Large N Gauge Theory on the Lightcone Worldsheet

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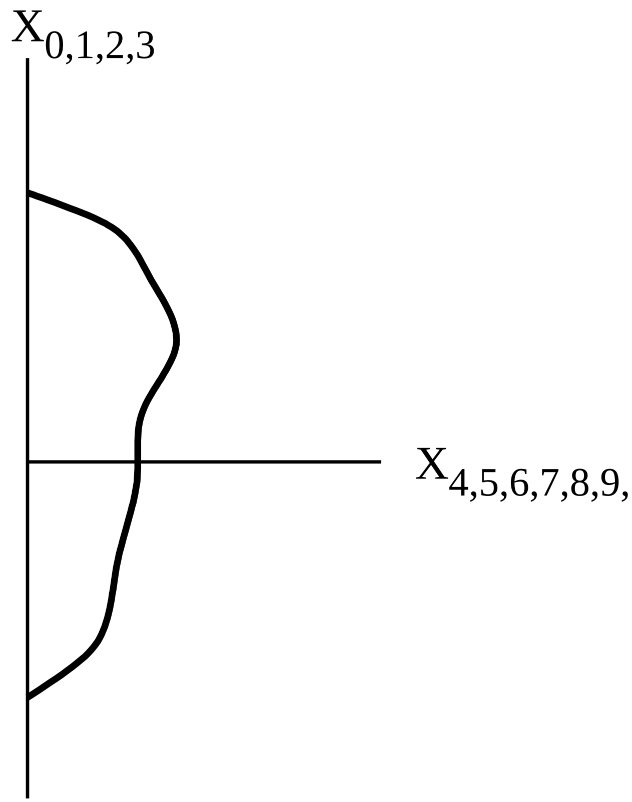
String Basis for Field/String Duality

Open String with SU(N) Chan-Paton $\underset{\alpha' \to 0}{\Longrightarrow}$ SU(N) Yang-Mills
(Scherk, Neveu and Scherk, 1971)

Left side a regulated version of right side

Open String Trees $\Rightarrow$ All String Tree and Loop Diagrams
($\alpha' > 0$) (1970)

D3-branes and 4D QFT (Polchinski, 1989)



$$x^M(\sigma,\tau) : \begin{cases} \text{Neumann b.c.}'\text{s} & M = 0,1,2,3 \equiv \mu \\ \text{Dirichlet b.c.}'\text{s} & M = 4,5,6,7,8,9 \end{cases}$$

't Hooft's $N \to \infty$:

$$\sum(\text{Planar Open String Loops})_{D3} \equiv \sum(\text{Closed String Trees})_{\text{bckgrnd}}$$

Left Side $\underset{\alpha' \to 0}{\Longrightarrow}$ $N = \infty$ Gauge Theory in 4d

Right Side $\underset{\substack{\alpha' \to 0 \\ g^2 N \to \infty}}{\Longrightarrow}$ Classical gravity

If $g^2 N = O(1)$, right side stays stringy as $\alpha' \to 0$.

Theme of this talk:

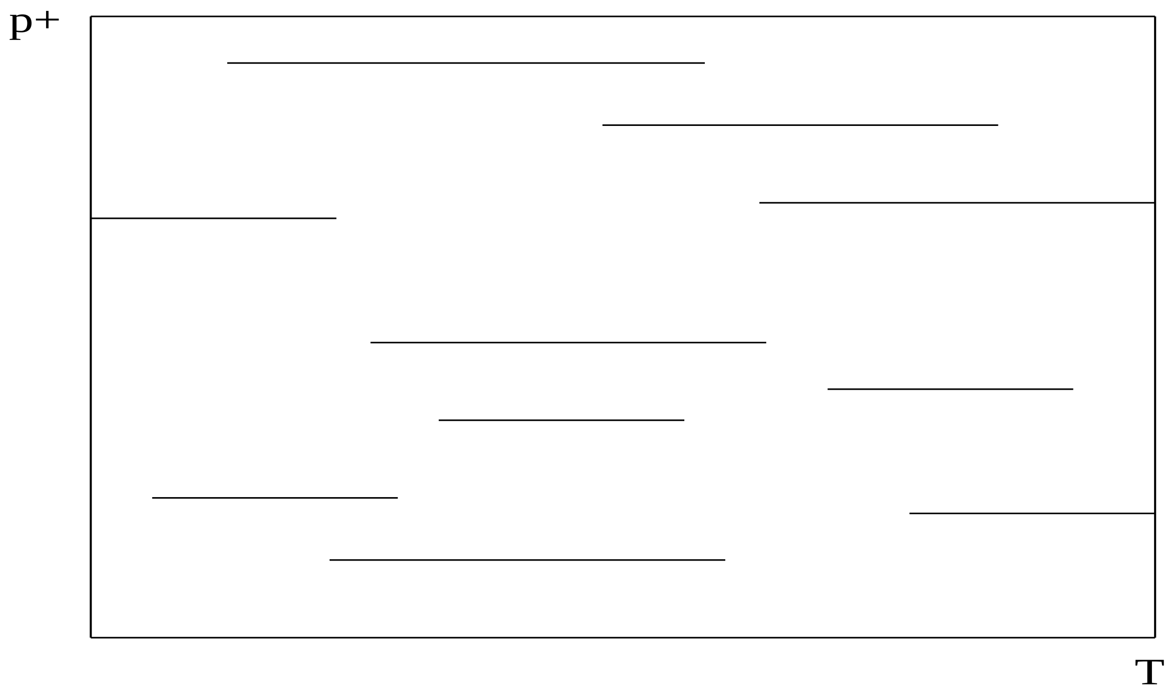
Why not try planar graph summation with $\alpha' > 0$?

