Critical transitions in anisotropic turbulence

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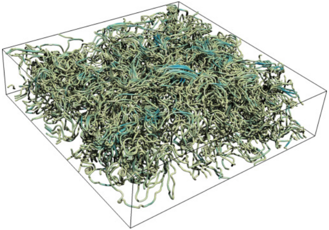
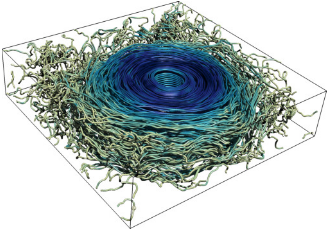




Outline

Introduction

Rotating Rayleigh-Bénard convection

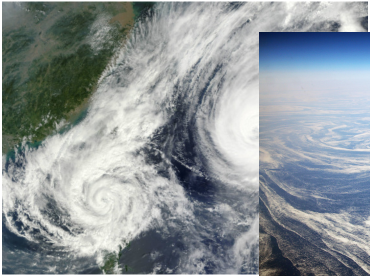
Finite amplitude perturbation and subcritical transition

Conclusions: vortices, jets, interfaces...

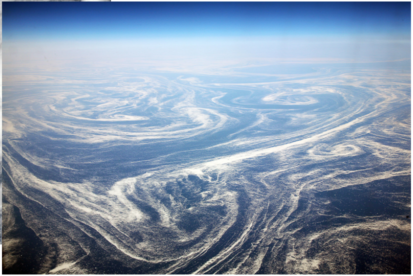
Motivations

NASA Earth observatory

Motivations

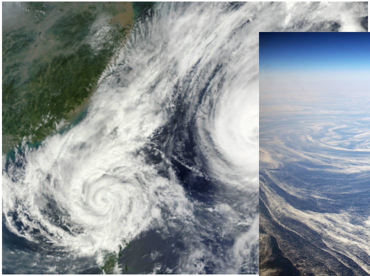


NASA Earth observatory



D. Schwen

Motivations



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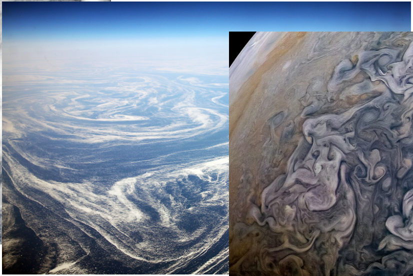
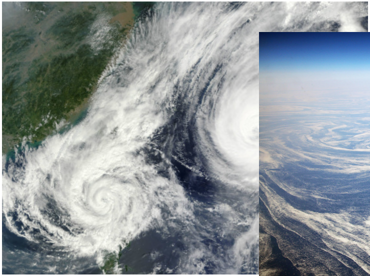


D. Schwen

NASA/JPL

Motivations



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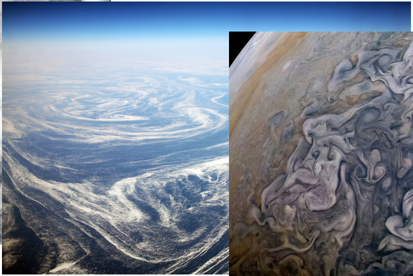

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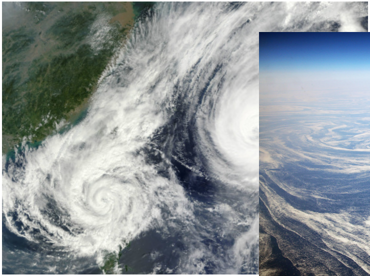
NASA/JPL

- Coexistence of large coherent flows and small-scale turbulence

Motivations



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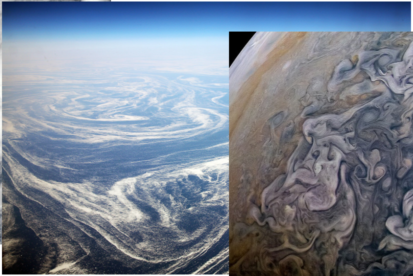

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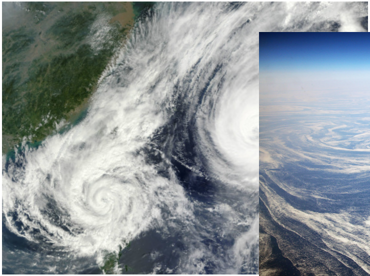
- Coexistence of large coherent flows and small-scale turbulence
- Broken scale invariance: large-scale quasi-2D and small-scale 3D?

Motivations



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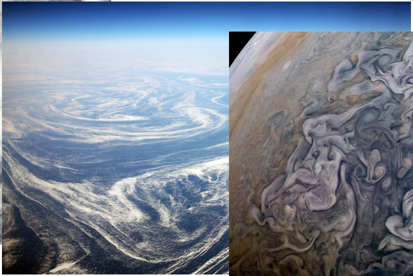

D. Schwen

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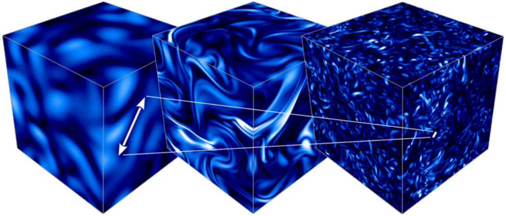
- Coexistence of large coherent flows and small-scale turbulence
- Broken scale invariance: large-scale quasi-2D and small-scale 3D?
- Nonlinear transfers and/or direct forcing?

Energy cascades: from 3D to 2D flows

3D Homogeneous Isotropic



Energy cascades: from 3D to 2D flows

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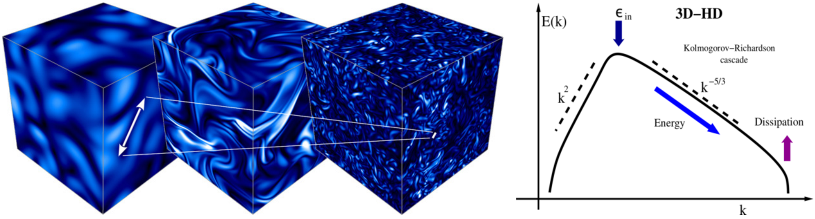

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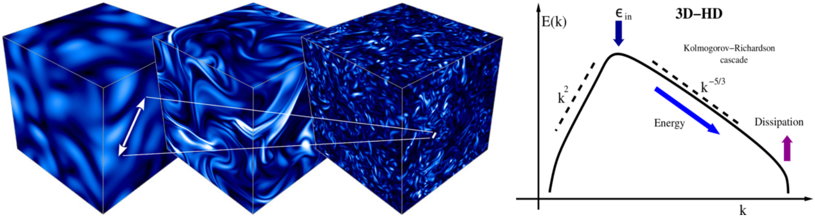
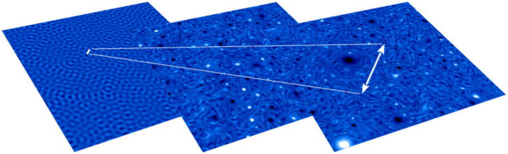


- Vortex stretching $\boldsymbol{\omega} \cdot \nabla \boldsymbol{u}$ leads to small-scale structures
- Dissipation anomaly: $\epsilon \to$ cste when $\nu \to 0$

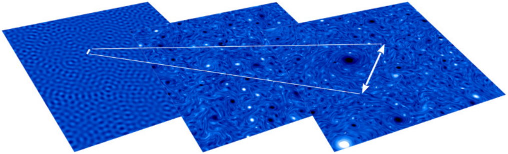
Energy cascades: from 3D to 2D flows

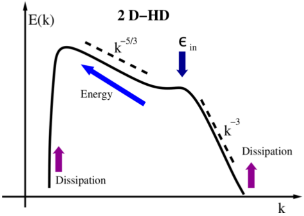
2D Homogeneous Isotropic



Energy cascades: from 3D to 2D flows

2D Homogeneous Isotropic

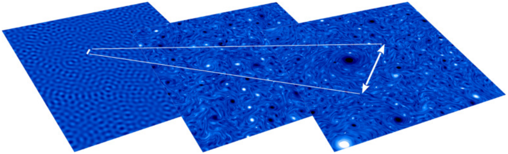




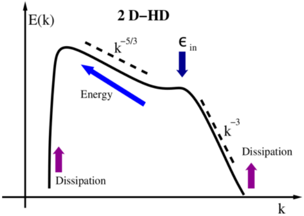
Kraichnan (1967)

Energy cascades: from 3D to 2D flows

2D Homogeneous Isotropic



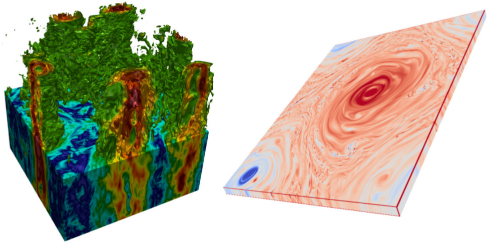
Kraichnan (1967)



- No vortex stretching $\boldsymbol{\omega} \cdot \nabla \boldsymbol{u} = 0$ leads to enstrophy conservation
- No dissipation anomaly: $\epsilon \to 0$ when $\nu \to 0$

Energy cascades: from 3D to 2D flows

3D anisotropic

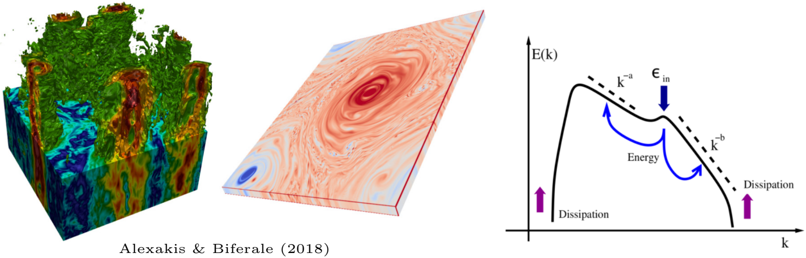


Alexakis & Biferale (2018)

- Rotating, stratified, MHD, thin-layer turbulence...

