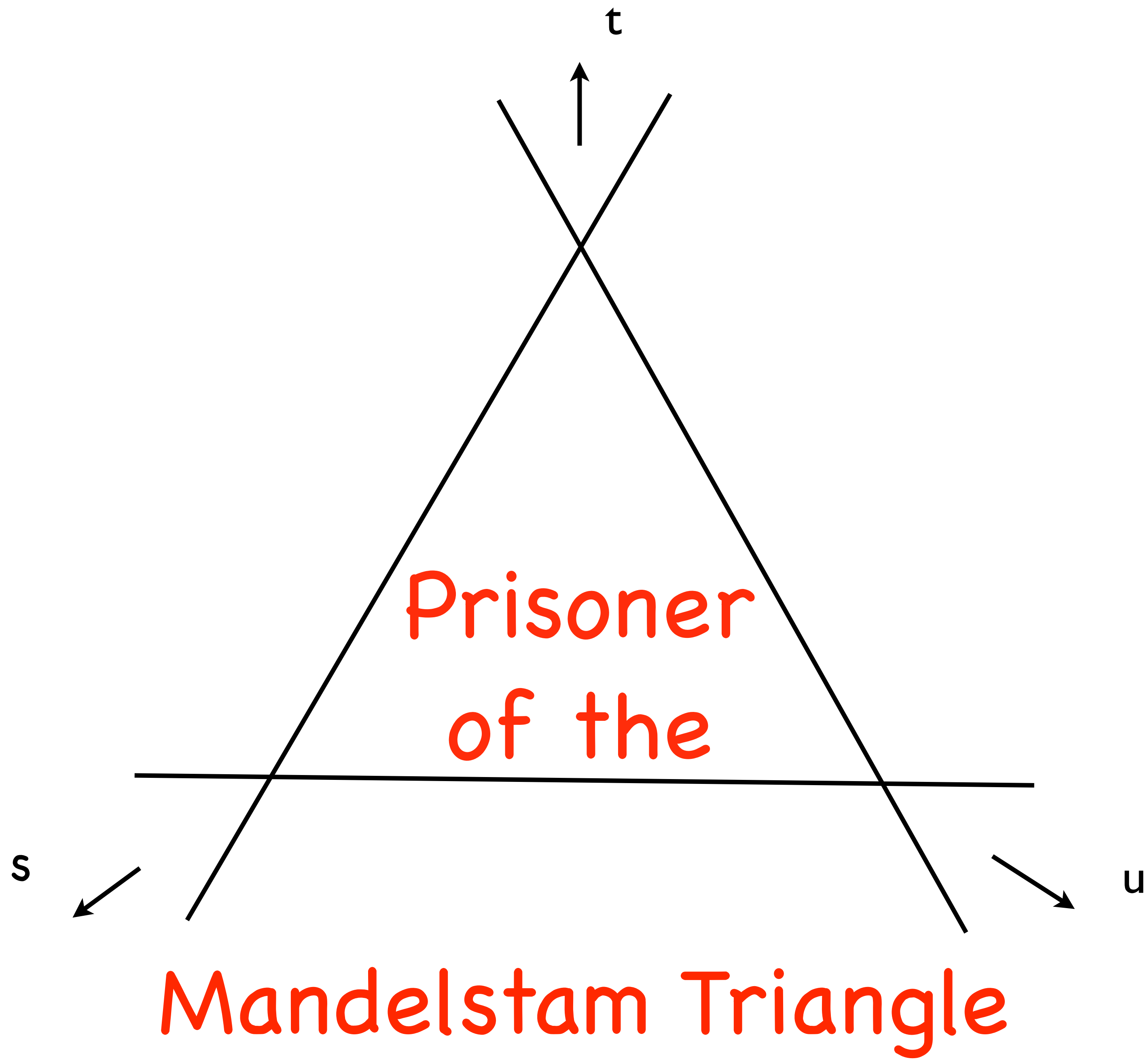
I knew very early of

Stanley Mandelstam

I started physics

as a


t
Prisoner of the
s
u
Mandelstam Triangle

I escaped
from the
Mandelstam Triangle

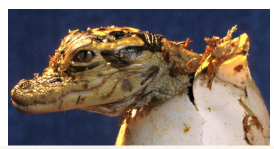
only to be ensnared
in
Light-Cone Superspace