Summing Planar Diagrams on the Lightcone

Mandelstam's lightcone path integral formalism provides a systematic approach to this problem

Potentially an effective numerical assault on the problem

Mandelstam Interacting String Diagram:



$$T = ix^+ = i(t+z)/\sqrt{2}, \qquad p^+ = (p^0 + p^z)/\sqrt{2}.$$

Diagram describes time evolution of a system of open strings, breaking and rejoining as shown by the horizontal lines.

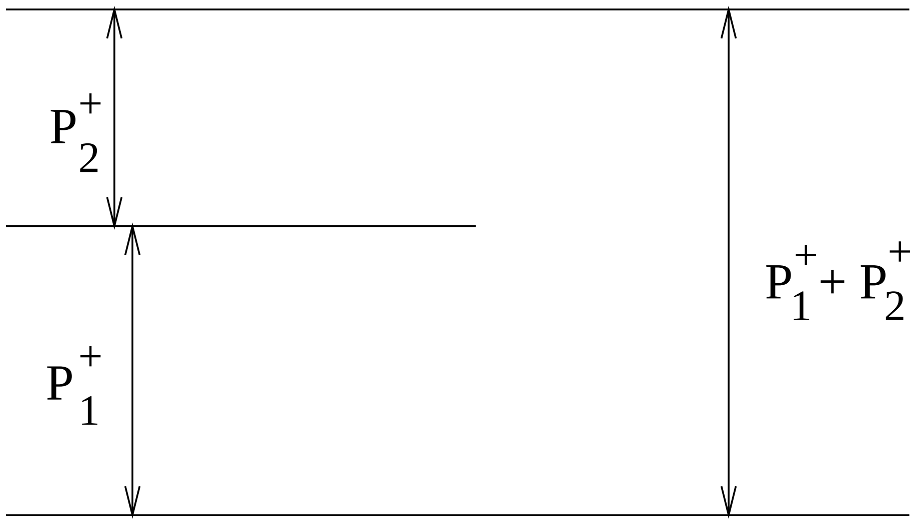
For **critical** open string, worldsheet path integral uses lightcone action for the free open string:

$$S_{l.c.} = \frac{1}{2}\int_0^T d\tau \int_0^{P^+} d\sigma \left[\left(\frac{\partial \boldsymbol{x}}{\partial \tau}\right)^2 + T_0^2 \left(\frac{\partial \boldsymbol{x}}{\partial \sigma}\right)^2\right]$$

$\sum$(planar diagrams)$=\sum$(#, length, location of horizontal lines).

For each beginning and end of a horizontal line there is a factor of string coupling $g\times$(prefactor).

Normalization



Lorentz covariance: Vertex $\sim \frac{1}{\sqrt{P_1^+ P_2^+ (P_1^+ + P_2^+)}}$

So under $P_i^+ \to \lambda P_i^+$, above diagram should scale as $\lambda^{-3/2}$.

