Duality and Confinement

Graduate work with Stanley Mandelstam from 1977 to 1980

Joe Polchinski
Stanleyfest
KITP, Feb. 13, 2009

Warmup problem (1977): *write down the action for a U(1) gauge field coupled to an electrically charged field* Φ_e *and a magnetically charged field* Φ_m.

One possible answer: this theory does not exist.

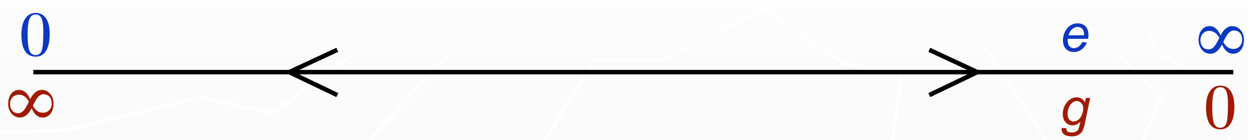
Why? Even U(1) plus an electrically charged field doesn't really exist because of the Landau pole: the coupling runs to infinity at a finite UV scale, where the theory breaks down.

Of course, we understand that for weak coupling this theory makes sense as an effective QFT, over a large range of scales. But with the magnetic charge, there is the Dirac quantization condition,

$$eg = 2\pi n$$

So *e* and *g* cannot both be small, and whichever is large diverges almost immediately in the UV.

Not so fast! Here's the picture:



The arrows indicate the flow toward the UV.

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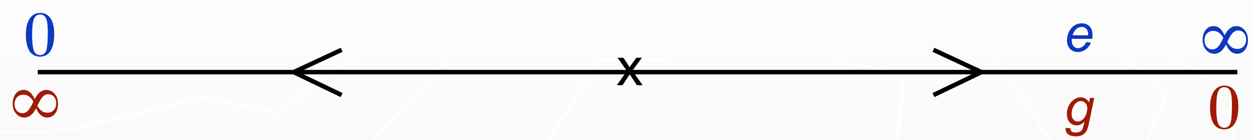


The arrows indicate the flow toward the UV. This strongly suggests that there is a fixed point, shown by the x. (IR attractive)

This is actually *better* than the purely electric theory, because it can exist as a continuum QFT, at the fixed point. *However,* it has no classical limit, and probably no action.

Argyres and Douglas (1995) have provided evidence that this theory actually exists, at least with $\mathcal{N} = 2$ supersymmetry. They start with an SU(3) gauge theory, and argue that its IR physics is a U(1) gauge field, with massless electric and magnetic charges along curves in moduli space:

The physics at the intersection seems to be the desired effective theory.

A & D also proposed an action for the effective theory. They began with the duality-invariant action for the free U(1) field (Schwarz & Sen, 1993):

$$S = -\frac{1}{2}\int d^4x\Big(B^{(\alpha)i}\mathcal{L}_{\alpha\beta}E_i^{(\beta)} + B^{(\alpha)i}B^{(\alpha)i}\Big)$$

$$E_i^{(\alpha)} = \partial_0 A_i^{(\alpha)} - \partial_i A_0^{(\alpha)} \qquad 1 \leq \alpha \leq 2$$

$$B^{(\alpha)i} = \epsilon^{ijk}\partial_j A_k^{(\alpha)} \qquad \mathcal{L} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Enlarged gauge symmetry $\delta A_0^{(\alpha)} = \Psi^{(\alpha)},\ \delta A_i^{(\alpha)} = \partial_i \Lambda^{(\alpha)}$ reduces to the proper degrees of freedom.

Electric/magnetic duality symmetry: $A_\mu^{(\alpha)} \to \mathcal{L}_{\alpha\beta}A_\mu^{(\beta)}$

A & D coupled $A_\mu^{(1)}$ to Φ_e and $A_\mu^{(2)}$ to Φ_m. But this does not respect the enlarged gauge symmetry:

$$\delta A_0^{(\alpha)} = \Psi^{(\alpha)}, \quad \delta A_i^{(\alpha)} = \partial_i \Lambda^{(\alpha)}$$

The equations of motion force the charge densities to vanish.

