

The Formation of Star Clusters



Jonathan Tan

University of Florida & KITP

In collaboration with:

Brent Buckalew (ERAU),
Michael Butler (UF u-grad),
Jayce Dowell (Indiana PhD),
Audra Hernandez (UF PhD),
Richard Klein (UCB),

Mark Krumholz (Princeton),
Elizabeth Lada (UF)
Christopher McKee (UCB),
Stella Offner (UCB PhD),
Elizabeth Tasker (UF)

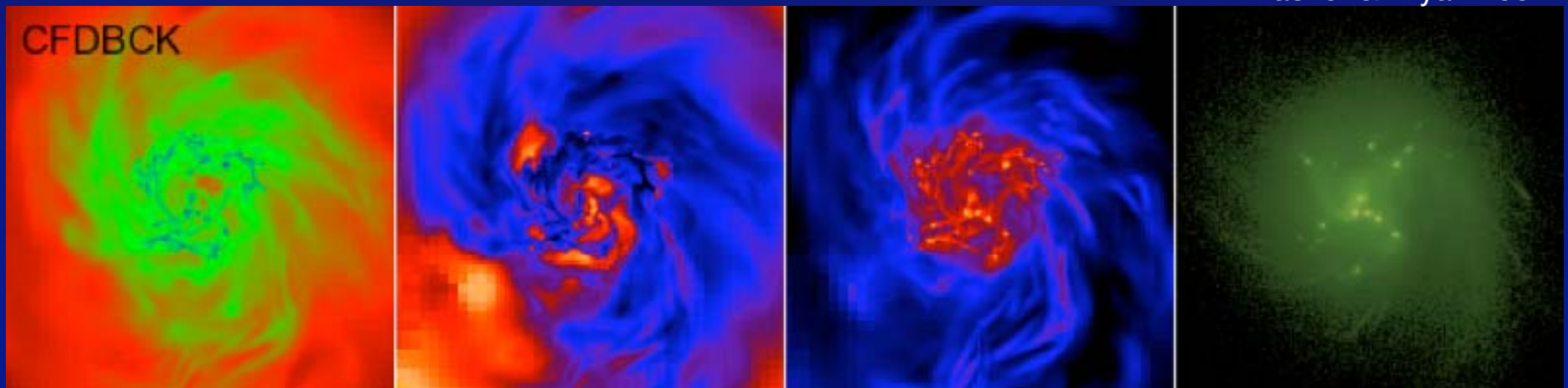
Star Cluster Formation

Important because most present-day star formation occurs in clusters, including essentially all massive star formation.

Creating the initial conditions for star cluster formation (i.e. clump formation rather than GMC formation) may be the rate limiting step of the Kennicutt-Schmidt law, since in the star-forming disk the gas mass fraction in GMCs is high.

Global galaxy simulations will need to resolve down to $< \sim \text{pc}$ scales, then use a sub-grid model for star cluster formation.

Tasker & Bryan 2007



Outline: star cluster formation

- Observed properties
- Initial conditions: quasi-equilibrium
- Formation timescale: long
- Mode of (massive) star formation: turbulent fragmentation
- Feedback: outflows + ionization

Star Formation: A complicated, nonlinear process

Physics:

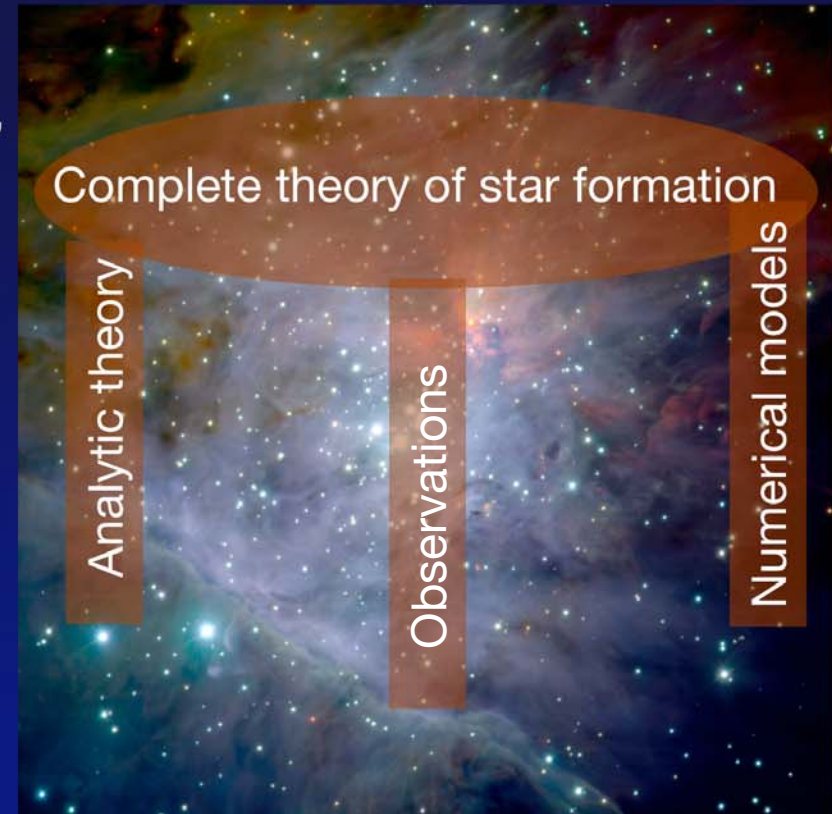
Gravity vs pressure (thermal, magnetic, turbulence, radiation, cosmic rays) and shear.

Heating and cooling, decay and sources of turbulence, diffusion of B-fields, generation of B-fields (dynamo), etc.

Chemical evolution of dust and gas.

Wide range of scales (~ 10 dex in space, time) and multidimensional.

Uncertain/unconstrained initial conditions/boundary conditions.

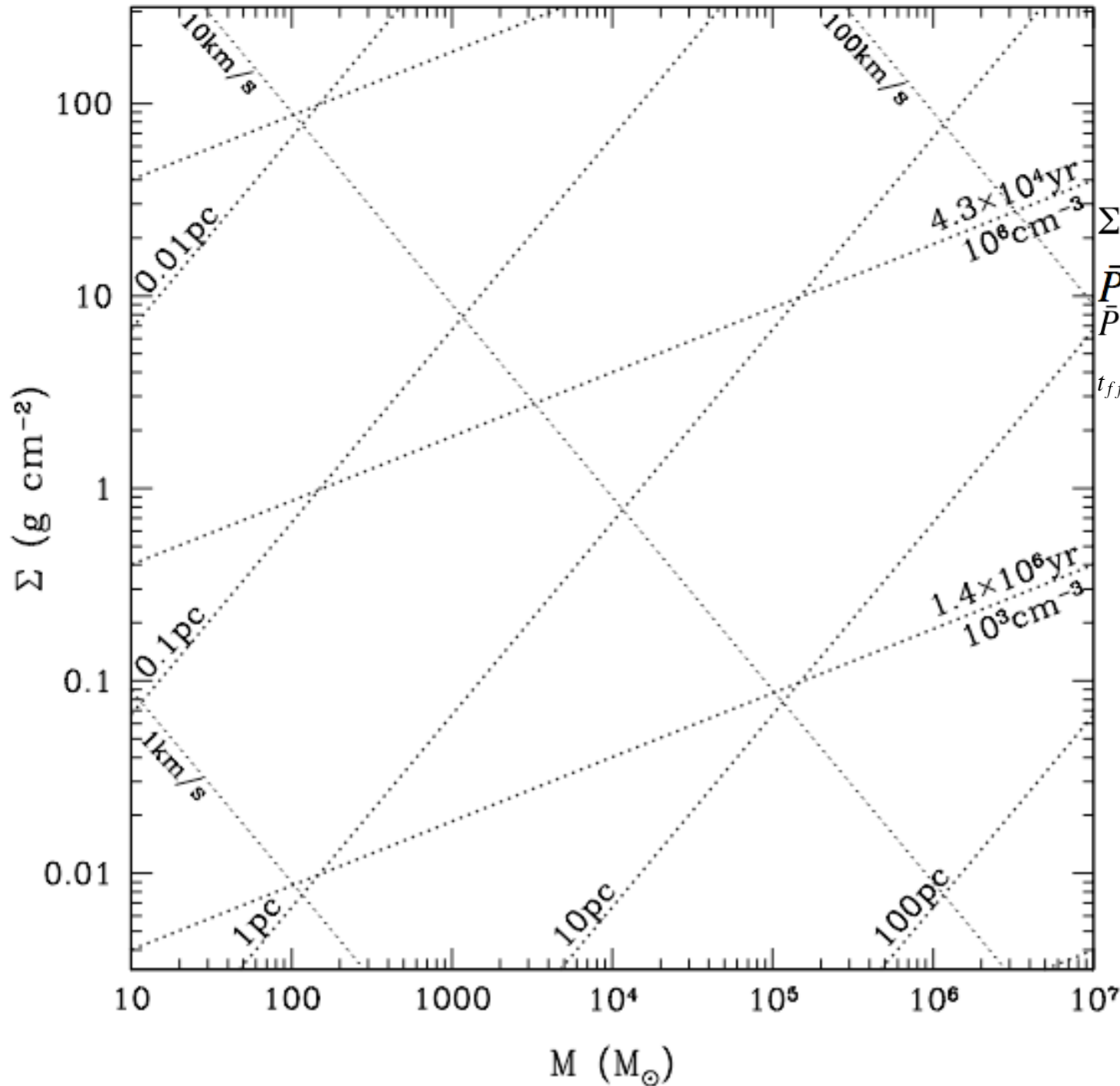


Star Formation: Open Questions

- Causation: external triggering or spontaneous gravitational instability?
- Initial conditions: how close to equilibrium?
- Accretion mechanism: turbulent fragmentation vs competitive accretion
- Timescale
- End result
 - Initial mass function (IMF)
 - Binary fraction and properties
 - Initial cluster mass function (ICMF)
 - Efficiency and Rate

How do these properties vary with environment?