A Note
of
Personal Gratitude

1971 NAL Visit

SuperConformal Theories

P. Ramond



UNIVERSITY OF FLORIDA

(with S. Ananth, D. Belyaev, L. Brink and S.-S. Kim)

Light-Cone Superspaces

N=4 Super Yang-Mills

N=8 SuperConformal

N=8 SuperGravity and $E_{7(7)}$

N=16 SuperGravity and $E_{8(8)}$

N=8 Light-Cone Superspace

houses

D=11: N=1 SuperGravity $SO(9)$; $F_4/SO(9)$

D=4: N=8 SuperGravity $SO(2)\times E_{7(7)}$

D=3: N=16 SuperGravity $E_{8(8)}$

D=2: N=16 Theory $E_{9(9)}$

N=4 Light-Cone Superspace

habitat for

D=10: N=1 Super Yang-Mills SO(8)

D=4: N=4 Super Yang-Mills PSU(2,2|4)

D=3: N=8 Super Conformal OSp(2,2|8)

LC2 Formalism

Light-Cone Coordinates:

$$x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^3) \qquad \partial^{\pm} = \frac{1}{\sqrt{2}}(-\partial^0 \pm \partial^3)$$

$$x = \frac{1}{\sqrt{2}}(x^1 + ix^2) \qquad \overline{x} = \frac{1}{\sqrt{2}}(x^1 - ix^2) \qquad \partial = \frac{1}{\sqrt{2}}(\partial^1 + i\partial^2) \qquad \overline{\partial} = \frac{1}{\sqrt{2}}(\partial^1 - i\partial^2)$$

Light-Cone Gauge:

$$A^+ = \frac{1}{\sqrt{2}}(A^0 + A^3) = 0$$

$$A^- = \frac{1}{\sqrt{2}}(A^0 - A^3)$$ is replaced through eqs. of motions

Physical Fields:

$$\bar{A} = \frac{1}{\sqrt{2}}(A^1 + iA^2) \qquad A = \frac{1}{\sqrt{2}}(A^1 - iA^2)$$

Four Complex Grassmann Variables

Chiral derivatives

$$d^m \equiv -\frac{\partial}{\partial \bar{\theta}_m} - \frac{i}{\sqrt{2}}\theta^m \partial^+ , \quad \bar{d}_m \equiv \frac{\partial}{\partial \theta^m} + \frac{i}{\sqrt{2}}\bar{\theta}_m \partial^+ ,$$

$$\{d^m , \bar{d}_n\} = -i\sqrt{2}\delta^m{}_n \partial^+$$

N=4: Home of the

Constrained Chiral Superfield

$$\varphi\,(y)^{a} \;=\; \frac{1}{\partial^+}\,A\,(y)^{a} + \frac{i}{\sqrt{2}}\,\theta^m\,\theta^n\,\overline{C}^{\,a}_{mn}\,(y) \;+\; \frac{1}{12}\,\theta^m\,\theta^n\,\theta^p\,\theta^q\,\epsilon_{mnpq}\,\partial^+\,\bar{A}\,(y)^{a}$$
$$+\;\frac{i}{\partial^+}\,\theta^m\,\bar{\chi}^{a}_m(y) + \frac{\sqrt{2}}{6}\theta^m\,\theta^n\,\theta^p\,\epsilon_{mnpq}\,\chi^q(y)^{a}$$

Chiral Constraint: $$d^m\,\varphi\,(y)^{a} \;=\; 0$$

$$y \;=\; (\,x,\,\bar{x},\,x^+,\,y^- \equiv x^- - \frac{i}{\sqrt{2}}\,\theta^m\,\bar{\theta}_m\,)$$

Inside-out Constraint: $$\bar{d}_m\,\bar{d}_n\,\varphi^{a} \;=\; \frac{1}{2}\,\epsilon_{mnpq}\,d^p\,d^q\,\overline{\varphi}^{a}$$