There is a deep reason why such things don't work. The quantization of Φ_m includes configurations such as

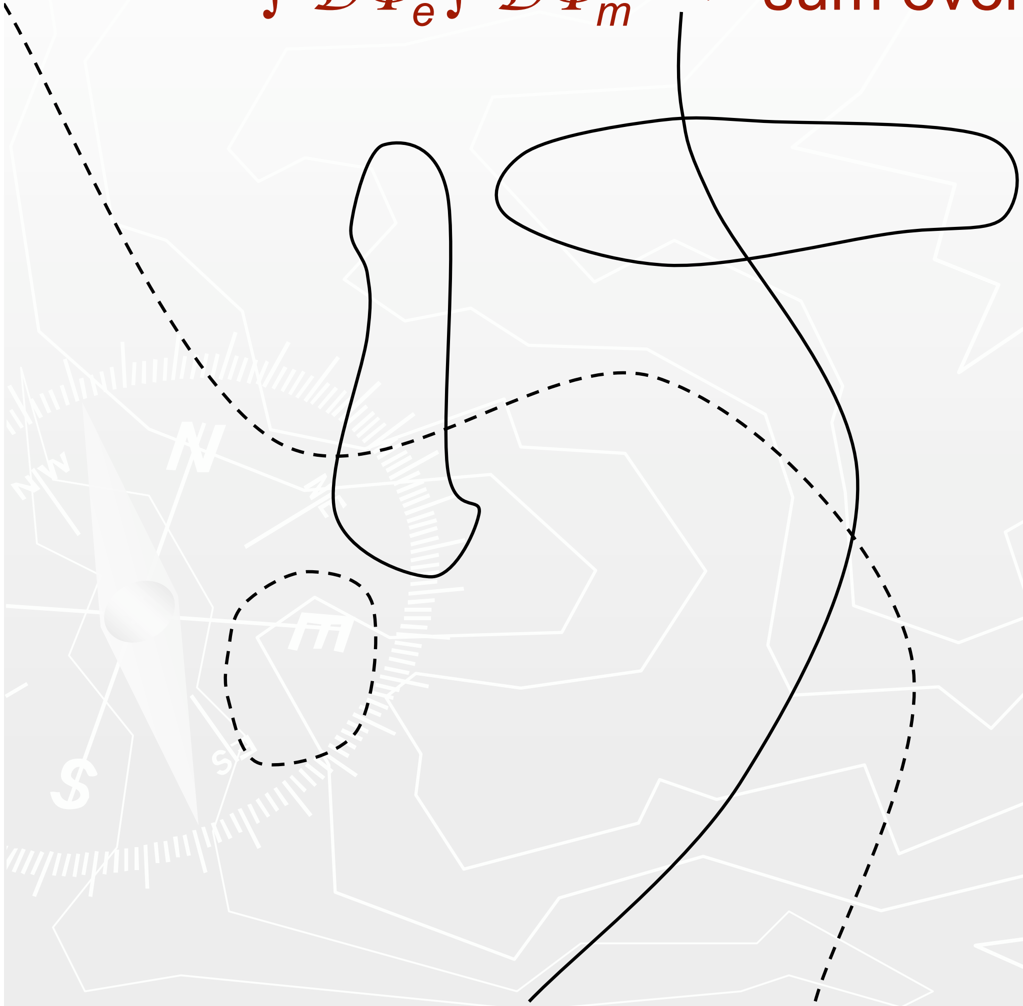
In the presence of these magnetic charges, Φ_e is not a function but a section of a bundle. The field space

$$\int \mathcal{D}\Phi_e \int \mathcal{D}\Phi_m$$

is not a product but much more complicated. Also, the action should only be defined mod 2π (Dirac quantization), so that e^{iS} is well-defined.

Brandt, Neri & Zwanziger did provide one solution. Rewrite the theory in first-quantized form,

$\int \mathcal{D}\Phi_e \int \mathcal{D}\Phi_m \rightarrow$ sum over charge & monopole paths



Action is now well-defined mod 2π, can integrate gauge field out.

Looks a bit formal, but should allow numerical (lattice) calculation (~compact QED with tuning).

Another solution is simply to use the $\mathcal{N}$=2 SU(3) AD theory (with soft SUSY breaking if desired) and flow to the fixed point. Indeed, many theories, such as multiple M2-branes in D=11, are defined only in this way…

The Big Picture:

Vortices and quark confinement in non-Abelian gauge theories

S. MANDELSTAM

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Berkeley, California 94720, U.S.A.

