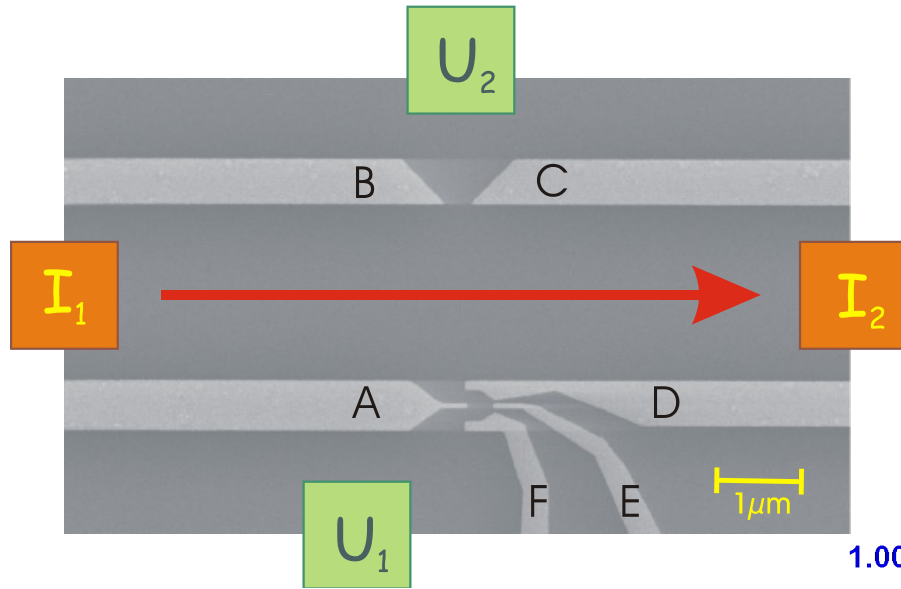
$$g = \frac{\hbar(G_l + G_r)}{2\pi e^2} \ll 1$$

No Coulomb energy is paid



$S_{max}$  is much smaller than Beenakker&Staring estimation, more consistent with the Mott's rule result  $eS \sim 1$  but enhanced compared to bulk  $eS \sim T/E_F \ll 1$

# From weak to strong coupling



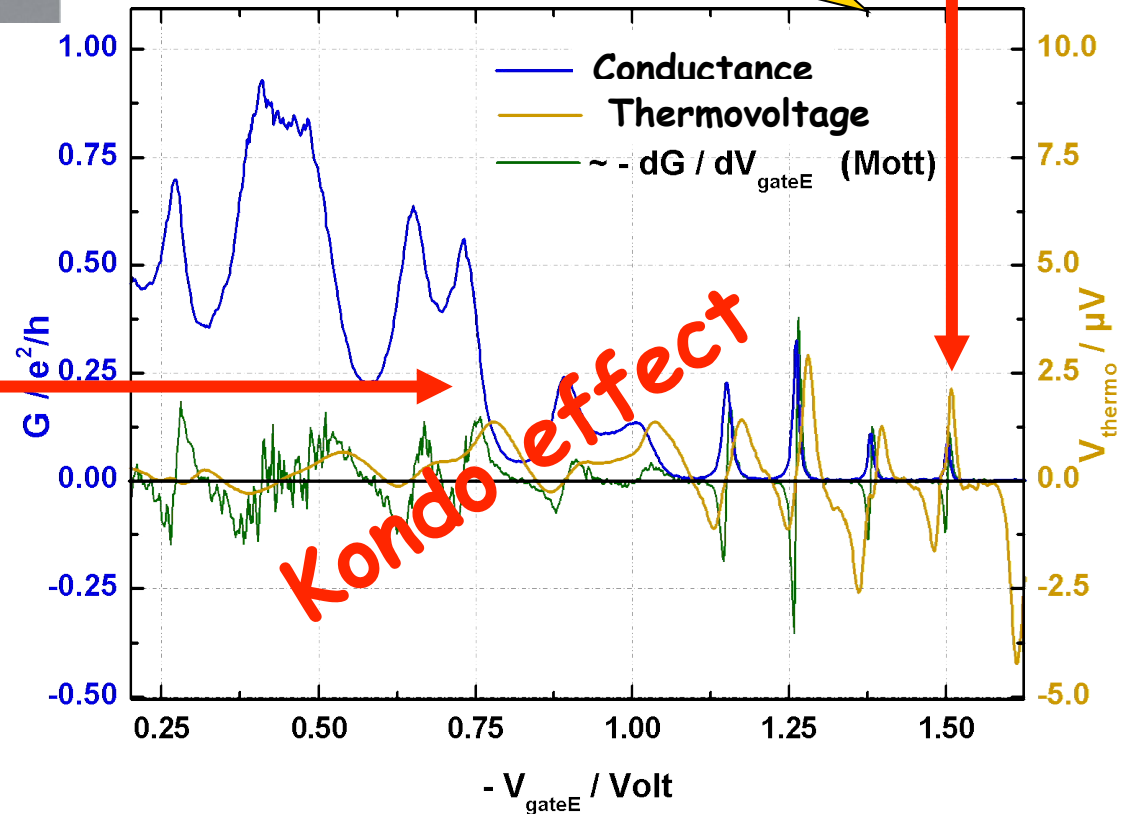
Weak coupling

Mott law is obeyed

$$S \sim \frac{\partial \ln G}{\partial V_g}$$

Strong coupling

Mott law is violated



Q1: How does the Kondo effect influence a thermoelectric transport through the nano-structures?

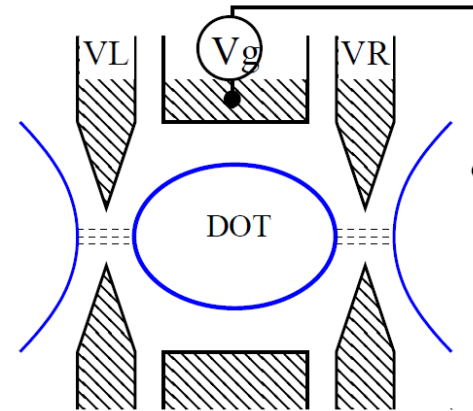
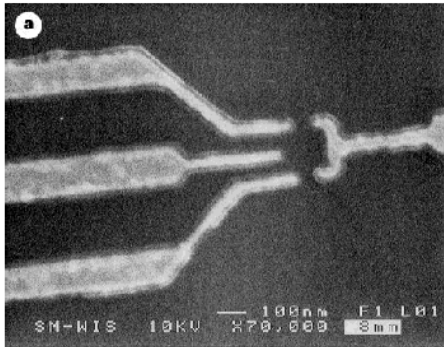
Q2: What are the manifestations of Kondo effect in the thermoelectric transport through the nanostructures?

Q3: Is there a room for NFL enhancement of thermopower in the nanostructures?

