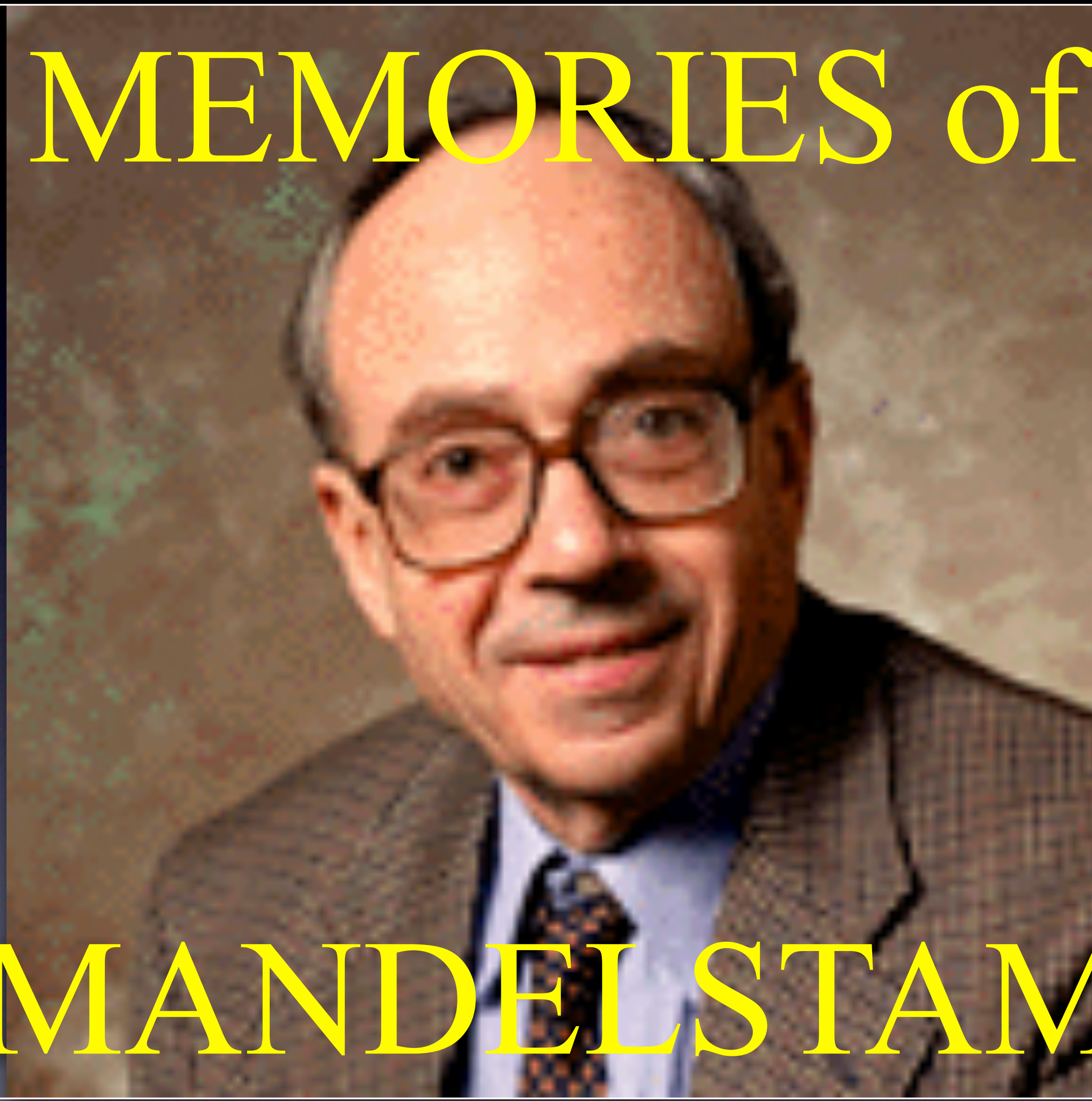
MEMORIES of MANDELSTAM



David Gross, KITP

Stanleyfest, KITP, Feb 13, 2009

THE MANDELSTAM REPRESENTATION

PHYSICAL REVIEW VOLUME 112, NUMBER 4 NOVEMBER 15, 1958

Determination of the Pion-Nucleon Scattering Amplitude from Dispersion Relations and Unitarity. General Theory

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(Received June 27, 1958)

A method is proposed for using relativistic dispersion relations, together with unitarity, to determine the pion-nucleon scattering amplitude. The usual dispersion relations by themselves are not sufficient, and we have to assume a representation which exhibits the analytic properties of the scattering amplitude as a function of the energy and the momentum transfer. Unitarity conditions for the two reactions $\pi+N \rightarrow \pi+N$ and $N+\bar{N} \rightarrow 2\pi$ will be required, and they will be approximated by neglecting states with more than two particles. The method makes use of an iteration procedure analogous to that used by Chew and Low for the corresponding problem in the static theory. One has to introduce two coupling constants; the pion-pion coupling constant can be found by fitting the sum of the threshold scattering lengths with experiment. It is hoped that this method avoids some of the formal difficulties of the Tamm-Dancoff and Bethe-Salpeter methods and, in particular, the existence of ghost states. The assumptions introduced are justified in perturbation theory.

