Spin supercurrents and torquing with majorana fermions

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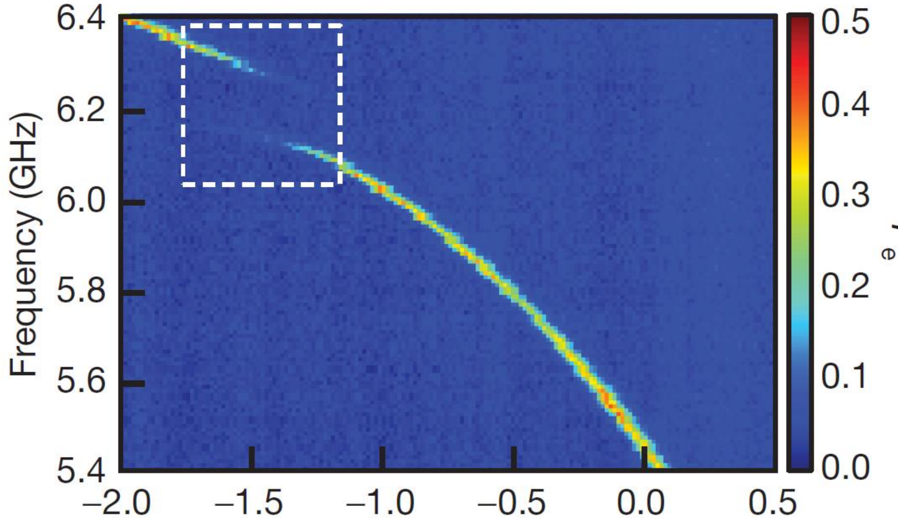
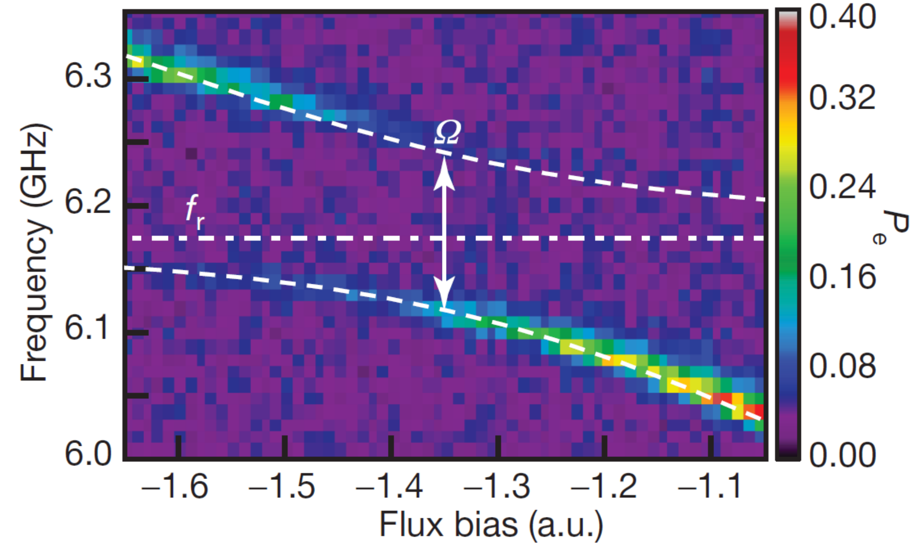
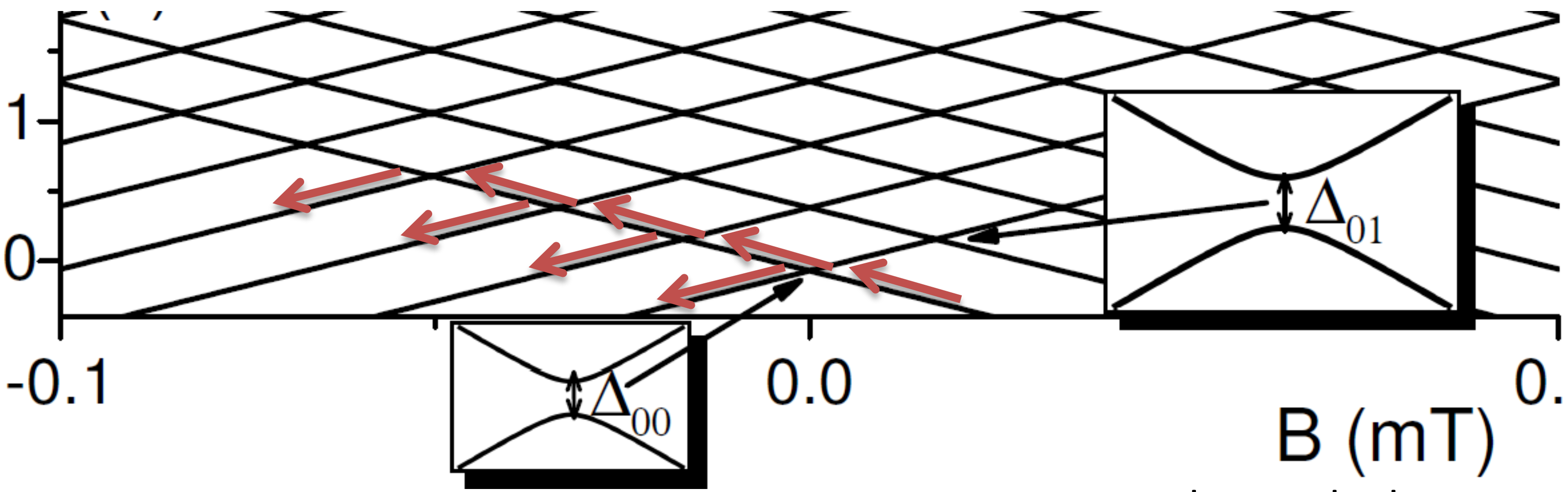
QUANTUM CONTROL ON A SINGLE PHONON LEVEL