Overview of Physical Scales



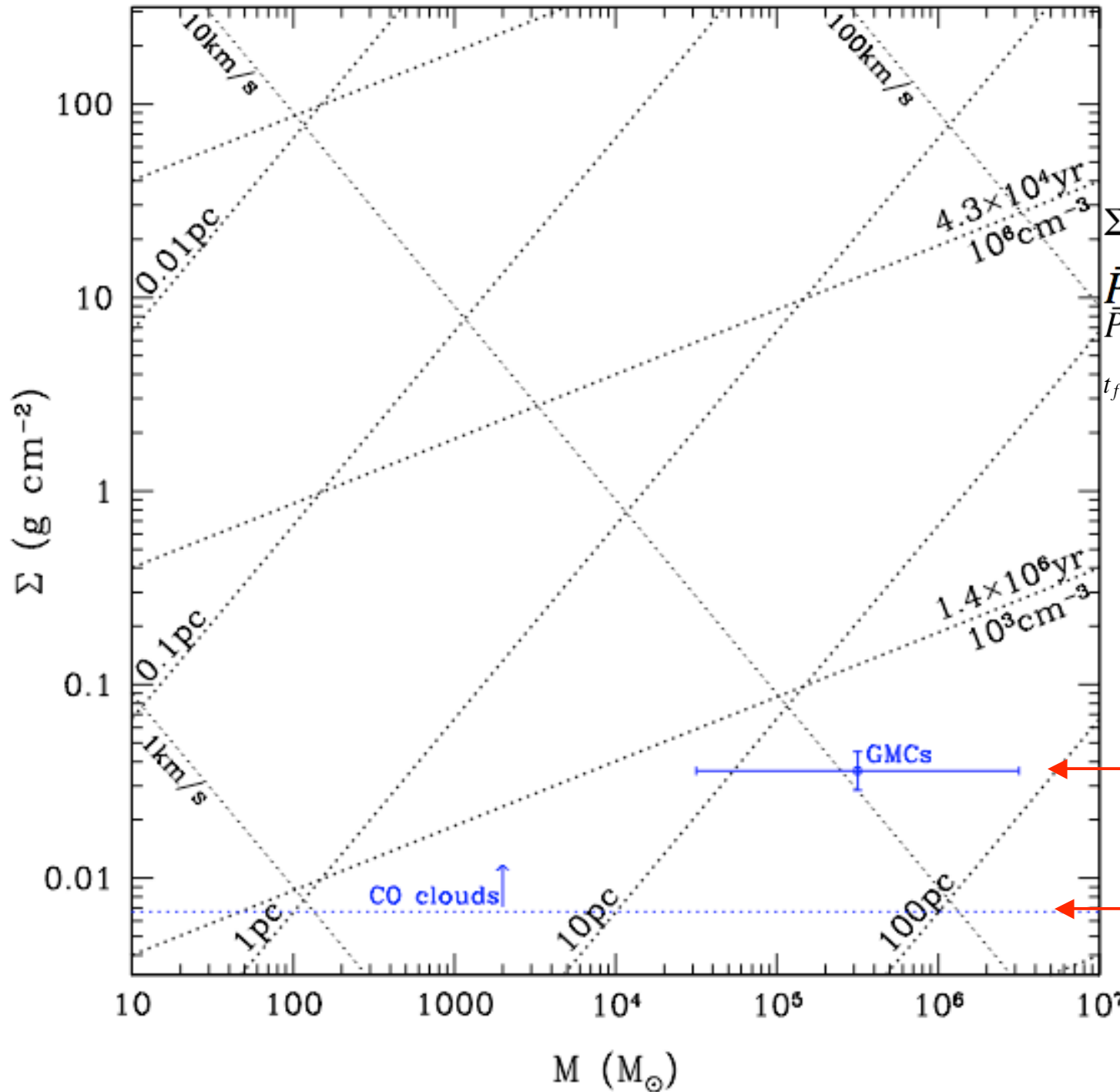
$$\Sigma \equiv \frac{M}{\pi R^2}$$

$$\bar{P} \simeq G \Sigma^2$$

$$\bar{P}/k = 4.3 \times 10^8 \Sigma^2 \text{ K cm}^{-3}$$

$$t_{ff} = \left(\frac{3\pi}{32G\rho} \right)^{1/2}$$

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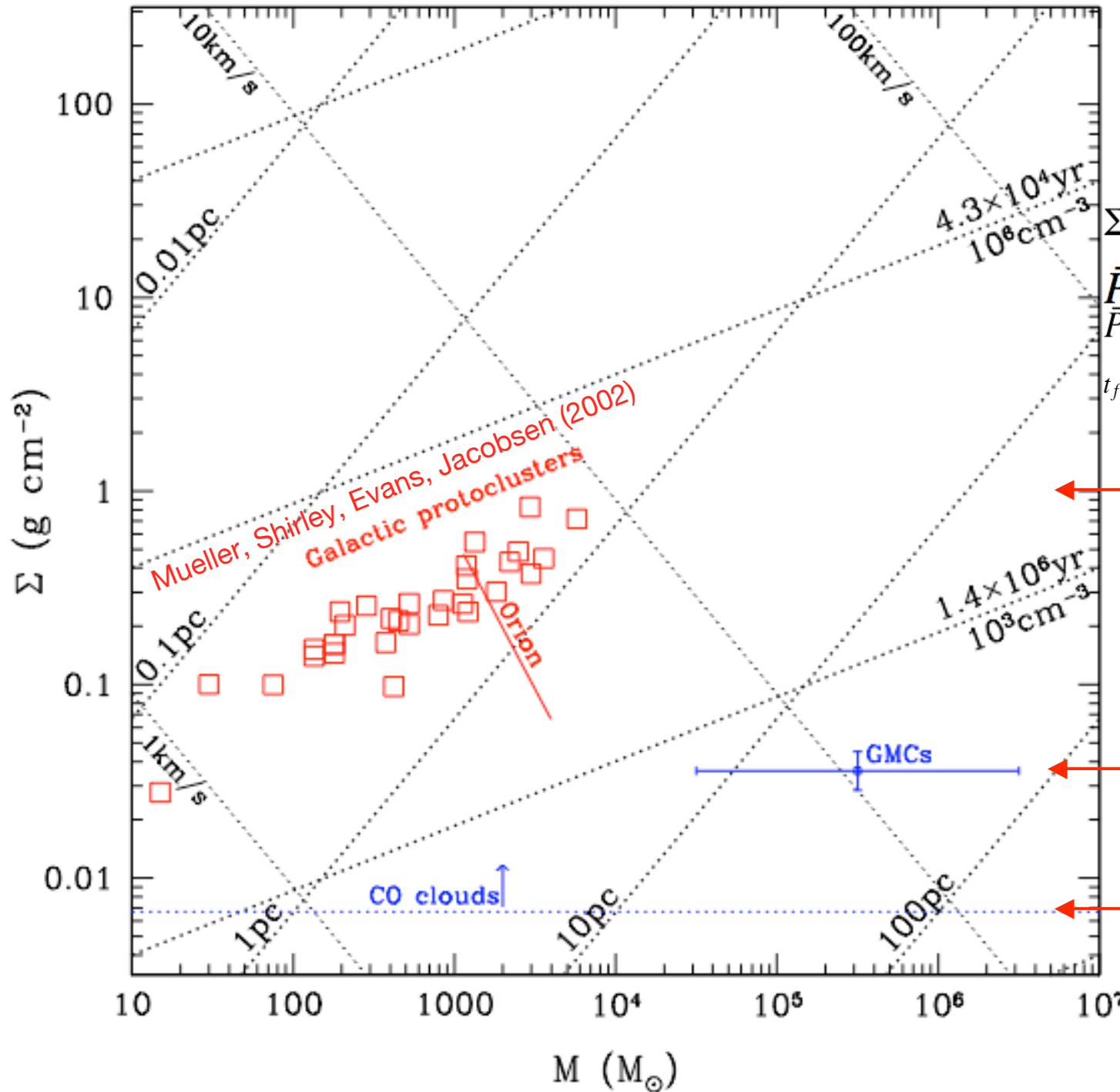
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 $N_H = 1.6 \times 10^{22} \text{ cm}^{-2}$
 $\Sigma = 180 M_\odot \text{ pc}^{-2}$

$A_V = 1.4$
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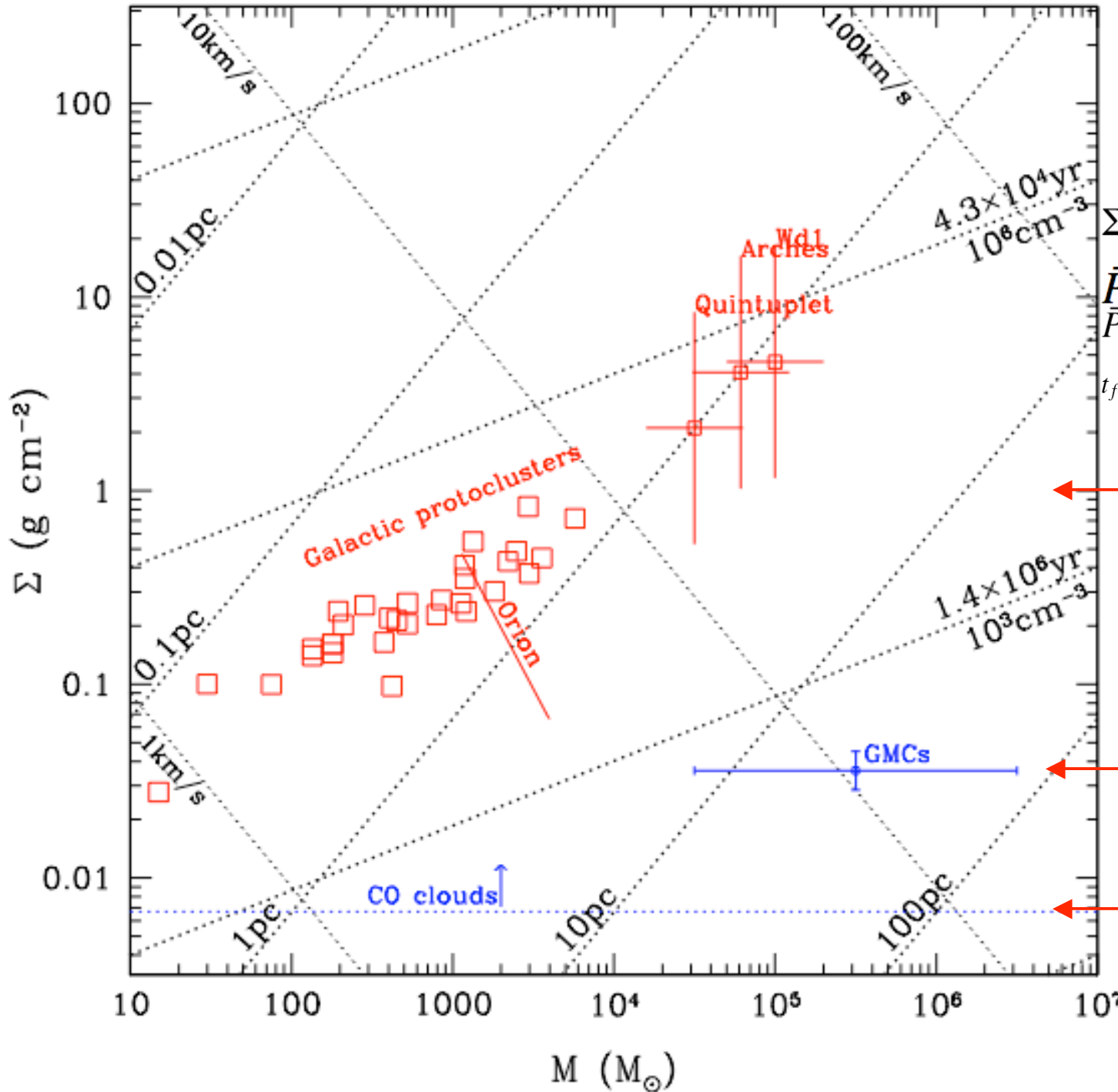
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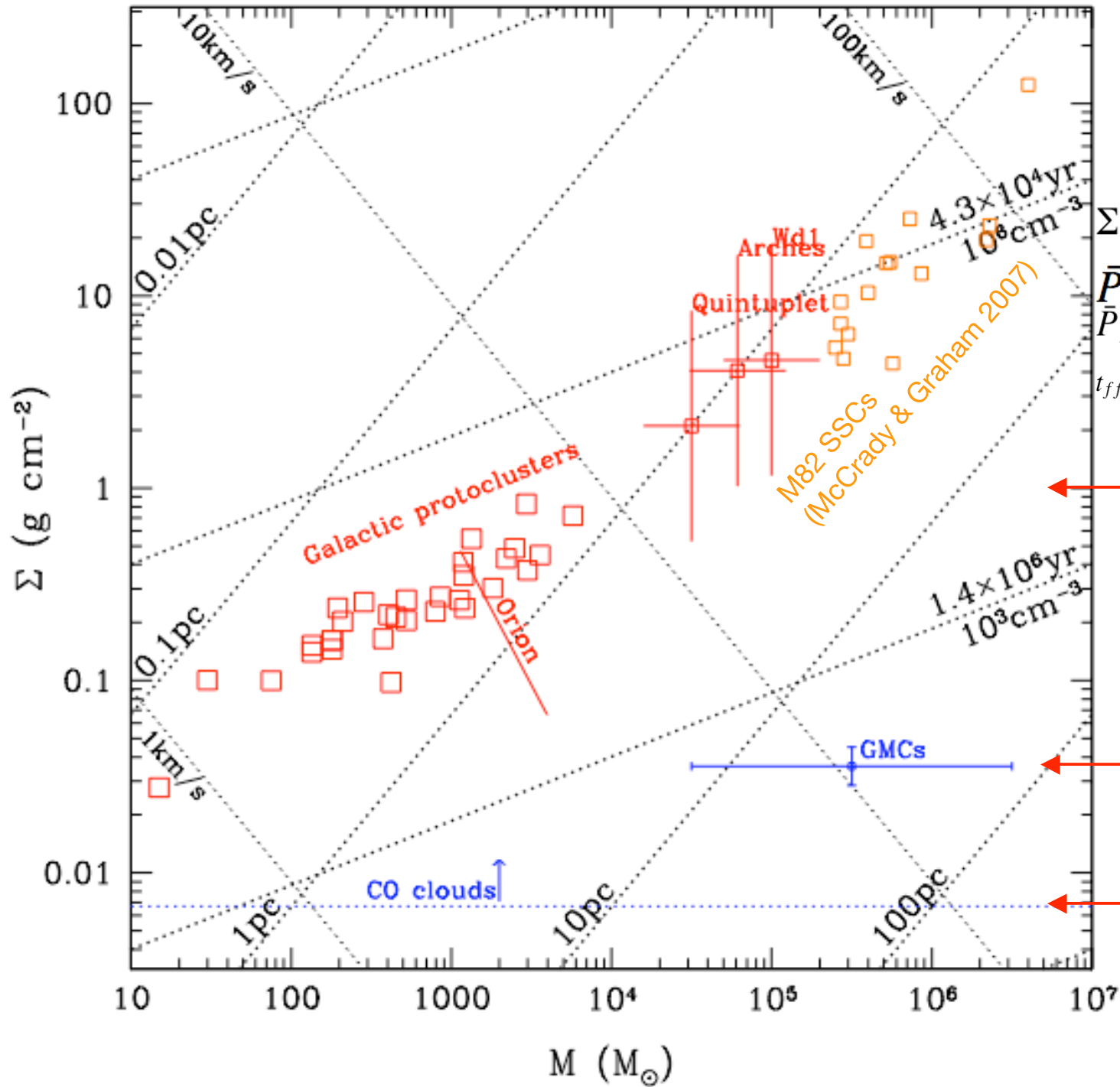
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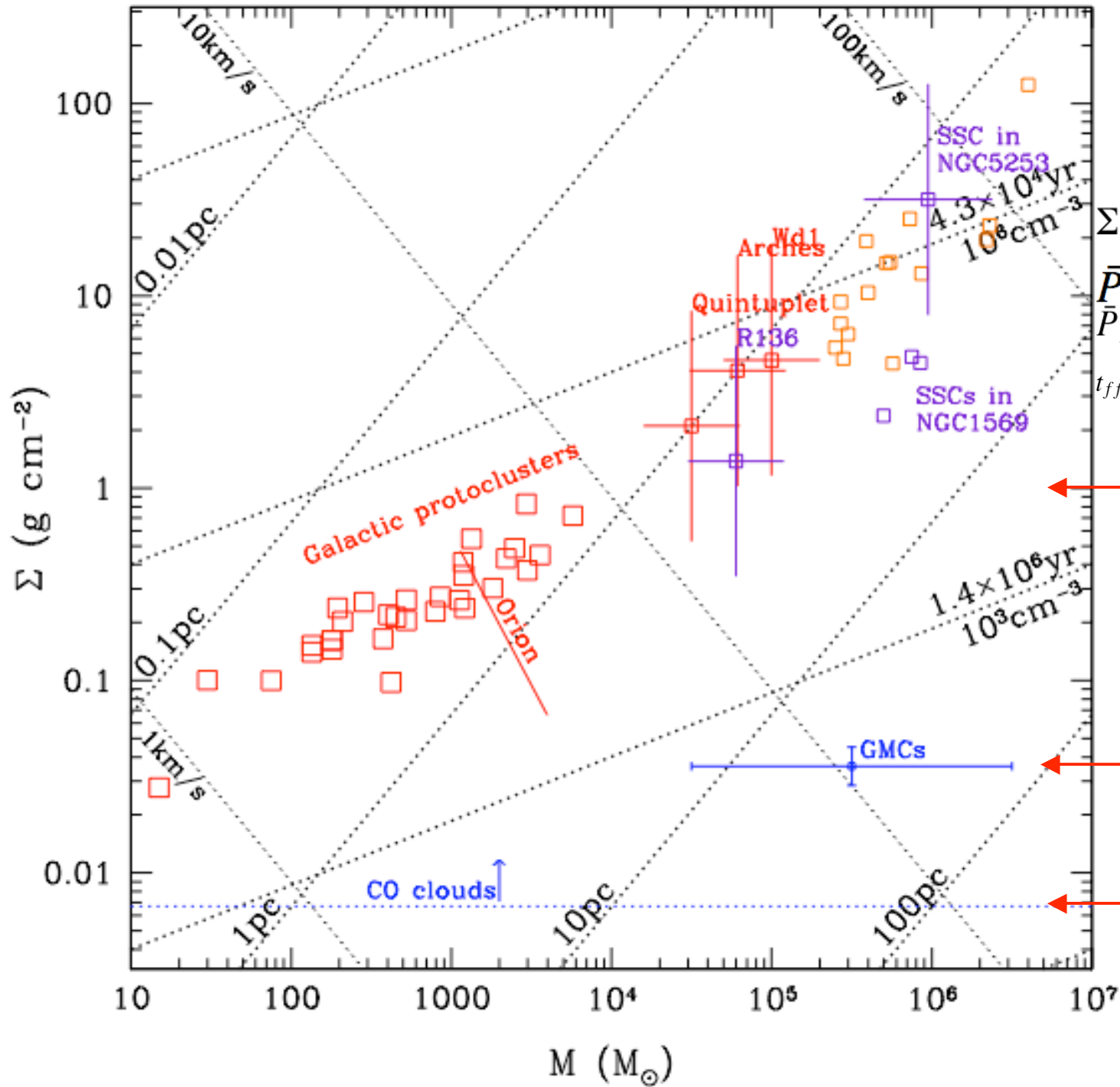
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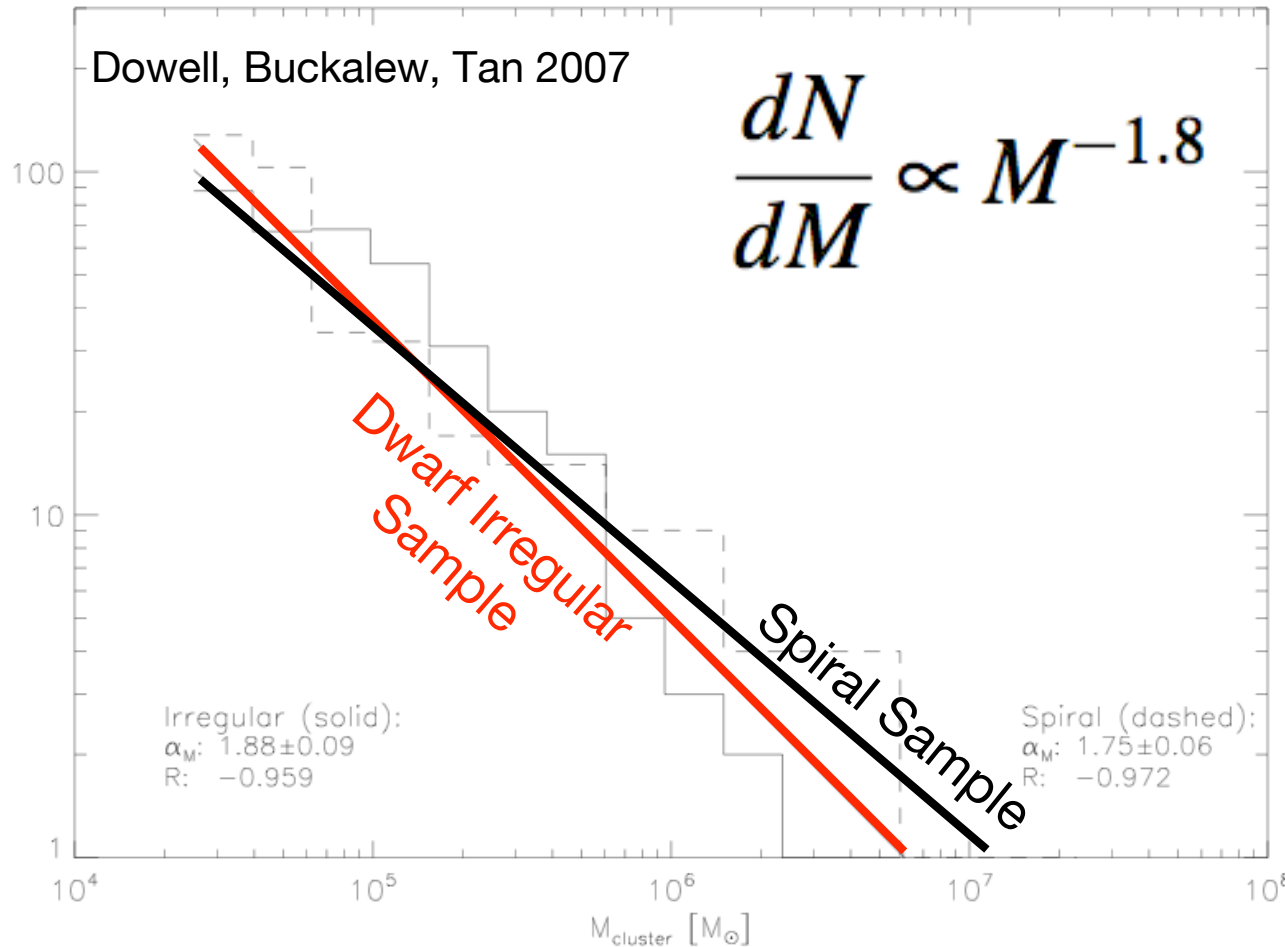
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Initial Cluster Mass Function

