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Interfacial spin-orbit coupling and chirality

Hyun-Woo Lee (POSTECH, KOREA)





COLLABORATORS







Kyoung-Whan Kim Postech, Korea Kyung-Jin Lee Korea Univ., Korea Mark Stiles NIST



Contents

- Short review of recent experimental data
 - Spin Hall effect
 - Interfacial spin-orbit coupling



- − Dzyaloshinski-Moriya interaction → Chiral magnetic structure
- Chirality in equilibrium
 - Interfacial spin-orbit coupling vs. DM interaction
- Chirality in nonequilibrium
 - Spin torque
 - Spin-dependent electromagnetic field (or spin motive force)
- Questions





SHORT REVIEW OF RECENT EXPERIMENTS

Magnetic bilayer – Domain wall motion

NM/FM = Pt/Co, Pt/CoFeB, Ta/CoFeB, W/CoFeB, ...



Miron et al., Nature Materials 10, 419 (2011)



Magnetic bilayer – Domain wall motion

NM/FM = Pt/Co, Pt/CoFeB, Ta/CoFeB, W/CoFeB, ...



Miron et al., Nature Materials 10, 419 (2011)



Magnetic bilayer – Magnetization switching



Interface spin-orbit coupling?

Bulk spin Hall effect?

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Magnetic bilayer – Domain wall motion

• Chirality?

Emori et al., Nature Materials 12, 611 (2013)



Ryu et al., Nature Nanotechnology 8, 527 (2013)



Dzyaloshinskii-Moriya interaction

 $H_{\rm DM} = \sum_{ij} \mathbf{D}_{ij} \cdot \left(\mathbf{S}_i \times \mathbf{S}_j \right)$

Dzyaloshinskii, Sov. Phys. JETP 5, 1259 (1957) Moriya, Phys. Rev. 120, 91 (1960)

 $H_{\rm DM}^{\rm 1D} = \int dx \Big[D\hat{\mathbf{y}} \cdot \left(\hat{\mathbf{m}} \times \partial_x \hat{\mathbf{m}} \right) \Big], \quad H_{\rm DM}^{\rm 2D} = \int dx dy \Big[D\hat{\mathbf{y}} \cdot \left(\hat{\mathbf{m}} \times \partial_x \hat{\mathbf{m}} \right) - D\hat{\mathbf{x}} \cdot \left(\hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}} \right) \Big]$

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Magnetic bilayers





CHIRALITY IN EQUILIBRIUM

Naïve analysis



• Interfacial spin-orbit coupling & no exchange coupling

$$H_{\rm R} = \boldsymbol{\sigma} \cdot \left[\frac{\alpha_{\rm R}}{\hbar} \left(\mathbf{p} \times \hat{\mathbf{z}} \right) \right]$$



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Naïve analysis

 Interfacial SOC & no exchange coupling $H_{\rm R} = \boldsymbol{\sigma} \cdot \left| \frac{\alpha_{\rm R}}{\hbar} (\mathbf{p} \times \hat{\mathbf{z}}) \right|$ $k > 0 \rightarrow B_{eff} \bullet$ Initial $k < 0 \rightarrow B_{\text{eff}} \bigotimes$ Final Initial • Let's turn on exchange coupling $H_{exc} = J\sigma \cdot \mathbf{m}$ - Case I - Case II Conduction electron energy lowered for chiral m POSTECH

Effecitive Hamiltonian for interface





Unitary transformation

- Unitary transformation $U = \exp\left[-ik_{\rm R}\boldsymbol{\sigma} \cdot \left(\mathbf{r} \times \hat{\mathbf{z}}\right)/2\right], \quad k_{\rm R} = \frac{2\alpha_{\rm R}m_e}{\hbar^2}$
 - Spin rotation by angle $k_{\rm R}r$ around ${\bf r} \times {\bf z}$ direction





