Location of L$\rightarrow$H Transition Point at a Tokamak Edge for Coupled Heat and Particle Fluxes

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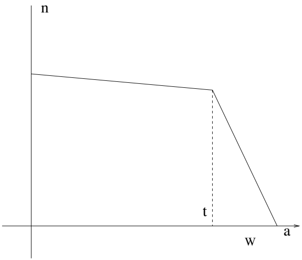
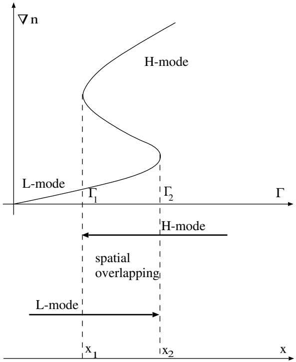
Outline

Transport barrier





- region of reduced (turbulent) transport relative to surroundings
- evident profile steepening
- Key issues:
  -threshold, pedestal width/extent

Brief History and Issues

- bistable flux model (minimal)

– Hinton ’91, Heat flux

$$-Q = \left(\chi_{nc} + \frac{\chi_T}{1 + \alpha \left(du_E / dx \right)^2} \right) \nabla T$$

χ_{nc} neoclassical –H-mode survivor, χ_{nc}, $\chi_{Turbulent}$ both const.
u_E- from radial force balance, $\alpha \sim 1/\gamma^2$

- two stable branches, H-mode gradient MHD-stability limited
- phase coexistence region
- transition may occur in *co-existence region* at any point
- key question: where (when) does *it actually* occur?
- flux suppression factor depends on both pressure and density gradients, suggests two field model at least (p, n)

Two field problem

- use two component model introduced by Hinton and Staebler, '93
- two equations for diffusive particle and energy transport
- flux suppression factors originating from $\mathbf{E} \times \mathbf{B}$ flow shear

$$\text{particles:} \quad \frac{\partial n}{\partial t} - \frac{\partial}{\partial x}\left[D_0 + \frac{D_1}{1+\alpha\,(dV_E/dx)^2}\right]\frac{\partial n}{\partial x} = S(x)$$

$$\text{heat:} \quad \frac{3}{2}\frac{\partial p}{\partial t} - \frac{\partial}{\partial x}\left[\chi_0 + \frac{\chi_1}{1+\alpha\,(dV_E/dx)^2}\right]\frac{\partial p}{\partial x} = H(x)$$

- S (fueling) is concentrated at the edge, $x \simeq a$ –edge fueling
- H (heating) at plasma center ($x = 0$) on -axis deposition
- equations are coupled because

$$V_E' \simeq \frac{c}{eB}\frac{\partial}{\partial x} n^{-1}(x,\tau)\frac{\partial}{\partial x} p(x,\tau) \to E_r' \textit{ coupling}$$

$\chi_{1,} D_1 \to$ pre-transition,
$\chi_0, D_0 \to$ post-transition (may be $\neq$ neo)

Reduction of the Model

$$g_1 = -\frac{dn}{dx},\; g_2 = -\frac{dp}{dx},$$

quasi-stationary state:
–*exact* relation between gradients of p and n (Γ and Q- integrated particle and heat sources)

$$g_2 = \frac{QD_1 g_1}{\chi_1 \Gamma - (D_0 \chi_1 - \chi_0 D_1) g_1}$$

arrive at effectively one field evolution

$$\frac{\partial g}{\partial t} = \frac{\partial^2}{\partial x^2}\left[g + \frac{\lambda g}{1 + g^4 (1+\theta g)^{-2}} - \Gamma_1(x)\right]$$

$g \propto g_1$, $\lambda = D_1/D_0$, $\theta = (\chi_0 D_1 - D_0 \chi_1)/\chi_1 \Gamma$, $\Gamma_1 = \Gamma/D_0$
-*decoupled equations*