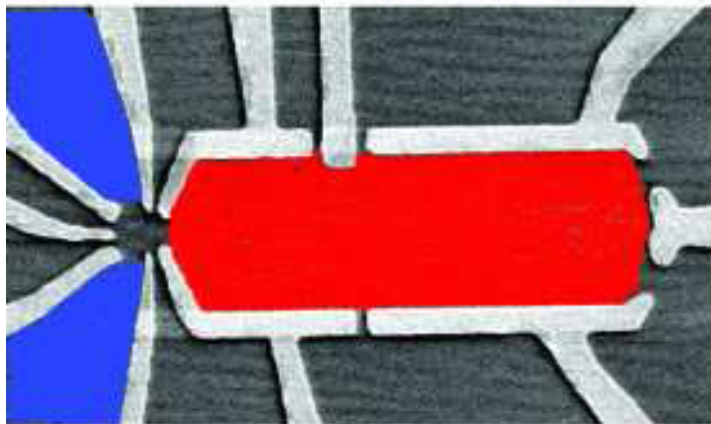


# Realization of Kondo-effect in nanostructures I

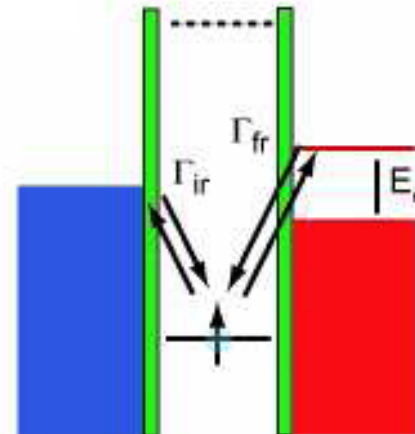
1CK



2CK

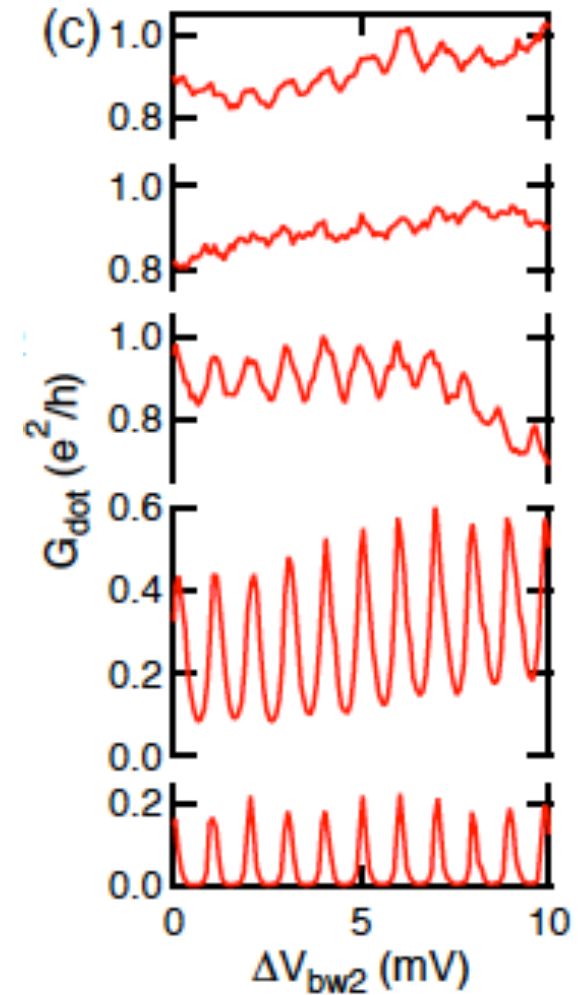
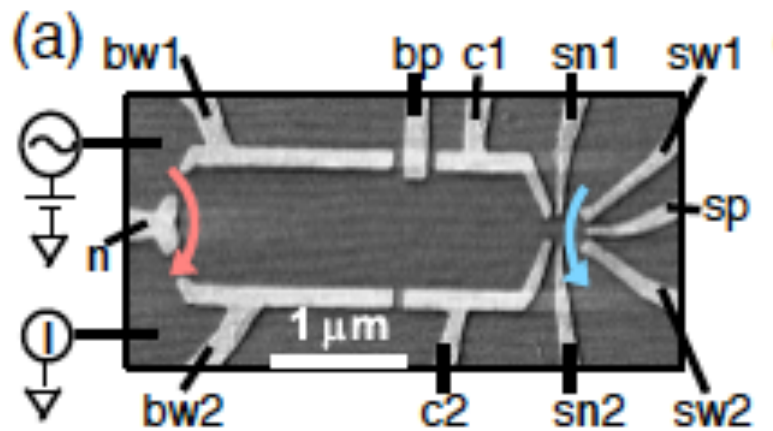
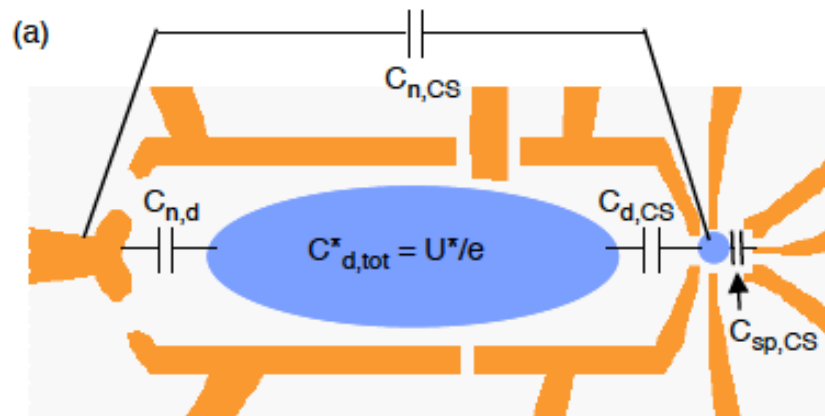


D.Goldhaber-Gordon et al, Nature, 1998

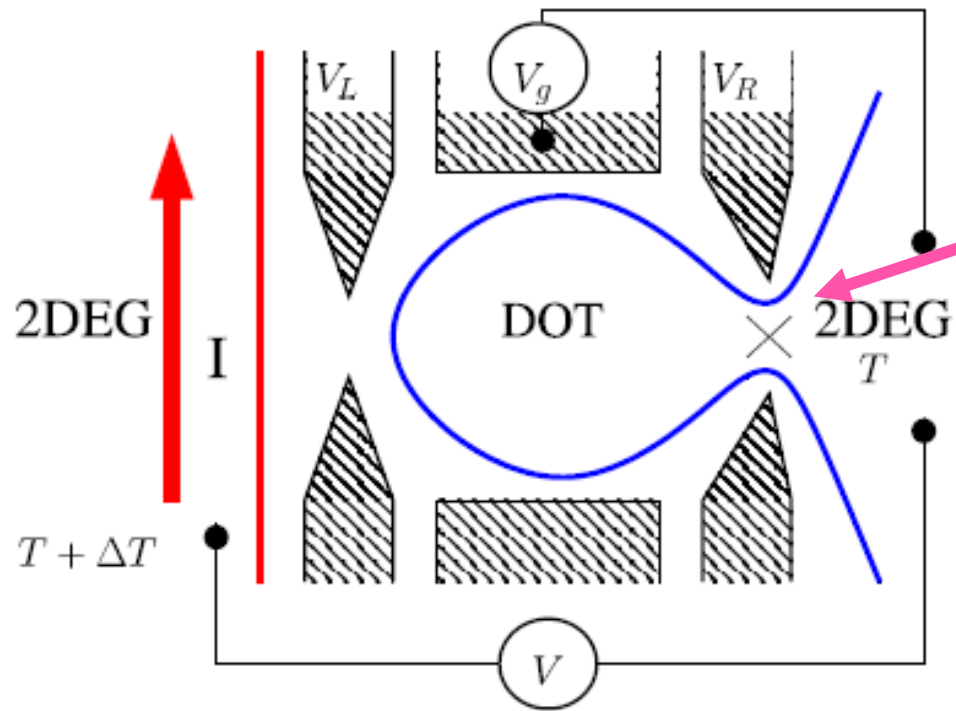


R.M. Potok et al, Nature, 2007

# Realization of Kondo-effect in nanostructures II



# Flensberg - Matveev - Furusaki setup

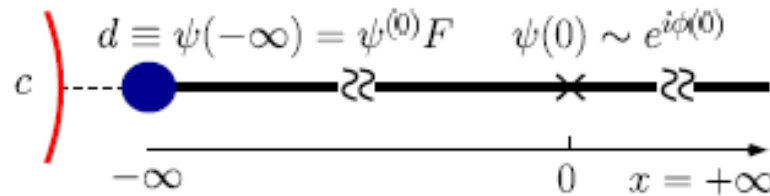


QPC

$$|r| \ll 1$$

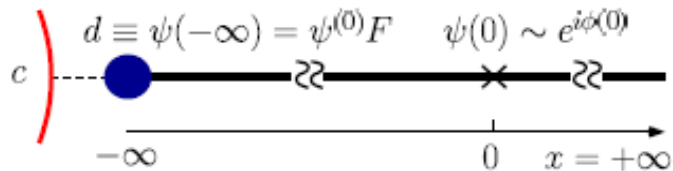
1CK - 2CK

$$T_K \sim E_C$$



# Strong coupling limit and effective model

Model



$$H = H_0 + H_L + H_R + H_C$$

$$H_0 = \sum_{k,\alpha} \epsilon_{k,\alpha} c_{k,\alpha}^\dagger c_{k,\alpha} + \sum_{\alpha} \epsilon_{\alpha} d_{\alpha}^\dagger d_{\alpha} + \sum_{\alpha} \frac{v_{F,\alpha}}{2\pi} \int_{-\infty}^{\infty} \{ [\Pi_{\alpha}(x)]^2 + [\partial_x \phi_{\alpha}(x)]^2 \} dx$$

$$H_L = \sum_{k,\alpha} (t_{k,\alpha} c_{k,\alpha}^\dagger d_{\alpha} + hc)$$

$$H_R = -\frac{D}{\pi} \sum_{\alpha} |r_{\alpha}| \cos[2\phi_{\alpha}(0)]$$

$$H_C = E_C \left[ \hat{n} + \frac{1}{\pi} \sum_{\alpha} \phi_{\alpha}(0) - N(V_g) \right]^2$$

Assumptions:

Strong coupling regime

$$T \ll E_C$$

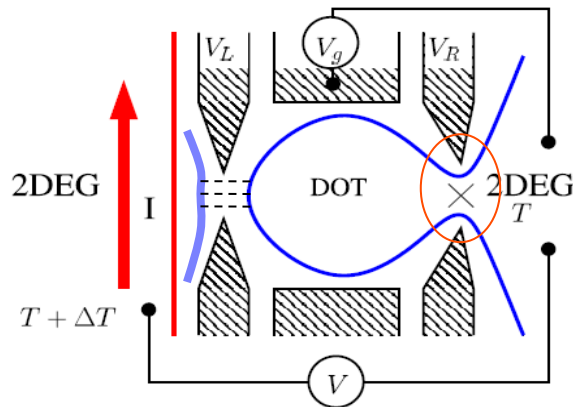
Weak Coulomb Blockade

$$|r_{\alpha}| \ll 1$$

Metallic regime

$$\delta \ll T$$

# Strong coupling and the Kondo physics (Matveev & Andreev, 2002)



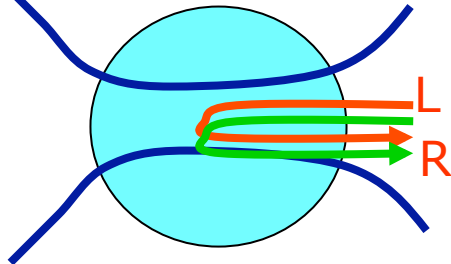
Ordinary (one-channel) Kondo

$$J c_{\sigma}^{\dagger} \vec{\sigma}_{\sigma\sigma'} c_{\sigma'} \vec{S}$$

N-channel Kondo:  
spin-1/2 impurity  
+ N orbital channels

$$J \sum_j c_{j\sigma}^{\dagger} \vec{\sigma}_{\sigma\sigma'} c_{j\sigma'} \vec{S}$$

Reflection plays a role of spin-flip

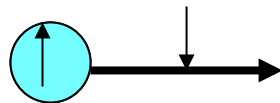


QPC is a quasi-spin-1/2 orbital “impurity” + two spin  $\uparrow$  and  $\downarrow$  channels: 2-channel Kondo. Symmetry of channels is protected by TRS