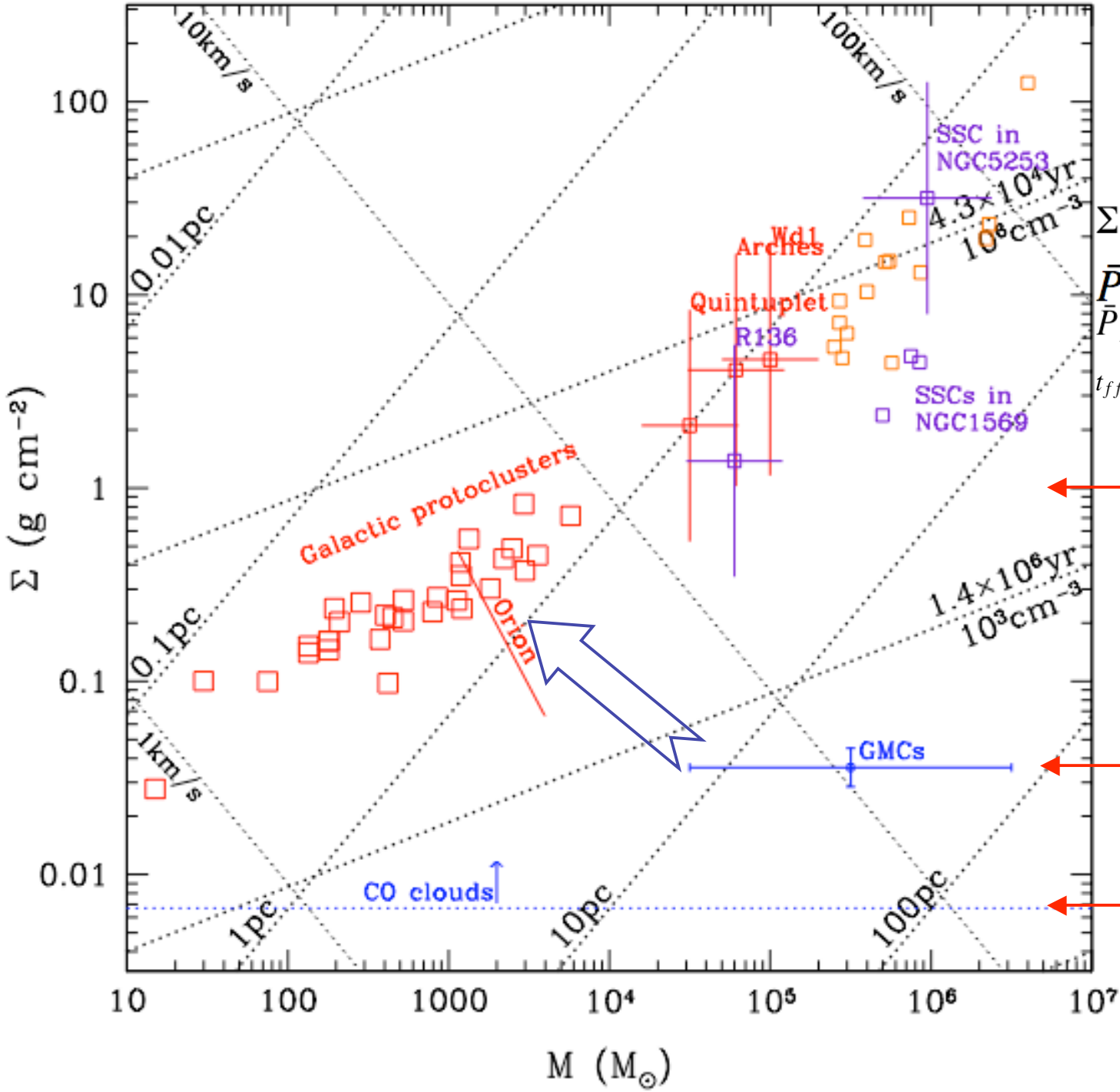
McKee & Williams 1997; Zhang & Fall 1999; Larson 2002; Billet et al. 2002; Lada & Lada 2003; Hunter et al. 2003



From SDSS data, ICMFs in dwarf irregular and spiral galaxies are statistically indistinguishable, in spite of different metallicities and galactic shear rates.

ICMF is set by processes operating on relatively small scales, decoupled from galactic shear, perhaps fragmentation in GMCs.

Initial Conditions



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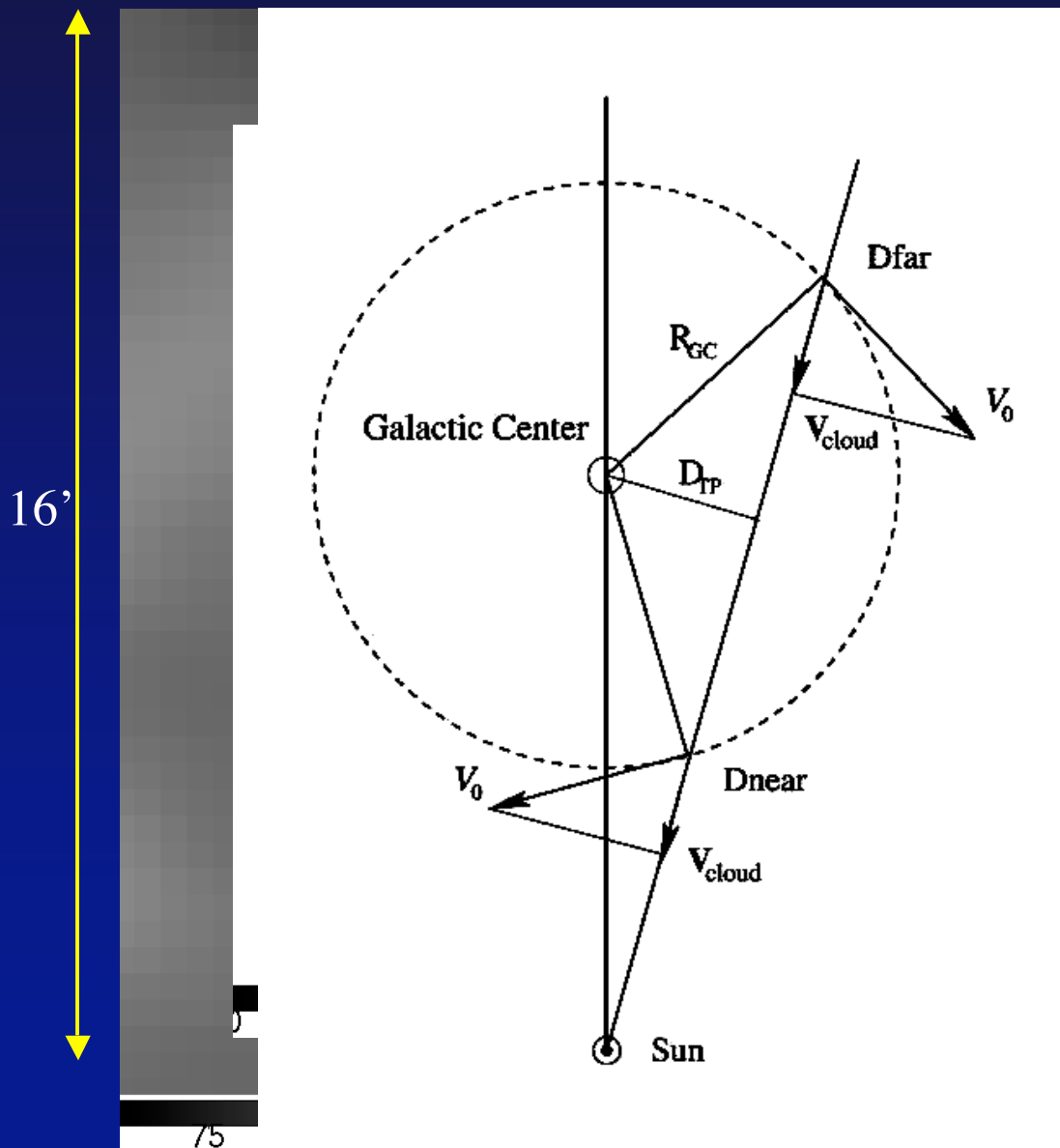
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Structure of Infrared Dark Clouds

with Butler, Hernandez, Krumholz, Offner, McKee, Klein in prep.