Unitary transformation

- Unitary transformation $U = \exp\left[-ik_{\rm R}\boldsymbol{\sigma} \cdot \left(\mathbf{r} \times \hat{\mathbf{z}}\right)/2\right], \quad k_{\rm R} = \frac{2\alpha_{\rm R}m_e}{\hbar^2}$
 - Spin rotation by angle $k_{\rm R}r$ around ${\bf r} \times {\bf z}$ direction
- Transformed Hamiltonian U^+HU $U^{\dagger}HU = \frac{\mathbf{p}^{2}}{2m_{e}} + J\mathbf{\sigma}\cdot\hat{\mathbf{m}}' + (\text{higher order terms})$ $\hat{\mathbf{m}}' = R^{-1}\hat{\mathbf{m}} \qquad J\mathbf{\sigma}\cdot\hat{\mathbf{m}}' \qquad J\mathbf{\sigma}\cdot\hat{\mathbf{m}}'$ -3×3 Rotation matrix $R(\mathbf{r})$ after before • Same rotation as U when acting on classical vector \mathbf{m} POSTPEH

Equilibrium energy

- Ground state energy of filled Fermi sea vs. ${f m}$

 $-k_{\rm R}=0$

• Energy density $\mathcal{E} = A \left(\partial_x \hat{\mathbf{m}} \cdot \partial_x \hat{\mathbf{m}} + \partial_y \hat{\mathbf{m}} \cdot \partial_y \hat{\mathbf{m}} \right)$

 $-k_{\rm R}\neq 0$

- Unitary transformation $H(k_{\rm R}, \{\hat{\mathbf{m}}\}) \Rightarrow H(k_{\rm R} = 0, \{\hat{\mathbf{m}}'\})$
- Energy density $\mathcal{E} = A \left(\partial_x \hat{\mathbf{m}}' \cdot \partial_x \hat{\mathbf{m}}' + \partial_y \hat{\mathbf{m}}' \cdot \partial_y \hat{\mathbf{m}}' \right)$

$$\partial_{x}\hat{\mathbf{m}}' = \partial_{x}\left(R^{-1}\hat{\mathbf{m}}\right) = R^{-1}\left(\partial_{x}\hat{\mathbf{m}} + k_{\mathrm{R}}\hat{\mathbf{y}}\times\hat{\mathbf{m}}\right) = R^{-1}\tilde{\partial}_{x}\hat{\mathbf{m}}$$
$$\partial_{y}\hat{\mathbf{m}}' = \partial_{y}\left(R^{-1}\hat{\mathbf{m}}\right) = R^{-1}\left(\partial_{y}\hat{\mathbf{m}} - k_{\mathrm{R}}\hat{\mathbf{x}}\times\hat{\mathbf{m}}\right) = R^{-1}\tilde{\partial}_{y}\hat{\mathbf{m}}$$



Equilibrium energy

- Ground state energy of filled Fermi sea vs. ${\bf m}$
 - $k_{\rm R} = 0$ • Energy density $\mathcal{E} = A \left(\partial_x \hat{\mathbf{m}} \cdot \partial_x \hat{\mathbf{m}} + \partial_y \hat{\mathbf{m}} \cdot \partial_y \hat{\mathbf{m}} \right)$ $\partial_u \hat{\mathbf{m}} \Rightarrow \tilde{\partial}_u \hat{\mathbf{m}} = \partial_u \hat{\mathbf{m}} + k_{\rm R} \left(\hat{\mathbf{z}} \times \hat{\mathbf{u}} \right) \times \hat{\mathbf{m}}$ • Unitary transformation $H(\alpha_{\rm R} \langle \hat{\mathbf{m}} \rangle) \Rightarrow H(\alpha_{\rm R} = 0 \langle \hat{\mathbf{m}}' \rangle)$
 - Unitary transformation $H(\alpha_{R}, \{\hat{\mathbf{m}}\}) \Rightarrow H(\alpha_{R} = 0, \{\hat{\mathbf{m}}'\})$ • Energy density $\mathcal{E} = A(\tilde{\partial}_{x}\hat{\mathbf{m}} \cdot \tilde{\partial}_{x}\hat{\mathbf{m}} + \tilde{\partial}_{y}\hat{\mathbf{m}} \cdot \tilde{\partial}_{y}\hat{\mathbf{m}})$

$$\partial_{x}\hat{\mathbf{m}}' = \partial_{x}\left(R^{-1}\hat{\mathbf{m}}\right) = R^{-1}\left(\partial_{x}\hat{\mathbf{m}} + k_{\mathrm{R}}\hat{\mathbf{y}}\times\hat{\mathbf{m}}\right) = R^{-1}\tilde{\partial}_{x}\hat{\mathbf{m}}$$
$$\partial_{y}\hat{\mathbf{m}}' = \partial_{y}\left(R^{-1}\hat{\mathbf{m}}\right) = R^{-1}\left(\partial_{y}\hat{\mathbf{m}} - k_{\mathrm{R}}\hat{\mathbf{x}}\times\hat{\mathbf{m}}\right) = R^{-1}\tilde{\partial}_{y}\hat{\mathbf{m}}$$