Energy cascades: from 3D to 2D flows

3D anisotropic

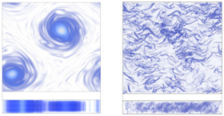


Alexakis & Biferale (2018)

- Rotating, stratified, MHD, thin-layer turbulence...
- Multiple energy cascade scenarii: both direct and inverse, sometimes simultaneously!

2D-3D mixed behaviour and split cascade: examples

Thin-layer flows



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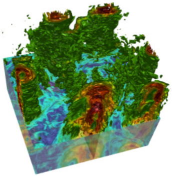
Rotating flows



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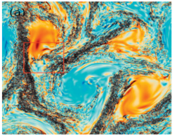
Rotating and stratified flows



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MHD flows

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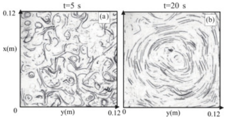
Experiments



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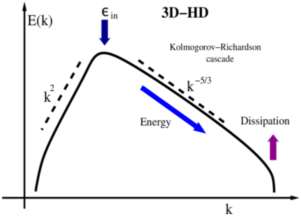
Critical transition from 3D to 2D dynamics



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Pictures from Alexakis & Biferale (2018)

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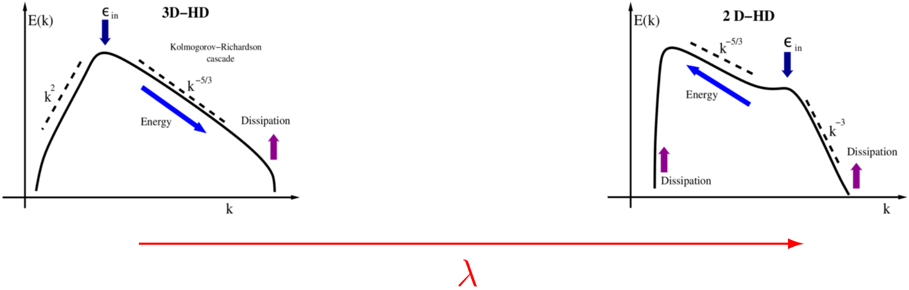

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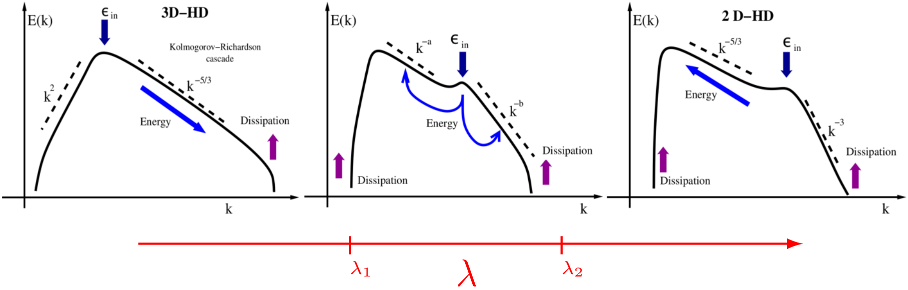

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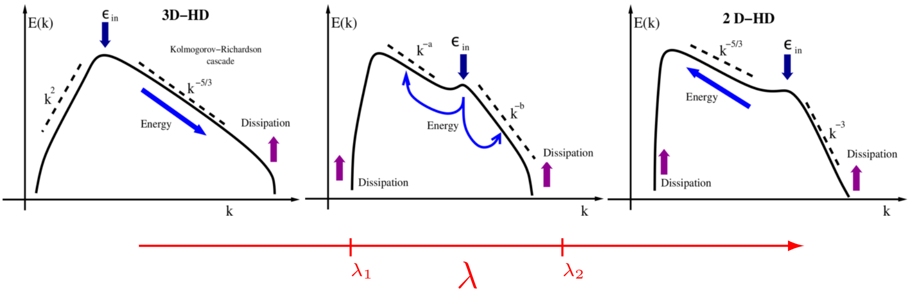

- All states are turbulent (*i.e.* $\lambda \neq Re$)

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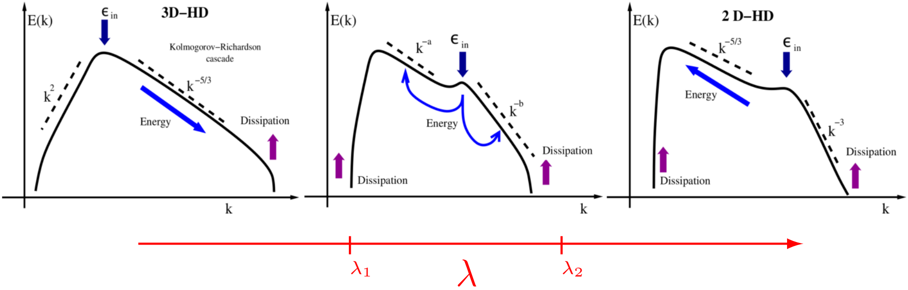

- All states are turbulent (*i.e.* $\lambda \neq Re$)
- Nature of the transition?

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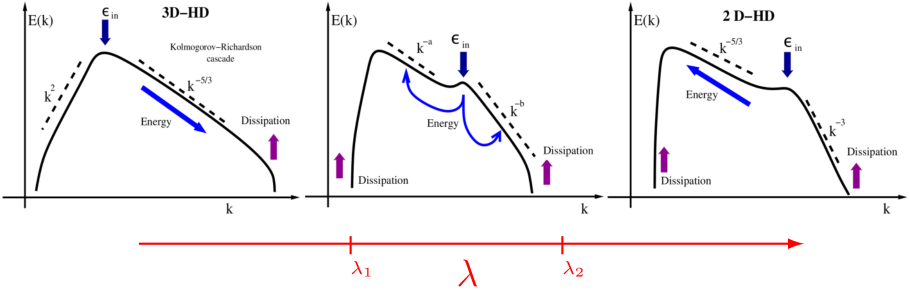


- All states are turbulent (*i.e.* $\lambda \neq Re$)
- Nature of the transition?
- Is the forcing playing any role? Lack of universality?

Pictures from Alexakis & Biferale (2018)

Outline

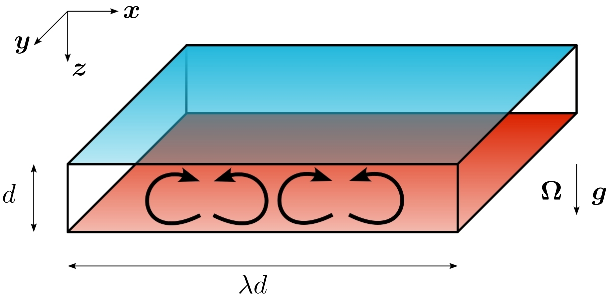
Introduction

Rotating Rayleigh-Bénard convection

Finite amplitude perturbation and subcritical transition

Conclusions: vortices, jets, interfaces...

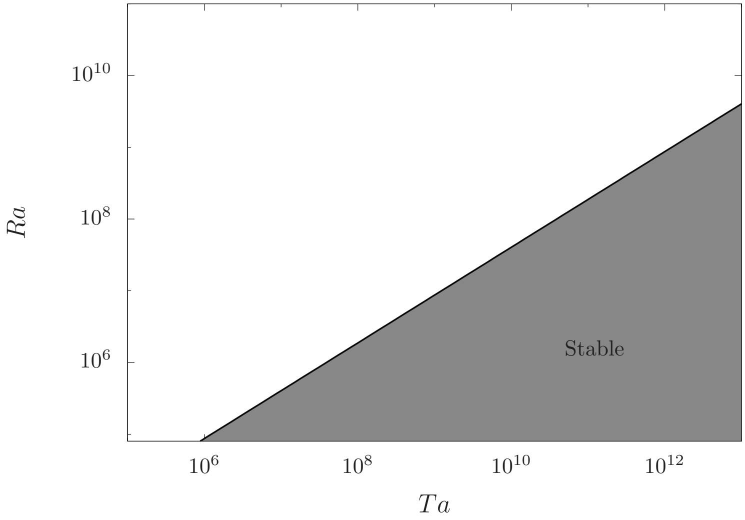
Rayleigh-Bénard Cartesian model



$$Pr = \frac{\nu}{\kappa} \qquad Ra = \frac{\alpha g \Delta T d^3}{\nu\kappa} \qquad Ta = \frac{4\Omega^2 d^2}{\nu^2}$$

- Periodic boundary conditions in the horizontal directions
- Fixed temperature T_0 at $z = 0$ and $T_0 + \Delta T$ at $z = d$
- Stress-free and impermeable $\partial_z u_x = \partial_z u_y = u_z = 0$ at $z = 0, d$

Regime diagram (rapid-rotation limit)



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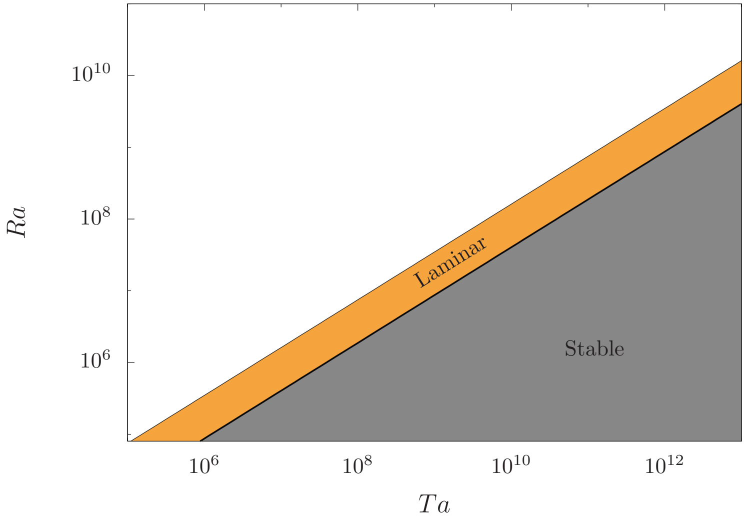