1. Quantum optical techniques for detection.

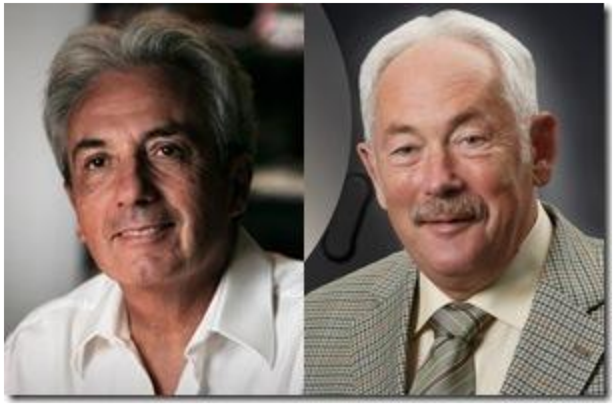
O'Connell et al., Nature 464, 697

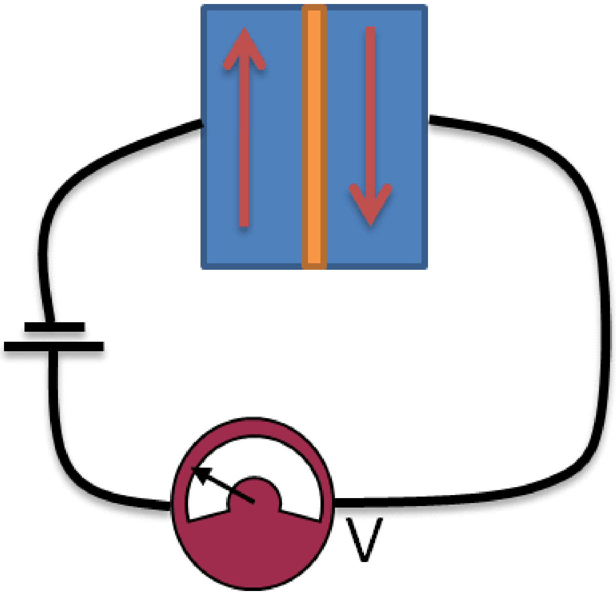


2. Detection by studying Landau-Zener transitions.

A.Kovalev et al. Phys. Rev. Lett. 106, 147203 (2011)

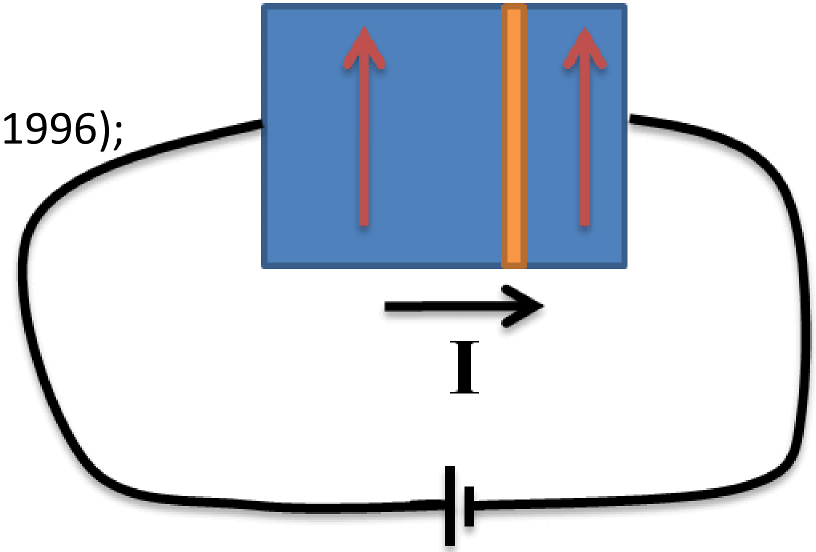
GMR EFFECT AND SPIN TRANSFER TORQUE





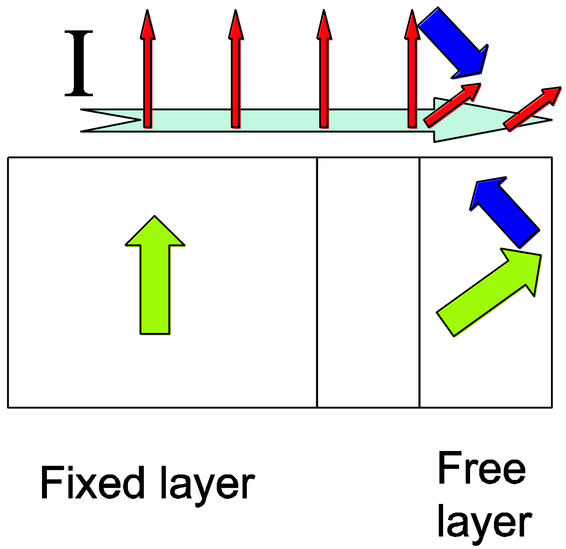
Albert Fert and Peter Grünberg discovered ***giant magnetoresistance effect*** in 1988,
Nobel Prize 2007.

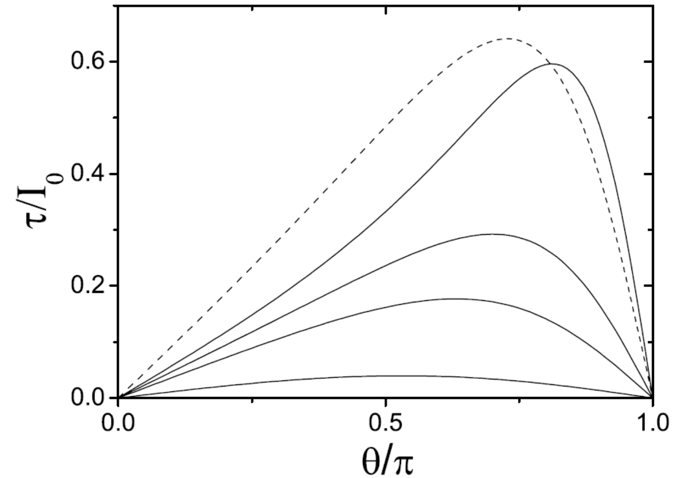
Opposite effect: ***current induced torque***.
J. C. Slonczewski, J. Magn. Magn. Mater. 159, L1 (1996);
L. Berger, Phys. Rev. B 54, 9353 (1996).



Spin torques with majoranas

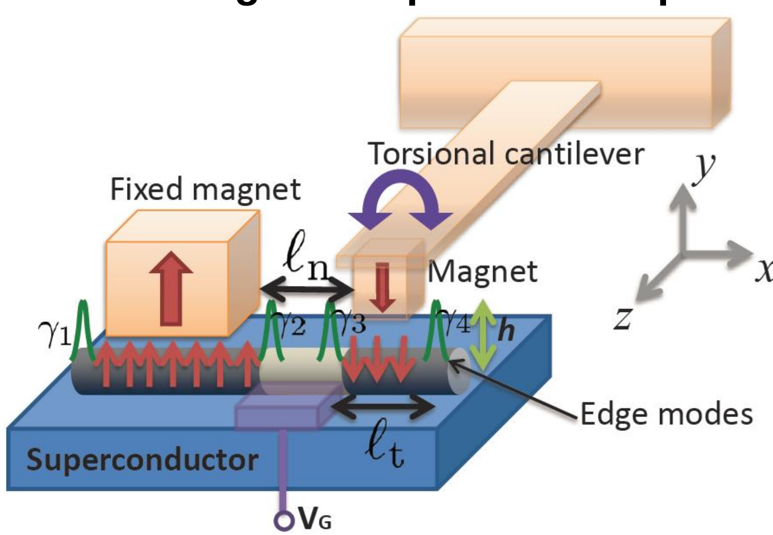
Current-induce spin-transfer torque is 2π periodic.