Equilibrium energy

- Ground state energy of filled Fermi sea vs. m
 - $-k_{\rm R}\neq 0$
 - Energy density

$$\varepsilon = A \Big(\partial_x \hat{\mathbf{m}} \cdot \partial_x \hat{\mathbf{m}} + \partial_y \hat{\mathbf{m}} \cdot \partial_y \hat{\mathbf{m}} \Big) + D \Big[\hat{\mathbf{y}} \cdot \Big(\hat{\mathbf{m}} \times \partial_x \hat{\mathbf{m}} \Big) - \hat{\mathbf{x}} \cdot \Big(\hat{\mathbf{m}} \times \partial_x \hat{\mathbf{m}} \Big) \Big]$$

Dzyaloshinskii-Moriya interaction

$$D = 2k_{\rm R}A = 2\frac{2\alpha_{\rm R}m_e}{\hbar^2}A$$

(*) Imamura, Bruno, and Utsumi, PRB 69, 121303 (2004)

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More realistic case ?

- Nonquadratic dispersion?
- Tight-binding version of *H*

$$H_{\rm kin} = -\frac{\hbar^2}{2m_e a^2} \sum_{ln\sigma} \left[C^{\dagger}_{l+1,n,\sigma} C_{l,n,\sigma} + C^{\dagger}_{l,n+1,\sigma} C_{l,n,\sigma} + {\rm h.c.} \right],$$

$$H_{\rm exc} = J \sum_{ln\sigma\sigma'} \left[C^{\dagger}_{l,n,\sigma}(\sigma)_{\sigma\sigma'} C_{l,n,\sigma'} \right] \cdot \hat{m}_{l,n},$$

$$H_{\rm R} = \frac{\alpha_{\rm R}}{2a} \sum_{ln\sigma\sigma'} \left[i C^{\dagger}_{l,n+1,\sigma}(\sigma_x)_{\sigma\sigma'} C_{l,n,\sigma} - i C^{\dagger}_{l+1,n,\sigma}(\sigma_y)_{\sigma\sigma'} C_{l,n,\sigma} + {\rm h.c.} \right]$$

- Nonquadratic dispersion

$$D = 2k_{\rm R}A = 2\frac{2\alpha_{\rm R}m_e}{\hbar^2}A$$





CHIRALITY IN NONEQUILIBRIUM

Effecitive Hamiltonian for interface





Unitary transformation

- Unitary transformation $U = \exp\left[-ik_{\rm R}\boldsymbol{\sigma} \cdot \left(\mathbf{r} \times \hat{\mathbf{z}}\right)/2\right], \quad k_{\rm R} = \frac{2\alpha_{\rm R}m_e}{\hbar^2}$
 - Spin rotation by angle $k_{\rm R}r$ around ${\bf r} \times {\bf z}$ direction
- Transformed Hamiltonian U^+HU $U^{\dagger}HU = \frac{\mathbf{p}^2}{2m_e} + J\mathbf{\sigma} \cdot \hat{\mathbf{m}}' + H'_{imp} + (\text{higher order terms})$ $\hat{\mathbf{m}}' = R^{-1}\hat{\mathbf{m}}$
 - -3×3 Rotation matrix $R(\mathbf{r})$
 - Same rotation as U when acting on classical vector ${f m}$