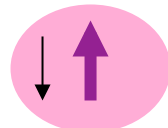
# FL and non-FL behavior

Enhancement by non-Fermi-liquid effects

$$Jc_{\sigma}^{+}\vec{\sigma}_{\sigma\sigma'}c_{\sigma'}\vec{S}$$



Kondo screening of the impurity spin



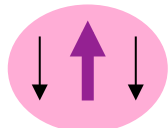
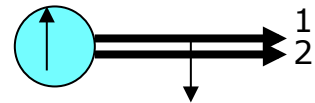
Complete screening below  $T_k$

Fermi-liquid behavior

Magnetic susceptibility  $\chi = \text{const}$  as  $T \rightarrow 0$

$$eS \sim T/T_k$$

$$\sum_j J_j c_{j\sigma}^{+}\vec{\sigma}_{\sigma\sigma'}c_{j\sigma'}\vec{S}$$



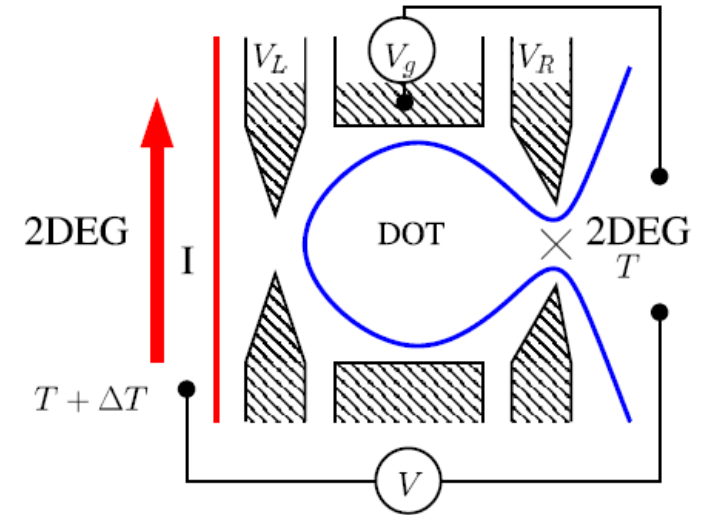
overscreening

Non-Fermi-liquid behavior

Magnetic susceptibility  $\chi \sim \ln(T_k/T)$

$$eS \sim (T/T_k)^{1/2} \ln(T_k/T)$$

# Two Kondo regimes



Spinless fermions:

QPC is fully spin-polarized: 1CK

Fermi liquid behavior:



$$S \propto -\frac{T}{E_C} |r| \sin(2\pi N)$$

**Enhancement of thermopower by electron-electron interaction !**

Spinful fermions:

QPC is non-polarized:  
isotropic 2CK

Non Fermi liquid behavior:



$$S \propto -|r|^2 \ln \frac{E_C}{T} \sin(2\pi N)$$

**Enhancement by non-Fermi-liquid effects**

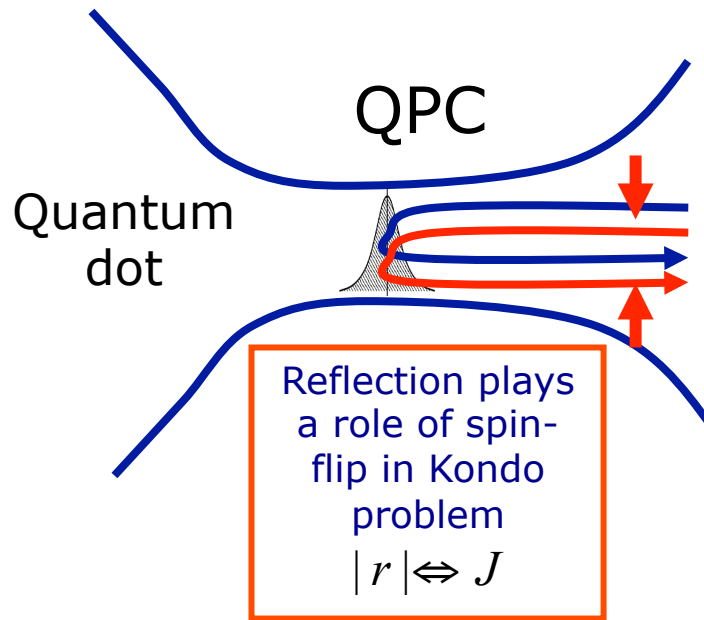
Q1: How does one regime crossover to another one?

Matveev, Andreev, 2001-2002

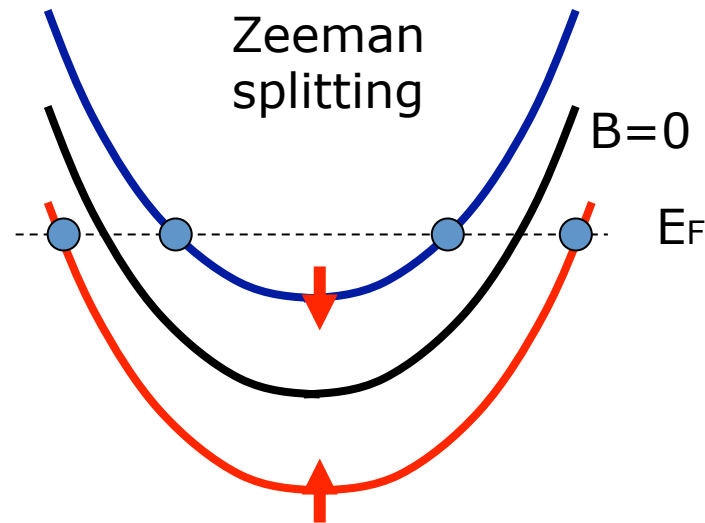
# How does magnetic field influence two Kondo regimes?



Parallel to the plane magnetic field



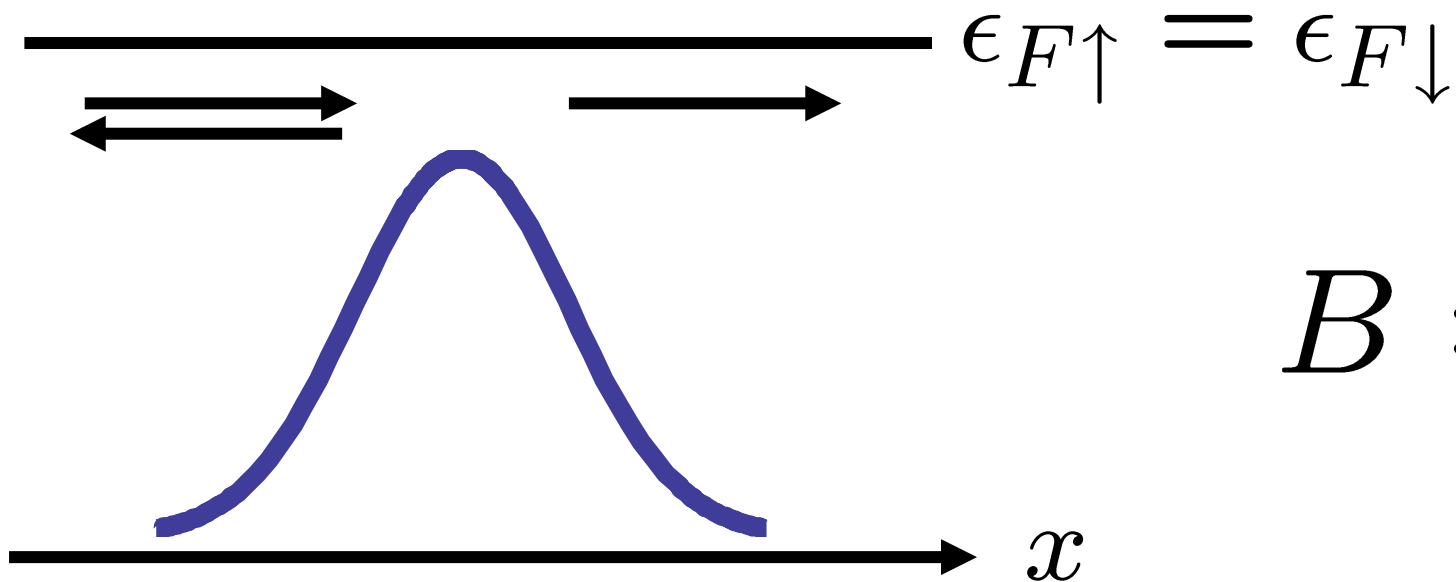
$$T_{\max} \sim |r|^2 E_C, \quad E_C = e^2 / (2C)$$