MSX

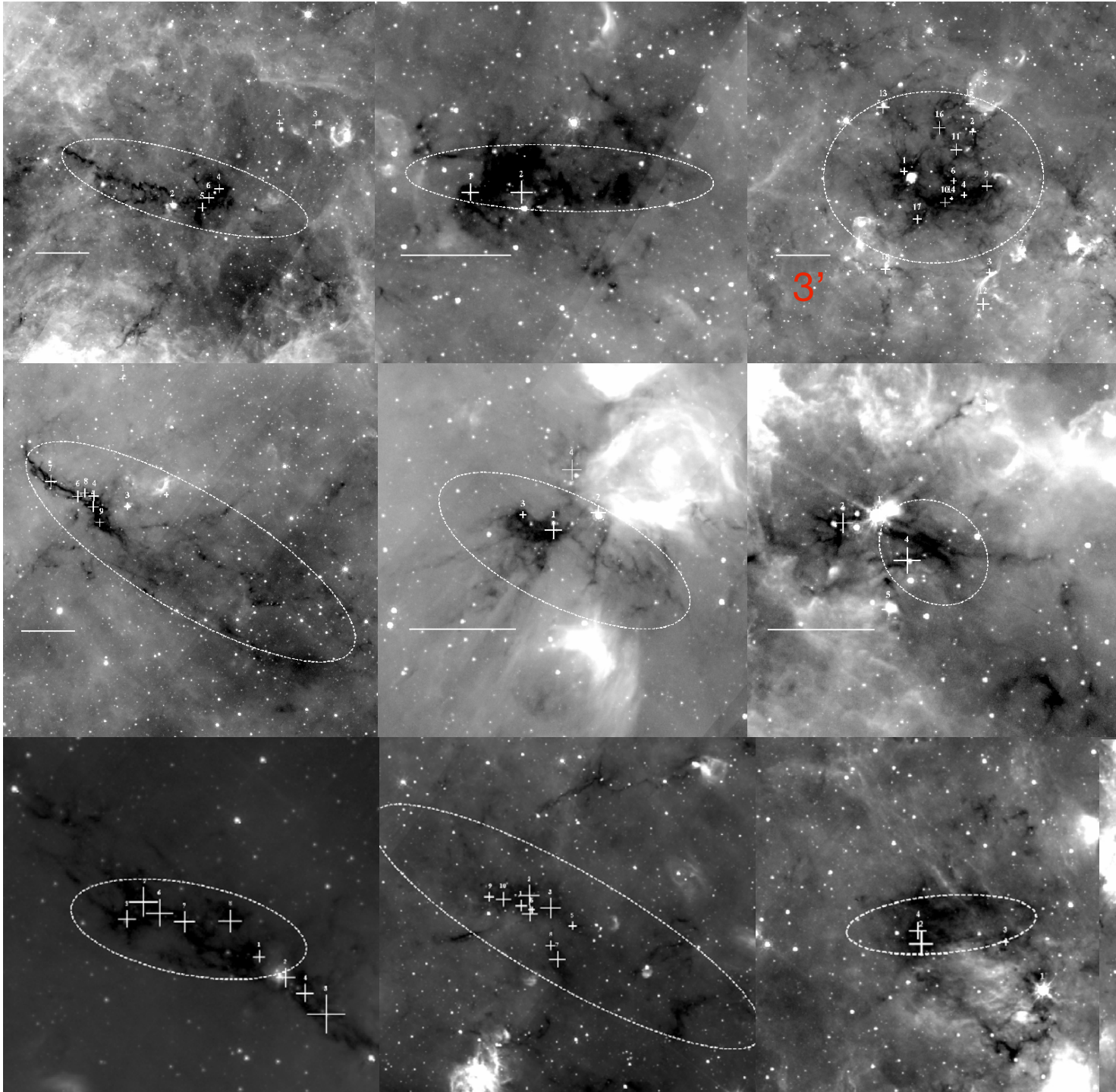


MSX IRDC sample from Rathborne et al. (2005); Simon et al. (2006).

Spitzer - IRAC $8\mu\text{m}$ (GLIMPSE)

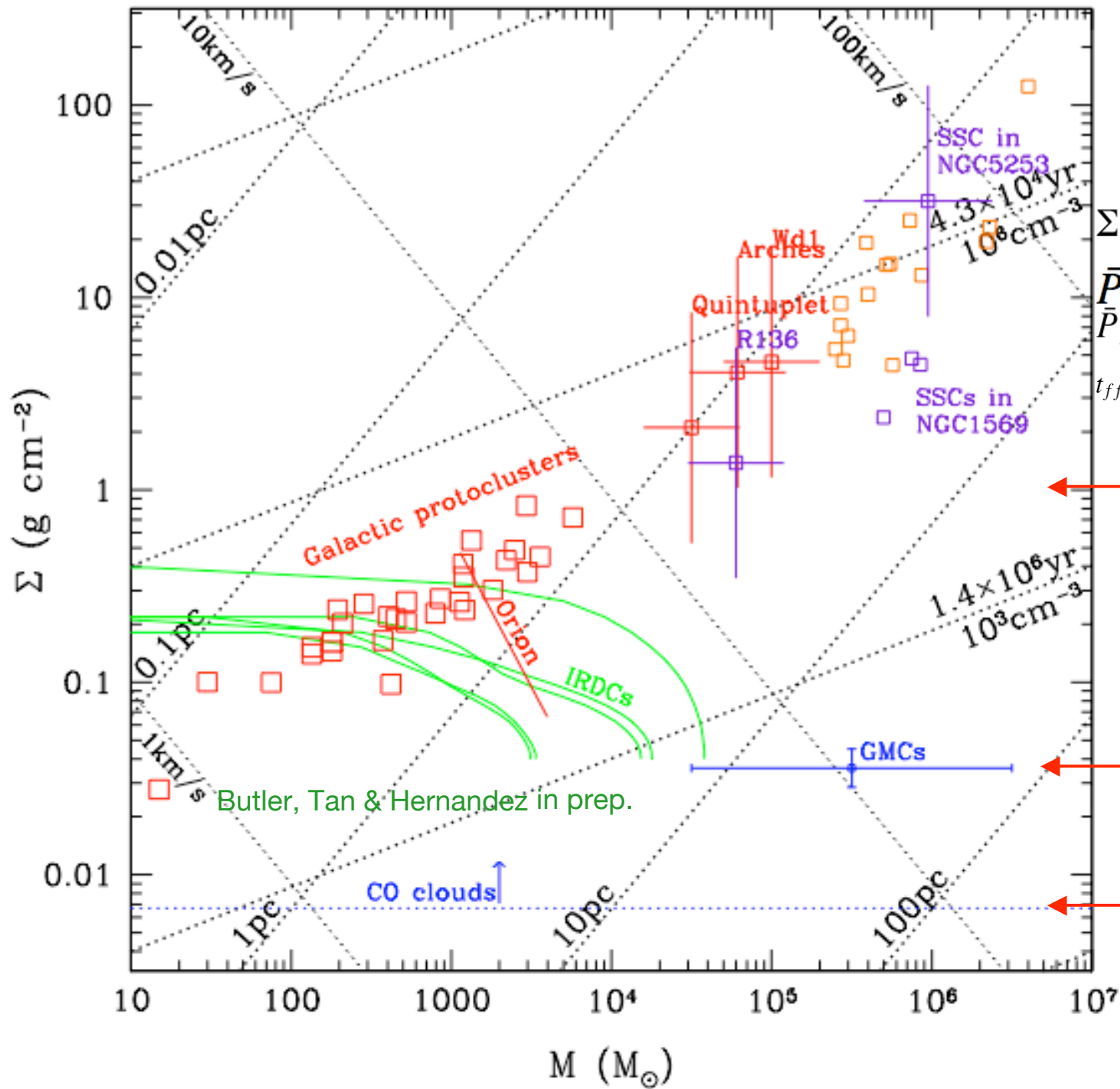
Extinction map to derive Σ

Distance from molecular line velocities (GRS) $\rightarrow M(\Sigma)$



Infrared Dark Clouds (IRDCs): initial conditions for star clusters (e.g. Carey, Jackson, Simon, Rathborne, Menten).

Spitzer IRAC (GLIMPSE) 8 μ m images of a sample of nearby IRDCs (Butler, Tan, Hernandez 2007, in prep.)