Spin torque $\partial_{\mathbf{r}} \hat{\mathbf{m}} = 0 \Longrightarrow \mathbf{T}_{adia} = \mathbf{T}_{non}$ = () $k_{\mathsf{R}}=0$ Adiabatic torque $v_{s} = \frac{Pjg\mu_{B}}{2eM_{s}}$ Nonadiabatic torque $\mathbf{T}_{non} = -\beta v_{s}\hat{\mathbf{m}} \times \partial_{x}\hat{\mathbf{m}}$ $\mathbf{T}_{adia} = v_{s} \partial_{r} \hat{\mathbf{m}}$ $\partial_x \hat{\mathbf{m}} \Rightarrow \tilde{\partial}_x \hat{\mathbf{m}} = \partial_x \hat{\mathbf{m}} + \mathbf{k}_{\mathrm{R}} \hat{\mathbf{y}} \times \hat{\mathbf{m}}$ $\tilde{\partial}_x \hat{\mathbf{m}} = 0 \Longrightarrow \mathbf{T}_{adia} + \mathbf{T}_{f} = \mathbf{T}_{non} + \mathbf{T}_{d} = 0$ $k_{\rm R} \neq 0$ $\mathbf{T}_{\text{adia}} \Rightarrow v_{\text{s}} \tilde{\partial}_{x} \hat{\mathbf{m}} \equiv \mathbf{T}_{\text{adia}} + \mathbf{T}_{\text{f}} \quad \mathbf{T}_{\text{non}} \Rightarrow -\beta v_{\text{s}} \hat{\mathbf{m}} \times \tilde{\partial}_{x} \hat{\mathbf{m}} = \mathbf{T}_{\text{non}} + \mathbf{T}_{\text{d}}$ $\mathbf{T}_{d} = -\beta k_{\mathrm{R}} v_{\mathrm{s}} \hat{\mathbf{m}} \times (\hat{\mathbf{y}} \times \hat{\mathbf{m}})$ $\mathbf{T}_{\rm f} = k_{\rm p} v_{\rm s} \hat{\mathbf{y}} \times \hat{\mathbf{m}}$ Damping-like torque (Slonczewski-like) Field-like torque

Manchon & Zhang, Obata & Tatara, Matos-Abiague & Rodriguez-Suarez, Hals, Brataas &Tserkovnyak



Wang & Manchon Kim, Seo, Ryu, Lee, **HWL** Pesin & MacDonald van der Vijl & Duine

Chirality in spin torque

$$\tilde{\partial}_{x}\hat{\mathbf{m}} = \partial_{x}\hat{\mathbf{m}} + k_{\mathrm{R}}\hat{\mathbf{y}} \times \hat{\mathbf{m}}$$

$$\tilde{\partial}_{x}\hat{\mathbf{m}} = 0$$

dx

 \rightarrow No current-induced torque if magnetization precesses at the same rate as the conduction electrons would due to RSOC



Theory vs. Experiment

Theory

DM interaction

$$D\left[\hat{\mathbf{y}}\cdot\left(\hat{\mathbf{m}}\times\partial_{x}\hat{\mathbf{m}}\right)-\hat{\mathbf{x}}\cdot\left(\hat{\mathbf{m}}\times\partial_{y}\hat{\mathbf{m}}\right)\right]$$
$$D=2k_{\mathrm{R}}A$$
$$- \Rightarrow k_{\mathrm{R}}=2.5\times10^{8}\,\mathrm{m}^{-1}$$

- Field-like torque $\mathbf{T}_{f} = -k_{R}v_{s}\hat{\mathbf{m}} \times \hat{\mathbf{y}} = -\gamma \hat{\mathbf{m}} \times \left(\frac{k_{R}v_{s}}{\gamma}\hat{\mathbf{y}}\right)$ $v_{s} = \frac{Pjg\,\mu_{B}}{2eM_{s}}$
 - 1.3 mT @ *j*=10¹¹ A/m²
- Damping-like torque $\mathbf{T}_{d} = -\beta k_{R} v_{s} \hat{\mathbf{m}} \times (\hat{\mathbf{y}} \times \hat{\mathbf{m}}) = -\gamma \hat{\mathbf{m}} \times \left(\frac{\beta k_{R} v_{s}}{\gamma} \hat{\mathbf{y}} \times \hat{\mathbf{m}}\right)$

Experiment (Emori, Pt/CoFe(0.6nm)/MgO)

- Chirality in domain wall
 - \rightarrow DM interaction
 - $D \sim 0.5 \text{ mJ/m}^2$, $A \sim 10^{-11} \text{ J/m}$
- Field-like torque
 - Effective field along y direction
 - $2 \text{ mT} @ j = 10^{11} \text{ A/m}^2$
- Damping-like torque
 - Effective field along $\mathbf{y} \times \mathbf{m}$ direction
 - $5 \text{ mT} @ j = 10^{11} \text{ A/m}^2$

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 $- \rightarrow \beta = 4$

Equilibrium vs. Nonequilibrium

Equilibrium (no current)