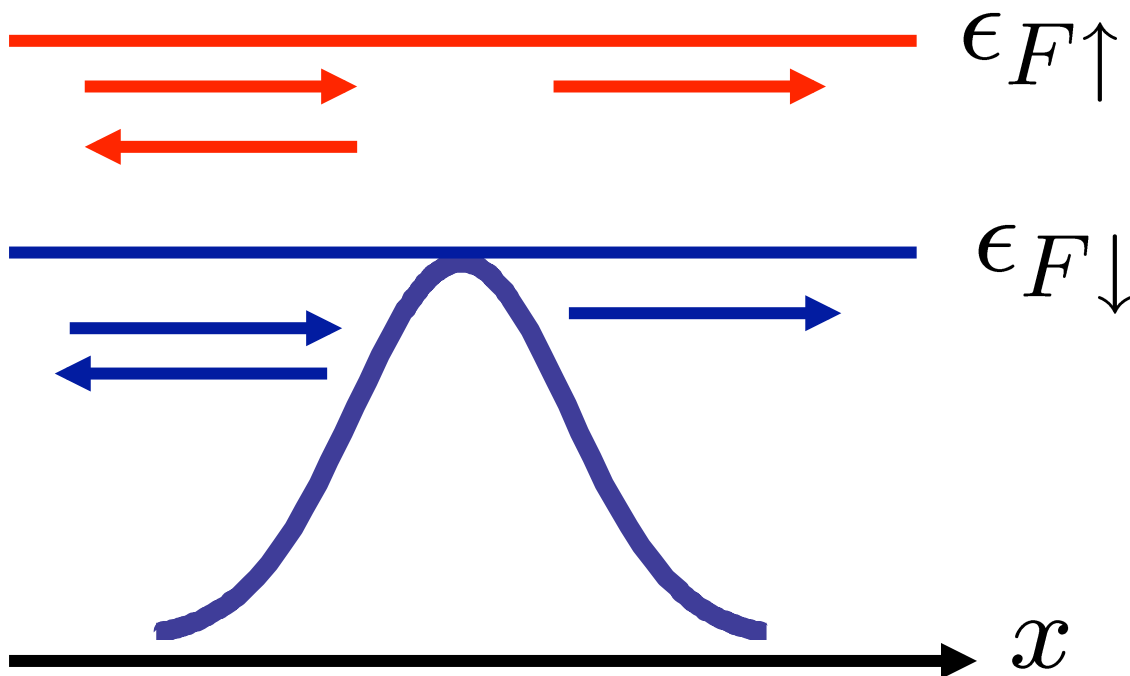
$$B \neq 0: \quad k_{F\uparrow} \neq k_{F\downarrow} \Rightarrow r_{\uparrow} \neq r_{\downarrow}$$

$$J_1 \neq J_2$$



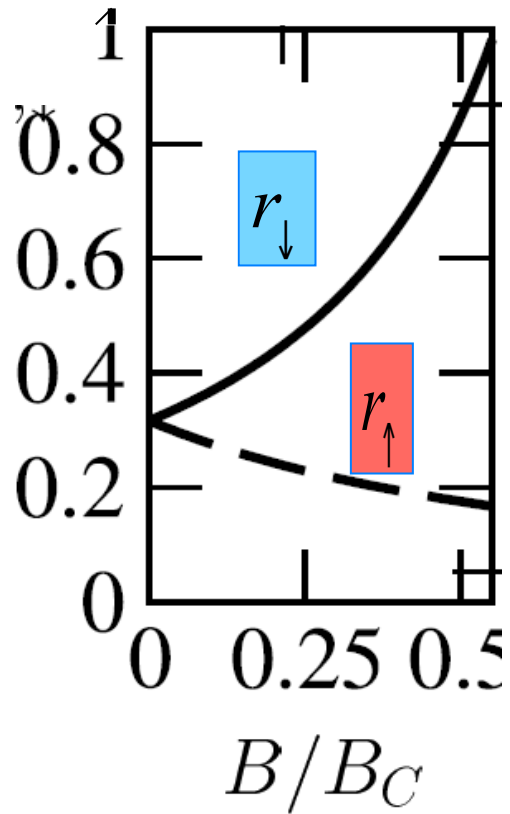


$$B = 0$$



$$B = B^*$$

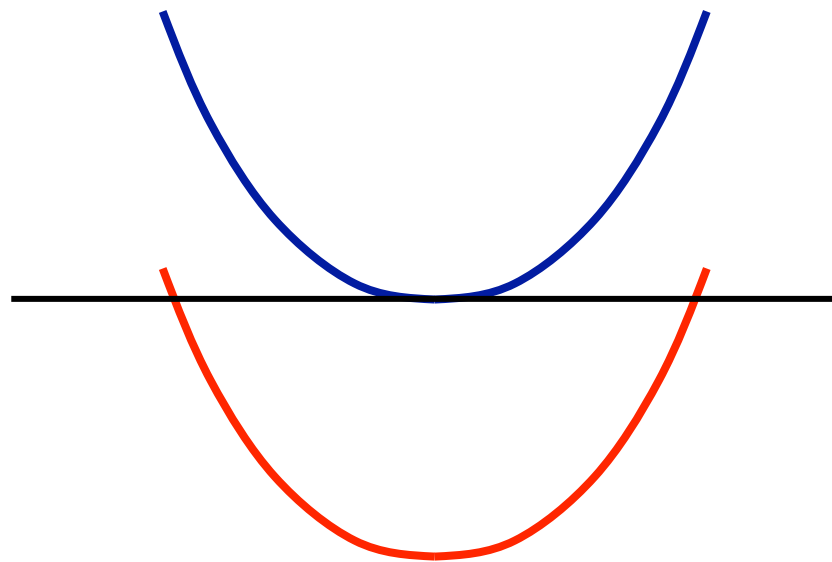
# Characteristic scales of magnetic field



Field  $B^* < B_c$  where spin-down  
electron is fully reflected

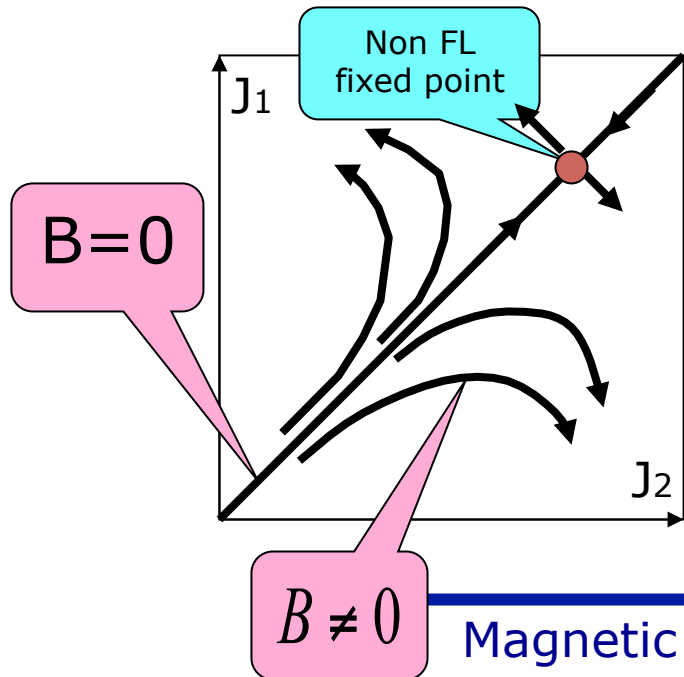
(model dependent)

$$E_{\uparrow,\downarrow}(B) - E_{\uparrow,\downarrow}(0) = \pm \frac{1}{2} g \mu_B B$$



Field of full polarization  $B_c$

# Instability of non-FL fixed point



$$\sum_j J_j c_{j\sigma}^+ \vec{\sigma}_{\sigma\sigma'} c_{j\sigma'} \vec{S}$$

The symmetric state  $J_1 = J_2$  and the non-FL fixed point is only stable if protected by the basic symmetry (Time Reversal Symmetry)

Magnetic field breaks TRS and drives system to the 1-channel Kondo with decreasing the temperature T

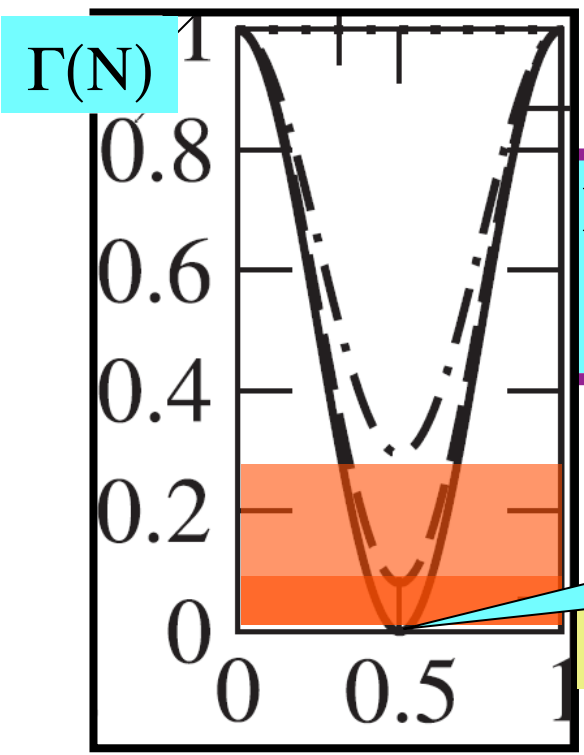
Suppression of thermopower by magnetic field