[Brink, Lindgren and Nilsson '82]

Super-Poincaré Group

Kinematical Generators

Translations, SO(2) rotations

$$p^{+} \;=\; -\,i\,\partial^{+}\,, \qquad p \;=\; -\,i\,\partial\,, \qquad \bar{p} \;=\; -\,i\,\bar{\partial}\,,$$

$$j \;=\; x\,\bar{\partial} - \bar{x}\,\partial + \frac{1}{2}\,(\,\theta^{m}\,\frac{\partial}{\partial\theta^{m}} \,-\, \bar{\theta}_{m}\,\frac{\partial}{\partial\bar{\theta}_{m}}\,) - \lambda$$

$$j^{+} \;=\; i\,x\,\partial^{+}\,, \qquad \bar{j}^{+} \;=\; i\,\bar{x}\,\partial^{+}\,, \qquad j^{+-} \;=\; i\,x^{-}\,\partial^{+} - \frac{i}{2}\,(\,\theta^{p}\bar{\partial}_{p} + \bar{\theta}_{p}\,\partial^{p}\,) + i\,.$$

Kinematical Susy

$$q^{m} \;=\; -\,\frac{\partial}{\partial\bar{\theta}} + \frac{i}{\sqrt{2}}\theta^{m}\,\partial^{+} \qquad\qquad \bar{q}_{m} \;=\; \frac{\partial}{\partial\theta} - \frac{i}{\sqrt{2}}\bar{\theta}_{m}\,\partial^{+}$$

$$\{\,q^{m}\,,\,\bar{q}_{n}\,\} \;=\; i\sqrt{2}\,\delta^{m}_{n}\,\partial^{+}$$

(respect chirality) $$\{\,d^{m}\,,\,\bar{q}_{n}\,\} \;=\; \{\,q^{m}\,,\,\bar{d}_{n}\,\} \;=\; 0$$

temperature?

Kinematical Generators act linearly on the fields

$$\delta^{\rm kin}_{\mathcal{O}}\,\varphi \;=\; \mathcal{O}\,\varphi$$

$$[\,\delta^{\rm kin}_{\mathcal{O}}\,,\,\delta^{\rm kin}_{\mathcal{O}'}\,]\,\varphi \;=\; \delta^{\rm kin}_{[\mathcal{O},\mathcal{O}']}\,\varphi$$

example: kinematical Susy

$$\delta^{\rm kin}_{s}\,\varphi \;=\; \epsilon^{m}\bar{q}_{m}\,\varphi \qquad\qquad \delta^{\rm kin}_{\bar{s}}\,\varphi \;=\; \bar{\epsilon}_{m}q^{m}\,\varphi$$

$$[\,\delta^{\rm kin}_{s}\,,\,\delta^{\rm kin}_{\bar{s}}\,]\,\varphi \;=\; \frac{i}{\sqrt{2}}\epsilon^{m}\bar{\epsilon}_{m}\partial^{+}\varphi \;=\; \frac{i}{\sqrt{2}}\,\delta^{\rm kin}_{t^{-}}\,\varphi$$

Dynamical Generators (linear only for free theories)

Hamiltonian

$$p^- \;=\; -\,i\frac{\partial\bar\partial}{\partial^+}$$

Boosts

$$j^- \;=\; i\,x\,\frac{\partial\bar\partial}{\partial^+}\;-\;i\,x^-\,\partial\;+\;i\left(\theta^p\bar\partial_p\;-\lambda-1\right)\frac{\partial}{\partial^+}\;\;,$$

$$\bar j^- \;=\; i\,\bar x\,\frac{\partial\bar\partial}{\partial^+}\;-\;i\,x^-\,\bar\partial\;+\;i\left(\bar\theta_p\partial^p+\lambda-1\right)\frac{\bar\partial}{\partial^+}\;\;.$$