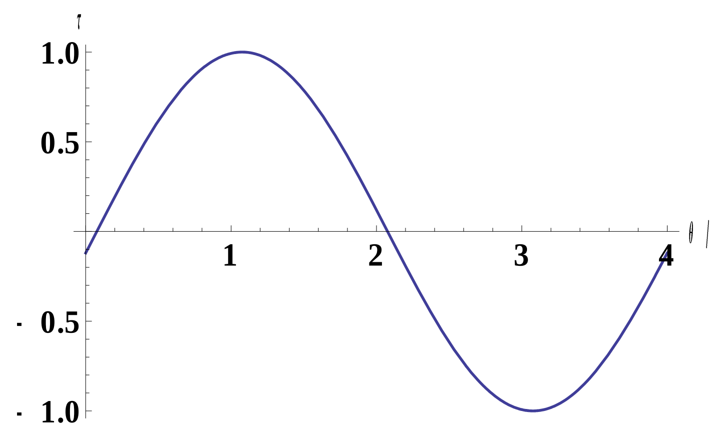




A.A. Kovalev, A. Brataas, G.E.W. Bauer, Phys. Rev. B 66, 224424 (2002)

In the presence of majorana fermions we can get 4π periodic torque!





P.Kotetes, A.Shnirman, G.Schön, arXiv:1207.2691

L. Jiang, D. Pekker, J. Alicea, G. Refael, Y. Oreg, A. Brataas, F. von Oppen, arXiv:1206.1581

MAJORANA FERMIONS

$$\begin{cases} \gamma_j^A = c_j + c_j^\dagger \\ \gamma_j^B = i(c_j - c_j^\dagger) \end{cases} \Rightarrow \begin{array}{c} (\gamma_j^\alpha)^\dagger = (\gamma_j^\alpha) \\ \left\{\gamma_i^\alpha, \gamma_j^\beta\right\} = 2\delta_{ij}\delta_{\alpha\beta} \end{array}$$

They are real and imaginary parts of a creation operator.

Can be realized in systems with interactions

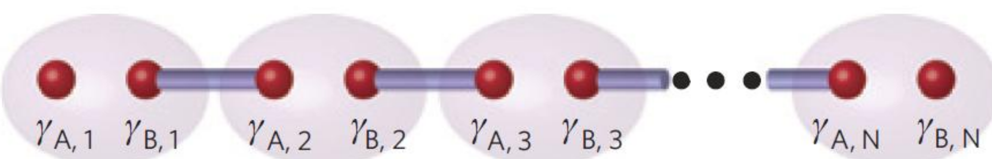
$$H = \sum_j -t(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j) - \Delta(c_j c_{j+1} + c_{j+1}^\dagger c_j^\dagger) - \mu c_j^\dagger c_j$$

A. Yu. Kitaev (2001)

$$\mu = 0,\ t = |\Delta|$$

$$H = -it \sum_j^{N-1} \gamma_{B,j}\gamma_{A,j+1}$$

$\gamma_{A,1},\ \gamma_{B,N}$ -- drop out from Hamiltonian and allow us to form an artificial fermion.

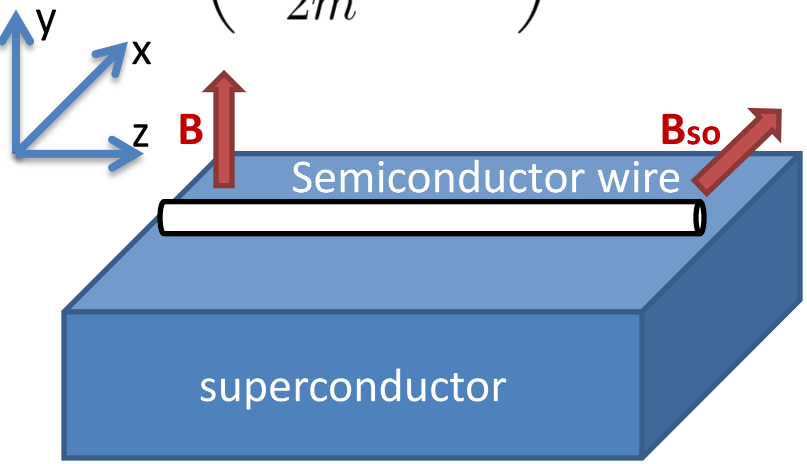


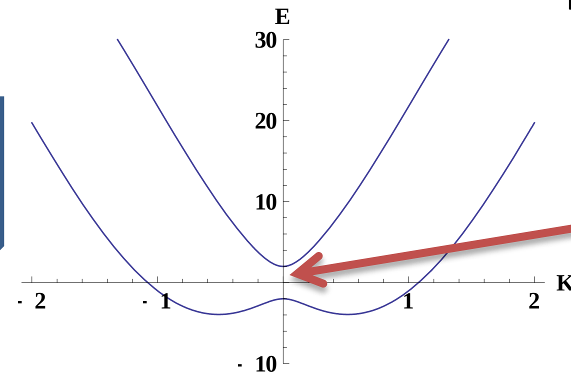
At low energies, the whole wire behaves as one fermion.