IRDCs are the Initial Conditions of Star Clusters

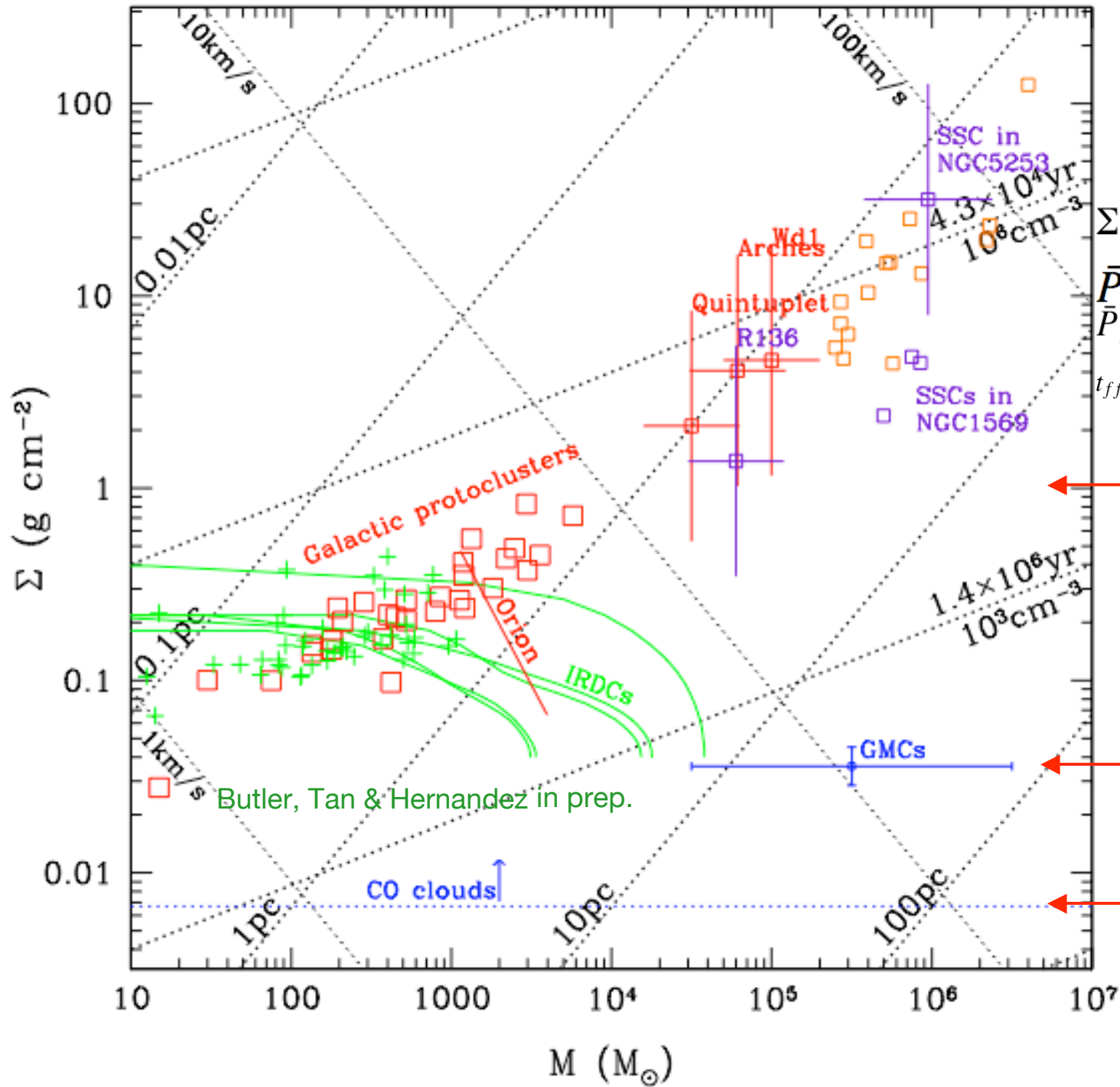
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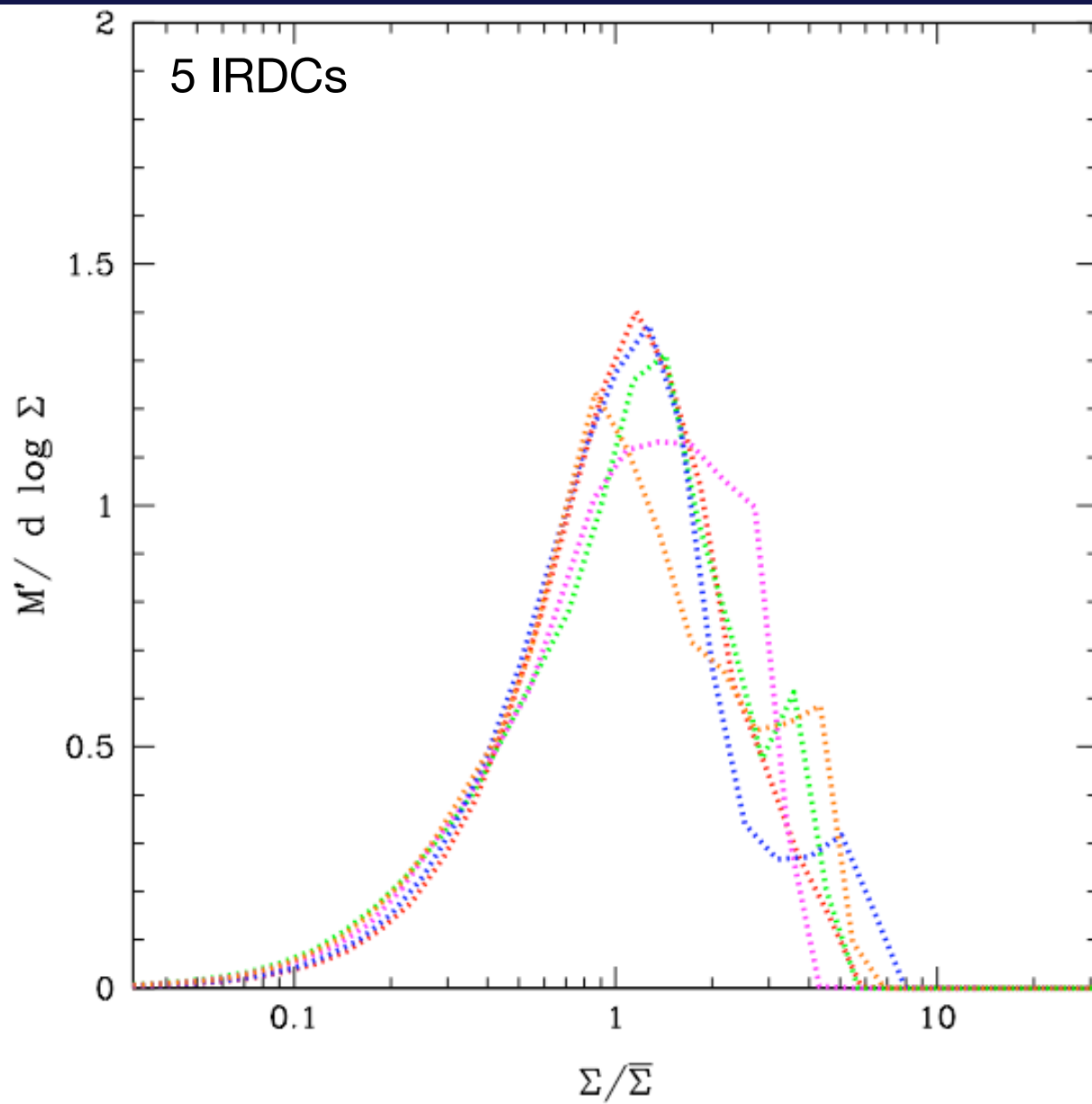
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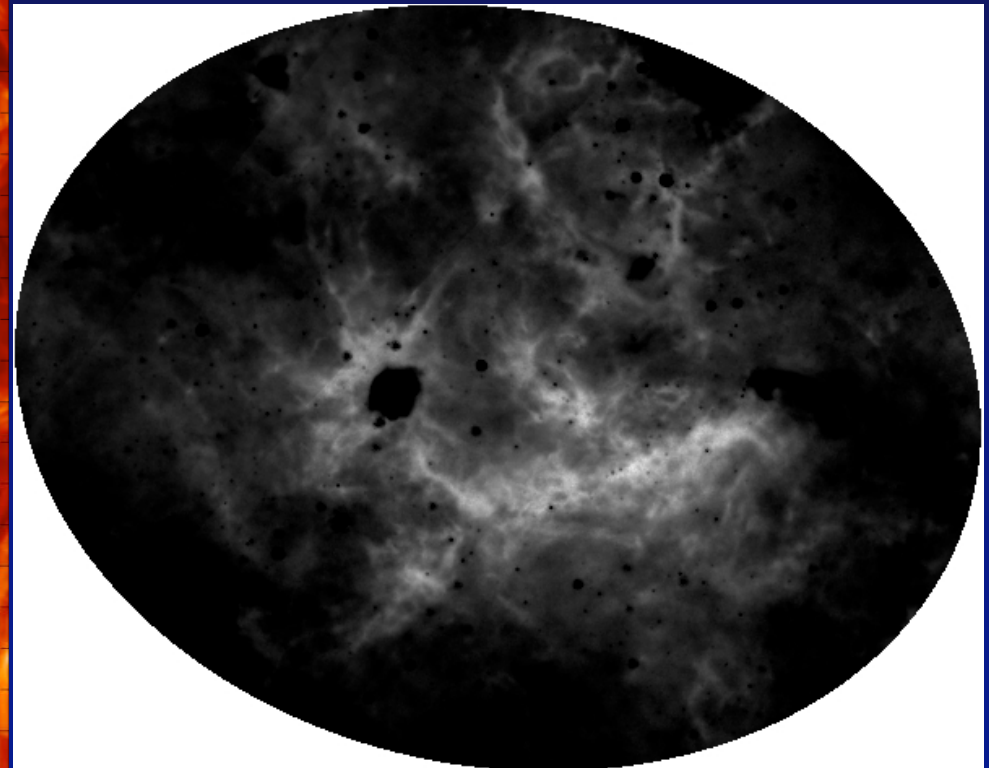
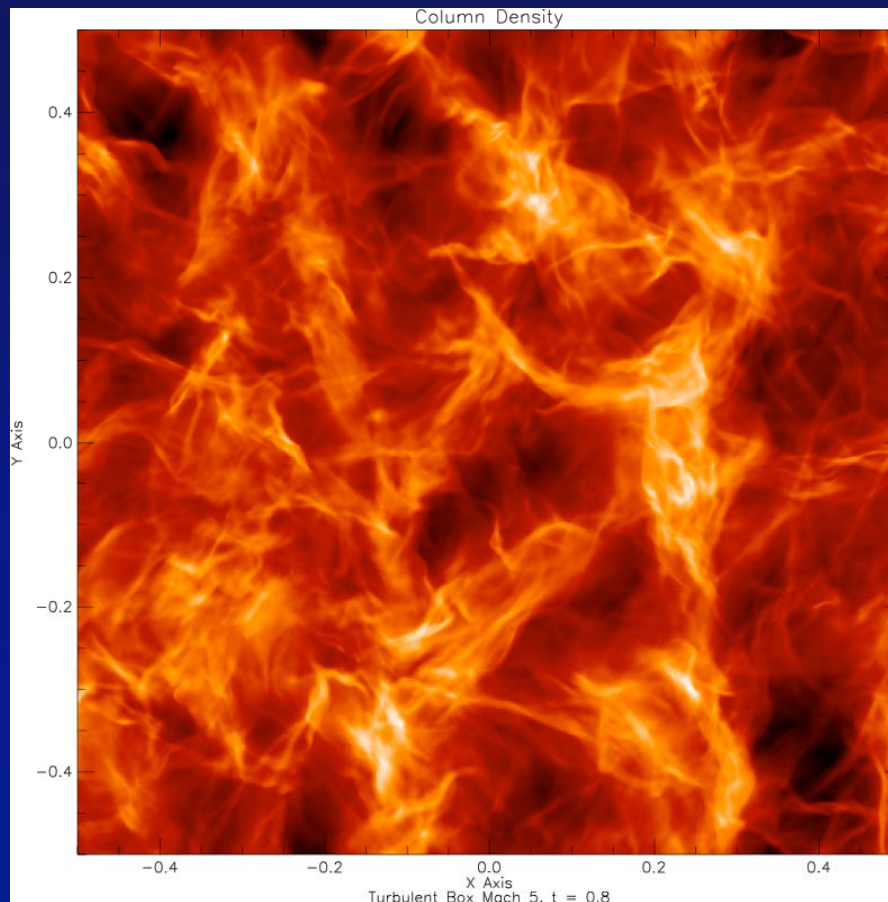
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Distribution of M with Σ