The plan:

- Construct ‘t Hooft operators
- Extract magnetic vector potential $A^{\vee}_{\mu}$ (group $G^{\vee}$)
- Construct Hamiltonian (or perhaps just an approximate vacuum) in terms of $A^{\vee}_{\mu}$
- Demonstrate confinement

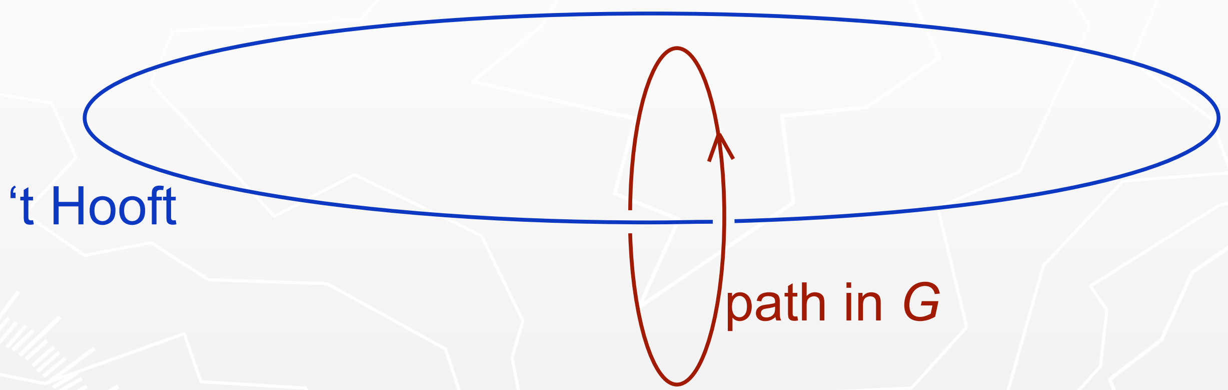
Soliton operators for the quantized sine-Gordon equation*

S. Mandelstam
Department of Physics, University of California, Berkeley, California 94720
(Received 24 February 1975)

$$\psi_1(x) = (c\mu/2\pi)^{1/2} \times e^{\mu/8\epsilon} : \exp\left[-2\pi i\beta^{-1}\int_{-\infty}^{x} d\xi\ \dot{\phi}(\xi) - \tfrac{1}{2}i\beta\phi(x)\right] : , \tag{2.8a}$$

$$\psi_2(x) = -i(c\mu/2\pi)^{1/2} \times e^{\mu/8\epsilon} : \exp\left[-2\pi i\beta^{-1}\int_{-\infty}^{x} d\xi\ \dot{\phi}(\xi) + \tfrac{1}{2}i\beta\phi(x)\right] : . \tag{2.8b}$$

Next problem: *Wilson loops are classified by representations of the gauge group G. ‘t Hooft loops are apparently classified only by an element of the center (i.e. the n-ality, for SU(n)):*



However, they should be classified by the representations of the dual group $G^\vee$, in the sense of Goddard, Nuyts & Olive (1977). Find this finer set of ‘t Hooft loops.

Solved by Kapustin in 2006:

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Wilson-'t Hooft operators in four-dimensional gauge theories and *S*-duality

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(Received 14 May 2006; published 7 July 2006)

We study operators in four-dimensional gauge theories which are localized on a straight line, create electric and magnetic flux, and in the UV limit break the conformal invariance in the minimal possible way. We call them Wilson-'t Hooft operators, since in the purely electric case they reduce to the well-known Wilson loops, while in general they may carry 't Hooft magnetic flux. We show that to any such operator one can associate a maximally symmetric boundary condition for gauge fields on $\mathrm{AdS}_E^2 \times S^2$. We show that Wilson-'t Hooft operators are classified by a pair of weights (electric and magnetic) for the gauge group and its magnetic dual, modulo the action of the Weyl group. If the magnetic weight does not belong to the coroot lattice of the gauge group, the corresponding operator is topologically nontrivial (carries nonvanishing 't Hooft magnetic flux). We explain how the spectrum of Wilson-'t Hooft operators transforms under the shift of the θ-angle by 2π. We show that, depending on the gauge group, either $SL(2, \mathbb{Z})$ or one of its congruence subgroups acts in a natural way on the set of Wilson-'t Hooft operators. This can be regarded as evidence for the S-duality of $N = 4$ super-Yang-Mills theory. We also compute the one-point function of the stress-energy tensor in the presence of a Wilson-'t Hooft operator at weak coupling.