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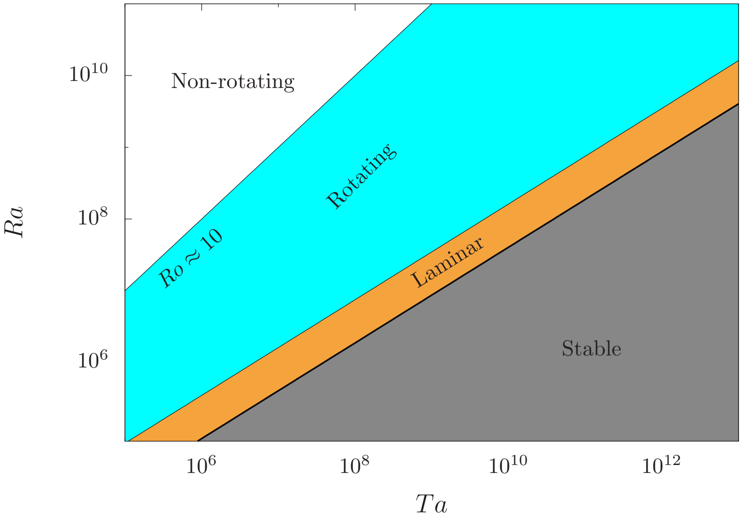

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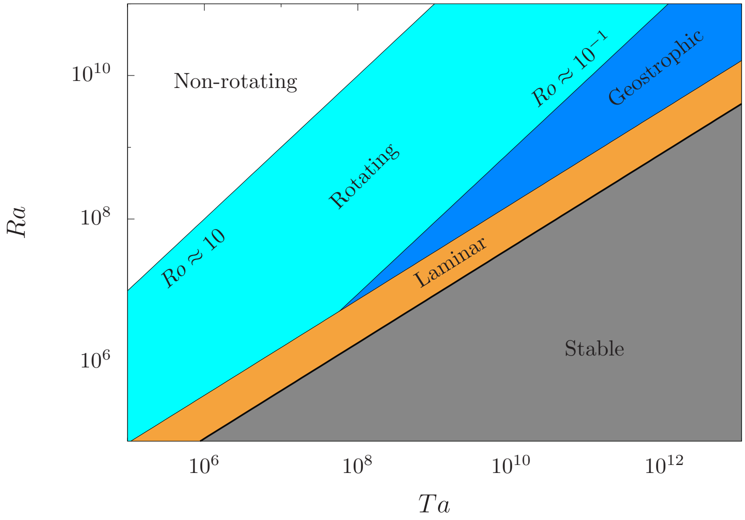

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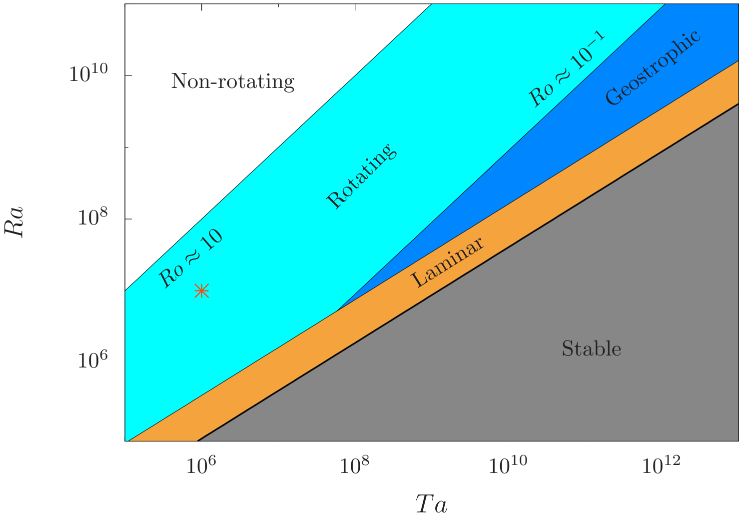
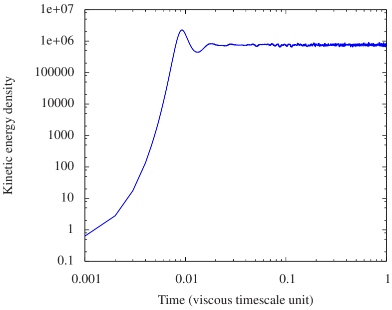
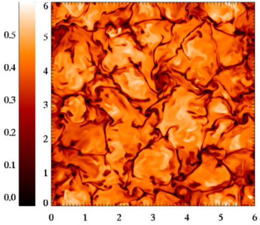


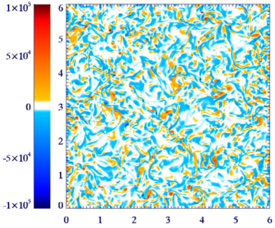
"Classical" rotating convection

Example with $Ta = 10^6$ and $Ra = 10^7$





Temperature



Vertical vorticity

Flow decomposition

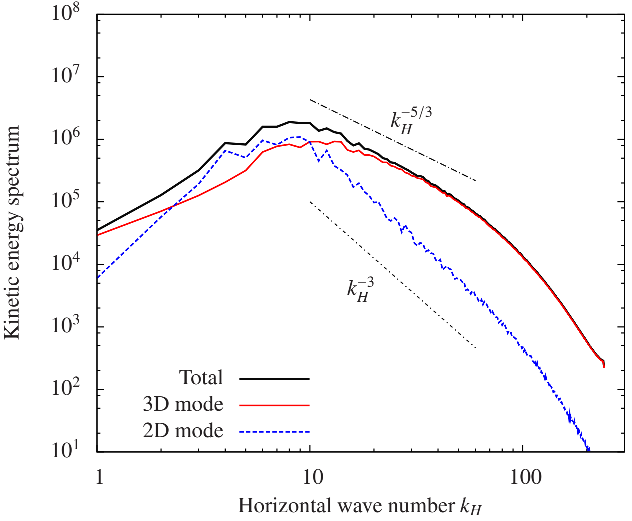
The horizontal flow can be decomposed as the slow 2D mode

$$\langle u\rangle_z(x,y,t) = \int_0^1 u(x,y,z,t)\,\mathrm{d}z$$
$$\langle v\rangle_z(x,y,t) = \int_0^1 v(x,y,z,t)\,\mathrm{d}z\ ,$$

and the fast 3D mode

$$u'(x,y,z,t) = u(x,y,z,t) - \langle u\rangle_z(x,y,t)$$
$$v'(x,y,z,t) = v(x,y,z,t) - \langle v\rangle_z(x,y,t)$$
$$w'(x,y,z,t) = w(x,y,z,t)$$

Energy spectra



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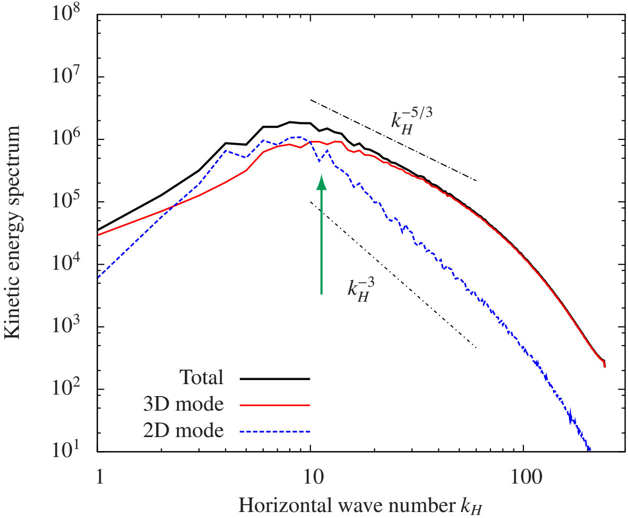
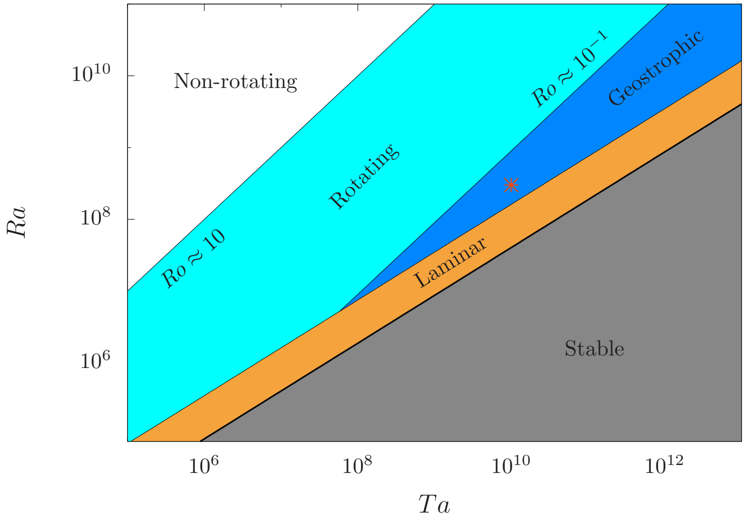
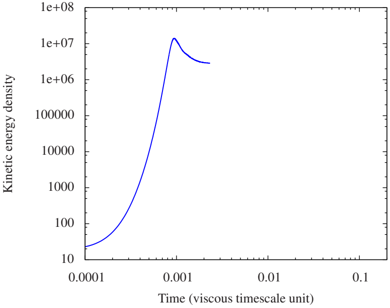