As an incidental result, we find the precise limits of the region for which the absorptive part of the scattering amplitude is an analytic function of the momentum transfer, and hence the boundaries of the region in which the partial-wave expansion is valid.

PHYSICAL REVIEW VOLUME 119, NUMBER 1 JULY 1, 1960

Theory of the Low-Energy Pion-Pion Interaction*

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(Received January 18, 1960)

The double-dispersion representation is applied to the problem of pion-pion scattering, and it is shown that, if inelastic effects are important only at very high energies and S-wave scattering dominates at low energy, a set of integral equations for the low-energy amplitudes can be derived. The solution of these equations depends on only one arbitrary real parameter, which may be defined as the pion-pion coupling constant. The order of magnitude of the new constant is established, and a procedure for solving the integral equations by iteration is outlined. If P-wave scattering is large the equations become singular and must be modified. Such a modification can be performed, at the expense of introducing an extra parameter, but is not considered here.

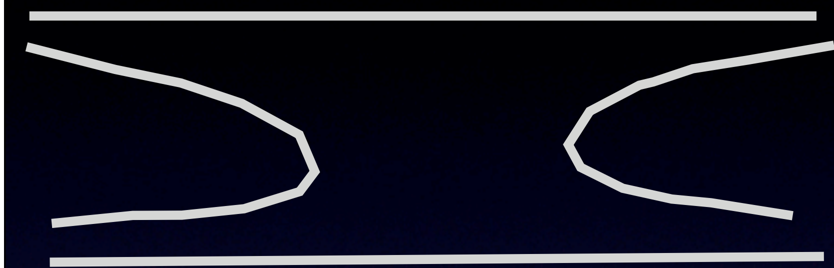
III. THE DOUBLE-DISPERSION REPRESENTATION

A prescription for extending the scattering amplitude to complex values of s, t, and u, subject to (II.3), has been given by one of us.[1] This rule is embodied by the representation[5]

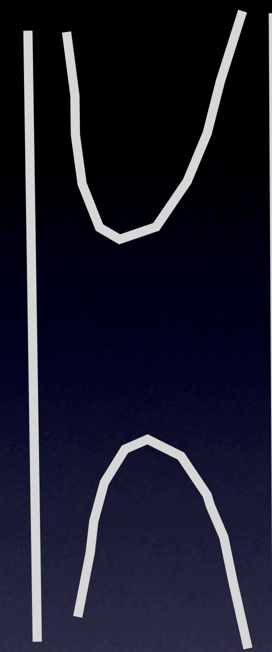
$$A(s,t,u)=\frac{1}{\pi^2}\int\int ds'dt'\frac{A_{st}(s',t')}{(s'-s)(t'-t)}+\frac{1}{\pi^2}\int\int ds'du'\frac{A_{su}(u',s')}{(s'-s)(u'-u)}+\frac{1}{\pi^2}\int\int dt'du'\frac{A_{tu}(t',u')}{(u'-u)(t'-t)}, \quad \text{(III.1)}$$

MANDELSTAM VARIABLES

Fubini, High Energy Conference, Berkeley, 1966



Poles in the s channel



Regge poles in the t channel

“ MANDELSTAM POLE ”

We shall apply crossing with the aid of the generalized superconvergence mentioned in Sec. II.[9] These relations are a consequence of Regge asymptotic behavior and the usual analyticity properties. We assume that a scattering amplitude $A(s,t)$ satisfies dispersion relations with only a right-hand cut and has the asymptotic behavior

$$A(s,t) \sim \sum_r \frac{\gamma_r(s)(-t)^{\alpha_r(s)}}{\sin\pi\alpha_r(s)}, \quad t \to \infty\,. \tag{5.1}$$

The following relation is then asymptotically true as N becomes large:

$$\int^N dt\, \mathrm{Im}A(s,t) \sim \sum_r \frac{\gamma_r(s) N^{\alpha_r(s)+1}}{\alpha_r(s)+1}. \tag{5.2}$$

By considering the functions $t^n A(s,t)$, we derive the further equations

$$\int^N dt\, t^n\, \mathrm{Im}A(s,t) \sim \sum_r \frac{\gamma_r(s) N^{\alpha_r(s)+n+1}}{\alpha_r(s)+n+1}. \tag{5.3}$$