Reduction cont'd

- 2 field $\rightarrow$ 1 field (but more complex functional form)
N.B. *Analytical Part* of Hinton - Stabler '93

$$\Rightarrow \frac{\chi_1}{\chi_0} = \frac{D_1}{D_0}$$

so, $g_2 = Q D_1 g_1 / \chi_1 \Gamma$
but

$$? \quad \frac{\chi_1}{\chi_0} = \frac{D_1}{D_0} \quad ?$$

For ES turbulence: $\chi_1 \sim D_1$
Post transition: $\chi_0 \sim \chi_{neo}$

$$D_1 \ll \chi_{neo}$$

- $\lambda = D_1 / D_0 > \lambda_{crit} \rightarrow 2 - 8$ depending on
$\theta = (\chi_0 D_1 - D_0 \chi_1) / \chi_1 \Gamma$
and physics of D_1 uncertain....
$\Rightarrow$ non-ELM particle transport in pedestal ??
- general case

$$g_2 = \frac{Q_s D_1 g_1}{\chi_1 \Gamma_s - (D_0 \chi_1 - \chi_0 D_1) g_1}$$

Phase coexistence

$\rightarrow$what is required for phase co-existence?
need to find roots of the equation

$$g + \frac{\lambda g}{1 + g^4 \left(1 + \theta g\right)^{-2}} = \Gamma_1$$

$\Rightarrow$phase coexistence criterion, simple for $\vartheta = 0$

$$\Pi_- < \sqrt{Q\Gamma} < \Pi_+$$

$$\Pi_\pm \equiv \left(\frac{y_\pm}{\alpha}\right)^{1/4} \frac{1 + \lambda + y_\pm}{1 + y_\pm} D_0 \sqrt{\frac{\chi_1}{D_1}}, \quad y_\pm = \frac{3\lambda}{2} - 1 \pm \frac{3}{2}\sqrt{\lambda\left(\lambda - \frac{16}{9}\right)}$$

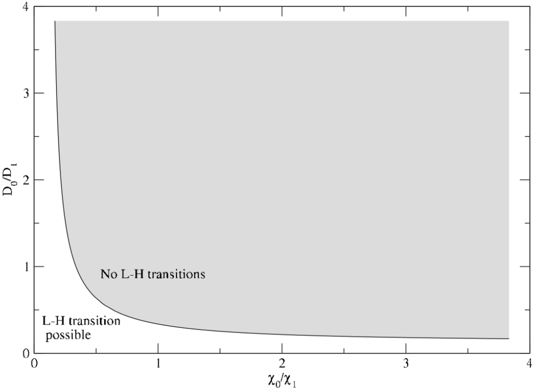
where **is** transition?

General Criterion for Coexistence

- arbitrary D's and χ's
- introduce:

$$A = \frac{D_0}{D_1} + \frac{\chi_0}{\chi_1}; \quad B = \frac{D_0 \chi_0}{D_1 \chi_1}$$

- coexistence condition



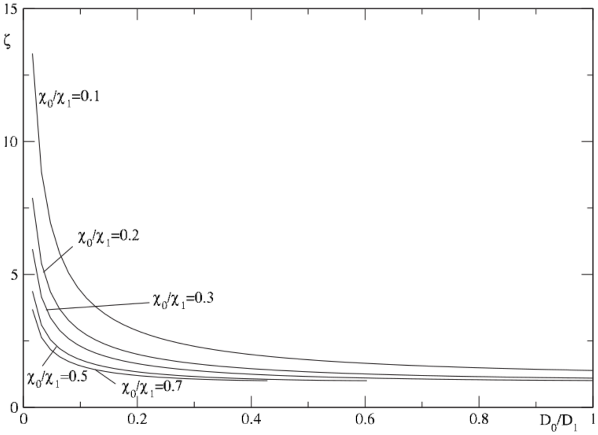
$$A^3(32B-4) - 4A^4 - 9A^2 + 108B(4B-1) < 0$$

Maximum Stregth of the Transition

- Bifurcation Depth
- introduce:
  $y = K^4 g_1^2 g_2^2 \geq 0$
  $K^2 = \sqrt{\alpha} c / eBn^2$

and

- $\zeta \equiv \frac{U_{max}}{U_{min}}$
- for a function



$$U(y) \equiv \sqrt{y}\left(1 + \frac{D_1/D_0}{1+y}\right)\left(1 + \frac{\chi_1/\chi_0}{1+y}\right) = K^2 \frac{\Gamma_s Q_s}{D_0 \chi_0} \equiv P$$