Dynamical Susy (square root of the Hamiltonian)

$$q^m_- \;\equiv\; i\,[\,\bar j^-\,,\,q^{\,m}\,]\;=\;\frac{\bar\partial}{\partial^+}\,q^{\,m}\,,\qquad \bar q_{\,-\,m}\;\equiv\; i\,[\,j^-\,,\,\bar q_{\,m}\,]\;=\;\frac{\partial}{\partial^+}\,\bar q_{\,m}$$

$$\{\,q^m_-\,,\,\bar q_{\,-\,n}\,\}\;=\;i\,\sqrt{2}\,\delta^{\,m}{}_{n}\,\frac{\partial\bar\partial}{\partial^+}$$

Dynamical Generators act non-linearly
(linear for free theories)

$$\delta^{\rm dyn}_{\mathcal{O}}\,\varphi \;=\; \delta^{\rm dyn,free}_{\mathcal{O}}\,\varphi + \delta^{\rm dyn,int}_{\mathcal{O}}\,\varphi \;=\; \mathcal{O}\,\varphi + {\rm non}-{\rm linear}$$

$$[\,\delta^{\rm kin}_{\mathcal{O}}\,,\,\delta^{\rm dyn}_{\mathcal{O}'}\,]\,\varphi \;=\; \delta^{\rm dyn}_{[\mathcal{O},\mathcal{O}']}\,\varphi$$

N=4 Super Yang-Mills

- Maximally Supersymmetric Theory with max spin 1
- 16 massless states: 8 Bosonic + 8 Fermionic
- helicity: 1 1/2 0 -1/2 -1

helicity:	1	1/2	0	-1/2	-1
# states:	1	4	6	4	1

SuperConformal Symmetry PSU(2,2|4)

(Conformal group SO(4,2) X SU(4) R-symmetry)

SuperConformal D=4 Kinematics

$$K^+ = 2i\,x\,\bar{x}\,\partial^+ \qquad [\,K^+ ,\, p^-\,] = -2i\,D + 2i\,j^{+-}$$

SuperConformal Susy

$$[\,K^+ \,,\, \bar{q}_{-\,n}\,] = \sqrt{2}\,\bar{s}_{+\,n} \qquad s_-^m = i\,[\,j^- \,,\, s_+^m\,]$$

$$\{q^m \,,\, \bar{s}_{-\,n}\} = -i\delta^m_n(D + j^{+-} + ij) + 2J^m{}_n$$

SU(4) R-symmetry

$$[\,J^m{}_n \,,\, J^p{}_q\,] = i\,\delta^m{}_q\,J^p{}_n - i\,\delta^p{}_q\,J^m{}_n - i\,\delta^m{}_n\,J^p{}_q + i\,\delta^p{}_n\,J^m{}_q$$

SuperConformal D=4 Dynamics

Dynamical Susy generates ALL dynamics

$$[\,\delta_s^{\rm dyn}\,,\,\delta_{\bar s}^{\rm dyn}\,]\,\varphi^a \;=\; -\sqrt{2}\,\delta_{p^-}^{\rm dyn}\,\varphi^a$$

$$[\,\delta_{\overline{K}}^{\rm kin}\,,\,\delta_{p^-}^{\rm dyn}\,]\,\varphi^a \;=\; -\,2\,i\,\delta_{\bar{j}^-}^{\rm dyn}\,\varphi^a$$

Commutation relations determine

unique dynamical Susy

SOLE Effect of Interactions

in N=4 SuperYang-Mills:

$$\bar{\partial}\,\delta^{ab} \quad \rightarrow \quad (\bar{\partial}\,\delta^{ab} - g f^{abc} \partial^{+}\,\varphi^{c}\,) \;\equiv\; \mathcal{D}^{ab}$$

Transverse Covariant Derivative

$$\delta_{\bar{s}}^{\mathrm{dyn}}\,\varphi^{a} \;=\; \frac{1}{\partial^{+}}\,\mathcal{D}^{ab}\,\epsilon^{m}\,\bar{q}_{m}\,\varphi^{c}$$

Incredibly Simple!