# The main result

At a finite B a gap in  $\Gamma(N)$  opens up at the degeneracy point  $N=1/2$

$$eS \sim -|r_{\uparrow} r_{\downarrow}| \left( \frac{T}{\Gamma(N)} \right) \ln \left( \frac{E_C}{T + \Gamma(N)} \right) \sin(2\pi N) F \left( \frac{\Gamma(N)}{T} \right)$$

$$F(x) = \begin{cases} x, & x \ll 1 \\ const, & x \gg 1 \end{cases}$$

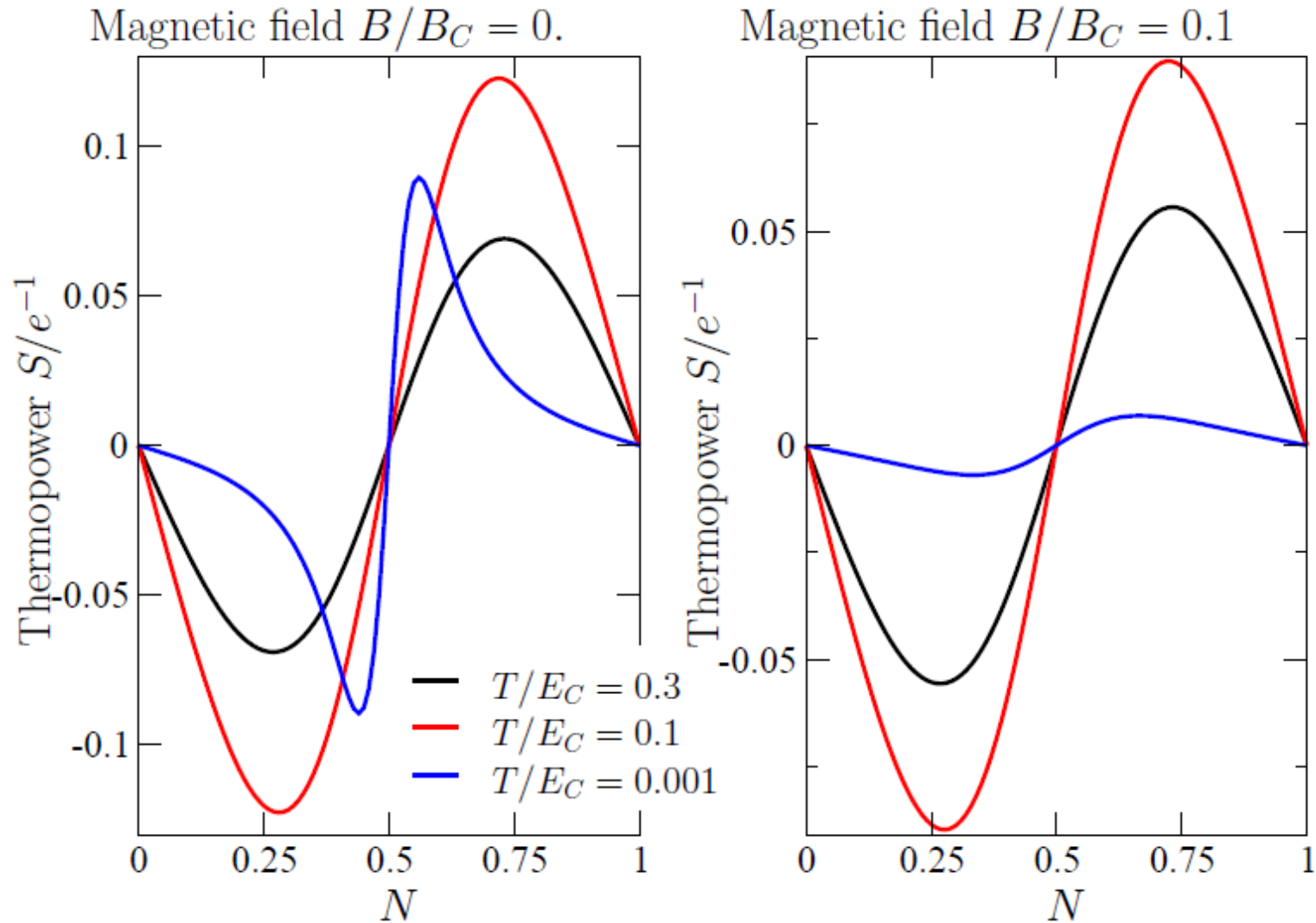


$$\Gamma(N) \sim E_C [ |r_{\uparrow} + r_{\downarrow}|^2 \cos^2(\pi N) + |r_{\uparrow} - r_{\downarrow}|^2 \sin^2(\pi N) ]$$

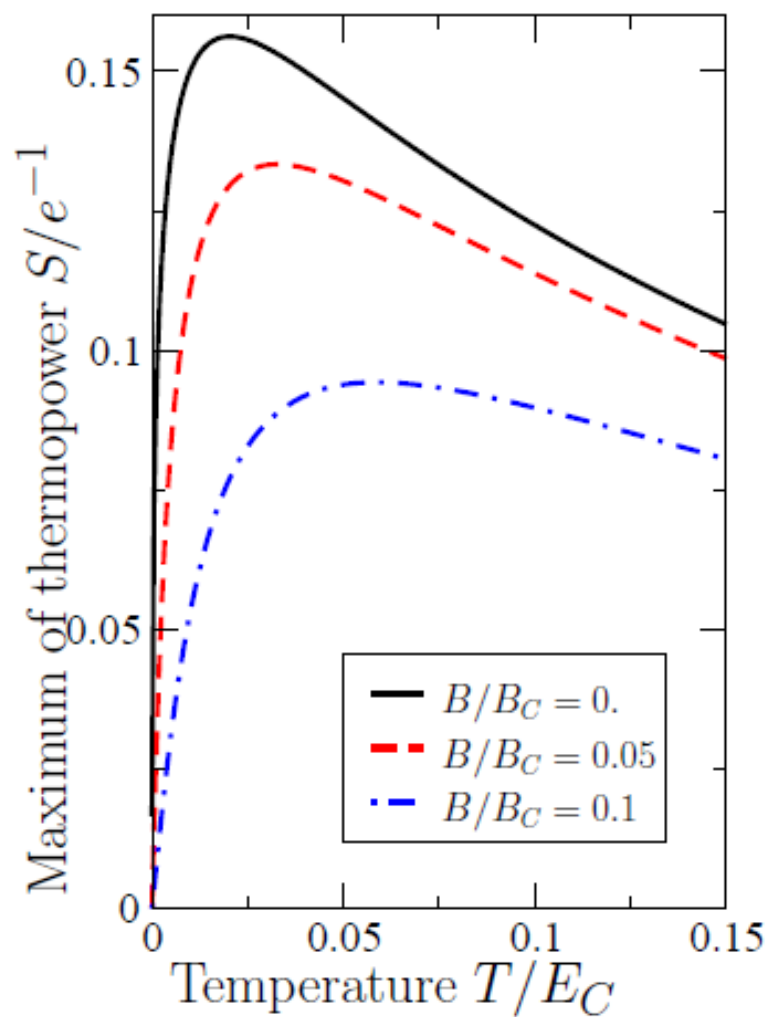
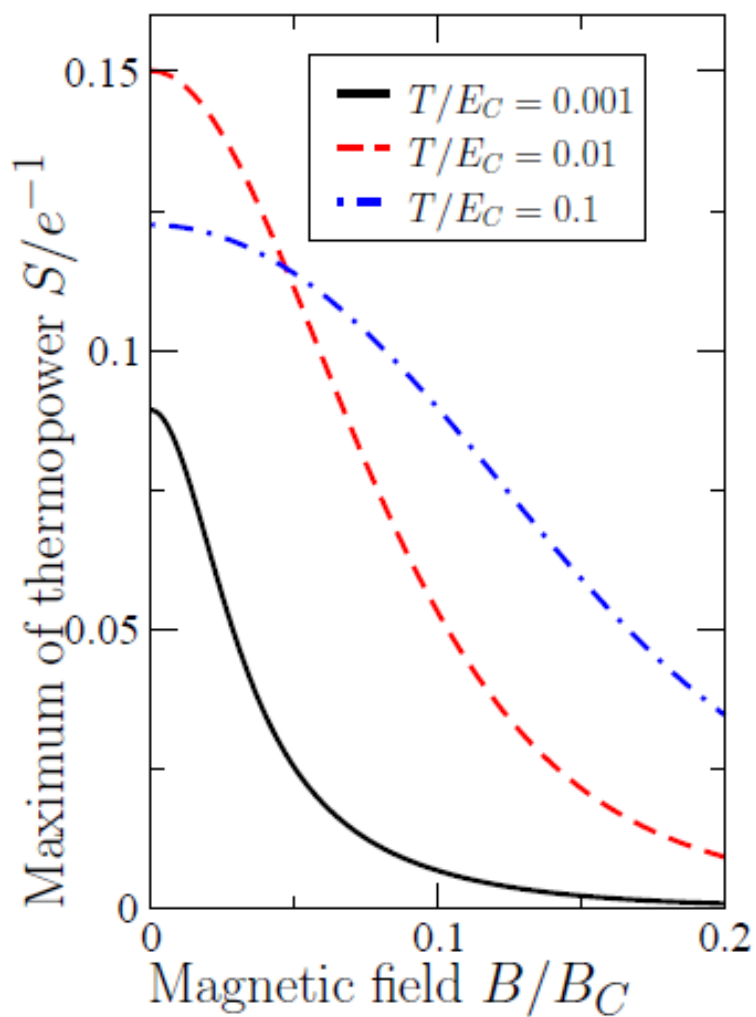
Coulomb blockade peak (degeneracy) point