PHYSICAL REVIEW VOLUME 166, NUMBER 5 25 FEBRUARY 1968

Dynamics Based on Rising Regge Trajectories*

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(Received 5 September 1967)

An outline is given of a dynamical scheme based on rising Regge trajectories. The fundamental approximation is that the scattering amplitude can be approximated by the contribution of a finite number of Regge poles. An additional simplifying assumption is that the Regge trajectories are straight lines or, equivalently, that the scattering amplitude is dominated by narrow resonances. Unitarity is introduced by means of the Cheng-Sharp equations, but, in the narrow-resonance approximation, we adopt a very trivial solution of these equations. Crossing is introduced by means of the generalized superconvergence relations due to Igi and to Horn and Schmid. Levinson's theorem is not used; the bootstrap condition is the absence of Kronecker-δ singularities in the J plane. It is hoped that this scheme avoids some of the disadvantages of conventional schemes. In the narrow-resonance approximation one has to solve numerical equations, not integral equations. The scheme is applied to the pseudoscalar, vector, and axial-vector nonets considered as bound states of the $N\bar{N}$ system. As only one channel is being examined, we have to introduce certain parameters from experiment, but we obtain reasonable values for the other parameters.

Narrow Resonance Approximation ~ Linear Regge Trajectories

Dynamics = Generalized Superconvergence Relations

BOOTSTRAP OF THE ρ REGGE TRAJECTORY

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(Received 12 October 1967)

The amplitude for $\omega+\pi\to\pi+\pi$ is considered within a dynamical scheme proposed by Mandelstam, based on rising Regge trajectories, the narrow-resonance approximation, and generalized superconvergence relations. The ρ trajectory is shown to qualitatively bootstrap itself. Also, a world consisting only of the particles on a vacuum trajectory is shown to be inconsistent within this approximation.

Mandelstam has recently proposed a dynamical scheme[1] based on approximating the amplitude by a finite number of Regge poles in all channels. Crossing is imposed by generalized superconvergence relations.[2,3] In the first approximation, which is essentially the narrow-resonance approximation (NRA), the trajectories are assumed to be straight lines and unitarity determines the Regge residue up to an entire function, which can then be approximated by a finite polynomial. This approximation, which can be systematically improved, allows one to derive algebraic relations between a finite number of parameters. The relations may be sufficient to determine these parameters self-consistently.

fixed t,[5]

$$\tilde{f}(s,t,u)=\csc\theta_t f_{0\lambda,00}{}^{t}(s,t)=\tilde{f}(u,t,s). \qquad (2)$$

The basic assumption is[1] that $G^J(t)$ can be approximated by one "ρ" trajectory

$$G_{0\lambda,00}{}^{J}(t)=\beta(t)/[J-\alpha(t)], \qquad (3)$$

and that $\alpha(t)=at+b$. Unitarity and analyticity in the NRA then determine the form of $\beta(t)$ up to an entire function $E(t)$,

$$\beta(t)=\left(\frac{4q_tq_{t'}a}{e}\right)^{at+b}\frac{t^{\frac{1}{2}}[(at+b)(at+b+1)]^{\frac{1}{2}}}{\Gamma(at+b+\frac{3}{2})}E(t). \qquad (4)$$

$$\frac{E(t)}{E[(1-b)/a]} = \frac{4(at+b+1)\Gamma(at+b)[t+2(1-b)/a-\Sigma]}{Ne}\left(\frac{e}{4Na}\right)^{at+b}. \qquad (7)$$

ven without saying anything more about $E(t)$, valuation of this equation at $t=-b/a$, $\alpha(t)=0$, nd at $t=(1\text{-}b)/a$, $\alpha(t)=1$ yields

$$(2-3b)/a = \Sigma; \quad Na = \sqrt{2}. \qquad (8)$$

 this form of a self-consistent bootstrap, is not arbitrary; it must surely lie above $1-b)/a$ and below $(3-b)/a$. Since we are ne- lecting daughter trajectories (clearly impor- ant here near $t \approx 0$) other trajectories with

of N! We have

$$\frac{E(t)}{E((1-b)/a)} = \frac{\Gamma(\alpha(t)+2)}{2}\left(\frac{e}{4\sqrt{2}}\right)^{\alpha(t)-1},$$

and $E(t)$ changes by only 4% as $\alpha(t)$ varies from 0 to 1. Note that in this lowest approximation we cannot say anything about the absolute value of E and, like all other bootstrap models, have no way of knowing whether the ρ trajectory will continue to be self-consistent when other channels and particles are added.

VENEZIANO FOUND THE ANSWER

Construction of a crossing - symmetric, Regge behaved amplitude for linearly rising trajectories **Nuovo Cim. A57:190-197,1968**

$$B(-s + m^2{}_r\, , -t + m^2{}_r)$$

STRING THEORY

HAPPY BIRTHDAY STANLEY