Apply Same Algebraic Techniques
to find the
Light-Cone Superspace Formulation
of the

Super-Conformal Theory in D=3

Bagger, Lambert, Gustavsson

Use the N=4 Superfield

$$\varphi(y)^{A} = \frac{1}{\partial^{+}} A(y)^{A} + \frac{i}{\sqrt{2}} \theta^m \theta^n \overline{C}_{mn}^{A}(y) + \frac{1}{12} \theta^m \theta^n \theta^p \theta^q \epsilon_{mnpq} \partial^{+} \bar{A}(y)^{A}$$
$$+ \frac{i}{\partial^{+}} \theta^m \bar{\chi}_m^{A}(y) + \frac{\sqrt{2}}{6} \theta^m \theta^n \theta^p \epsilon_{mnpq} \chi^q(y)^{A}$$

since

OSp(2,2|8)

closes linearly (free theory)

on φ(y)

(A is some label)

$$SO(4,2) \longrightarrow Sp(2,2) \sim SO(3,2)$$

$$SU(4) \longrightarrow SO(8)$$

$$PSU(2,2|4) \longrightarrow OSp(2,2|8)$$

SO(8) ⊃ SU(4)XU(1)

U(1) generator

(was helicity)

$$J \;=\; -\,\frac{i}{4\sqrt{2}}\,(\,q^l\,\bar{q}_l\;-\;\bar{q}_l\,q^l\,)\,\frac{1}{\partial^+}$$

Coset generators

$$J^{mn} \;=\; \{\,q^m_-\,,\,s^n_+\,\}\;=\;\frac{i}{\sqrt{2}}\,q^m\,q^n\,\frac{1}{\partial^+}$$

$$\bar{J}_{mn} \;=\; \{\,\bar{q}_{-m}\,,\,\bar{s}_{+n}\,\}\;=\;\frac{i}{\sqrt{2}}\,\bar{q}_m\,\bar{q}_n\,\frac{1}{\partial^+}$$

Action on the superfield

$$\delta_{U(1)}\,\varphi^a \;=\; \omega J\,\varphi^a$$

$$\delta_{coset}\,\varphi^a \;=\; \frac{i}{2\sqrt{2}}\,\omega^{mn}\bar{q}_m\,\bar{q}_n\,\frac{1}{\partial^+}\,\varphi^a$$

Constraints on Susy transformations

$$\delta^{\rm dyn}_{\bar{s}}\,\varphi^a \;=\; \frac{i}{\sqrt{2}}\frac{\partial}{\partial^+}\epsilon^m\bar{q}_m\,\varphi^a + \delta^{\rm int}_{\bar{s}}\,\varphi^a$$

$$[\,\delta_{U(1)}\,,\,\delta_s\,]\,\varphi^a \;=\; \frac{\omega}{2}\,\delta_s\varphi^a$$

SO(8) vector:

$$[\,\delta_{coset}\,,\,\delta_s\,]\,\varphi^a \;=\; \delta'_{\bar{s}}\,\varphi^a \qquad\qquad \epsilon^{m'} \;=\; \omega^{mn}\,\bar{\epsilon}_n$$

$$[\,\delta_{\overline{coset}}\,,\,\delta_s\,]\,\varphi^a \;=\; 0$$

Desperately Seeking Susy

$$\delta_{\bar{s}}^{\mathrm{dyn}}\,\varphi^a \;=\; \frac{i}{\sqrt{2}}\,\frac{\partial}{\partial^+}\,\epsilon^m\,\bar{q}_m\,\varphi^a \quad + \;\delta_{susy}^{int}\varphi^a\,?$$

(free theory)