Effect of B

## Theoretical predictions: gate voltage dependence



## Theoretical predictions: B and T -dependences

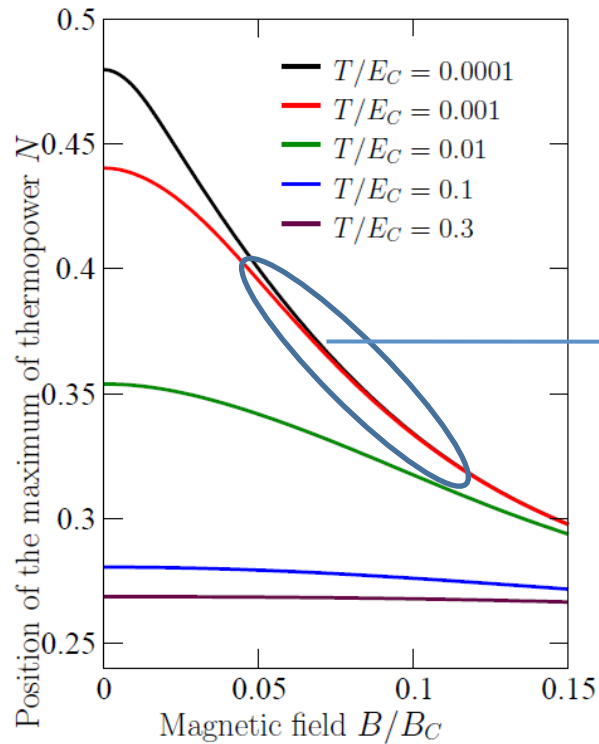


# Effects of magnetic field on thermopower

$$|r_{\downarrow} r_{\uparrow}| \xrightarrow{B \uparrow} |r_{\uparrow}|$$

For  $T \ll T_{\min}$ :

$$S \propto -\frac{1}{e} |r_{\uparrow} r_{\downarrow}| \frac{T}{\Gamma(N)} \ln \frac{E_C}{\Gamma(N)} \sin(2\pi N)$$



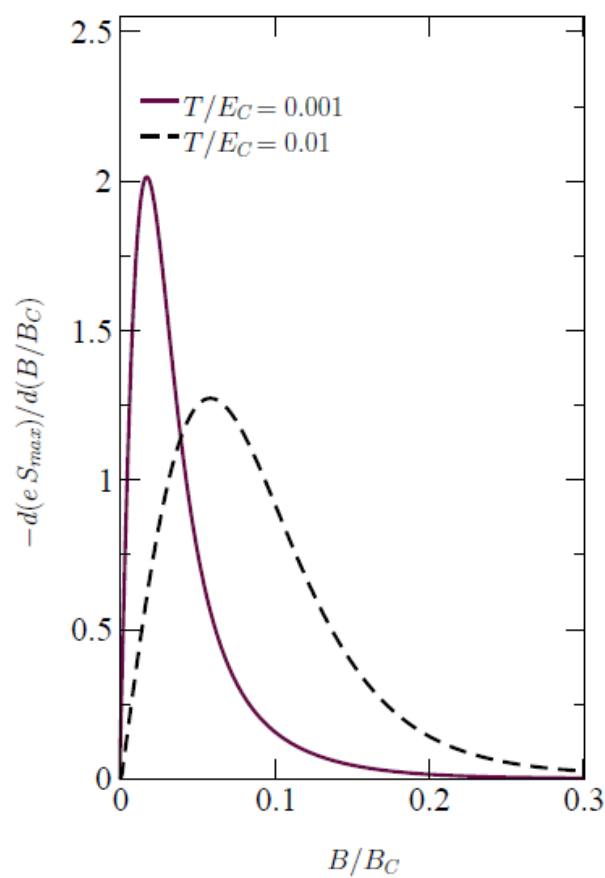
$$eS_{\max} \propto \frac{T}{E_0}$$

$$E_0 = E_C \left| \frac{B}{B_C} \right| \ln^{-1} \left[ \frac{B_C}{B|r|} \right]$$

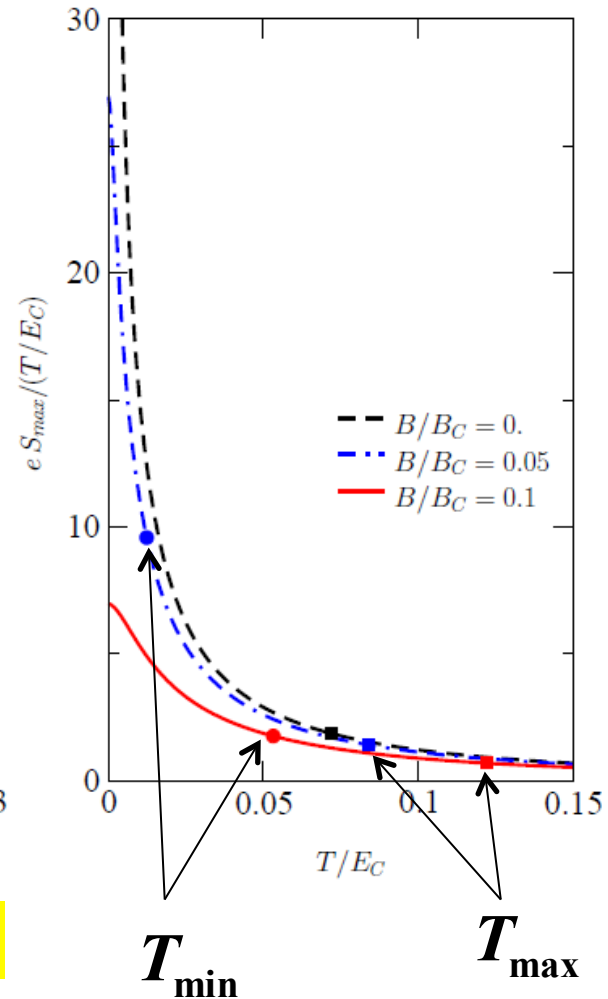
**“Giant Fermi energy”**

**Giant Fermi-liquid behavior in magnetic field**

# Theoretical predictions: derivatives



**Existence of maximum in dS/dB**

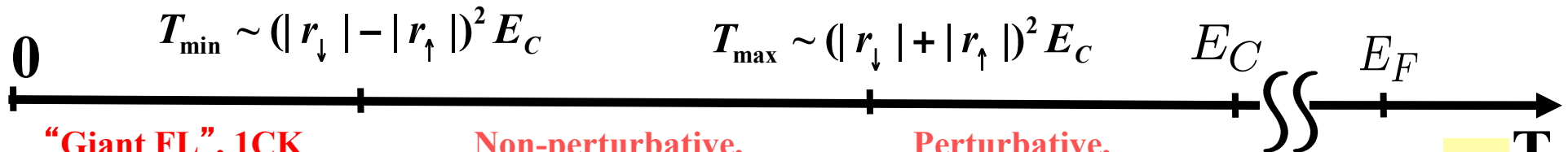


**$B=0$ :  $S_{max}/T$  diverges at  $T = 0$ ;  
 Finite  $B$ :  $S_{max}/T$  saturates below  $T_{min}$**



# Message to take home

$$T_{min} \sim r_0^2 E_C (B/B_c)^2$$



**“Giant FL”, 1CK**

**Non-perturbative, strong NFL, 2CK**

**Perturbative, weak NFL, 2CK**

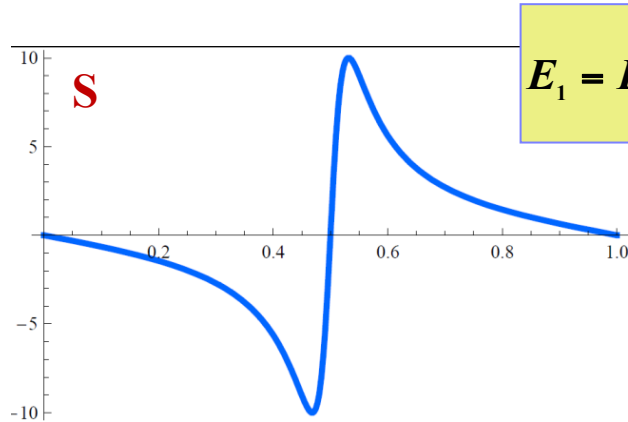
**BULK**

$$eS^{max} \sim \frac{T}{E_0}$$

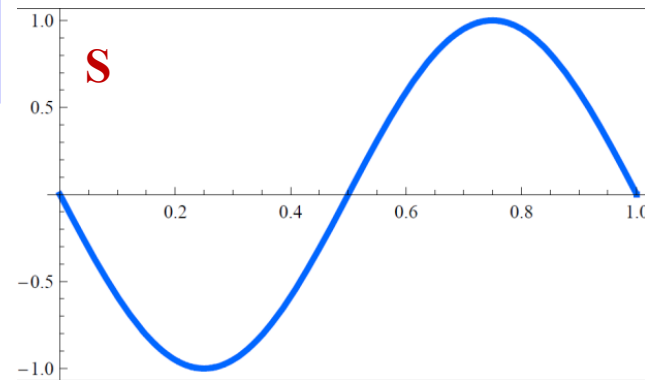
$$eS^{max} \sim \sqrt{\frac{T}{E_1}} \ln\left(\frac{E_C}{T}\right)$$