J. Alicea, Y. Oreg, G. Refael, F. von Oppen & M. P. A. Fisher Nature Physics 7, 412–417 (2011)

IMPLEMENTATIONS BASED ON SEMICONDUCTOR WIRES

$$H=\left(-\frac{\hbar^2}{2m}\partial_x^2-\mu\right)\tau_z+B_y\sigma_y+B_z\sigma_z+i\alpha_R\partial_x\sigma_y\tau_z+\Delta\tau_x$$

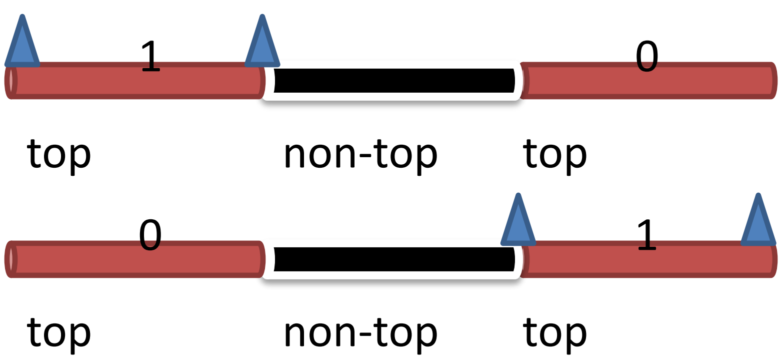


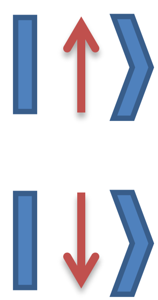


Nambu and spin spinor

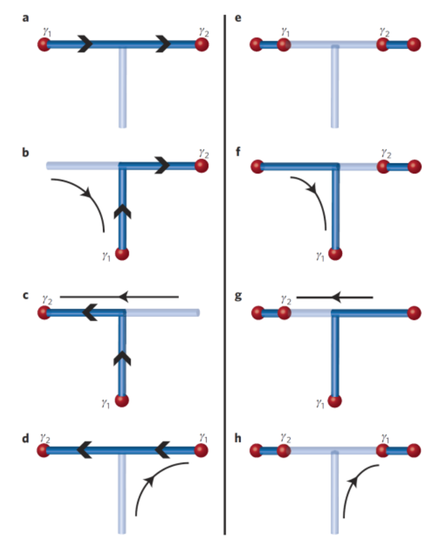
Majorana is obtained by tuning magnetic field and chemical potential in the gap

Due to conservation of parity we need 4 edge modes to form a qubit



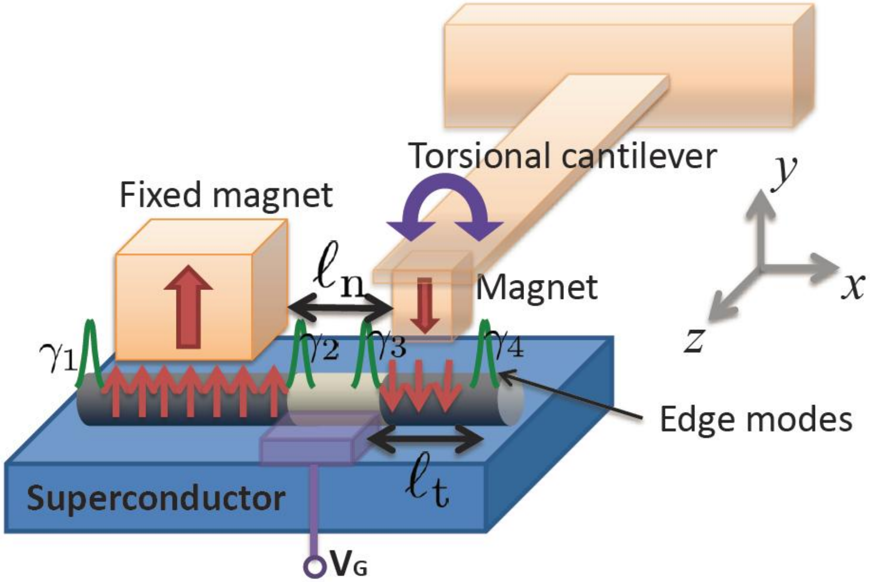


Some gates can be realized by braiding of edge modes.



J. Alicea, Y. Oreg, G. Refael, F. von Oppen & M. P. A. Fisher, Nature Physics 7, 412–417 (2011)

SPIN CURRENTS AND TORQUES WITH MAJORANAS



Magneto-Josephson effect leads to spin current

$$j^S = \frac{\partial \langle H \rangle}{\partial \theta}$$

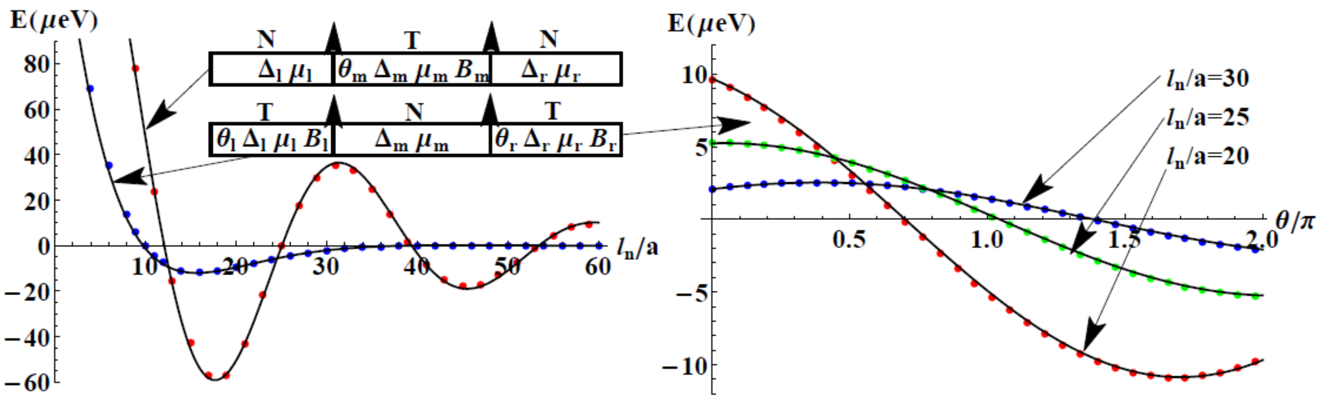
The torque is further transferred to magnets!