In D=3 Dim φ is half-odd integer

$$\delta_{susy}^{int}\varphi^a \;\sim\; \varphi^b\varphi^c\varphi^d$$

$$\longrightarrow \quad f^{abcd}$$

Technology

$$\widehat{\mathcal{O}} \rightarrow \frac{1}{\partial^+}\mathcal{O}$$

coherent states

$$E_\eta = e^{\eta\cdot\hat{\bar{d}}} \qquad E_\epsilon = e^{\epsilon\cdot\hat{\bar{q}}}$$

$$E_{\bar{\epsilon}\eta} = e^{\bar{\epsilon}\cdot\hat{q}}\,E_\eta \qquad E_z = e^{z\,\hat{\partial}}$$

d- & q-eigenstates

$$d^m\,E_\eta\,\varphi = i\sqrt{2}\,\eta^m\varphi$$

$$q^m\,E_\epsilon\,\varphi = -\,i\sqrt{2}\,\epsilon^m\varphi$$

chiral engineering

two chiral fields

$$Z^{bc(\eta)} \;=\; \frac{1}{\partial^{+A}}\left\{ E_\eta \partial^{+B}\, \varphi^b\, E_\eta^{-1} \partial^{+C}\, \varphi^c \right\}$$

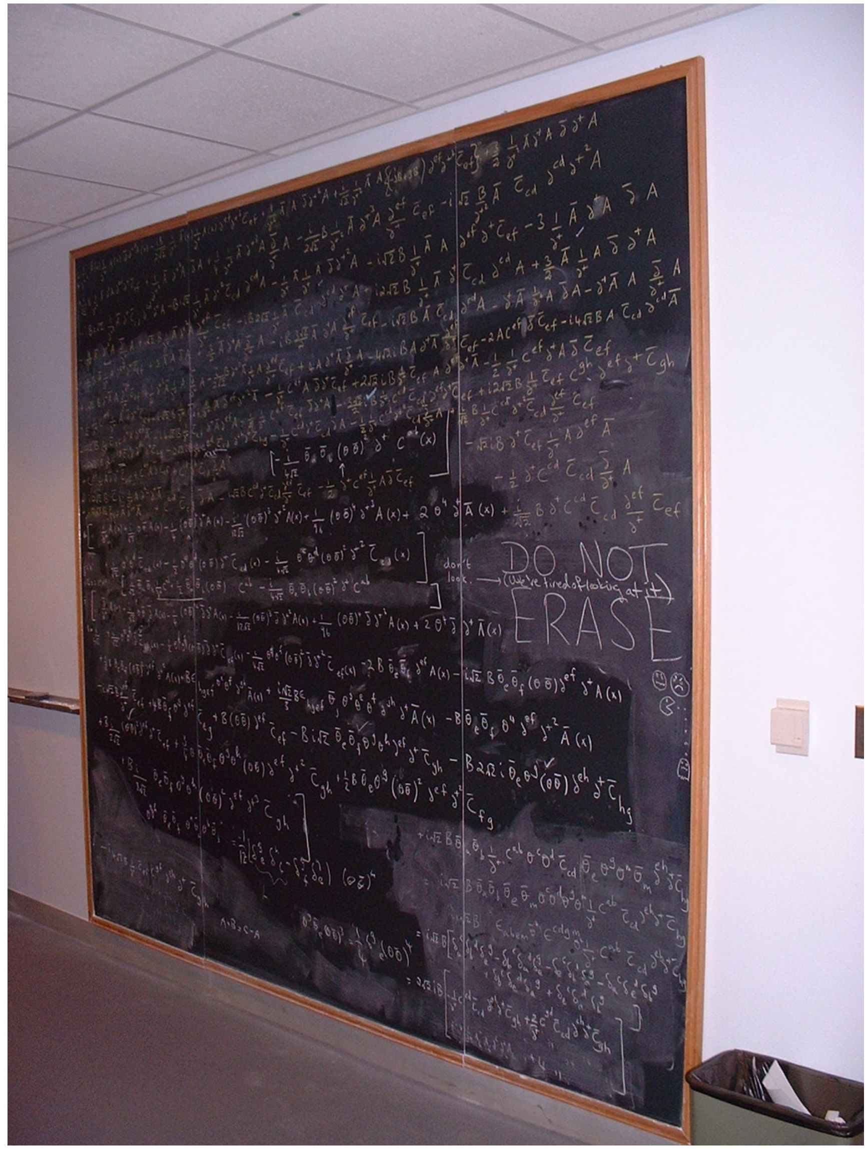
$$d^m\, Z^{bc(\eta)} \;=\; 0$$

three chiral fields: nested construction

$$\Delta_\alpha^{a\,(\bar\epsilon\eta,\zeta)} \;=\; \frac{f^{abcd}}{\partial^{+A_\alpha}}\left\{ E_{\bar\epsilon\,\eta} \partial^{+B_\alpha}\, \varphi^b\, E_{\bar\epsilon\,\eta}^{-1} \frac{1}{\partial^{+M_\alpha}}\left[E_\zeta \partial^{+C_\alpha}\, \varphi^c\, E_\zeta^{-1} \partial^{+D_\alpha} \varphi^d \right] \right\}$$

$$d^m\, \Delta_\alpha^{a\,(\bar\epsilon\eta,\zeta)} \;=\; 0$$

after some elementary manipulations...



two independent ansätze

both transform as $\underset{\sim}{8}$ of SO(8)

$$\delta_{\bar{s}}^{\rm int}\,\varphi^a \;=\; \sum_{\alpha=1/2,-1/2}(-1)^\alpha\,\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}}\,\Delta_\alpha^{a\,(\bar\epsilon\eta,\zeta)}\,|_{\eta=\zeta=0}$$

and/or

$$\sum_{\alpha=-1,0,1}(-1)^\alpha\,\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}}\,\Delta_\alpha^{a\,(\bar\epsilon\eta,\zeta)}\,|_{\eta=\zeta=0}$$