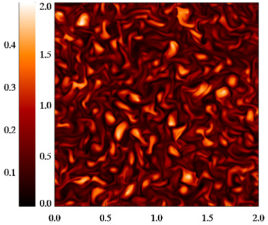
Regime diagram



Rotating convection with inverse cascade

Example with $Ta = 10^{10}$ and $Ra = 2 \times 10^8$





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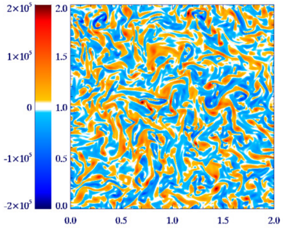

Vertical vorticity

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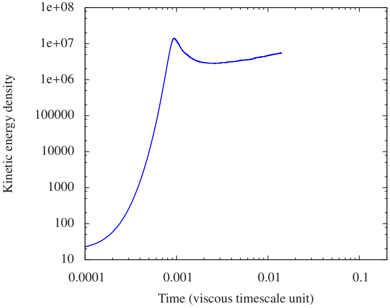
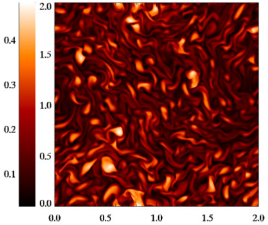
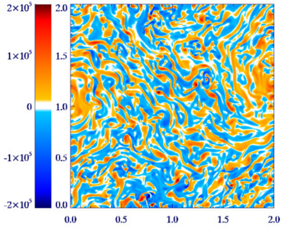




Temperature



Vertical vorticity

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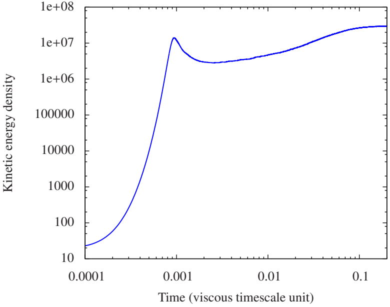

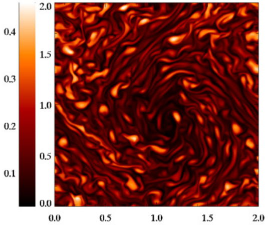

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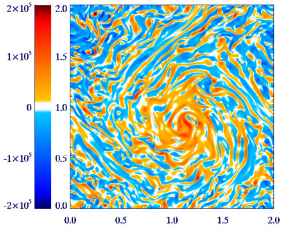
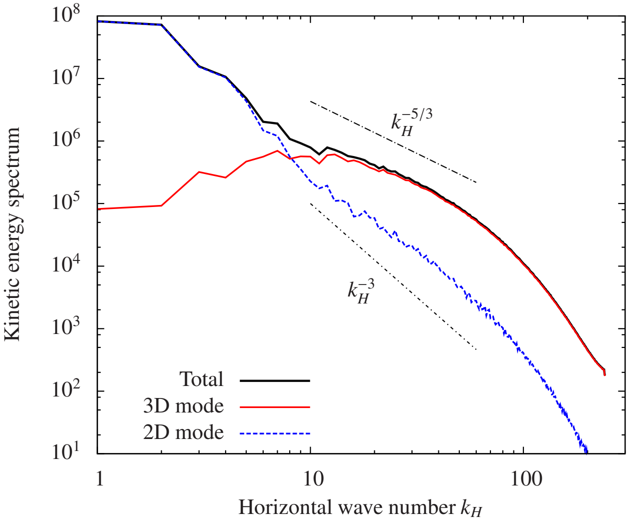


Vertical vorticity

Energy spectra



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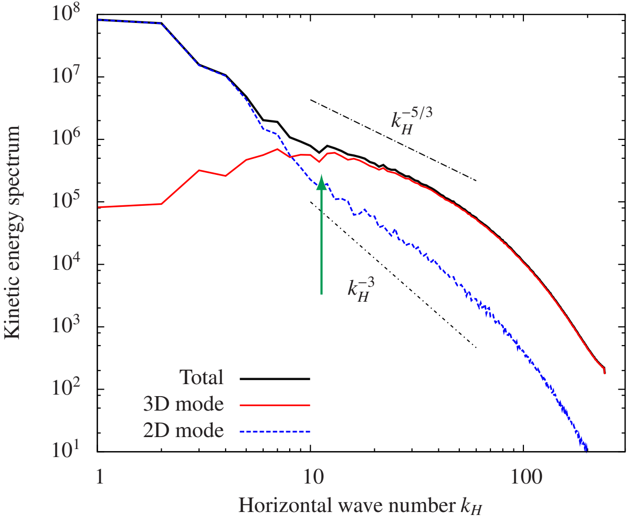
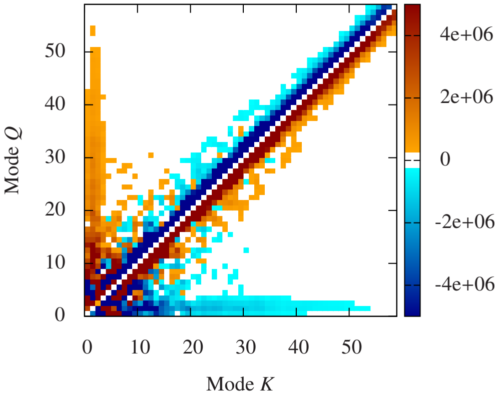
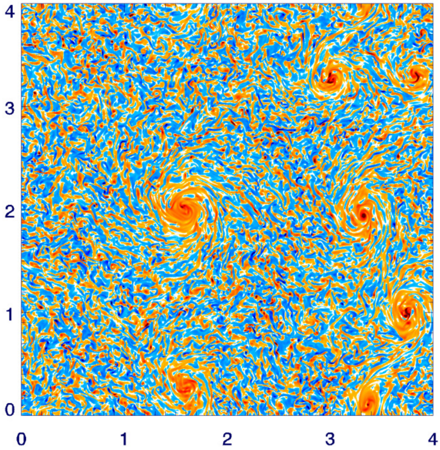


Non-local energy transfer

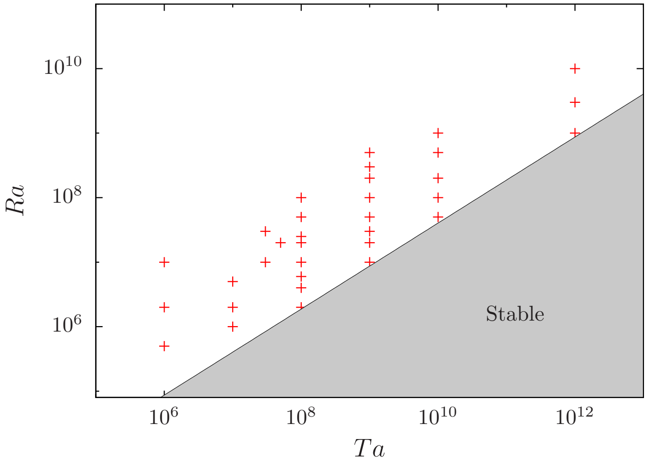


$$\mathcal{T}(Q,K) = -\int_V \boldsymbol{u}_K \cdot \left(\boldsymbol{u} \cdot \nabla \boldsymbol{u}_Q\right) \mathrm{d}V$$

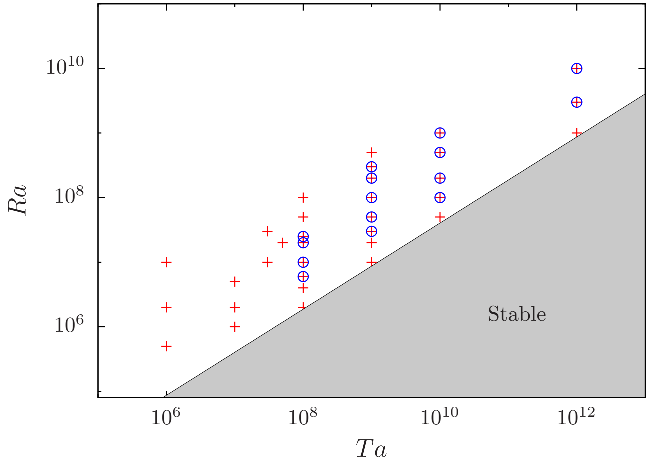
Vortex merging



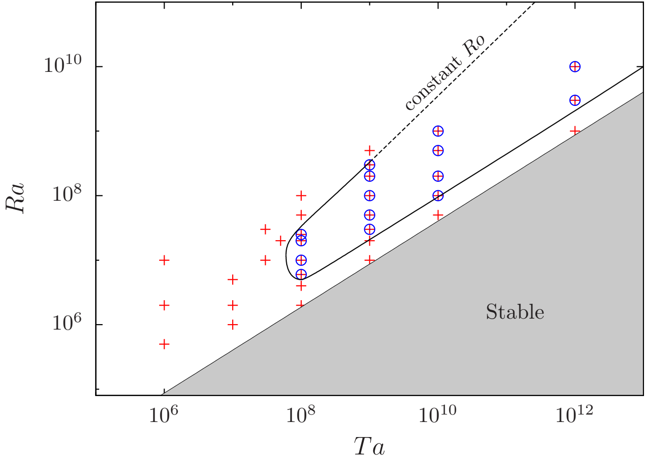
Conditions for inverse transfers



Conditions for inverse transfers



Conditions for inverse transfers



Julien et al. (2012), Rubio et al. (2014), Favier et al. (2014), Guervilly et al. (2014)