P. Kotetes, A. Shnirman, G. Schön, arXiv:1207.2691
L. Jiang, D. Pekker, J. Alicea, G. Refael, Y. Oreg,
A. Brataas, F. von Oppen, arXiv:1206.1581

$$H = \left(-\frac{\hbar^2}{2m}\partial_x^2 - \mu\right)\tau_z + B_y\sigma_y + B_z\sigma_z + i\alpha_R\partial_x\sigma_y\tau_z + \Delta\tau_x$$

This result can be obtained by exact diagonalization of tight-binding version of Hamiltonian or by considering evanescent solutions of the form $\Psi(x) = e^{\kappa x}\bar{\Psi}(\kappa)$

Hybridization energies and spin currents



Hybridization energies of two Majorana bound states over topological and non-topological regions in a semiconductor wire as a function of the hybridization region length (left) and relative angle of magnetic fields (right), only for T–N–T structures. The circles represent the corresponding numerical results.

$$\frac{E^{\mathrm{n}}}{E_0^{\mathrm{n}}} \approx e^{-\ell_{\mathrm{n}} \mathfrak{Re}(\kappa_2^{\mathrm{n}})} \cos\left[\frac{\theta}{2} + \Phi_0 + \ell_{\mathrm{n}}\, \mathfrak{Im}(\kappa_2^{\mathrm{n}})\right]$$ **- Hybridization for non-topological region**

$$\kappa_2^n = m^*/\hbar^2\left(i\alpha_{\mathrm{so}} - i\sqrt{2(i\Delta+\mu)\hbar^2/m^* + \alpha_{\mathrm{so}}^2}\right)$$

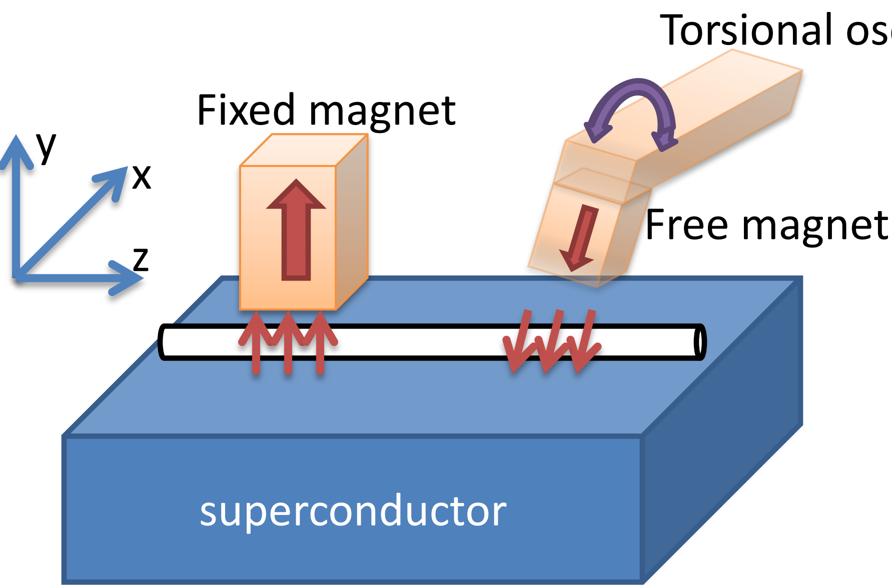
$$\frac{E^{\mathrm{t}}}{E_0^{\mathrm{t}}} \approx e^{-\ell_{\mathrm{t}}\kappa_2^{\mathrm{t}}} + |A_0|\, e^{-\ell_{\mathrm{t}}\mathfrak{Re}(\kappa_1^{\mathrm{t}})} \cos\left[\arg A_0 + \ell_{\mathrm{t}}\, \mathfrak{Im}(\kappa_1^{\mathrm{t}})\right]$$ **- Hybridization for topological region**

$\kappa_{1,2}^{\mathrm{t}}$ -- positive solutions of $\sqrt{B^2 - [\kappa^2(\hbar^2/2m)^2 + \mu]^2} = \Delta + \alpha_{\mathrm{so}}\kappa$

$$j_s^z = \mathfrak{Re}\left[\Psi^\dagger(x)\hat{\sigma}^z\hat{v}\Psi(x)\right] = \pm\frac{\partial E^{\mathrm{n}}(\theta)}{\partial\theta}$$ **- Spin-current in non-topological region**

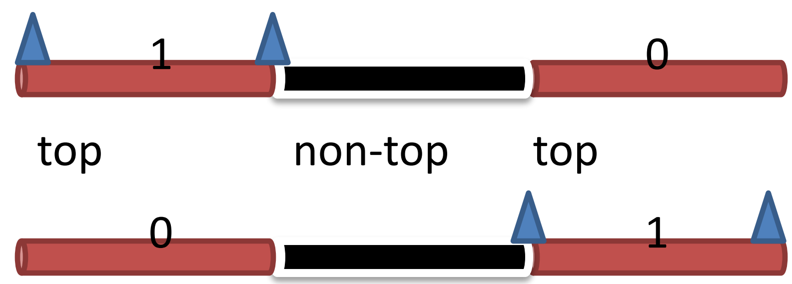
SENSING MAJORANAS WITH MAGNETIC RESONANCE FORCE MICROSCOPY

Magnetic tips with fields 0.1 Tesla are available. Sufficient to drive wire into topological state.

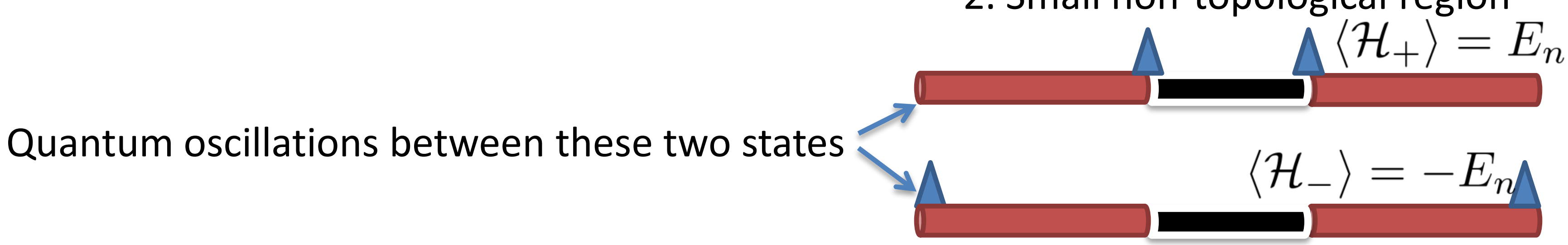


The mechanism of coupling:

1. Large non-topological region

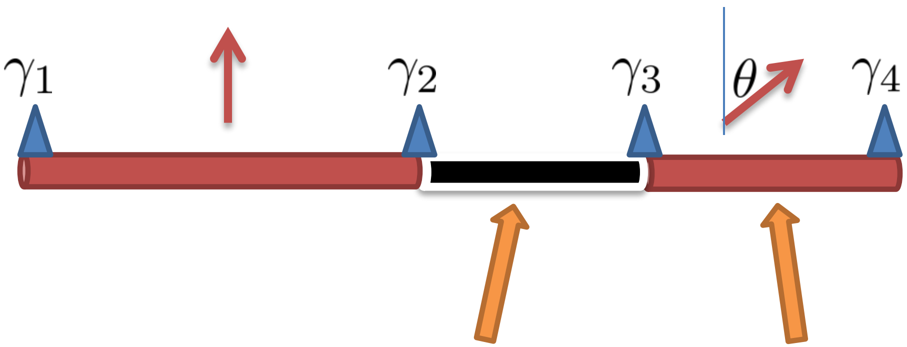


2. Small non-topological region



The state $\langle \mathcal{H} \rangle = \pm E_n$ is associated with spin current and torque. Thus, AC torque acts on resonator. Resonant case: resonator frequency is equal to frequency of quantum oscillations.