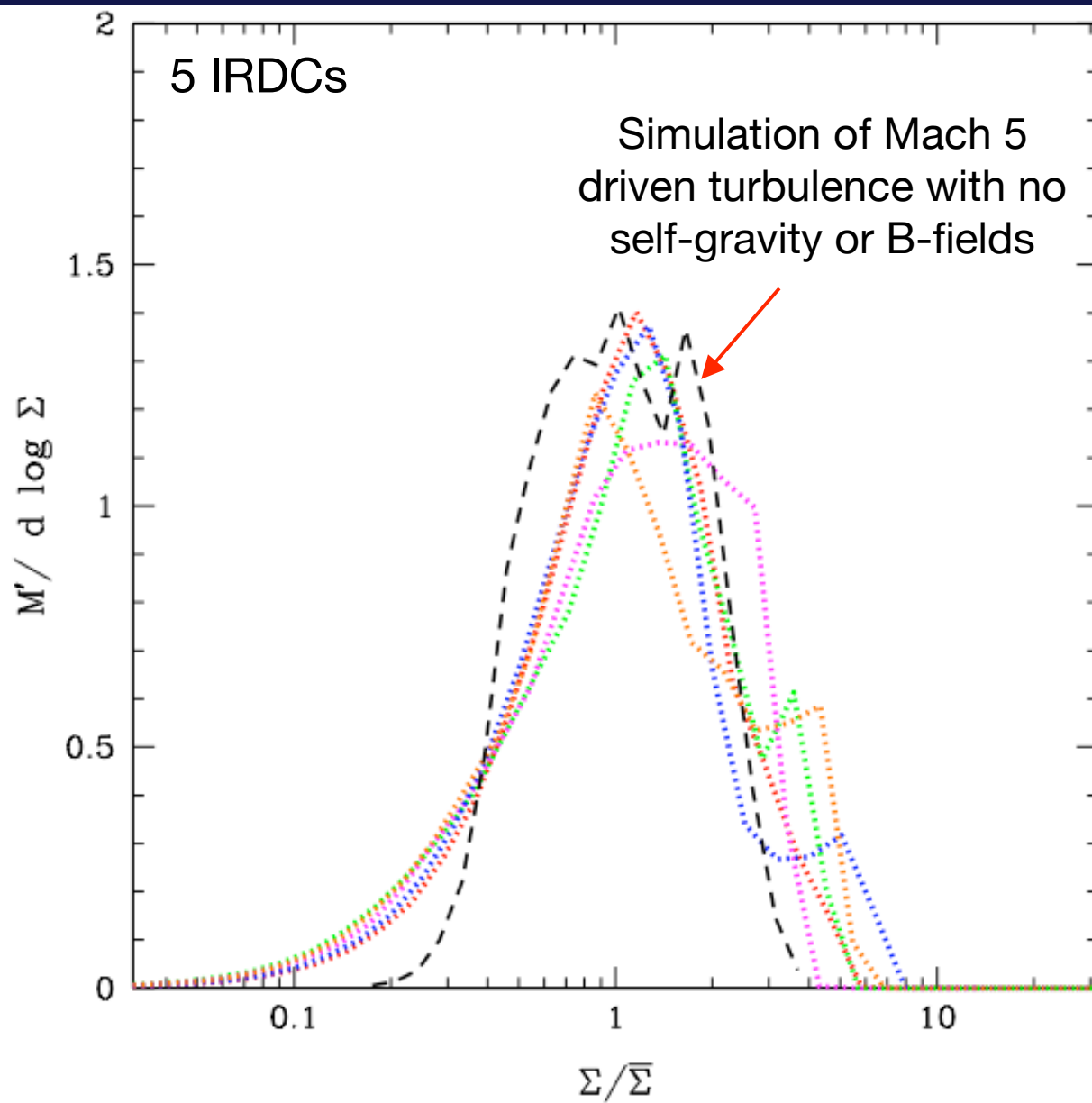


Comparison to Numerical Simulations of Turbulence

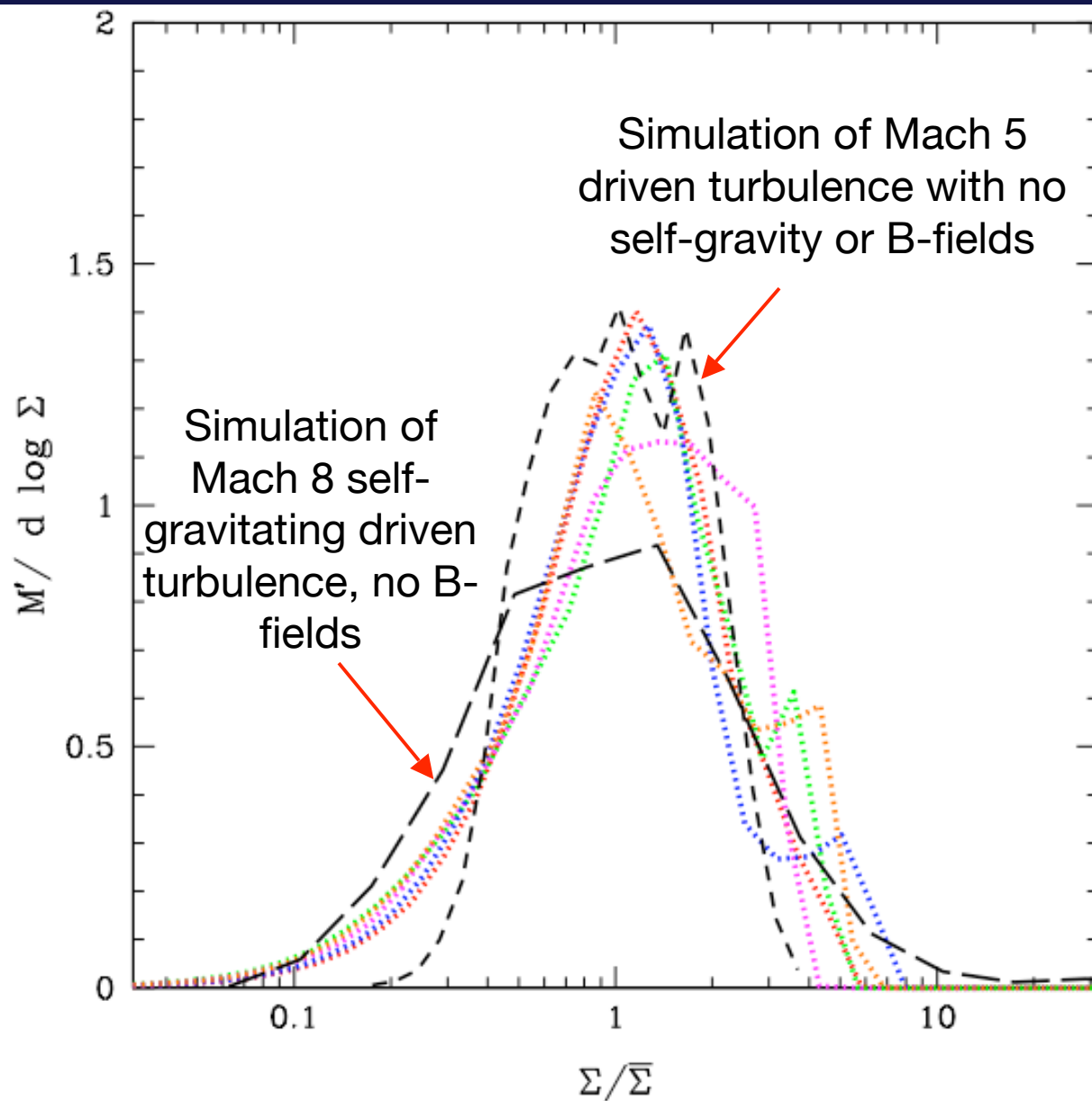
Eulerian - AMR code
Driven turbulence
(Offner, Krumholz, Klein, McKee)



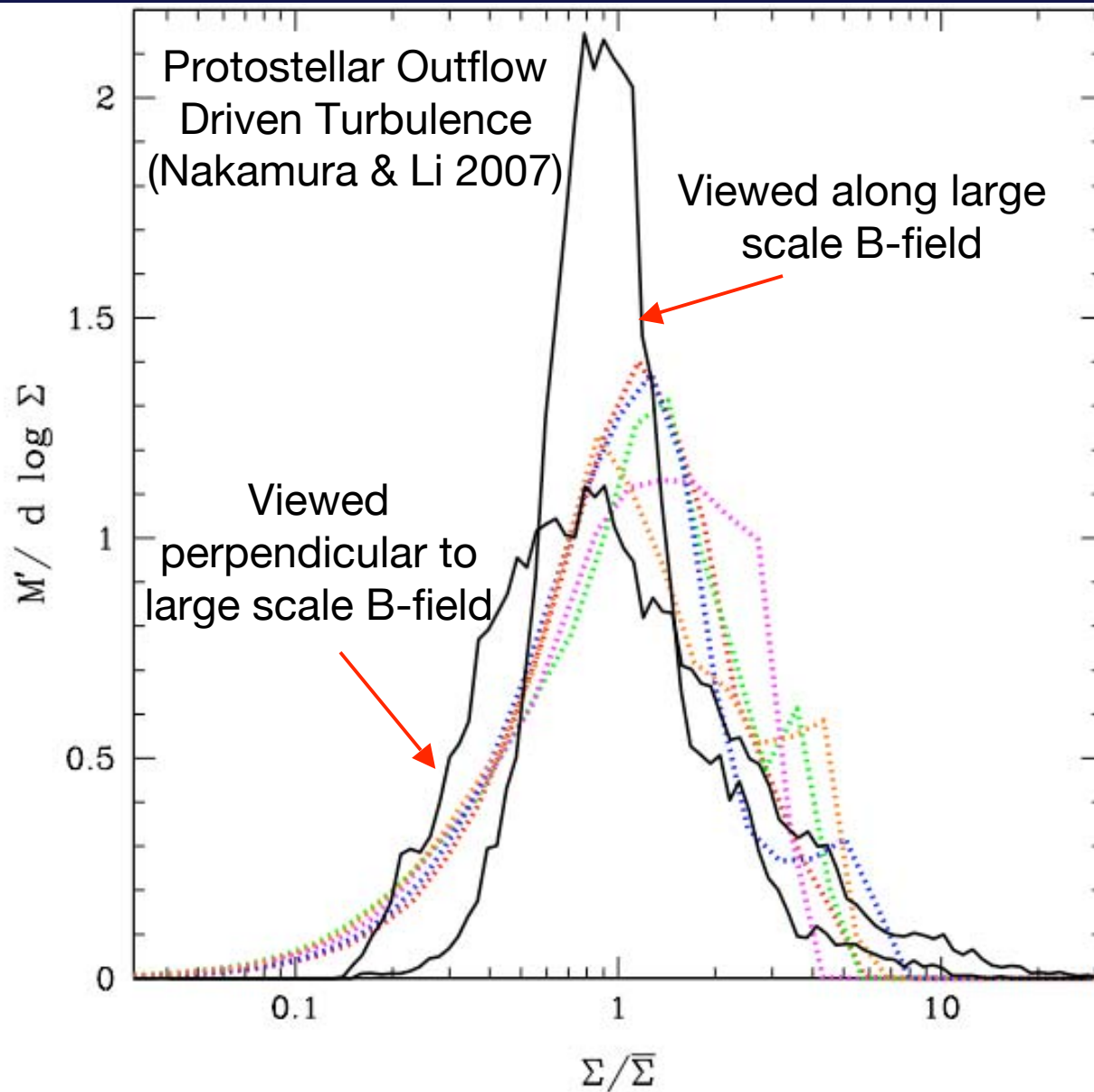
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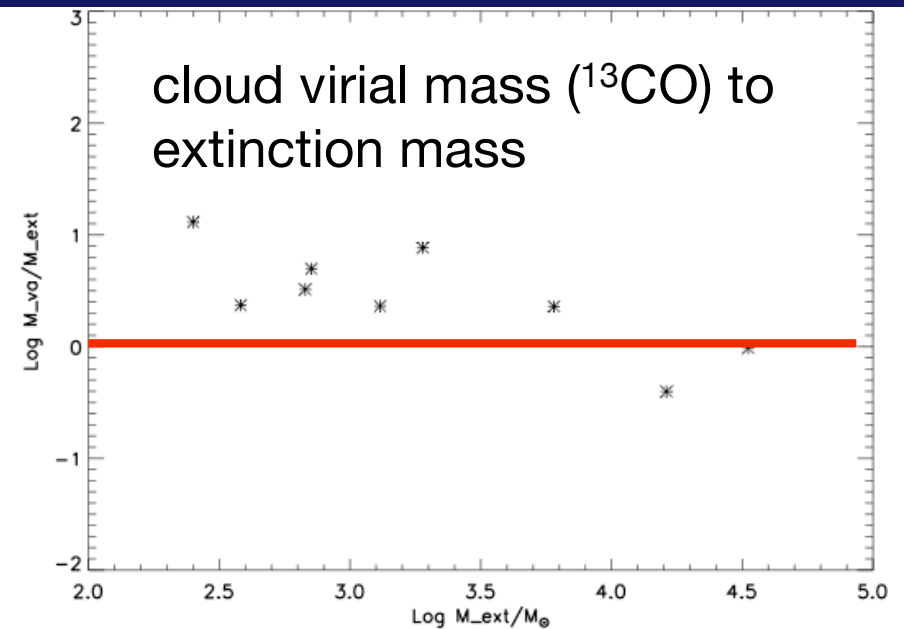
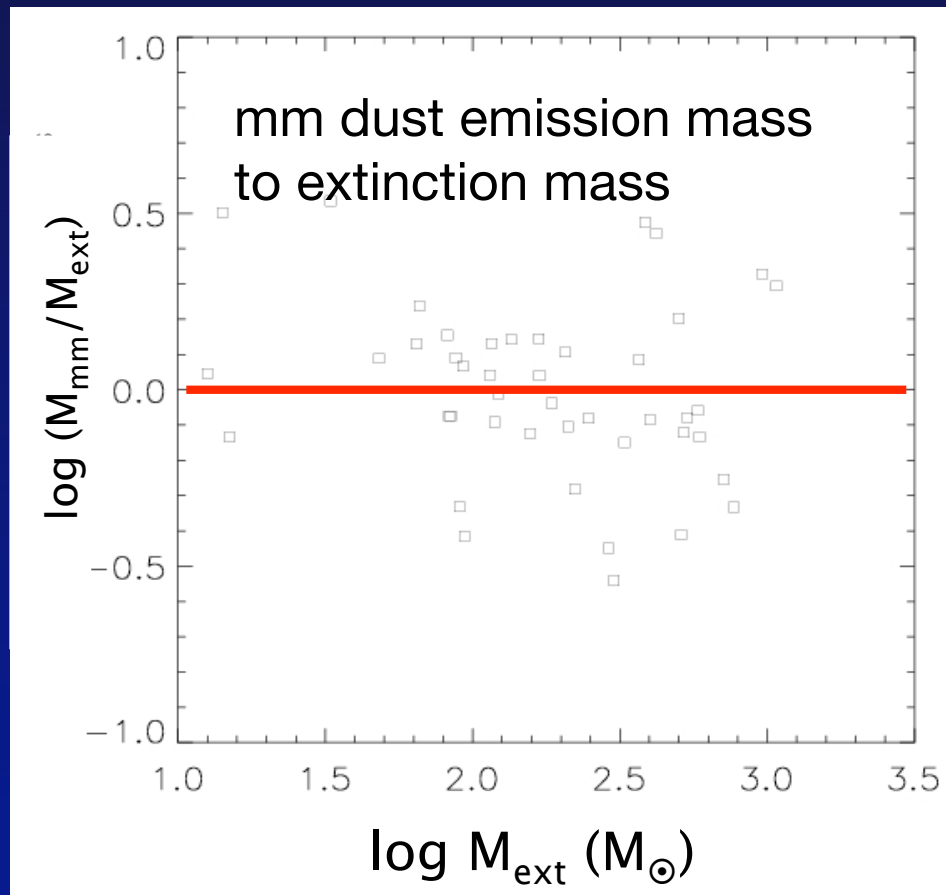


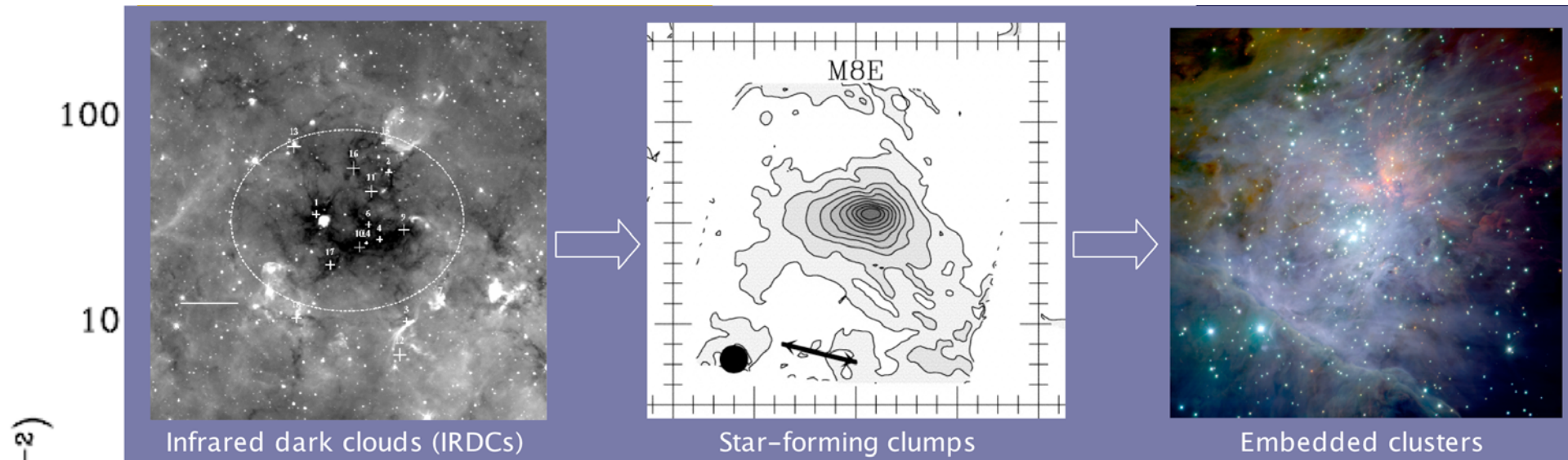
Distribution of M with Σ



Mass comparisons

Hernandez ea. in prep





Σ (g cm^{-2})

