RECENT PROGRESS IN AdS/CFT

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I was a graduate student in UC Berkeley for the four-year period 1962-66. This was an exciting time to be there, both scientifically and politically.

My advisor was Geoffrey Chew, but Stanley Mandelstam was also important in my education. For example, I took an excellent and challenging course in Quantum Field Theory from Stanley.

Geoff was quite skeptical about the possibility of QFT describing the strong interactions. Stanley helped to provide a more balanced picture.

Some of Stanley's Major Contributions

- Analytic S Matrix and Regge theory: Double dispersion relations (Mandelstam representation), Regge cuts, etc.
- Light-cone superspace and finiteness of $N = 4$ SYM
- Fermion-fermion scattering in RNS model
- The interacting-string picture in light-cone gauge

Stanley's work has been discussed already by other speakers, so I will now describe briefly some recent progress concerning holographic gauge theory/string theory equivalences (also known as AdS/CFT duality).

Some of you have heard me speak about this previously. This will be an abridged version.

Introduction

There are three maximally supersymmetric examples of AdS/CFT duality:

- **M2-brane Duality:** M theory on $AdS_4 \times S^7$ is dual to a maximally supersymmetric SCFT in 3d.
- **D3-brane Duality:** Type IIB superstring theory on $AdS_5 \times S^5$ is dual to a maximally supersymmetric SCFT in 4d ($\mathcal{N} = 4$ SYM theory).
- **M5-brane Duality:** M theory on $AdS_7 \times S^4$ is dual to a maximally supersymmetric SCFT in 6d.

The Type IIB / $\mathcal{N} = 4$ SYM Example

The planar (large N, fixed $\lambda = g^2 N$) approximation to $SU(N)$ $\mathcal{N} = 4$ SYM theory is believed to be given by an integrable spin-chain model. Also, type IIB superstring theory on $AdS_5 \times S^5$ is described in tree approximation by an integrable world-sheet action.

As a result of a great deal of ingenious work by many people, the equivalence of these two integrable theories has been largely, but not completely, demonstrated.

The two M-theory dualities are much tougher: M-theory is less well understood than type IIB superstring theory, since it does not have a perturbation expansion. Also, the dual SCFTs in 3d and 6d are much more elusive.

Within the past year or so, there has been a great deal of progress in formulating the 3d SCFT that is dual to M-theory on $AdS_4 \times S^7$, as well as certain orbifold generalizations. This talk will discuss these developments.

SCFT in Three Dimensions

In 2004 I proposed that the 3d SCFTs that are dual to M-theory on $AdS_4 \times K_7$ should be gauge theories in which the YM fields appear in Chern–Simons terms

$$S_{\rm CS} = \frac{k}{4\pi} \int \omega_3,$$

since these have dimension 3. Recall that the CS 3-form ω_3 satisfies $d\omega_3 = {\rm tr}(F \wedge F)$, and the integer k is called the level. There should be no kinetic term for the gauge fields, since it would have dimension 4.

In the case of maximal supersymmetry ($K_7 = S^7$), the SCFT should also contain matter fields consisting of an equal number of scalars X and spinors Ψ. These belong to inequivalent **8**s of the $Spin(8)$ R-symmetry group.

The dimension of X is $1/2$ and the dimension of Ψ is 1, so the possible dimension-3 terms are schematically:

$$(DX)^2, \quad \Psi D\Psi, \quad X^2\Psi^2, \quad X^6.$$

even though I was on the right track I did not find any SCFTs with $\mathcal{N} = 8$ susy. My mistake was to assume that the gauge group should be $SU(N)$ and that the matter should belong to the adjoint representation. These assumptions were based on analogy with $\mathcal{N} = 4$ SYM.

The floodgates opened following a breakthrough in November 2007 by Bagger and Lambert and by Gustavsson. The definitive paper by Aharony, Bergman, Jafferis, and Maldacena appeared in June 2008.

The BLG Construction

BLG associate an $\mathcal{N} = 8$ SCFT to every 3-algebra $\mathcal{A}$, whose generators, T^A, have a totally antisymmetric bracket

$$[T^A, T^B, T^C] = f^{ABC}{}_D T^D.$$

The structure constants $f^{ABC}{}_D$ must satisfy a generalization of the Jacobi identity. There must also be a metric (nondegenerate and symmetric) h^{AB}, such that

$$f^{ABCD} = h^{DE} f^{ABC}{}_E$$

has total antisymmetry.

Theorem (Papadopoulos; Gauntlett, Gutowski):

There is a unique nontrivial 3-algebra with a positive-definite metric. It has four generators. The structure constants and metric are given by

$$f^{ABCD} = \varepsilon^{ABCD} \quad \text{and} \quad h^{AB} = \delta^{AB}.$$

The BLG $SO(4)$ Theory

The four-generator 3-algebra gives a BLG theory with $SO(4)$ gauge symmetry. Identifying $SO(4) = SU(2) \times SU(2)$, the 'twisted' Chern–Simons term in the BLG construction is actually the difference of the Chern–Simons terms for the two $SU(2)$ factors. There is a quantized coupling constant $g^2 = 1/k$, where k is the CS level.