$$eS^{max} \sim |r_{\uparrow} r_{\downarrow}| \ln\left(\frac{E_C}{T}\right)$$

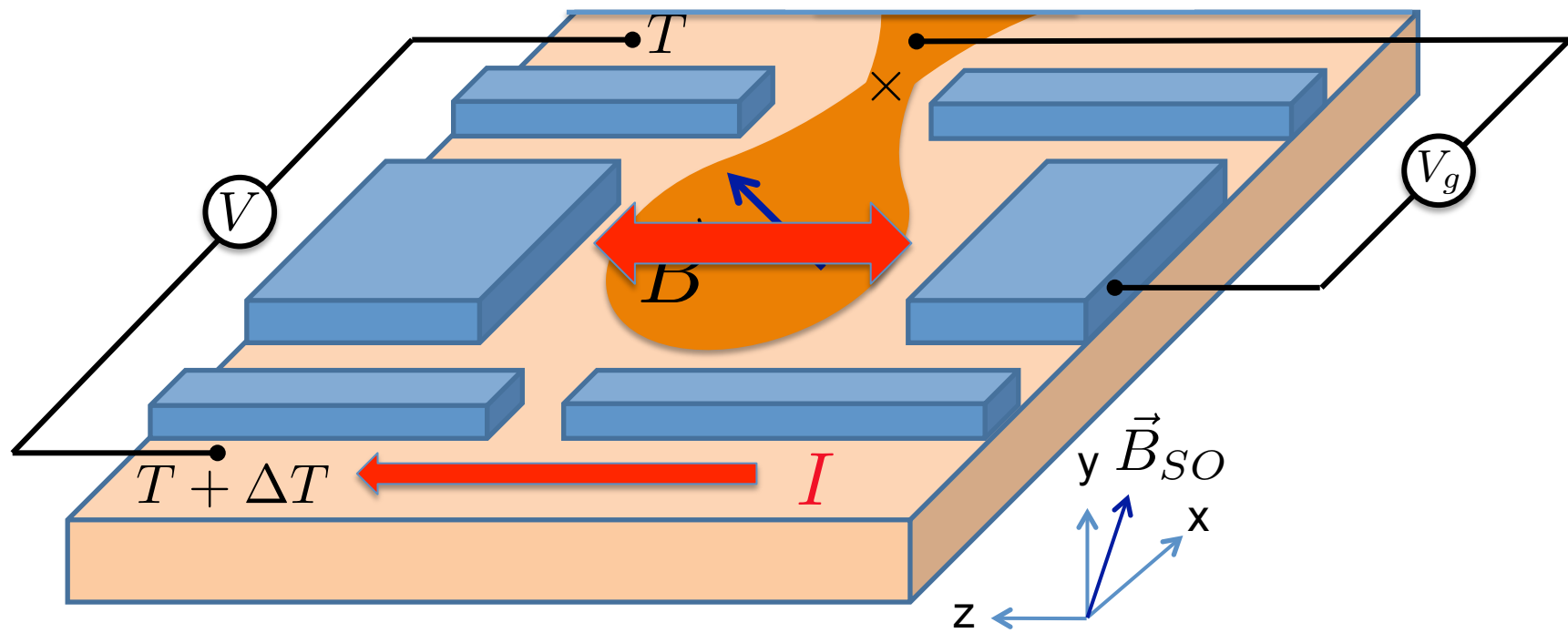
$$S \sim \frac{T}{E_F}$$



$$E_1 = E_C \left( \frac{1}{r_{\uparrow}} + \frac{1}{r_{\downarrow}} \right)^2$$



Flensberg - Matveev - Furusaki setup in external parallel magnetic field in the presence of SOI



$$L \ll l_{SO}$$

## Strong Zeeman effect vs strong spin-orbit interaction

$$H = H_0 + H_{LL} + H_{\text{tun}} + H_Z + H_{BS} + H_C + H_{SO}$$

$$H_0 = \sum_{k,\sigma} \epsilon_{p,\sigma} c_{p,\sigma}^\dagger c_{p,\sigma} + \sum_{\sigma} \epsilon_{\sigma} d_{\sigma}^\dagger d_{\sigma} \quad H_{LL} = i v_F \sum_{\gamma=\pm, \sigma=\uparrow, \downarrow} \gamma \int dx \Psi_{\gamma,\sigma}^\dagger \partial_x \Psi_{\gamma,\sigma}$$

$$H_{\text{tun}} = \sum_{k,\sigma} (t_{k,\sigma} c_{k,\sigma}^\dagger d_{\sigma} + \text{h.c.}) \quad H_Z = g\mu_B \vec{B} \cdot (\vec{s}_{\text{leads}} + \vec{S}_{\text{dot}})$$

$$H_{BS} = -v_F \sum_{\sigma} |r_{\sigma}| \left[ \Psi_{L,\sigma}^\dagger(0) \Psi_{R,\sigma}(0) + \text{h.c.} \right]$$

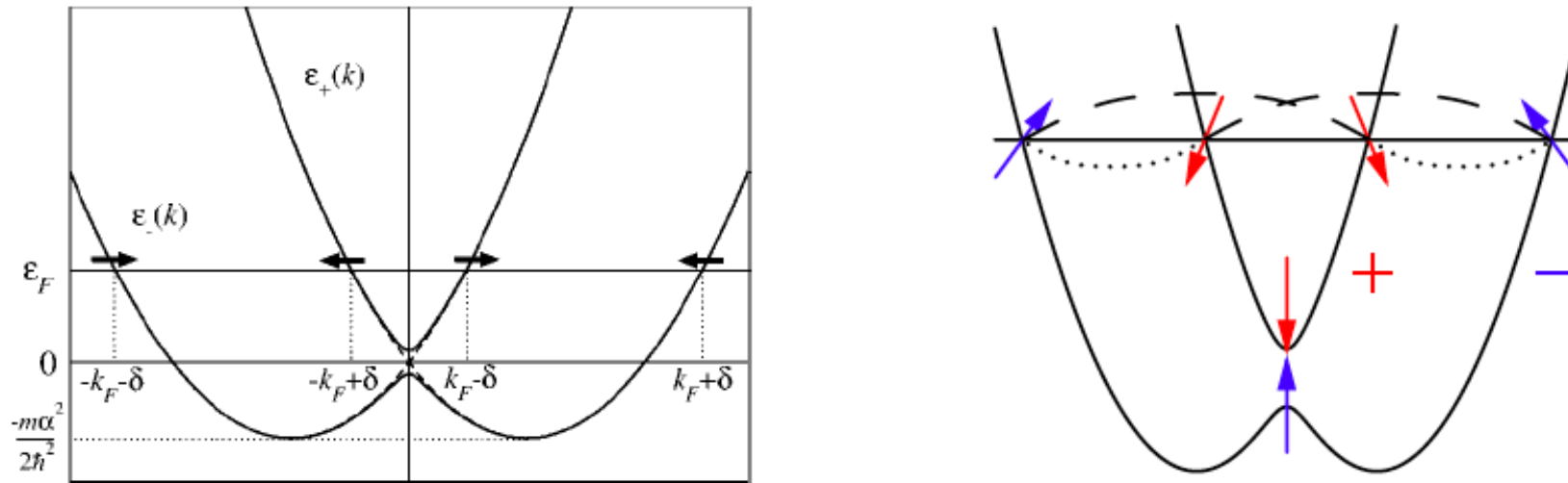
$$H_C = E_C \left[ \hat{n} + \int_0^{\infty} \sum_{\gamma,\sigma} \Psi_{\gamma,\sigma}^\dagger(x) \Psi_{\gamma,\sigma}(x) dx - N(V_g) \right]^2$$

$$H_{SO} = \alpha_R [\vec{p} \times \vec{n}_z] \cdot \vec{\sigma} + \alpha_D p_x \sigma_x$$

$$H_R = i\alpha_R k_F \sum_{\gamma=\pm} \gamma \int dx \left[ \Psi_{\gamma,\uparrow}^\dagger \Psi_{\gamma,\downarrow} - \Psi_{\gamma,\downarrow}^\dagger \Psi_{\gamma,\uparrow} \right]$$

$$H_D = \alpha_D k_F \sum_{\gamma=\pm} \gamma \int dx \left[ \Psi_{\gamma,\uparrow}^\dagger \Psi_{\gamma,\downarrow} + \Psi_{\gamma,\downarrow}^\dagger \Psi_{\gamma,\uparrow} \right]$$

## Strong spin-orbit interaction at $B \ll B_{SO} = \alpha p_F$



### Spectra

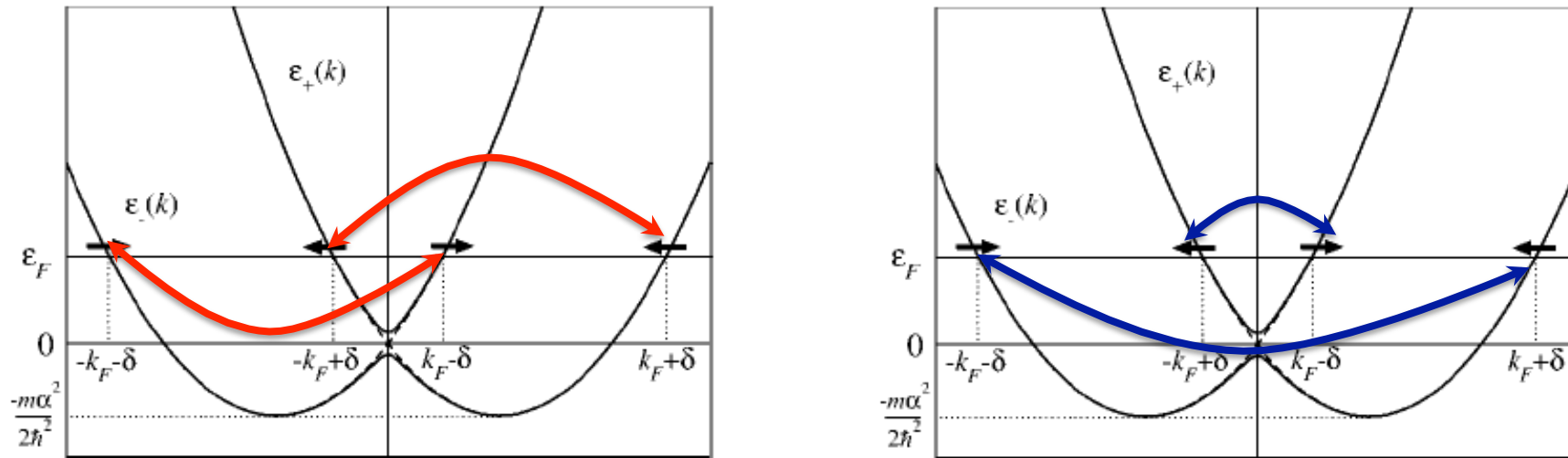
R-SOI 
$$E_{\pm}(p) = \frac{p^2}{2m} \pm \sqrt{(\alpha_R p)^2 + (g\mu_B B/2)^2}$$

$$E_{\pm}(B) - E_{\pm}(0) = \pm \frac{g\mu_B B^2}{4B_{SO}}$$

D-SOI 
$$E_{\pm}(p) = \frac{p^2}{2m} \pm \sqrt{(\alpha_D p)^2 + (g\mu_B B/2)^2 + \alpha_D p g\mu_B B \cos \theta}$$

$$E_{\pm}(B) - E_{\pm}(0) = \pm g\mu_B/2 \left( B \cos \theta + \frac{B^2}{2B_{SO}} \sin^2 \theta \right)$$