Infrared dark clouds (IRDCs)

Star-forming clumps

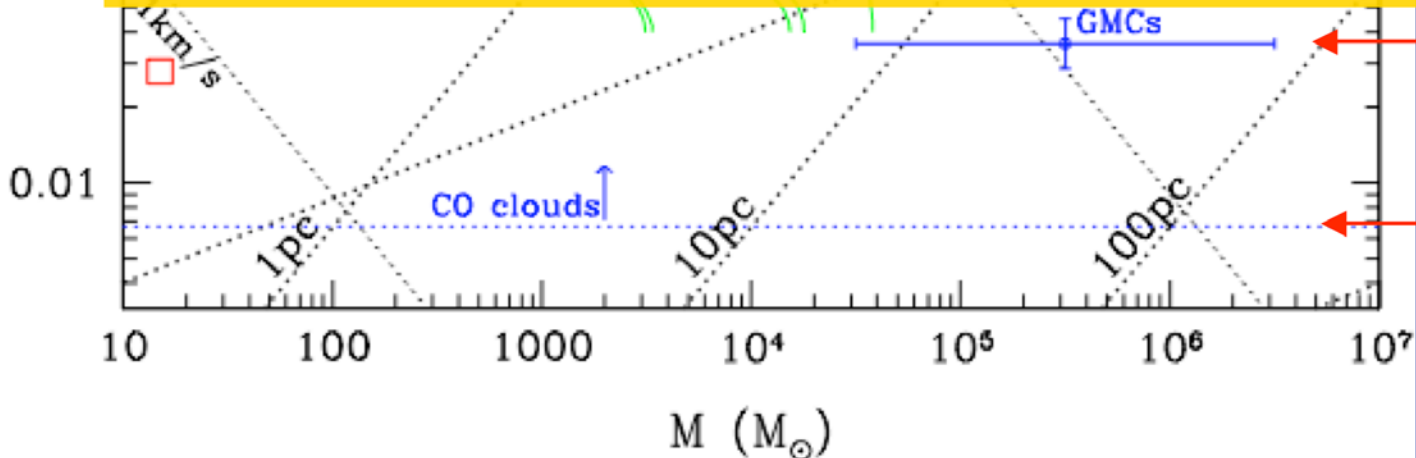
Embedded clusters

Timescale?
 Mode of Star Formation?
 Feedback?

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$M (M_\odot)$

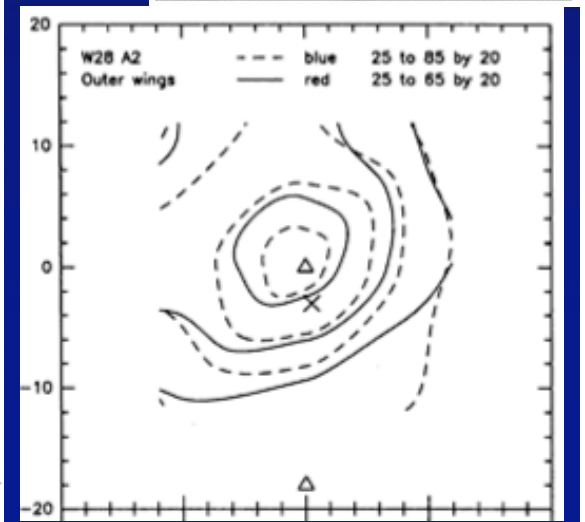
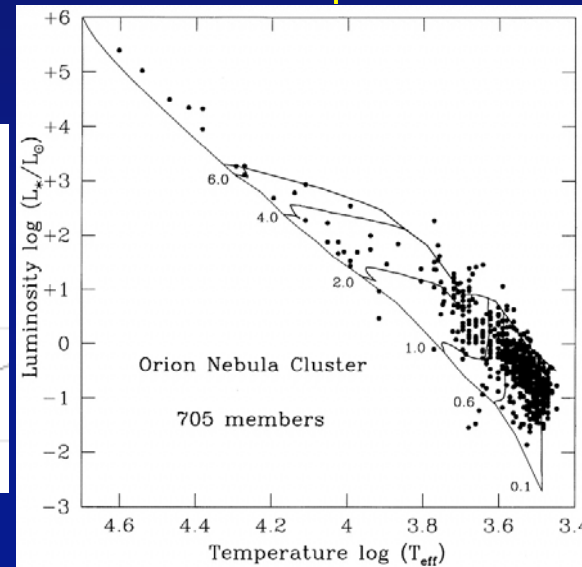
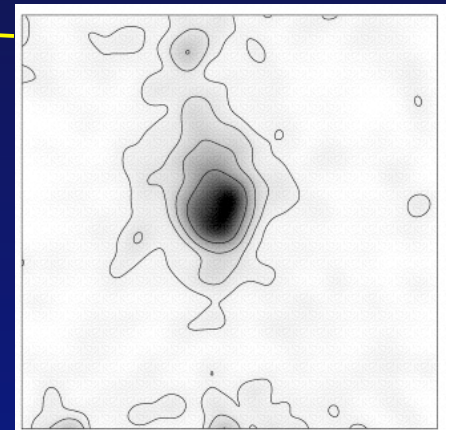
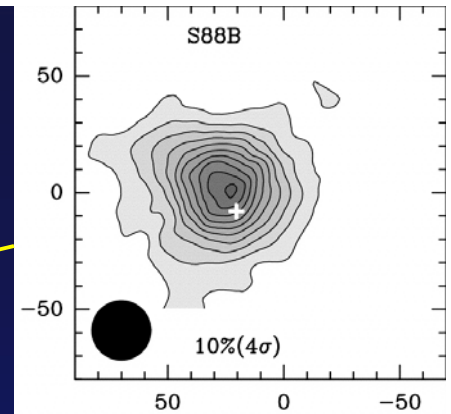
Timescale: Slow, Equilibrium Star Cluster Formation

(Tan, Krumholz, McKee 2006)

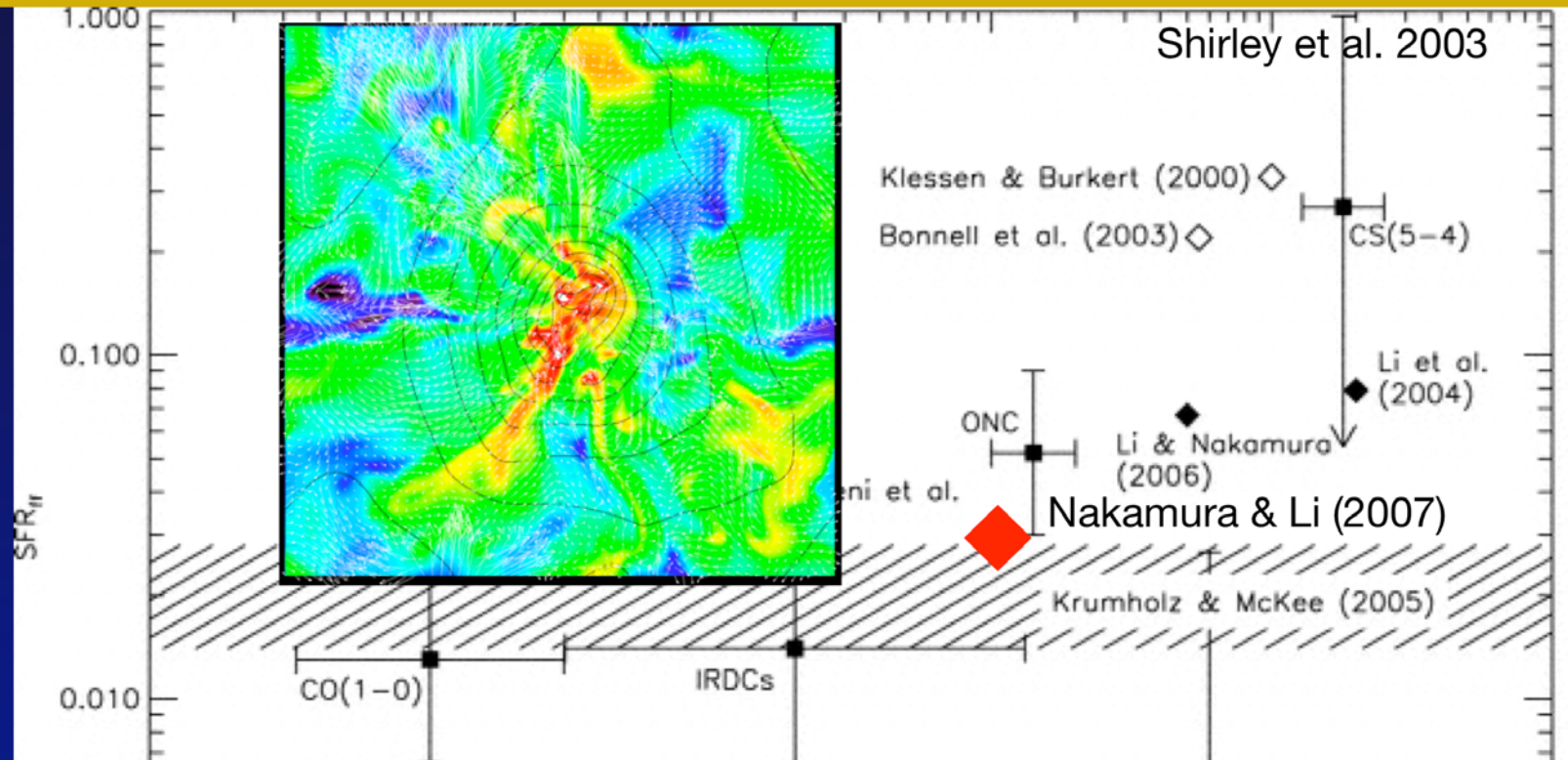
Formation time long relative to free-fall time for rich (high SFE) clusters

Observational evidence:

- ROUND Clump morphologies
- SMOOTH Substructure of young stars
- SMALL Momentum flux of outflows
- LARGE Age spreads of cluster stars
- OLD Age of ONC ejection event



If one accepts the theoretical and observational evidence for low SF efficiency per free-fall time ($SFR_{ff} \sim 0.03$) time from turbulent gas, then the observed high overall SFE of rich clusters (up to $\sim 30-50\%$) require long formation times. (Krumholz & McKee 2005; Krumholz & Tan 2007, Nakamura & Li 2007).



- Implications:
1. Star formation in rich clusters is a local process regulated by turbulence rather than global collapse (turbulent fragmentation rather than competitive accretion)
 2. Turbulence must be driven and maintained [probably by outflows]
 3. Mass segregation of massive stars: more time available in gas rich phase

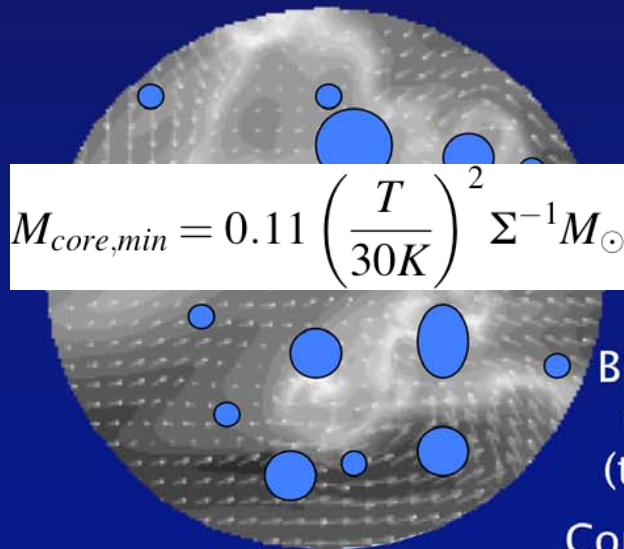
Mode of star formation in star clusters

Two different models:

Turbulent Fragmentation into Cores

Padoan & Nordlund (2002); McKee & Tan 2003;
Vázquez-Semadeni et al. 2004;

Stars form from “cores”, $M_{\text{core}} \sim m_*$,
that fragment from the clump



$$M_{\text{core},\text{min}} = 0.11 \left(\frac{T}{30\text{K}} \right)^2 \Sigma^{-1} M_{\odot}$$

$$\bar{P} = \phi_P G \Sigma^2$$

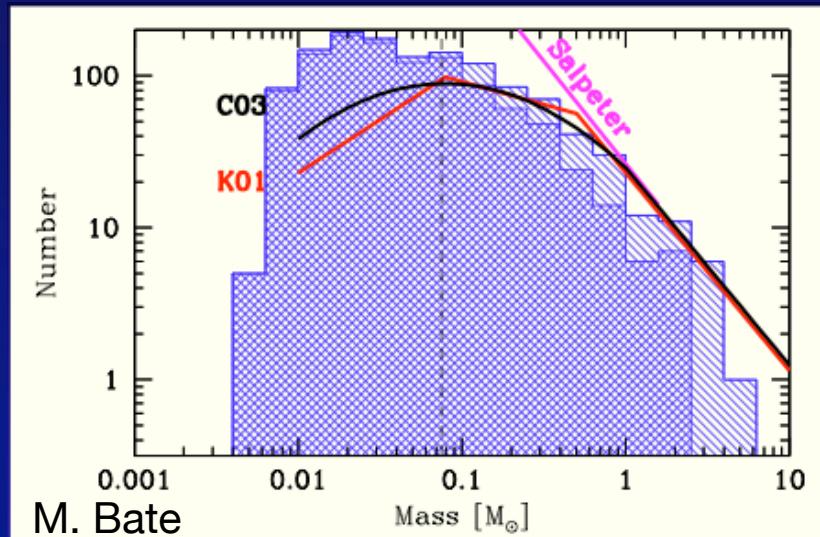
If in equilibrium,
then self-gravity
is balanced by
internal pressure:
B-field, turbulence,
radiation pressure
(thermal P is small)

Cores form from this
turbulent medium: at any given time there
is a small mass fraction in unstable cores.
These cores collapse quickly to form
individual stars or binaries.