QUANTUM OSCILLATIONS AND TORQUES

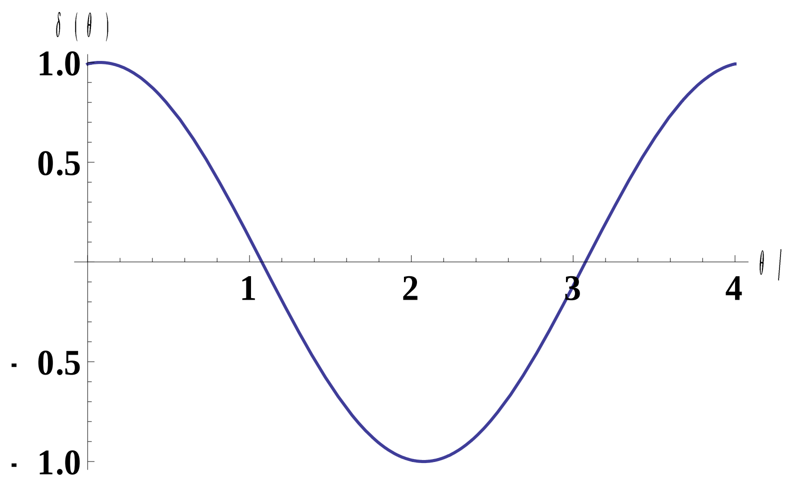


$$V_1 = 4iE_n(\theta)\gamma_2\gamma_3 \quad V_2 = 4iE_t\gamma_3\gamma_4$$

$$c_1 = \gamma_2 + i\gamma_3$$
$$c_1^\dagger = \gamma_2 - i\gamma_3$$
$$c_2 = \gamma_4 + i\gamma_1$$
$$c_2^\dagger = \gamma_4 - i\gamma_1$$

Fix electron parity of two fermions,
Hilbert space of (0,1) and (1,0) states

$$\hat{H} = \begin{bmatrix} E_n(\theta) & E_t \\ E_t & -E_n(\theta) \end{bmatrix}$$

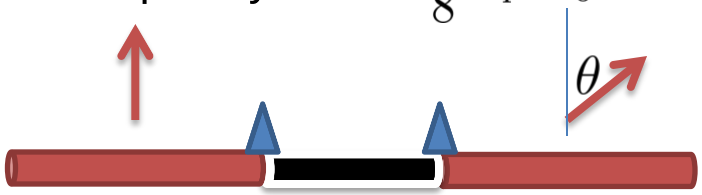


QED WITH MAJORANAS AND RESONATOR

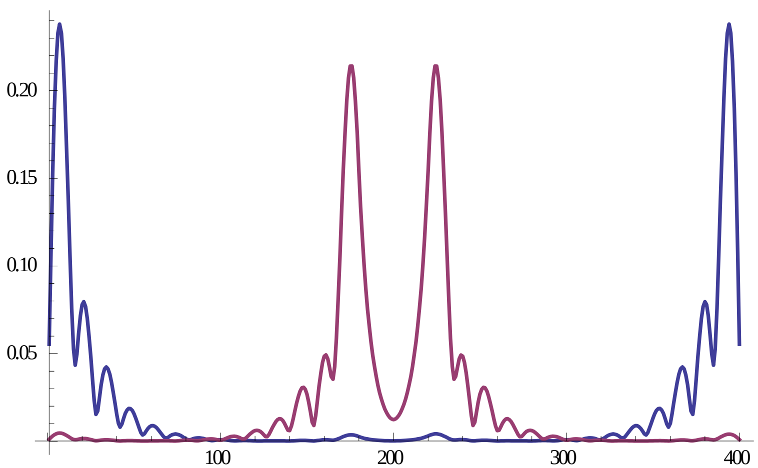
Effective low energy description: $\mathcal{H}=\hbar\omega_{\rm r}a^\dagger a+iE^{\rm n}(\theta)\gamma_2\gamma_3+iE^{\rm t}\gamma_3\gamma_4$

Can reduce to Rabi Hamiltonian! $\mathcal{H}=\hbar\omega_{\rm r}a^\dagger a+\left[\frac{E^{\rm n}(\theta_0)}{4}+\frac{\partial E^{\rm n}}{\partial\theta}\frac{\theta_{\rm zpf}}{4}(a^\dagger+a)\right]\sigma_x+\frac{E^{\rm t}}{4}\sigma_z$

Rabi frequency: $\mathfrak{g}=\frac{1}{8}\theta_{\rm zpf}E_0^{\rm n}e^{-\ell_{\rm n}\mathfrak{Re}(\kappa_2^{\rm n})}$



$$E_n(\theta)=\delta_0\cos(\theta/2-\psi_0)$$



Angular displacement of mechanical resonator:

$$\delta\theta=\theta_{zpf}(a+a^\dagger)\quad\theta_{zpf}=\sqrt{\hbar/I_x\omega_r}$$

θ_{zpf} angle of zero-point fluctuations

Strong coupling can be realized when: $\omega_r/Q<g$

$Q\sim 50$

Should be accessible with carbon nanotubes: J. C. Meyer, M. Paillet, and S. Roth, Science 309, 1539 (2005)