Worldsheet Lattice calculation gives $\lambda^{-(D-2)/16}$ for bosonic string
$D = 26$ for Lorentz covariance (Giles-CBT,1977)

Hearing the Shape of a (Polygonal) Drum (Mark Kac, 1967)

$$\mathrm{Tr}\; e^{t\nabla^2/2} \sim \frac{\text{Area}}{2\pi t} - \frac{\text{Perimeter}}{4\sqrt{2\pi t}} + \sum_{\text{corners}} \frac{1}{24}\left(\frac{\pi}{\theta} - \frac{\theta}{\pi}\right) + o(1)$$

Lightcone vertex is a 360° corner. Putting $\theta = 2\pi$,

$$\frac{1}{24}\left(\frac{\pi}{\theta} - \frac{\theta}{\pi}\right) \to -\frac{1}{16}$$

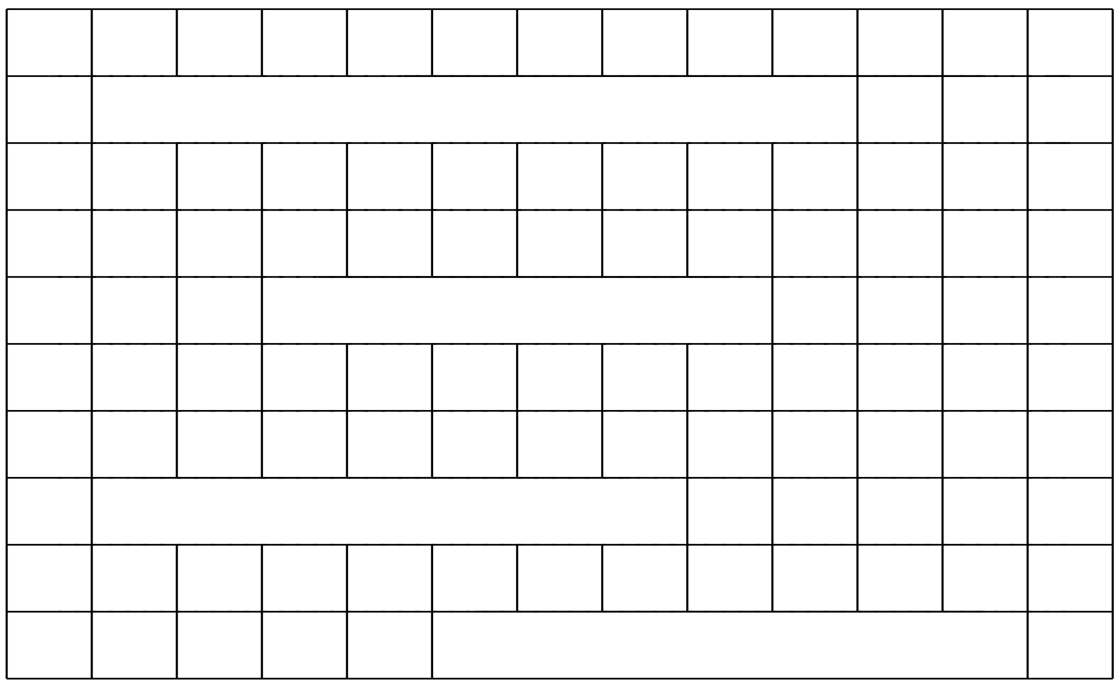
Notice that rounding the corner would spoil this nice result

A case where smoother is NOT better!

Lightcone Worldsheet for Planar Sum

Mark the presence or absence of a horizontal line at any point by an Ising spin variable $S(\sigma, \tau) = 1, 0$.

Worldsheet Lattice (Bosonic String, Giles-CBT, 1977):