# Strong spin-orbit interaction at $B \ll B_{\text{SO}} = \alpha p_F$



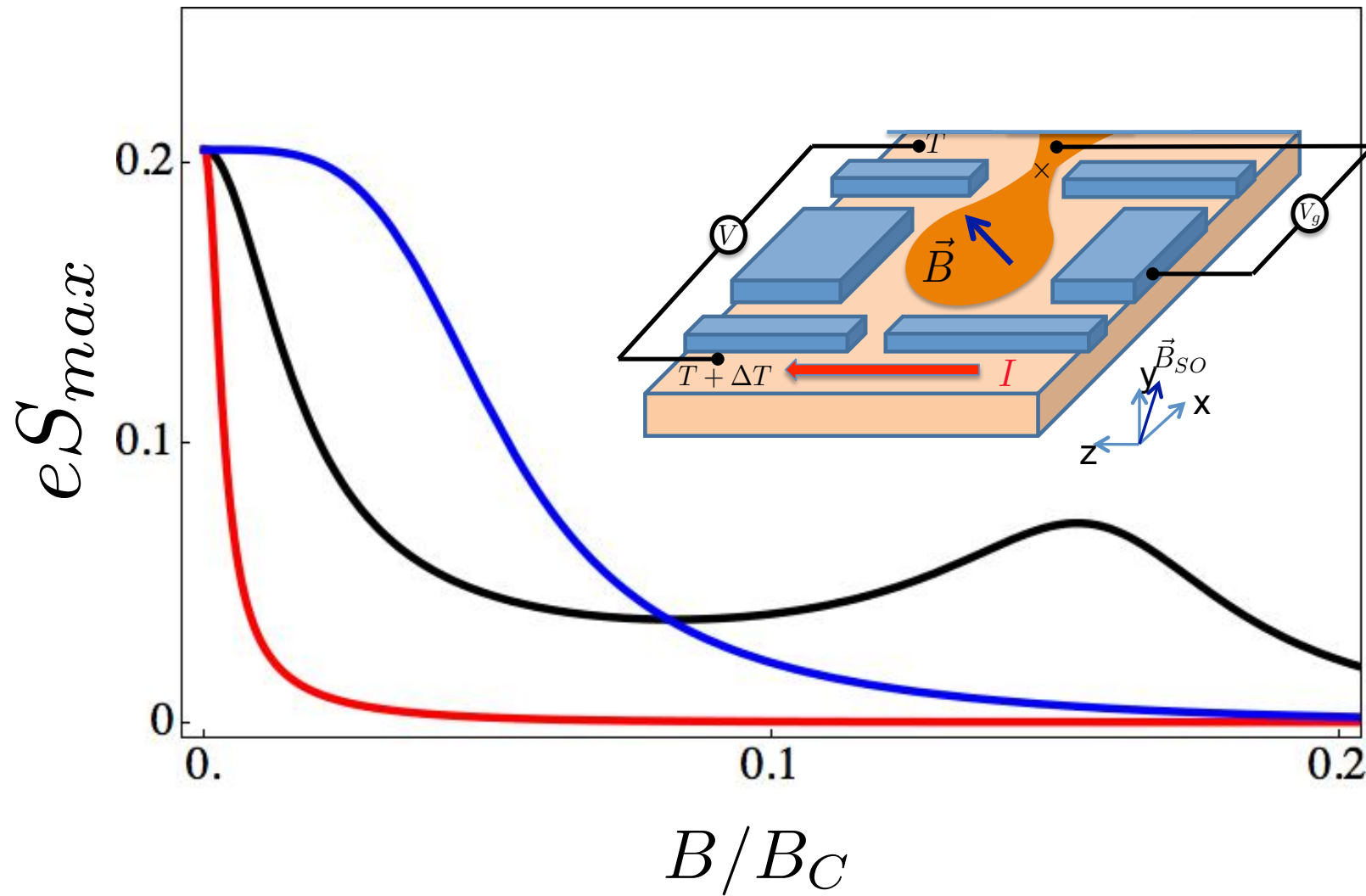
## Scattering

Spin conserving  $H_{BS} = -v_F \sum_{\sigma=\uparrow\downarrow} |r_\sigma| \left[ \Psi_{L,\sigma}^\dagger(0) \Psi_{R,\sigma}(0) + \text{h.c.} \right]$

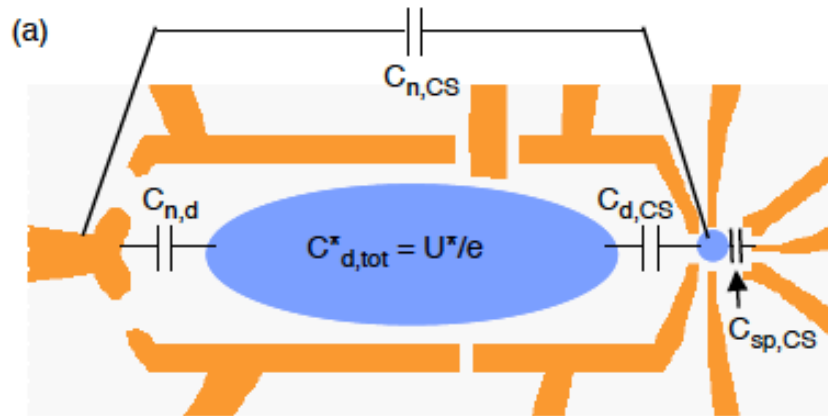
Spin flip  $H_{BS} = -v_F \sum_{\lambda=\pm} |r_\lambda| \left[ \Psi_{L,\lambda}^\dagger(0) \Psi_{R,\lambda}(0) + \text{h.c.} \right]$

$$|r_\pm| \sim r_0 \frac{B}{\alpha p_{F\pm}}$$

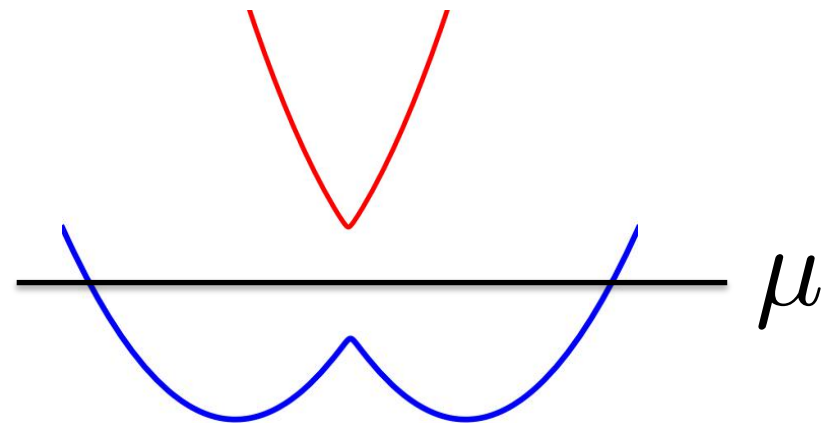
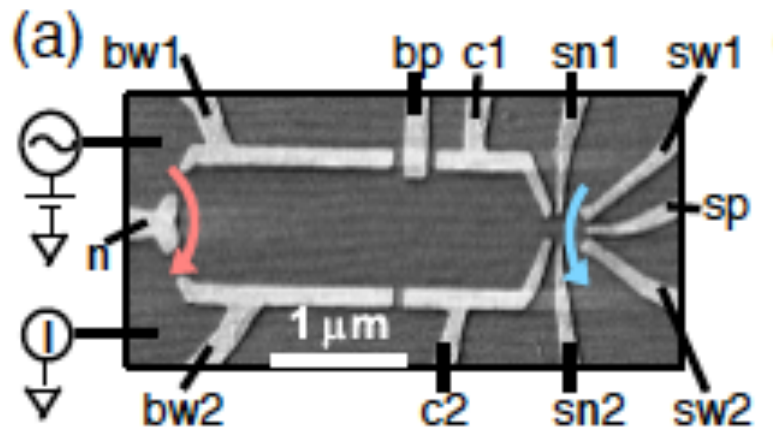
# Theoretical predictions: strong spin-orbit



# Perspectives:



- Multi-channel Kondo effect
- Influence of noise
- Influence of finite s-d voltage
- Quenches with the gate and s-d voltage
- SOI in quantum dot



# Conclusions



- Thermopower of a quantum dot can be much larger than in the bulk  $eS_{\text{BULK}} \sim T/E_F$
- Kondo physics in thermopower of an open dot; magnetic field leads to crossover from 2CK to 1CK
- Magnetic field suppresses thermopower and restores FL behavior at  $T < T_{\text{min}}$
- Spin - orbit interaction "protects" the NFL at magnetic fields  $B < B_{\text{SO}}$
- Interplay between linear and quadratic Zeeman effect leads to non-monotonic B- behavior of thermopower

**T.K.T. Nguyen, MK and V.E. Kravtsov, PRB 82, (2010)**  
**MK and Z. Ratiani (2013)**