Location of Transition Point

1. Hyperdiffusion regularization- Reduced Transition Model

$$\frac{\partial g}{\partial t} = \frac{\partial^2}{\partial x^2}\left[g + \frac{\lambda g}{1+\beta(x)g^4\left(1+\theta g\right)^{-2}} - \Gamma_1(x) - \varepsilon^2\frac{\partial^2 g}{\partial x^2}\right]$$

$\rightarrow$ **Maxwell**

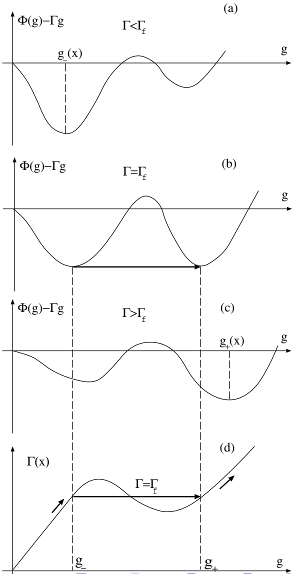
2. Variational approach: construct $\Phi(g)$ such that

$$\frac{\partial g}{\partial t} = \frac{\partial^2}{\partial x^2}\frac{\delta\Lambda}{\delta g}, \quad \Lambda = \int\left[\Phi(g) - \Gamma_1 g\right]dx$$

One can verify that

$$\frac{d\Lambda}{dt} \leq 0$$

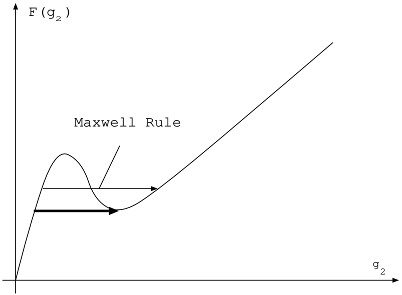
so that the “true” stationary solution requires a global minimum of Λ. This leads to the **Maxwell** rule, again



Curvature effects of the pressure profile

- second derivative of the pressure profile

$$\frac{dV_E}{dx} \simeq -\frac{c}{eBn^2}\frac{\partial n}{\partial x}\frac{\partial p}{\partial x} + \frac{c}{eBn}\frac{\partial^2 p}{\partial x^2}$$



- bifurcation problem (reduction to one field still works)

$$F(g_2,\mu) \equiv$$

$$\chi_0 g_2 + \frac{\chi_1 g_2}{1+\left(\frac{\sigma g_2^2}{1+\kappa g_2} + \mu\frac{dg_2}{dx}\right)^2} = Q(x)$$

$$\sigma = \sqrt{\alpha}\frac{c}{eBn^2}\frac{\Gamma\chi_1}{QD_1};$$

$$\kappa = \frac{D_0\chi_1 - \chi_0 D_1}{D_1 Q};\ \mu = \sqrt{\alpha}\frac{c}{eBn}$$

Conclusions

- $\Rightarrow$hyperdiffusion regularization, variational principle and noise lead to the Maxwell rule
- $\Rightarrow$new rule for barrier location is established: in the finite pressure curvature case it occurs at the lowest possible value of thermal flux (for coexistence)
- in the core plasma, the curvature of the pressure profile is shown to be able to produce an L$\rightarrow$H transition even if the density profile is flat (i.e. stable)
- $\triangle$curvature driven transition is different from the standard case in which the density and pressure barriers are coupled

Discussion

$\Rightarrow$What Does this All Mean, in Practice....
$\Rightarrow$for "standard" minimal model:
- $\exists$ co-existence region
- scale$\leftrightarrow$ λ_N (*tiny* in ITER)
- P_{crit} $\leftrightarrow$ D_0 (*very poorly understood*)
- hysteresis $O(1/2)$ expectation i.e. Maxwell back-transition naive back transition (not good news)$\Rightarrow$including pressure curvature:
-transition for $Q_0 = Q_{min}$
-hysteresis uncertain
-transition possible for weak flat $\nabla n \rightarrow$ beat λ_N??
-*dynamics* require further study.