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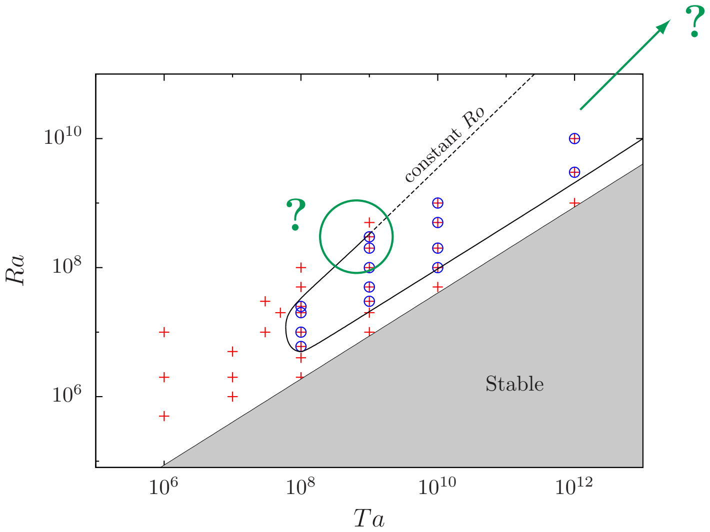


Julien et al. (2012), Rubio et al. (2014), Favier et al. (2014), Guervilly et al. (2014)

Outline

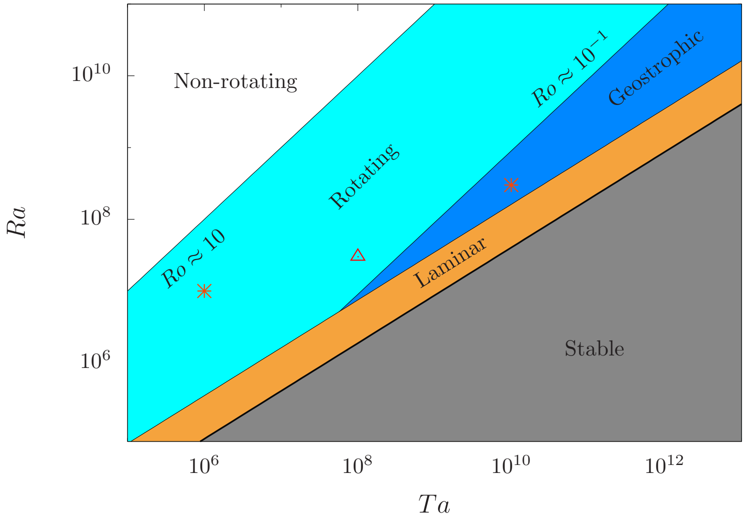
Introduction

Rotating Rayleigh-Bénard convection

Finite amplitude perturbation and subcritical transition

Conclusions: vortices, jets, interfaces...

Reference solution



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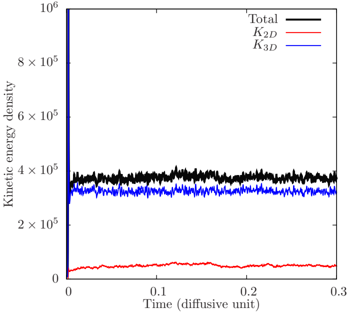
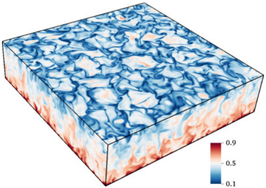
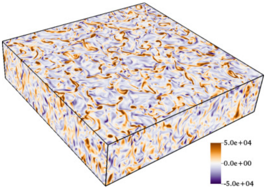


Control parameters:

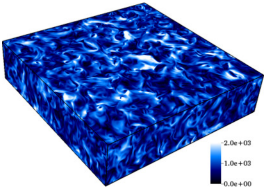
$$Ta = 10^8$$
$$Ra = 3 \times 10^7$$
$$Pr = 1$$
$$\lambda = 4$$
$$(Ro \approx 1)$$



Temperature



Vertical vorticity

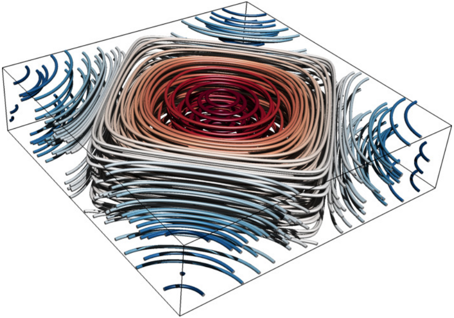


Velocity amplitude

Finite amplitude initial conditions

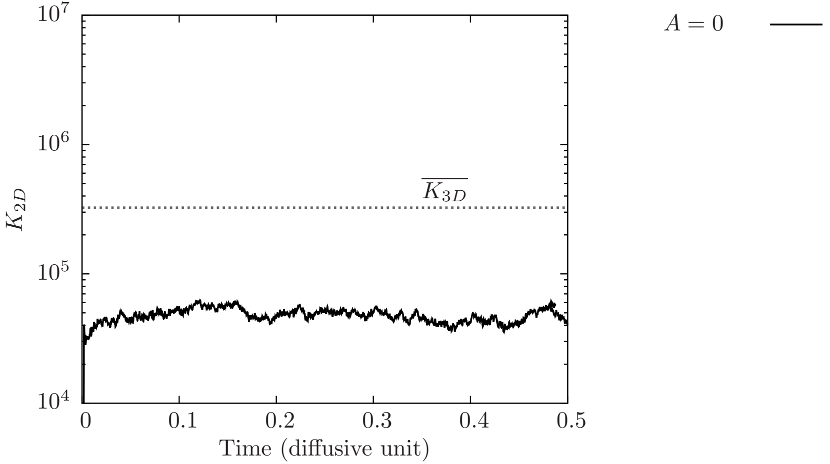
We consider the aribtrary initial conditions given by

$$\boldsymbol{u}(t=0) = \left(A\sin\left(\frac{2\pi y}{\lambda}\right), -A\sin\left(\frac{2\pi x}{\lambda}\right), 0\right) \qquad \theta(t=0)=0$$

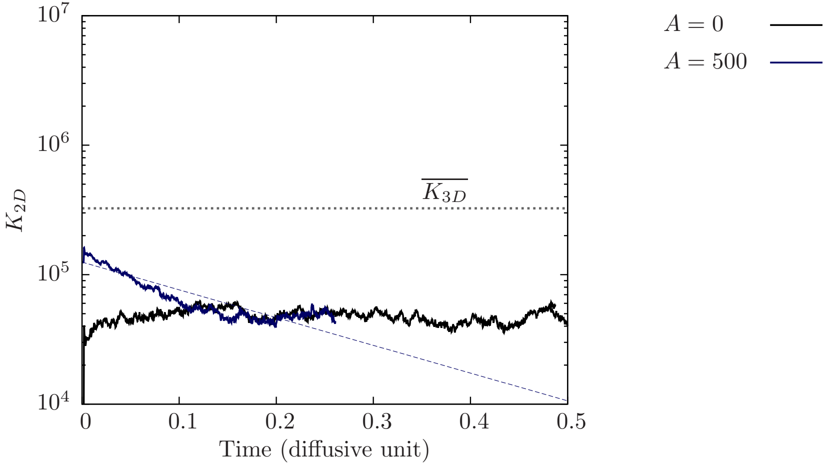


$$K_0 = \frac{1}{V}\int_V \frac{1}{2}\boldsymbol{u}^2 \mathrm{d}V = \frac{A^2}{2} \qquad K(t) = K_0 \exp(-8\pi^2 Pr\ t/\lambda^2)$$

Varying the vortex dipole amplitude A



Varying the vortex dipole amplitude A



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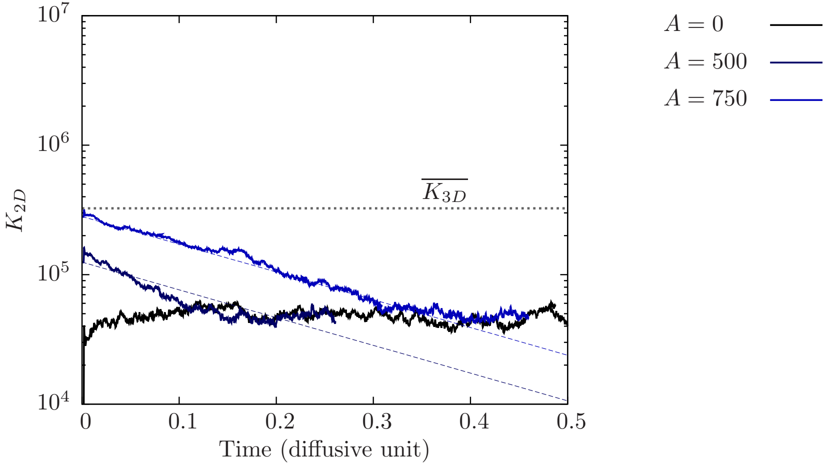

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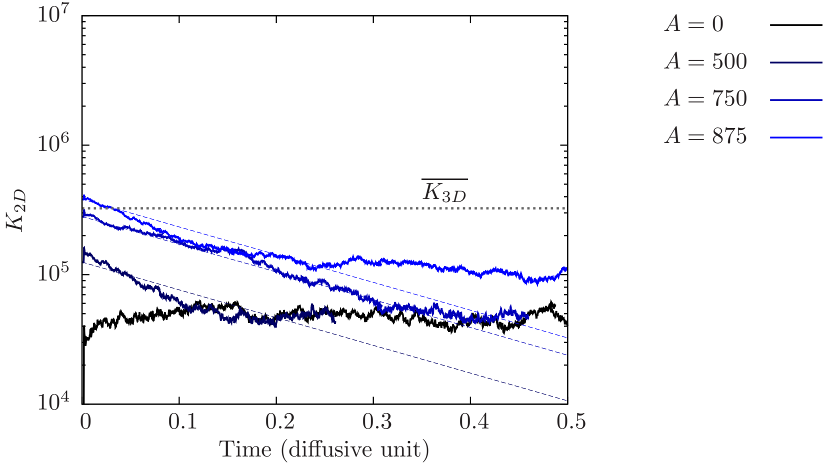