 $k_{\rm R} \neq 0$

- Exchange energy $J \boldsymbol{\sigma} \cdot \hat{\boldsymbol{m}} \Rightarrow A (\partial_x \hat{\boldsymbol{m}} \cdot \partial_x \hat{\boldsymbol{m}})$
 - Reactive torque $\mathbf{H}_{eff} = -\frac{1}{M_s} \frac{\delta E[\hat{\mathbf{m}}]}{\delta \hat{\mathbf{m}}} \Rightarrow -\gamma \hat{\mathbf{m}} \times \mathbf{H}_{eff}$ - Dissipative torque (Landau-Lifshitz) $-\alpha \hat{\mathbf{m}} \times (\hat{\mathbf{m}} \times \gamma \mathbf{H}_{eff})$
- DM interaction $D\left[\hat{\mathbf{y}} \cdot (\hat{\mathbf{m}} \times \partial_x \hat{\mathbf{m}})\right]$
 - Reactive torque

$$\mathbf{H}_{\rm eff}^{\rm DM} = -\frac{1}{M_{\rm s}} \frac{\delta E^{\rm DM} \left[\hat{\mathbf{m}} \right]}{\delta \hat{\mathbf{m}}} \Longrightarrow -\gamma \hat{\mathbf{m}} \times \mathbf{H}_{\rm eff}^{\rm DM}$$

Dissipative torque

$$-\alpha \hat{\mathbf{m}} \times (\hat{\mathbf{m}} \times \gamma \mathbf{H}_{eff}^{DM})$$

Xiao, Zangwill, Stiles

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Nonequilibrium (finite current)

- $\hat{\mathbf{m}} \cdot \partial_x \hat{\mathbf{m}}$ Correction to equilibrium torques
 - Correction to reactive torque
 - \rightarrow Adiabatic torque $\mathbf{T}_{adia} = v_s \partial_x \hat{\mathbf{m}}$
 - Correction to dissipative torque
 - \rightarrow Nonadiabatic torque $\mathbf{T}_{non} = -\beta v_s \hat{\mathbf{m}} \times \partial_x \hat{\mathbf{m}}$
 - Correction to equilibrium torques
 - Correction to reactive torque
 - \rightarrow Field-like torque $\mathbf{T}_{f} = -k_{R}v_{s}\hat{\mathbf{m}} \times \hat{\mathbf{y}}$
 - Correction to dissipative torque
 - → Damping-like torque

 $\mathbf{T}_{\mathrm{d}} = -\beta k_{\mathrm{R}} v_{\mathrm{s}} \hat{\mathbf{m}} \times \left(\hat{\mathbf{y}} \times \hat{\mathbf{m}} \right)$

Garate, Gilmore, Stiles, MacDonald

Spin-dependent electric field

Adiabatic electric field

$$\mathbf{E}_{\text{adia}}^{\pm} = \pm \frac{\hbar}{2e} \sum_{i=x,y} \hat{\mathbf{e}}_i \left(\partial_i \hat{\mathbf{m}} \times \hat{\mathbf{m}} \cdot \partial_i \hat{\mathbf{m}} \right)$$

Berger, PRB 1986 Volovik, JPC 1987 Barnes & Maekawa, PRL 2007 Nonadiabatic electric field

$$\mathbf{E}_{\text{non}}^{\pm} = \pm \beta \frac{\hbar}{2e} \sum_{i=x,y} \hat{\mathbf{e}}_i \left(\partial_i \hat{\mathbf{m}} \times \partial_t \hat{\mathbf{m}} \right)$$

Duine, PRB 2008, 2009 Tserkovnyak & Mecklenburg, PRB 2008

$$\tilde{\mathbf{E}}_{adia}^{\pm} = \pm \frac{\hbar}{2e} \sum_{i=x,y} \hat{\mathbf{e}}_i \left(\tilde{\partial}_i \hat{\mathbf{m}} \times \hat{\mathbf{m}} \cdot \partial_t \hat{\mathbf{m}} \right)$$

$$\tilde{\mathbf{E}}_{adia}^{\pm} = \pm \beta \frac{\hbar}{2e} \sum_{i=x,y} \hat{\mathbf{e}}_i \left(\tilde{\partial}_i \hat{\mathbf{m}} \times \hat{\mathbf{m}} \cdot \partial_t \hat{\mathbf{m}} \right)$$

$$= \mathbf{E}_{adia}^{\pm} + \mathbf{E}_{f}^{\pm}$$

$$= \mathbf{E}_{non}^{\pm} + \mathbf{E}_{d}^{\pm}$$

"Field-like" electric field

$$\mathbf{E}_{\mathrm{f}}^{\pm} = \pm \frac{\hbar}{2e} \frac{2\alpha_{\mathrm{R}} m_{e}}{\hbar^{2}} \hat{\mathbf{z}} \times \partial_{t} \hat{\mathbf{m}}$$