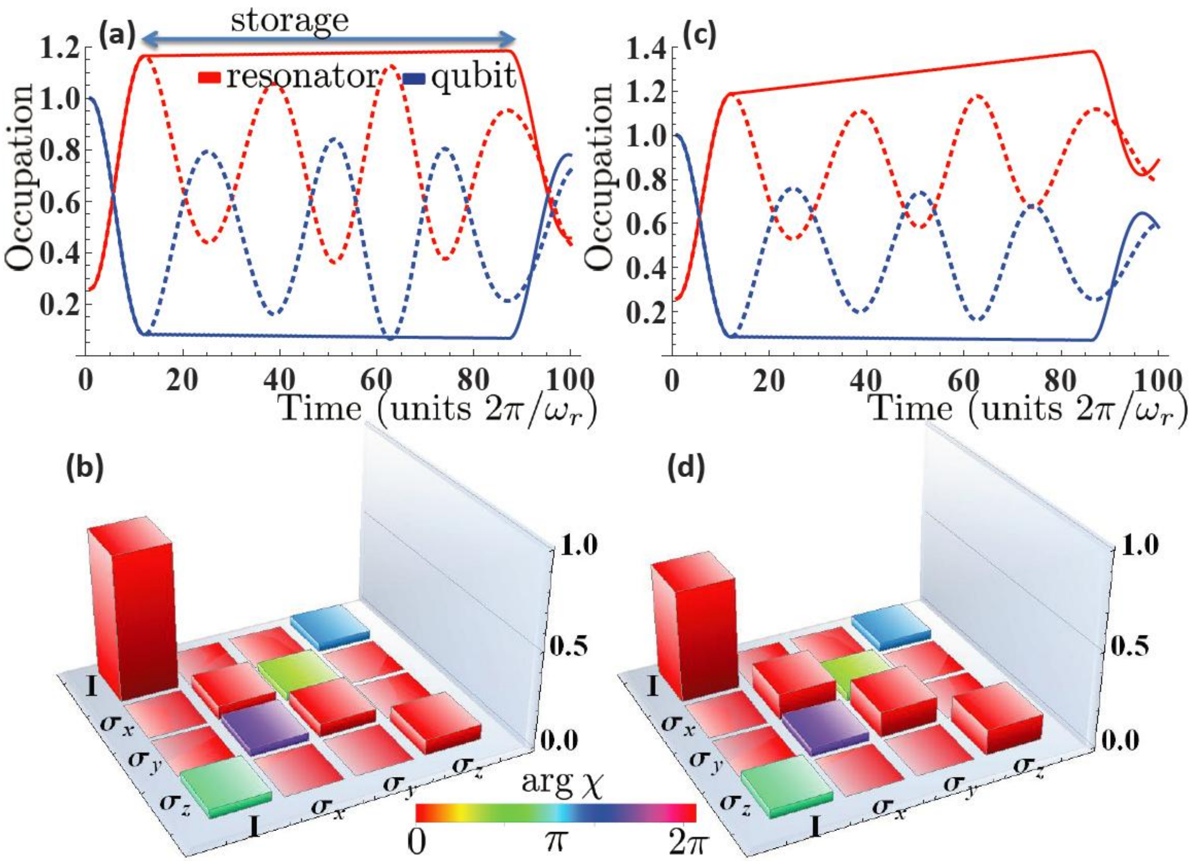
Dissipative dynamics and transfer of quantum information

Effective low energy description: $\mathcal{H} = \hbar\omega_{\rm r} a^\dagger a + iE^{\rm n}(\theta)\gamma_2\gamma_3 + iE^{\rm t}\gamma_3\gamma_4$

Can reduce to Rabi Hamiltonian! $\mathcal{H} = \hbar\omega_{\rm r} a^\dagger a + \left[\frac{E^{\rm n}(\theta_0)}{4} + \frac{\partial E^{\rm n}}{\partial\theta}\frac{\theta_{\rm zpf}}{4}(a^\dagger + a)\right]\sigma_x + \frac{E^{\rm t}}{4}\sigma_z$

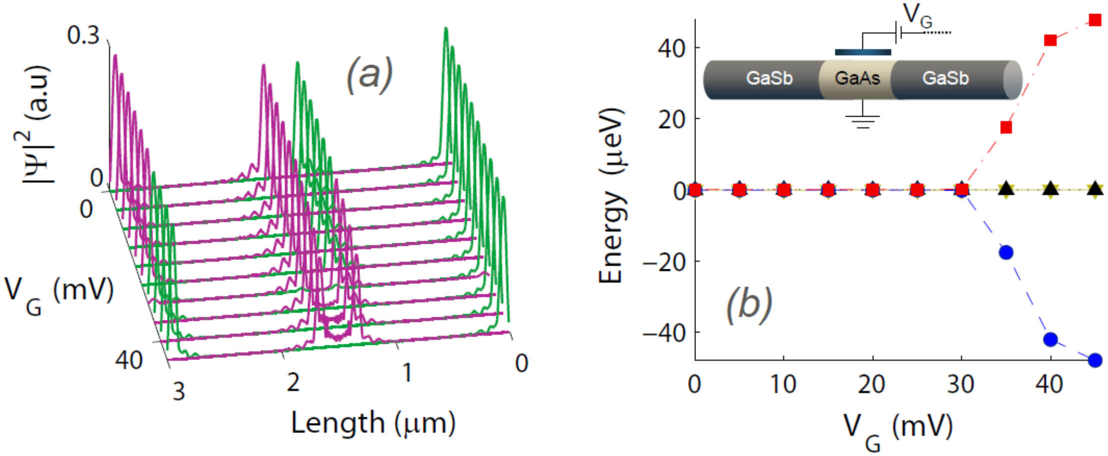
Rabi frequency: $\mathfrak{g} = \frac{1}{8}\theta_{\rm zpf} E_0^{\rm n} e^{-\ell_{\rm n}\mathfrak{Re}(\kappa_2^{\rm n})}$

Dynamics simulated with Lindblad master equation: $\dot{\rho}(t) = -\frac{i}{\hbar}[H(t),\rho] + \frac{1}{2}\sum_k \left[\mathcal{L}_k, \rho(t)\mathcal{L}_k^\dagger\right] + \left[\mathcal{L}_k\rho(t), \mathcal{L}_k^\dagger\right]$

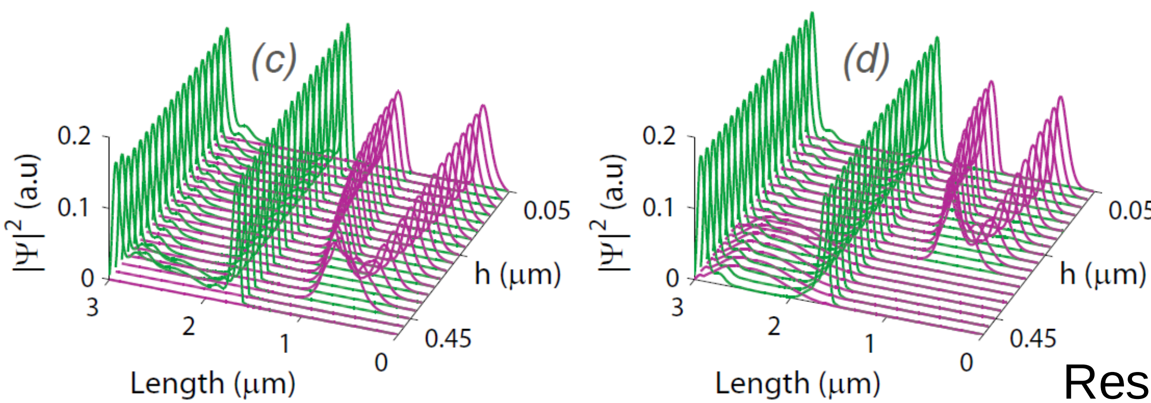


(a) Rabi oscillations of a Majorana qubit coupled to a mechanical resonator.
(b) The quantum process tomography of a process in which qubit state is transferred to the resonator, then stored in the resonator while the systems are detuned, and finally transferred back to the qubit.
(c) and **(d)** same as (a) and (b) but for a resonator with smaller quality factor.