What I tried: $V(C)$ = function of $\vec{A}^a$, $\vec{E}^a$.
Problem with regularization on C

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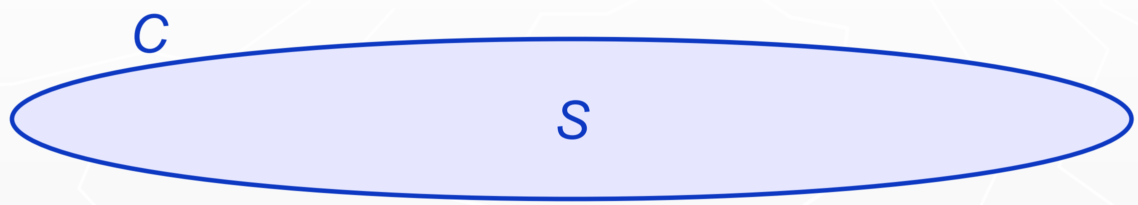

Abelian case:

$$V_p'(C,S) = \exp\left\{\frac{\pi p Z_3}{e_r}\int_S d\sigma^y_{\alpha\beta}\tilde{F}_{r\alpha\beta}(y)\right\}$$

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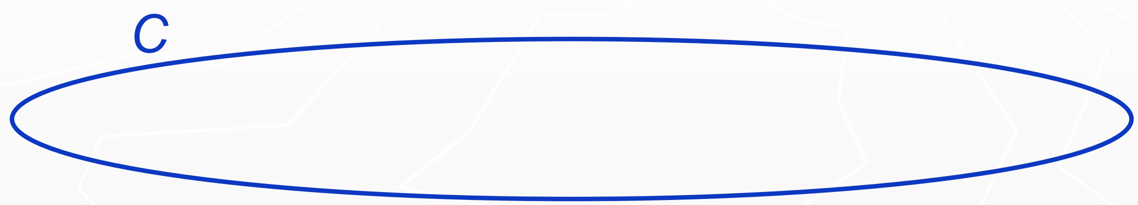


Abelian case:

$$V_p'(C,S) = \exp\{-\frac{\pi p Z_3}{e_r}\int_S d\sigma^y_{\alpha\beta}\tilde{F}_{r\alpha\beta}(y)\}$$

Kapustin: excise C and put boundary conditions there (specify singularities in the fields).

One more recollection: $D = 11$ supergravity!?

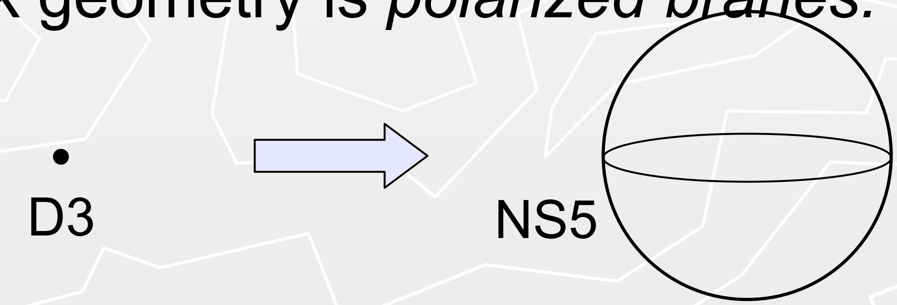
Thesis problem: *demonstrate confinement via monopole condensation in Yang-Mills theory, possibly using supergravity.*

March 2000

The String Dual of a Confining Four-Dimensional Gauge Theory

Joseph Polchinski * and Matthew J. Strassler †

- $\mathcal{N} = 4$ Yang-Mills (conformal, non-confining) has a supergravity dual.
- Adding explicit mass terms for all but the gauge fields gives pure Yang-Mills spectrum, and we expect confinement. Use dual description to solve.
- Mass terms correspond to perturbation of boundary condition for 3-form fluxes, naïve continuation into bulk give unphysical singularity. Correct bulk geometry is *polarized branes:*



• Finite radius of D5's replaces horizon with hard wall, infinite redshift with finite.

• Potential between external quarks now confining rather than Coulombic:

• Same theory has a Higgs phase (replace NS5 with D5), which is explicitly S-dual to the confining phase: the latter is due to dual (monopole) Higgs condensate.

Our academic lineage (thanks to Vishnu Jejjala, Djordje Minic & AMS/Clay/NDSU Mathematics Geneology Project)

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Johann Pfaff 1786 — Joseph Louis Lagrange

Abraham Kästner 1739 — Leonhard Euler 1726

Christian Hausen 1713 — Johann Bernoulli 1690

Johann Wichmannshausen 1685 — Jacob Bernoulli 1684

Otto Mencke 1666 — Gottfried Wilhelm Leibniz 1666

Christiaan Huygens 1647

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Marin Mersenne 1611

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