Kim, Moon, Lee & HWL, PRL 2012

"Damping-like" electric field $\mathbf{E}_{d}^{\pm} = \mp \beta \frac{\hbar}{2e} \frac{2\alpha_{R}m_{e}}{\hbar^{2}} \hat{\mathbf{z}} \times (\hat{\mathbf{m}} \times \partial_{t}\hat{\mathbf{m}})$ Tatara, Nakabayashi & Lee, PRB 2013



Spin-dependent magnetic field

$$\mathbf{B}_{\pm} = \mp \hat{\mathbf{z}} \frac{\hbar}{2e} \Big(\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}} \Big) \cdot \hat{\mathbf{m}}$$

Volovik, JPC 1987



$$\tilde{\mathbf{B}}_{\pm} = \mp \hat{\mathbf{z}} \frac{\hbar}{2e} \Big(\tilde{\partial}_{x} \hat{\mathbf{m}} \times \tilde{\partial}_{y} \hat{\mathbf{m}} \Big) \cdot \hat{\mathbf{m}} \\ = \mp \hat{\mathbf{z}} \frac{\hbar}{2e} \Big(\partial_{x} \hat{\mathbf{m}} \times \partial_{y} \hat{\mathbf{m}} \Big) \cdot \hat{\mathbf{m}} \pm \frac{\hbar}{2e} k_{\mathrm{R}} \nabla \times \Big(\hat{\mathbf{z}} \times \hat{\mathbf{m}} \Big) + O\Big(k_{\mathrm{R}}^{2} \Big)$$



Chiral magnet & skyrmion

Single atomic layer of Mn on W

Vol 447 10 May 2007 doi:10.1038/nature05802

nature

LETTERS

Chiral magnetic order at surfaces driven by inversion asymmetry

M. Bode¹[†], M. Heide², K. von Bergmann¹, P. Ferriani¹, S. Heinze¹, G. Bihlmayer², A. Kubetzka¹, O. Pietzsch¹, S. Blügel² & R. Wiesendanger¹

Single atomic layer of Fe on Ir

nature physics

ARTICLES PUBLISHED ONLINE: 31 JULY 2011 | DOI:10.1038/NPHYS2045

Spontaneous atomic-scale magnetic skyrmion lattice in two dimensions

Stefan Heinze¹*[†], Kirsten von Bergmann²*[†], Matthias Menzel^{2†}, Jens Brede², André Kubetzka², Roland Wiesendanger², Gustav Bihlmayer^{3†} and Stefan Blügel³







Chiral magnet & skyrmion

$$\tilde{\mathbf{B}}_{\pm} = \mp \hat{\mathbf{z}} \frac{\hbar}{2e} \left(\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}} \right) \cdot \hat{\mathbf{m}} \pm \frac{\hbar}{2e} k_{\mathrm{R}} \nabla \times \left(\hat{\mathbf{z}} \times \hat{\mathbf{m}} \right)$$

$$0 \qquad 140 \,\mathrm{T}$$

Single atomic layer of Fe on Ir

$$\tilde{\mathbf{B}}_{\pm} = \mp \hat{\mathbf{z}} \frac{\hbar}{2e} \left(\partial_x \hat{\mathbf{m}} \times \partial_y \hat{\mathbf{m}} \right) \cdot \hat{\mathbf{m}} \pm \frac{\hbar}{2e} k_{\mathrm{R}} \nabla \times \left(\hat{\mathbf{z}} \times \hat{\mathbf{m}} \right)$$

$$10^4 \,\mathrm{T} \qquad ? \,\mathrm{T}$$





Summary

- Interfacial spin-orbit coupling
 - → Chirality in equilibrium energy
 - → Chirality in nonequilibrium properties (spin torque, spindependent electromagnetic fields)
 - → Chiral derivative
 - \rightarrow Chiral corrections can be important
 - Spin-orbit torque
 - Spin-dependent magnetic field in chiral magnet & skyrmion