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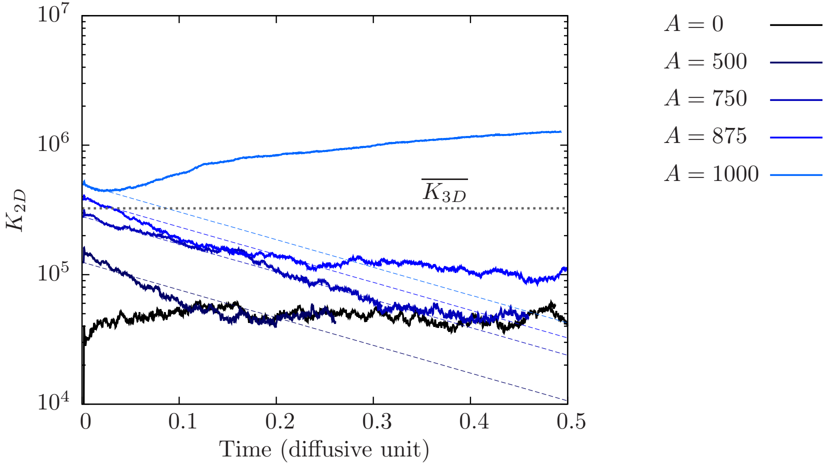

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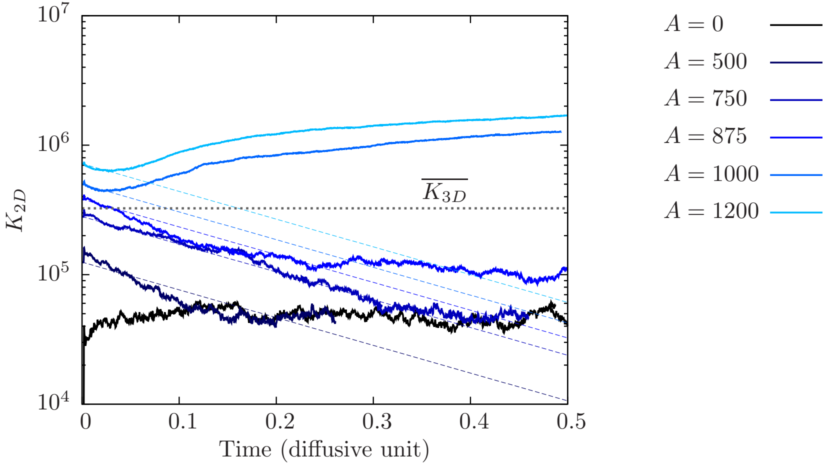

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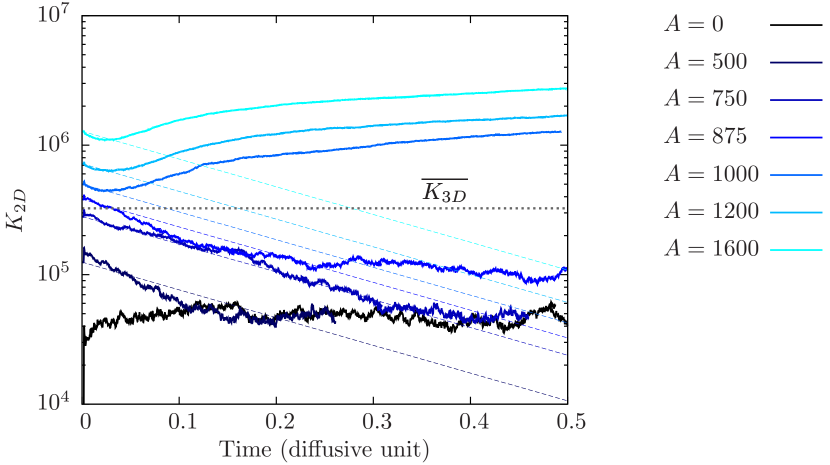

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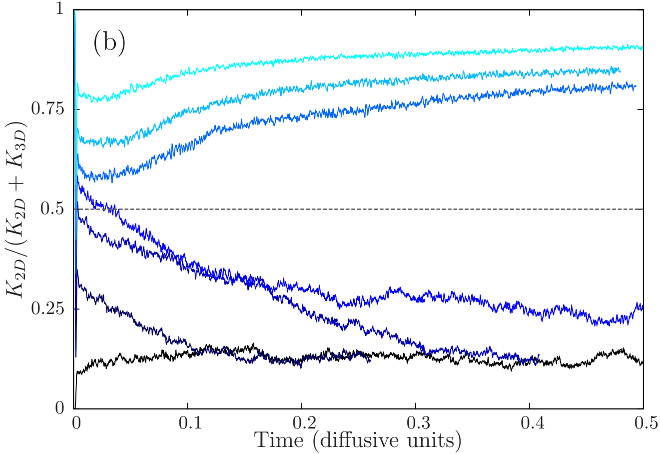
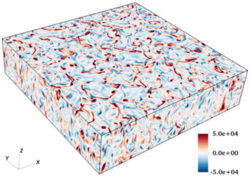


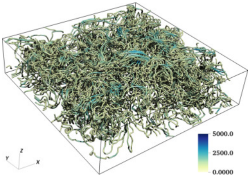
Bistability between two turbulent states!

Bi-stable states

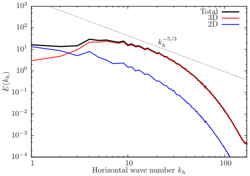
Vertical vorticity



Streamlines

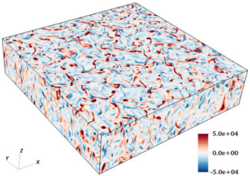


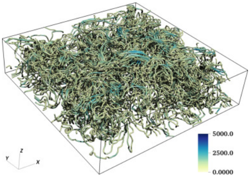
Energy spectra

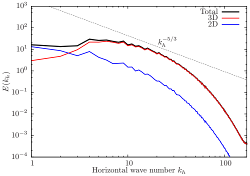


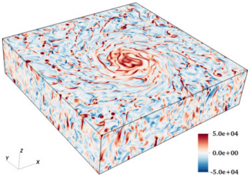
Bi-stable states

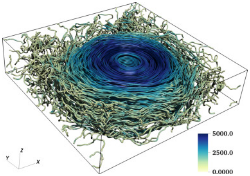
Vertical vorticity | Streamlines | Energy spectra

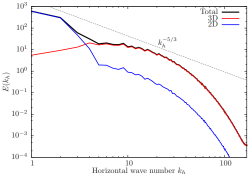












Energy balance

Vertically-averaged Navier-Stokes equations:

$$\frac{\partial \langle \boldsymbol{u} \rangle_z}{\partial t} + \langle \boldsymbol{u} \rangle_z \cdot \nabla_h \langle \boldsymbol{u} \rangle_z = -\nabla_h \langle p \rangle_z + Pr \nabla_h^2 \langle \boldsymbol{u} \rangle_z \underbrace{- \langle \nabla \cdot \boldsymbol{u}' \boldsymbol{u}' \rangle_z}_{\text{Reynolds stresses}}$$

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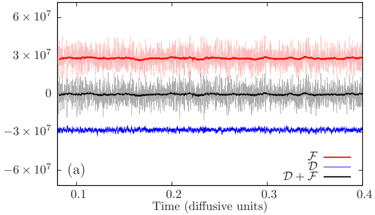

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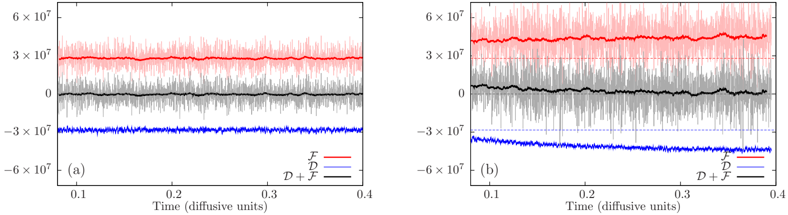
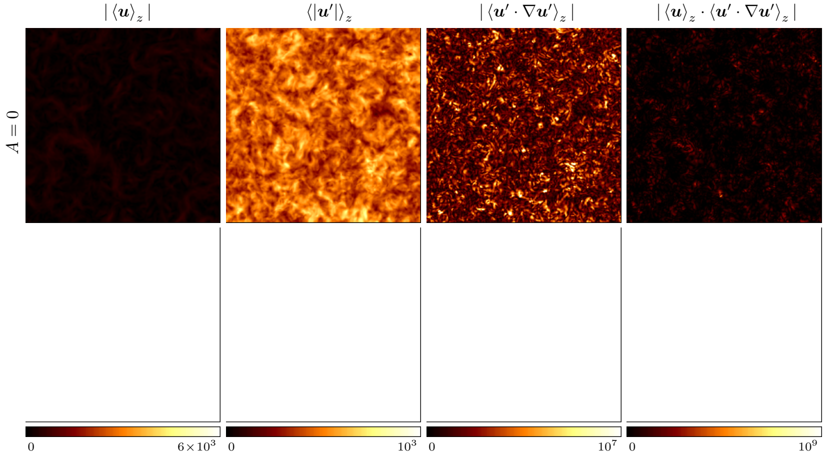


Positive feedback



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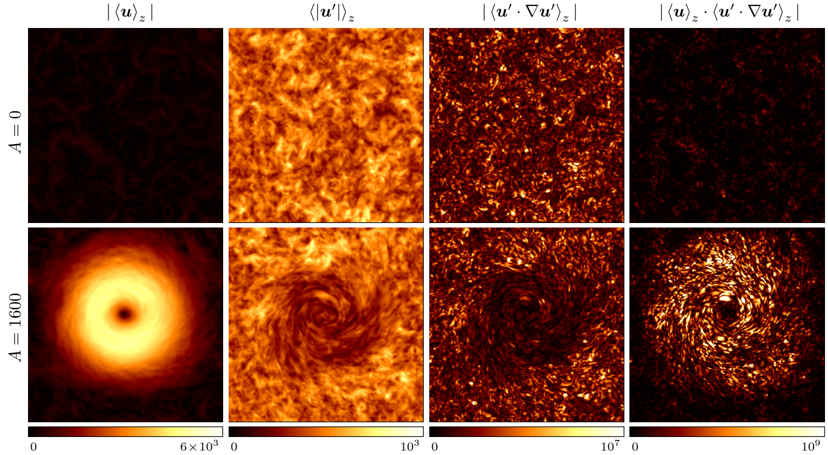