This theory is parity conserving. Parity combines interchange of the two $SU(2)$s with spatial reflection.

Lorentzian 3-Algebras

In May 2008 three groups introduced new BLG theories based on Lorentzian signature 3-algebras that are extensions of compact semisimple Lie algebras.

The 3-algebra has two generators T^+ and T^- in addition to the generators T^a of a Lie algebra $\mathfrak{g}$ with structure constants ${f^{ab}}_c$. The structure constants are given by

$$ {f^{+ab}}_c = {f^{ab}}_c \,, $$

with all other nonzero components of ${f^{ABC}}_D$ related by permuting, raising, or lowering indices.

The invariant metric of the 3-algebra is given by

$$h^{+-} = -1, \qquad h^{++} = 0,$$
$$h^{--} = 0, \qquad h^{ab} = \delta^{ab}.$$

As originally formulated, the Lorentzian BL theory contains propagating ghost degrees of freedom, which makes the theory unsatisfactory.

Gauging the Shift Symmetries

The Lorentzian theory is invariant under constant shifts of X_{-}^{I} and Ψ_{-}^{α}. Bandres, Lipstein, and I suggested modifying the theory by adding new fields and interactions that makes the global shift symmetries into local symmetries. This preserves the full $\mathcal{N} = 8$ superconformal symmetry.

It turns out that the modified theory is precisely equivalent to $\mathcal{N} = 8$ super Yang-Mills theory in 3D. This is correct, but disappointing, because the original goal was to discover a dual description of the IR fixed-point $g_{\rm YM} \to \infty$.

Orbifold Theories

Distler, Mukhi, Papageorgakis, Van Raamsdonk suggested that the $SO(4)$ BLG theory at level k describes two M2-branes at a $\mathbb{C}^4/\mathbb{Z}_k$ orbifold singularity. This idea is basically correct, though the BLG theory isn't exactly the right one.

The orbifold action breaks the $SO(8)$ R-symmetry to an $SU(4) \sim SO(6)$ subgroup when $k > 2$. For $k = 1, 2$ the $SO(8)$ R-symmetry is unbroken. Thus there is only $\mathcal{N} = 6$ for $k > 2$. The same is required of the dual SCFTs.

The ABJM Theories

ABJM constructed $\mathcal{N} = 6$ superconformal Chern–Simons theories with the gauge group $U(N) \times U(N)$ and bifundamental matter. N corresponds to the number of flux units $\int *F_4$ on $S^7/\mathbb{Z}_k$.

Bandres, Lipstein, and I proved that the ABJM theories have the required $\mathcal{N} = 6$ superconformal symmetry. (Only Poincaré supersymmetry and scale invariance were established by ABJM.)

Type IIA Formulation

It is useful to express S^7 as an S^1 bundle over CP^3. The orbifold group acts only on the S^1 fibre – reducing the radius of the circle by a factor of k: $R_{11} = R/k$

M-theory compactified on a circle is type IIA superstring theory with $g_s \sim R_{11}^{3/2}$ (in 11d Planck units). So, since $R_{11} = R/k$, we have weakly coupled string theory on $AdS_4 \times CP^3$ for $k \gg N^{1/5}$. A supergravity approximation is valid if one also has $N \gg k$.

The ABJM SCFT has a perturbation expansion in powers of $1/k$. Moreover, for fixed 't Hooft parameter

$$\lambda = N/k,$$

it has a large-N expansion in powers of $1/N$. This has the usual topological interpretation: The leading term is the planar approximation, etc.

Just as in the familiar case of AdS5/CFT4, the large-N SCFT is weakly coupled for small λ, and stringy effects of the AdS theory are suppressed for large λ.

Other Related Dualities

It was noted by Hosomichi, Lee, Lee, Lee, and Park that $\mathcal{N} = 6$ SCFTs also exist for $U(M)_k \times U(N)_{-k}$ and that $\mathcal{N} = 5$ SCFTs exist for $O(M)_{2k} \times USp(2N)_{-k}$.

The relevant dualities for these cases were explained by Aharony, Bergman, and Jafferis (ABJ).

Conclusions

Various $\mathcal{N} = 5, 6$ 3d SCFTs with $k \gg N^{1/5}$ are dual to weakly coupled Type IIA superstring theory on $AdS_4 \times CP^3$ with appropriate fluxes.

Just as in the case of AdS5/CFT4, it is plausible that these SCFTs are exactly solvable for all $\lambda = N/k$ in the planar (large-N) limit and that the dual superstring world-sheet theories are also exactly solvable. There is already some evidence to that effect.

Happy Birthday

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