ANALOGY TO SPIN TRANSISTOR



Gate voltage can control the spin supercurrent and hybridization in spin-transistor type architecture.

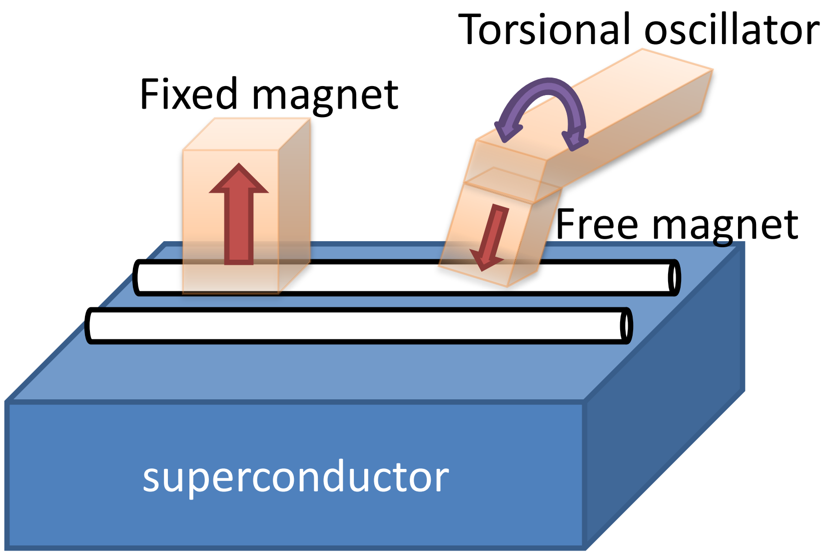


Left: for a field of a dipole; Right: field of thin disk

Results are obtained by numerical diagonalization of Hamiltonian:

$$H = \sum_{i,\sigma,\sigma'} \left[c^\dagger_{i+1\sigma}(-t_0\sigma_0 + i\frac{\alpha_i}{2}\sigma_z)_{\sigma\sigma'} c_{i\sigma'} + H.c. \right] + \sum_{i,\sigma}(2t_0 - \mu_i)c^\dagger_{i\sigma}c_{i\sigma} + \sum_i (\widetilde{\Delta}_i c^\dagger_{i\uparrow}c^\dagger_{i\downarrow} + H.c.) + \sum_i (\widetilde{B}_i c^\dagger_{i\uparrow}c_{i\downarrow} + H.c.)$$

MIGHT BE USEFUL FOR QUANTUM COMPUTING AND SENSORS



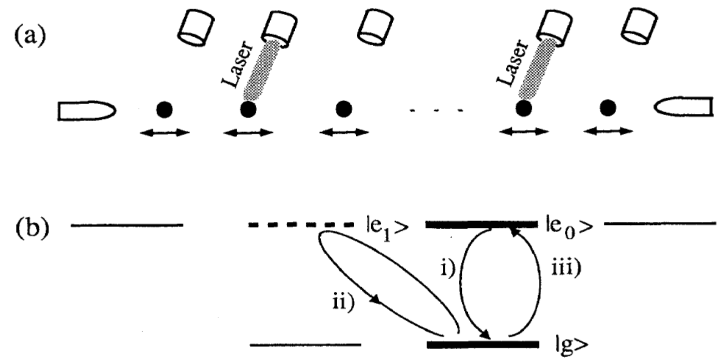
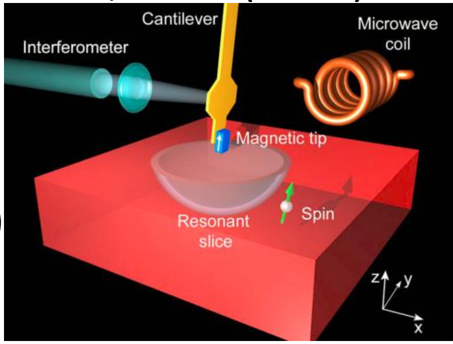
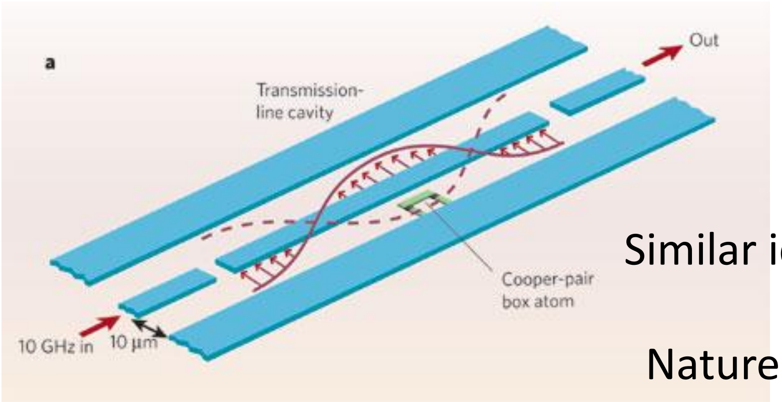


FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

J. I. Cirac and P. Zoller, PRL **74**, 4091 (1995).

D. Rugar, R.Budakian, H. J. Mamin and B. W. Chui, Nature 430, 329-332 (2004)





Similar ideas with transmission line due to coupling via Rashba

Nature 451, 664-669(2008)

Conclusions

- ***Conservation of angular momentum*** in nanoscale devices with spin supercurrents can lead to mechanical torques.
- I suggested coupling between ***majorana modes*** and mechanical modes in the quantum regime, i.e. ***quantum spin torques***.
- Results can be potentially useful for detection of majorana modes by ***magnetic resonance force microscopy***.
- Described phenomena can be potentially useful for ***quantum computing*** as well as observing fundamental quantum mechanical effects in macroscopic objects.