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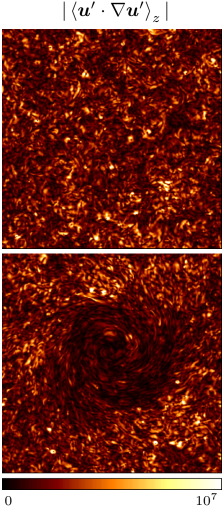
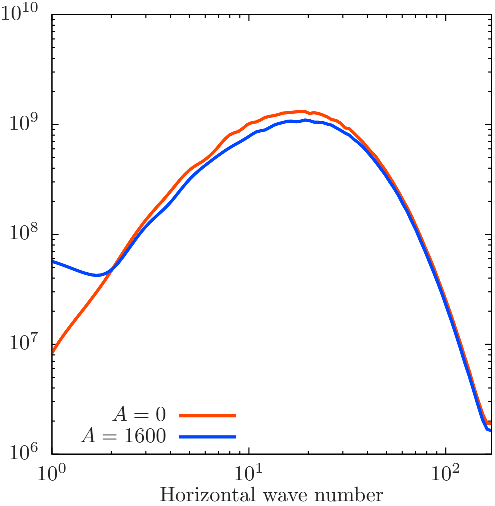


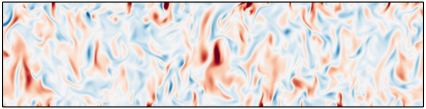


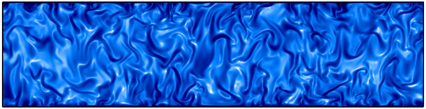
Positive feedback

Vertical vorticity ω_z

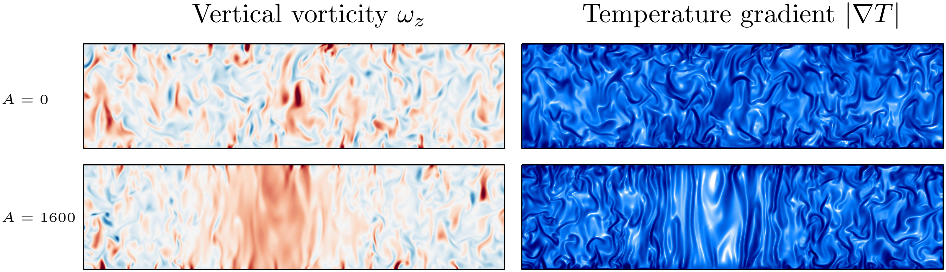
Temperature gradient $|\nabla T|$

$A = 0$





Positive feedback



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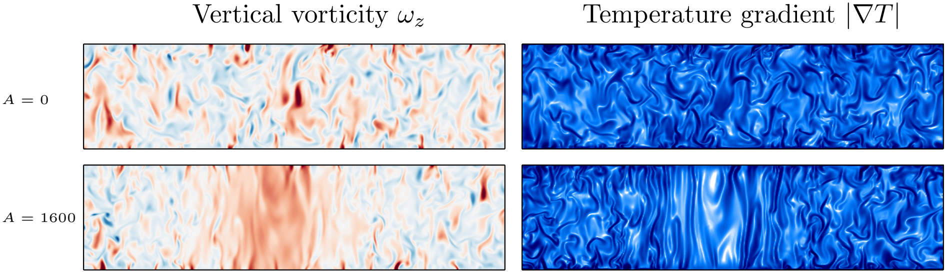

- Small-scale anisotropy induced by large-scale vorticity? Large-scale shear?
- Increase in the small-scale phase correlation?

Outline

Conclusions

- Coexistence of two numerically stable turbulent states at identical control parameter values, one with a large-scale vortex structure and one without.

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- This is a new example of multi-stability in turbulent flows
  - Rotating homogeneous turbulence (Yokoyama & Takaoka 2017)
  - Turbulent Couette flows (Mujica & Lathrop 2006, Zimmerman et al. 2011, Huisman et al. 2014, Xia et al. 2018)
  - von Kármán flows (Ravelet et al. 2004)
  - Thin-layer turbulence (van Kan & Alexakis 2019)
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  - ...

- Positive feedback of the vortex on the 3D fluctuations, leading to anti-diffusive effects and an enhanced energy transfer towards the 2D manifold.

Open questions

- Is it possible to observe this bifurcation in rotating (Yokoyama & Takaoka 2017), MHD (Alexakis 2011) or thin-layer turbulence (Benavides & Alexakis 2017)?

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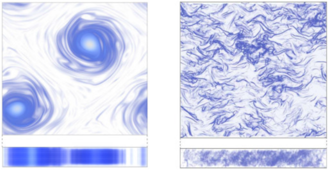
- Can we hope to find optimal perturbations? Genetic algorithms? Direct adjoint methods?
- Does the forcing play any role?

Could the forcing play a role?

- Similar transitions are observed in thin-layer turbulence...



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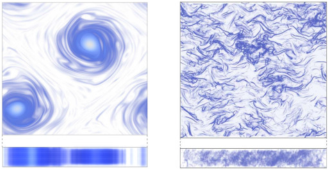
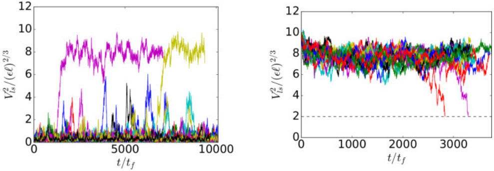


- ... but the nature of the bifurcation is different! (van Kan & Alexakis 2019)



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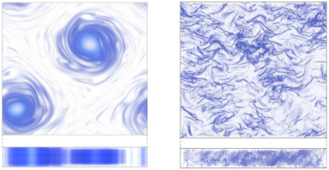

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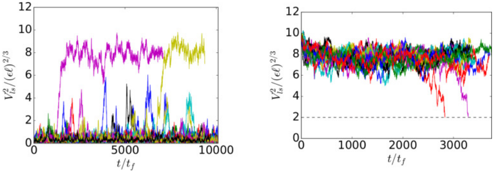
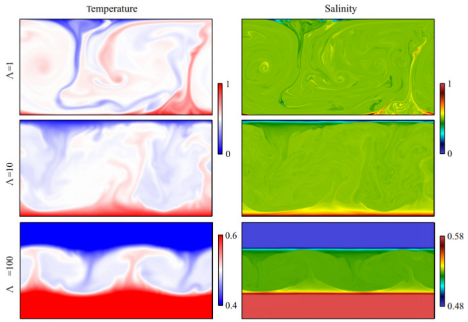


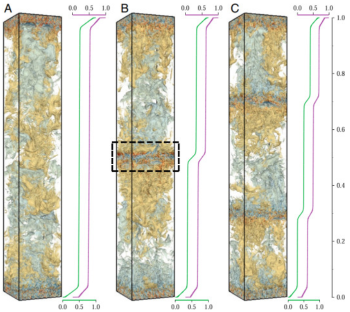
- Could it be due to the difference in forcing?
  - 3D stochastic forcing $\boldsymbol{f}_{3D}$ independant of the solution $\boldsymbol{u}$?
  - 2D stochastic forcing $\boldsymbol{f}_{2D}$ independant of the solution $\boldsymbol{u}$?
  - Instability $\boldsymbol{f}(\boldsymbol{u})$?

“Subcritical” layering

Nonlinear double-diffusive convection

(Veronis (1965), Huppert & Moore (1976), Knobloch & Proctor (1981), ...)

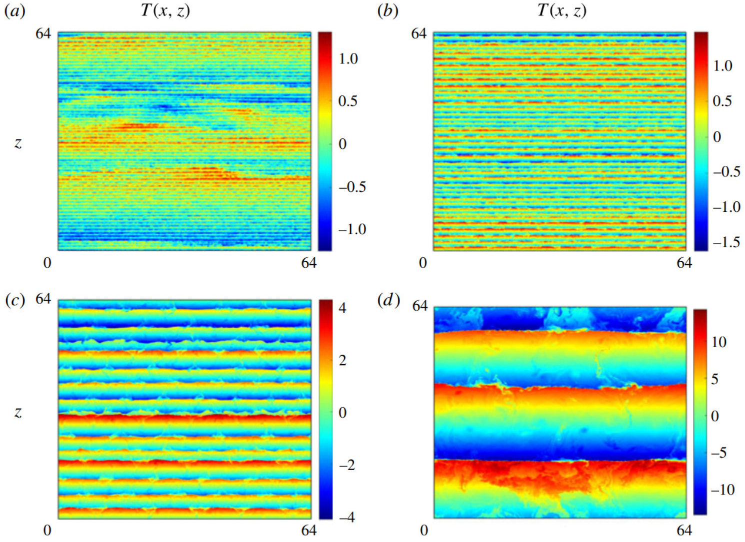




Chong, Yang, Yang, Verzicco & Lohse, JFM (2020)

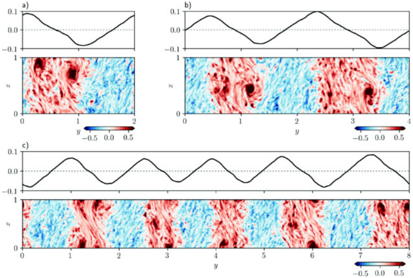
Yang, Chen, Verzicco & Lohse, PNAS (2020)