triality

$$\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}}\;\equiv\;\frac{\epsilon^{i_1\cdots i_{2-2\alpha}\cdots i_4}}{(2+2\alpha)!(2-2\alpha)!}\,\frac{\partial}{\partial\eta^{i_1\cdots i_{2-2\alpha}}}\,\frac{\partial}{\partial\zeta^{i_{3-2\alpha}\cdots i_4}}$$

$$A_{\alpha-1}=A_\alpha+1\qquad B_{\alpha-1}=B_\alpha+1\qquad M_{\alpha-1}=M_\alpha-2$$

$$C_{\alpha-1}=C_\alpha-1\qquad D_{\alpha-1}=D_\alpha-1$$

Light-Cone Hamiltonian

$$[\,\delta_s^{\rm free}+\delta_s^{\rm int}\,,\,\delta_{\bar s}^{\rm free}+\delta_{\bar s}^{\rm int}\,]\varphi^a \;=\; \delta_{p^-}^{\rm dyn}\,\varphi^a$$

terms linear in f^{abcd}

$$[\,\delta_s^{\rm free}\,,\,\delta_{\bar s}^{\rm int}\,]\varphi^a+[\,\delta_s^{\rm int}\,,\,\delta_{\bar s}^{\rm free}\,]\varphi^a$$

$$\delta_{p^-}\,\varphi^a \;=$$

$$i\bar{\epsilon}\cdot\epsilon\frac{\partial}{\partial z}\left\{\sum_{\alpha=\frac{1}{2},-\frac{1}{2}}(-1)^{\alpha}\,\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}}\Delta_{\alpha}^{a\,(z\eta,\zeta)}+\sum_{\alpha=-1,0,1}(-1)^{\alpha}\,\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}}\Delta_{\frac{1}{2}+\alpha}^{a\,(\eta,z\zeta)}\right\}_{z=\eta=\zeta=0}$$

$$E_z \;=\; e^{z\,\hat{\partial}} \qquad\qquad E_{z\eta} \;= E_z E_\eta$$

Conformal generator

$$K \;=\; -\,2i\,x\,(\frac{1}{2}\,x\,\partial - x^-\,\partial^+ + \theta^m\,\frac{\partial}{\partial\theta^m} + \bar{\theta}_m\,\frac{\partial}{\partial\bar{\theta}_m})$$

From Hamiltonian to Boost

$$[\,K\,,\,p^-\,] \;=\; -\,2i\,j^-$$

$$[\,j^-\,,\,p^-\,] \;=\; 0$$

yet to be checked

Consistency being checked (with Belyaev, Brink and S-S Kim)