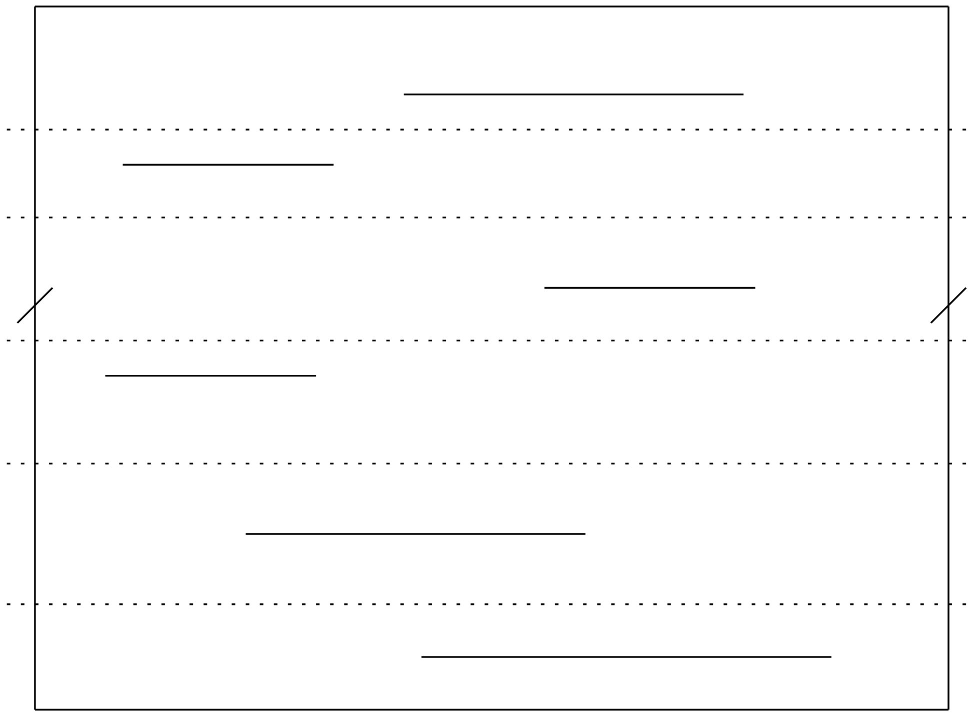


$$
\begin{aligned}
S \quad \to \quad & \frac{1}{2}\sum_{ij}\left[(x_i^{j+1}-x_i^j)^2+T_0^2 S_i^j (x_{i+1}^j - x_i^j)^2\right] \\
& - \sum_{ij}\left[S_i^j(1-S_i^{j+1})+S_i^{j+1}(1-S_i^j)\right]\ln g
\end{aligned}
$$

Monte Carlo simulations very feasible for bosonic string (Peter Orland).

Less promising with worldsheet fermions (NSR, Superstring)

Alternate Approach to Planar Sum (Kruczenski)



A geometric sum: adds a hole operator to the free closed string Hamiltonian.

String dual for given QFT

AdS/CFT Paradigm:

Lift $\mathcal{N} = 4$ Yang-Mills to NSR/GSO Open String ending on D3-branes in 10D Minkowski space-time

Broken 10D translation invariance:
p^μ has 4 space-time components.

Bulk of open string vibrates in all 10 space-time dimensions.

Massless states:

- an adjoint vector: vibrations $\parallel$ D3-branes,
- 6 adjoint scalars: vibrations $\perp$ D3-branes,
- 4 Majorana fermions.

An Open String for Pure 4D Yang-Mills?

Delete fermionic states (no R sector)

Even G-parity sector of Neveu-Schwarz (NS+) open string

Simplest choice for YM: NS+ model in 4D

no extra scalars and massive would-be graviton –but ...

Conformal anomaly $\Rightarrow$ technical complications in closed sector

Keeping 10D $\Rightarrow$ conformal anomaly cancels.

But: Usual D3-brane trick $\Rightarrow$ 6 massless scalars in the 4D theory

Consider NS open string in $D = 9$:

$$Z(w) = (1 - w^{1/2})\frac{\prod_r (1 + w^r)^8}{\prod_n (1 - w^n)^8}$$

Compared to 10D NS open string ending on a D8-brane:

$$Z(w) = \frac{\prod_r (1 + w^r)^8}{\prod_n (1 - w^n)^8}$$

First case: 7 massless states (9D gluon)

Second case: 8=7+1 massless states (9D gluon + 1 scalar)

Goal: Modify D8 b.c.'s to achieve same spectrum

T-dual D-brane conditions

For a D8-brane at $x^9 = 0$:

$$x^9(0,\tau) \quad = \quad x^9(\pi,\tau) = 0$$

T-dual transform $x^9(\sigma,\tau) \rightarrow y^9(\sigma,\tau)$:

$$\frac{\partial y^9}{\partial\sigma}(0,\tau) = \frac{\partial y^9}{\partial\sigma}(\pi,\tau) = 0.$$

Then the zero mode of y^9:

$$p_0^9 \equiv \int d\sigma \dot{y}^9(\sigma,\tau) = x^9(\pi,\tau) - x^9(0,\tau) = 0$$

SU(2) Invariance

Interpret y^9 as a $c = 1$ conformal scalar field, compactified on a circle: $p_0^9 = 2\pi n/R$.