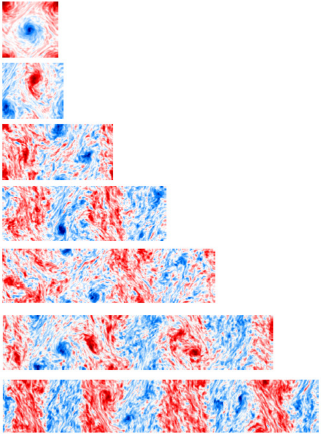
Shear-induced double-diffusive layering



Radko, JFM 805 (2016)

Jets in anisotropic boxes



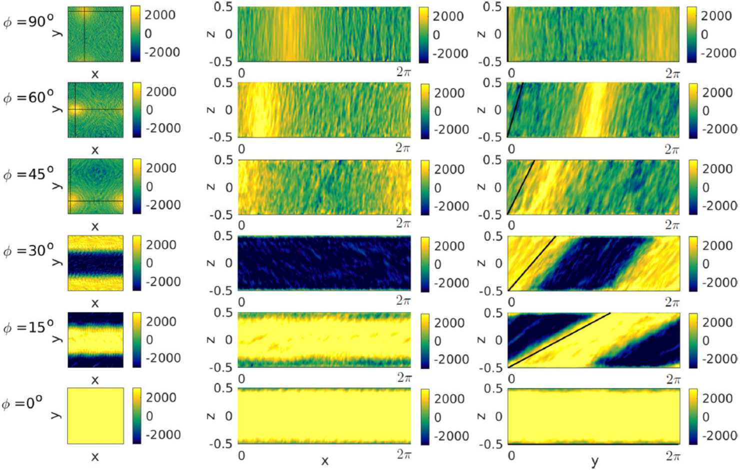


(b) $\langle\zeta\rangle$

Guervilly & Hughes, PRF 2 (2017)

Julien, Knobloch & Plumley, JFM 837 (2018)

Jets in inclined boxes

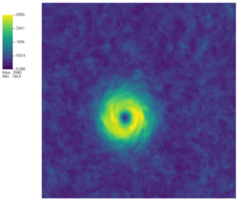


Novi, Hardenberg, Hughes, Provenzale & Spiegel, PRE 99 (2019)

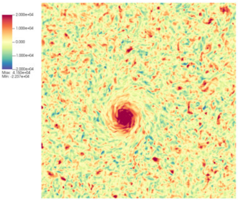
Localised coherent structures

Initial perturbation is a localised shielded monopole:

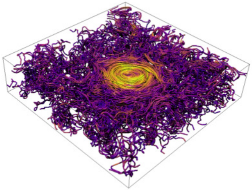
$$\omega_z(t=0) = \omega_0 \left(1 - \frac{r^2}{r_0^2}\right) \exp(-r^2/r_0^2)$$



Velocity amplitude



Vertical vorticity



Streamlines

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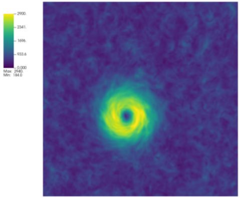
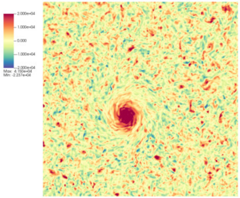
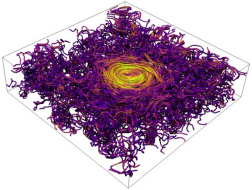


Velocity amplitude



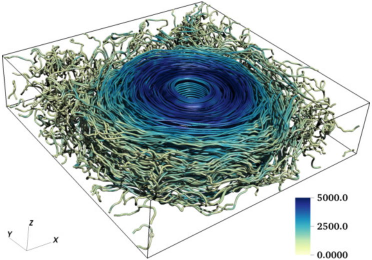
Vertical vorticity



Streamlines

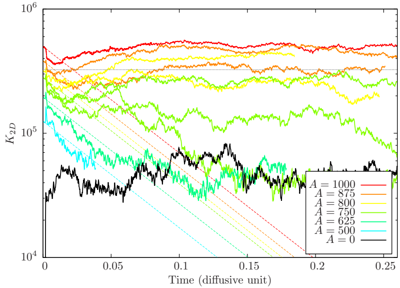
This coherent structure remains stable (and might even grow to the box scale) by locally feeding on the small-scale perturbations.

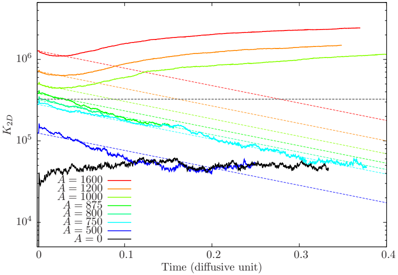
Thank you for your attention!



B. Favier, C. Guervilly & E. Knobloch, Subcritical turbulent condensate in rapidly rotating Rayleigh-Bénard convection, *J. Fluid Mech.* **864** R1 (2019)

Role of the aspect ratio λ





Governing equations in the Boussinesq approximation

- Momentum equation:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \sigma\sqrt{Ta}\,\hat{\boldsymbol{z}} \times \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla p + \sigma Ra\theta\,\hat{\boldsymbol{z}} + \sigma\nabla^2\boldsymbol{u}$$

- Incompressibility condition:

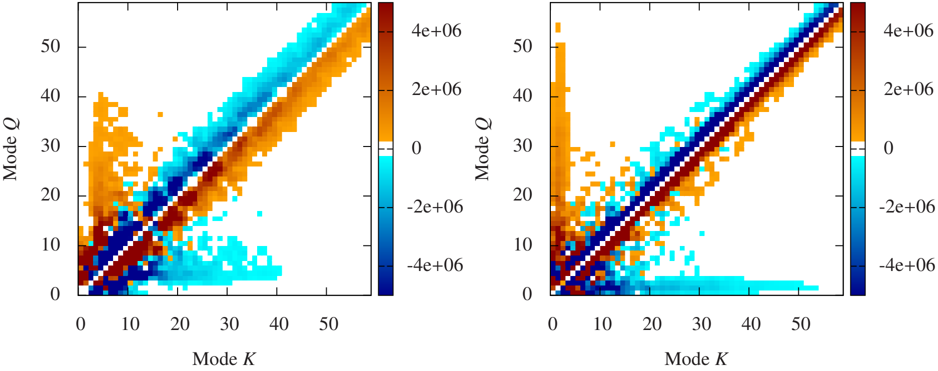
$$\nabla\cdot\boldsymbol{u} = 0$$

- Heat equation:

$$\frac{\partial \theta}{\partial t} + \boldsymbol{u}\cdot\nabla\theta = u_z + \nabla^2\theta$$

$$\sigma = \frac{\nu}{\kappa}, \quad Ra = \frac{\alpha g \Delta T d^3}{\nu\kappa} \quad \text{and} \quad Ta = \frac{4\Omega^2 d^2}{\nu^2}$$

Non-local energy transfer



$$\mathcal{T}(Q,K) = -\int_V \boldsymbol{u}_K \cdot \left(\boldsymbol{u} \cdot \nabla \boldsymbol{u}_Q\right) \mathrm{d}V$$

Flow decomposition

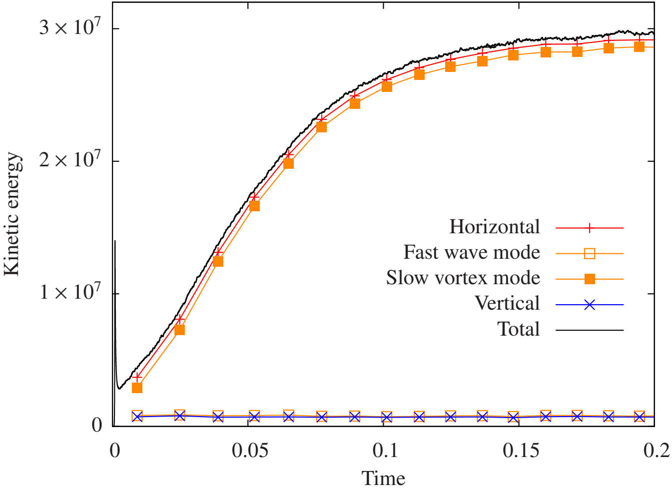
The horizontal flow can be decomposed as the slow 2D ("vortex") mode

$$\langle u \rangle_z (x, y) = \int_0^1 u(x, y, z)\,\mathrm{d}z$$
$$\langle v \rangle_z (x, y) = \int_0^1 v(x, y, z)\,\mathrm{d}z \ ,$$

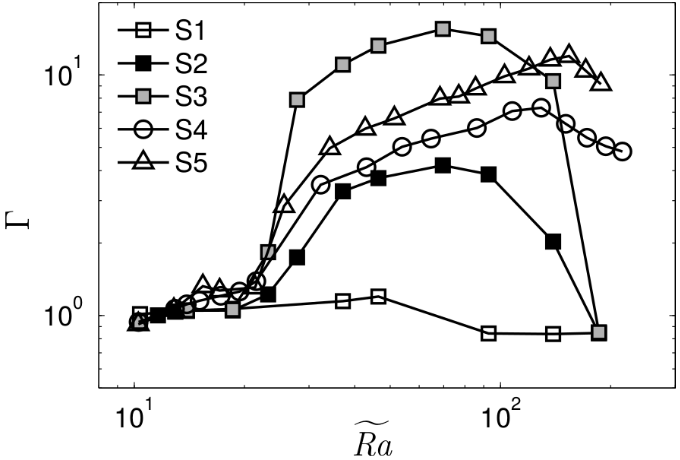
and the fast 3D ("wave") mode

$$u'(x, y, z) = u(x, y, z) - \langle u \rangle_z (x, y)$$
$$v'(x, y, z) = v(x, y, z) - \langle v \rangle_z (x, y)$$
$$w'(x, y, z) = w(x, y, z)$$

Flow decomposition



Details on the transition



$$\Gamma = \frac{\left\langle u_x^2 + u_y^2 + u_z^2 \right\rangle}{3 \left\langle u_z^2 \right\rangle}$$

Guervilly *et al.* (2014)