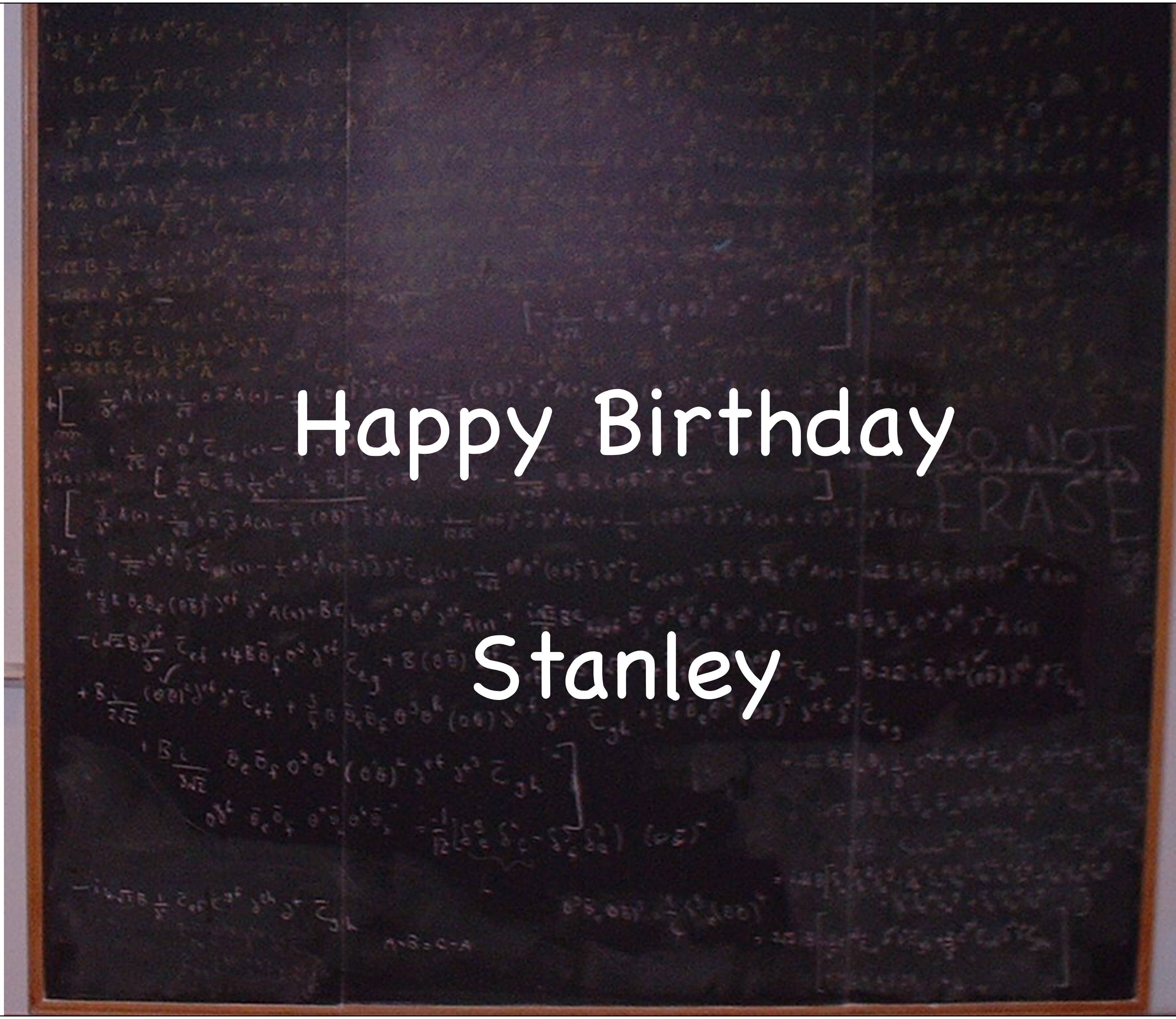
Work in Progress

Check boost and Hamiltonian commute

Compute f-f term

Properties of f^{abcd} from commutation

Path Integral measure over φ



Happy Birthday

Stanley