SU(2) symmetry emerges when R is such that $|0, \pm 2\pi/R\rangle$ are massless.
$(b^9_{-1/2}|0\rangle, |0, \pm 2\pi/R\rangle)$ transform as a vector

Invariance under SU(2) $\Rightarrow n = 0$ **and** projects out $b^9_{-1/2}|0\rangle$

Repeat for x^8, x^7, x^6, x^5, x^4:

SU(2) invariance for each of 6 extra coordinates, projects out all massless scalars in open string state space.

Vertex operator construction of SU(2) generators

$$\begin{aligned} J_3 &= p_0^I \sqrt{2\alpha'} \\ J_\pm &= \sqrt{2} \oint \frac{dz}{2\pi i z} H^I(z) : e^{\pm i y^I(z)/\sqrt{2\alpha'}} : \end{aligned}$$

$: e^{iy^I(z)/\sqrt{2\alpha'}} :$ is a bosonized fermionic field,
when acting on the states with $p_0^I \in \mathbb{Z}/\sqrt{2\alpha'}$.

$[J_+, J_-] = 2J_3$.

Essential point:

$[J_\pm, G_r] = [J_\pm, L_n] = 0$ on this subspace,

so the SU(2) commutes with physical state conditions

Manifestly O(3) Invariant Description

Replace each bosonic y^I with a pair of fermion fields H_1^I, H_2^I

Call original $H^I \equiv H_3^I$.

Then H_a transform as a vector under $O(3)$ with generators

$$J^a = \epsilon^{abc} \oint \frac{dz}{2\pi i z} H_b(z) H_c(z).$$

Nonabelian D-brane condition: $J^a |\text{Phys}\rangle = 0$.

Scattering Amplitudes

Trees are subset of 10D NS trees:

Take external strings only in even G-parity states invariant under 6 SU(2)'s associated with the 6 extra dimensions.

Noninvariant states decouple, including the massless scalars:

$b^I_{-1/2}|0,p\rangle$, $I = 4, 5, \ldots, 9$

One Loop and Closed Strings

Loops require projectors onto SU(2) invariant states.

At one loop simply multiplies partition function by $(1 \mp w^{1/2})^6$
Also loop momentum integral only over p^μ, $\mu = 0, 1, 2, 3$

After Jacobi transformation to cylinder variables $\ln q = 2\pi^2 / \ln w$, these differences produce the extra factors

$$\left[\sqrt{\frac{-\pi}{\ln q}}(1 \mp w^{1/2})\right]^6 = \begin{cases} \left[\int d\mu q^{\mu^2/4} \sin^2 \frac{\mu}{2\sqrt{2}}\right]^6 \\ \left[\int d\mu q^{\mu^2/4} \cos^2 \frac{\mu}{2\sqrt{2}}\right]^6 \end{cases}$$

relative to the usual one loop integrand in $D = 10$.

O(3) Projectors

Projector for the Ith SU(2):

$$P_I = \int dR e^{i\theta_a J_I^a}, \qquad J_I^a = \int d\sigma \mathcal{J}_I^a(\sigma)$$

dR is the O(3) invariant Haar measure.

Apply $P = \prod_{I=4}^{9} P_I$ to each open string propagator participating in a loop.

Gauging the O(3)'s

Since $P^2 = P$ we can put a projector on each time-slice of each string propagator. Introduce independent R's for each point σ, t:

$$
\begin{aligned}
P_I &= \prod_t P_I = \int \prod_t dR(t) e^{i\theta_a(t) J_I^a} \\
&= \int \prod_t \prod_\sigma dR(\sigma, t) e^{i \int d\sigma \theta_a(\sigma,t) \mathcal{J}_I^a} \delta(R'(\sigma, t))
\end{aligned}
$$

Delete the $\delta(R'(\sigma, t))$ factor whenever σ sits on a horizontal line.

We have gauged each O(3) symmetry on the worldsheet, replacing the factor $e^{-\int F^2/4}$ with $\prod_{\sigma,t} \delta(F(\sigma, \tau))$.

Worldsheet local prescription for inserting projectors.

Summary

- Open String ($\alpha' > 0$) determines/regulates gauge theory
- Open/Closed Duality $\underset{\alpha' \to 0}{\Longrightarrow}$ Field/String Duality
- Open String for 4D Yang-Mills:
  10D NS+ with nonabelian D3-brane b.c.'s on 6 dimensions
- Lightcone path integrals: a non-perturbative formulation for $\sum$(Planar Diagrams)

Happy Birthday, Stanley!