Competitive Accretion

Bonnell, Vine, & Bate 2004
Schmeja & Klessen 2004

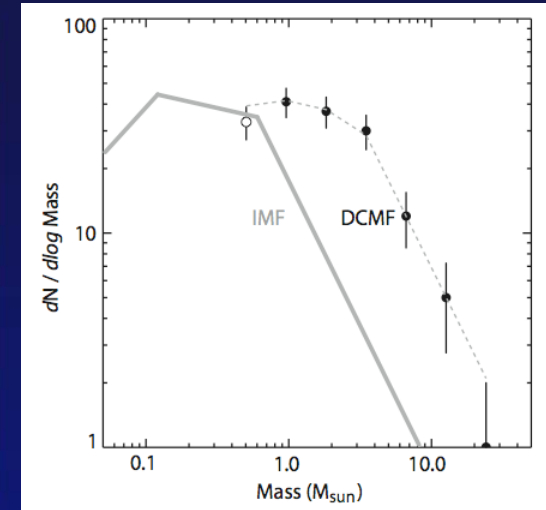
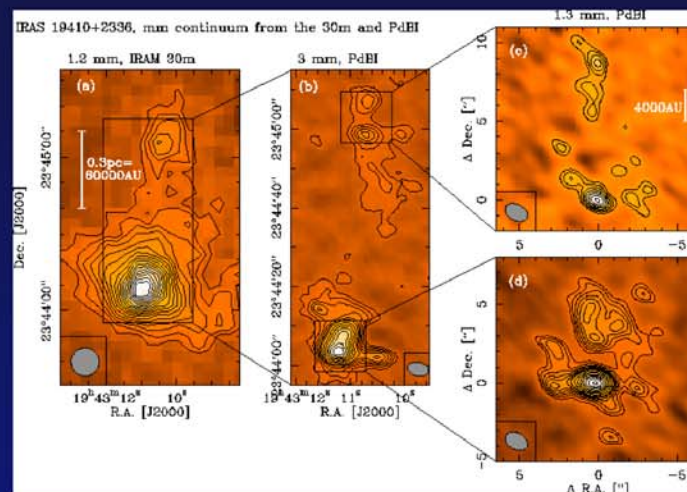
Stars gain most mass by Bondi-
Hoyle accretion of ambient gas



Based on SPH simulations
with sink particles

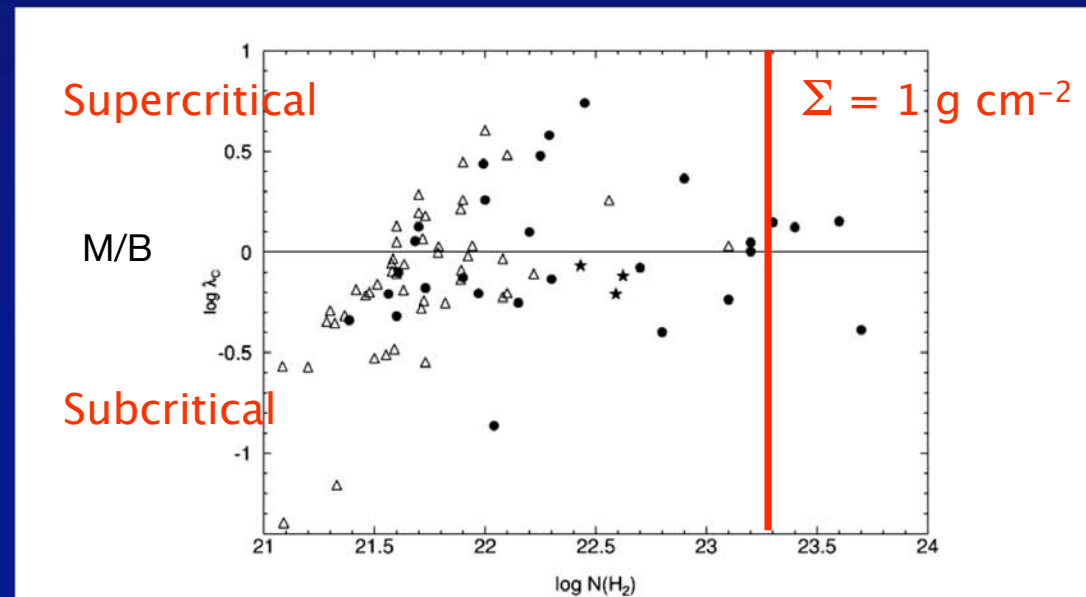
Observed Cores: Mass Function; Turbulent Motions; Magnetic Fields

Cores are seen, both with and without stars. Mass function of cores appears similar to stellar IMF (Motte et al. 2001; Beuther & Schilke 2004; Mike Reid & Wilson 2005; Alves et al. 2007)



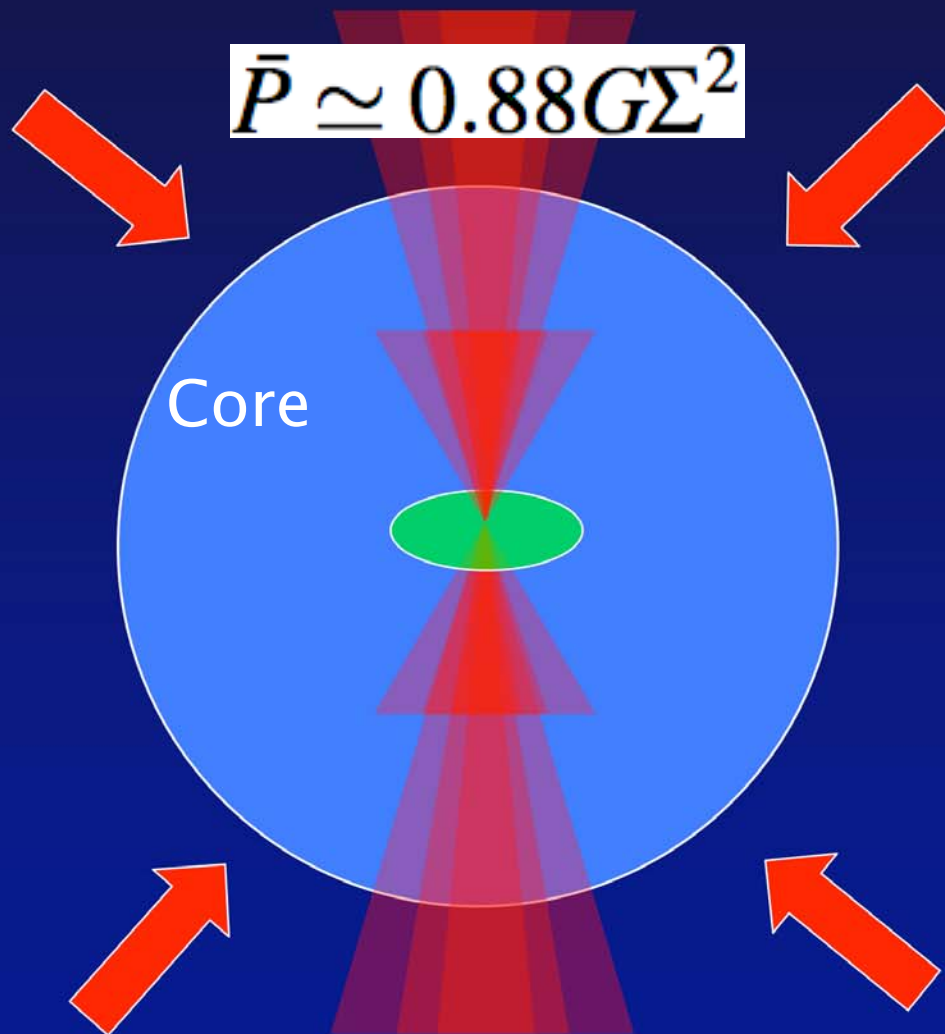
Larger cores have line widths that are much broader than thermal (e.g. Caselli & Myers 1995)

Strength of B-field vs. Σ (Crutcher 2005)



What are the initial conditions for individual massive star formation?

Theory: core surrounded by pressure of clump



$$r_{core} = 0.06 \left(\frac{M_{core}}{60M_{\odot}} \right)^{\frac{1}{2}} \Sigma^{-\frac{1}{2}} pc$$

$$r_{disk} = 1200 \frac{\beta}{0.02} \left(\frac{M_{core}}{60M_{\odot}} \right)^{\frac{1}{2}} \Sigma^{-\frac{1}{2}} AU$$

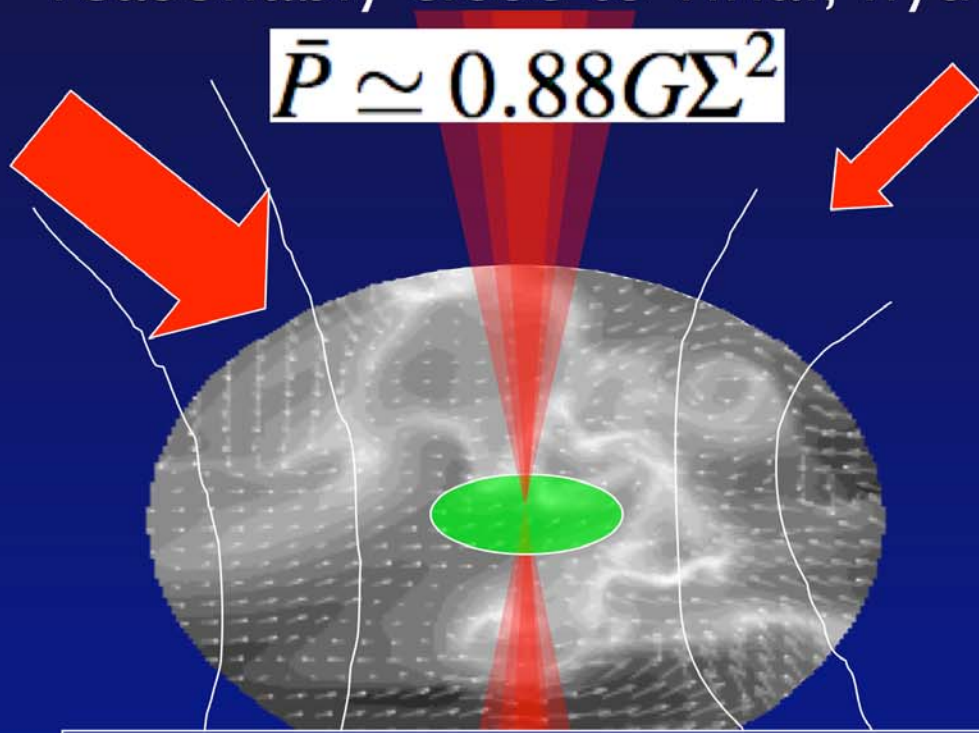
$$t_{*f} = 1.3 \times 10^5 \left(\frac{M_{core}}{60M_{\odot}} \right)^{\frac{1}{4}} \Sigma^{-\frac{3}{4}} yr$$

Final mass accretion rate

$$\dot{m}_* = 4.6 \times 10^{-4} \left(\frac{M_{core}}{60M_{\odot}} \right)^{\frac{3}{4}} \Sigma^{\frac{3}{4}} M_{\odot} yr^{-1}$$

What are the initial conditions for individual massive star formation?
 Turbulent cores, fragmenting from a turbulent medium,
 reasonably close to virial, hydrostatic equilibrium

$$\bar{P} \simeq 0.88 G \Sigma^2$$



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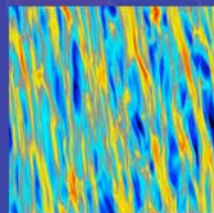
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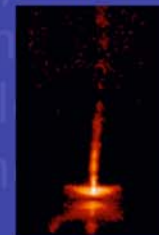
Protostellar evolution



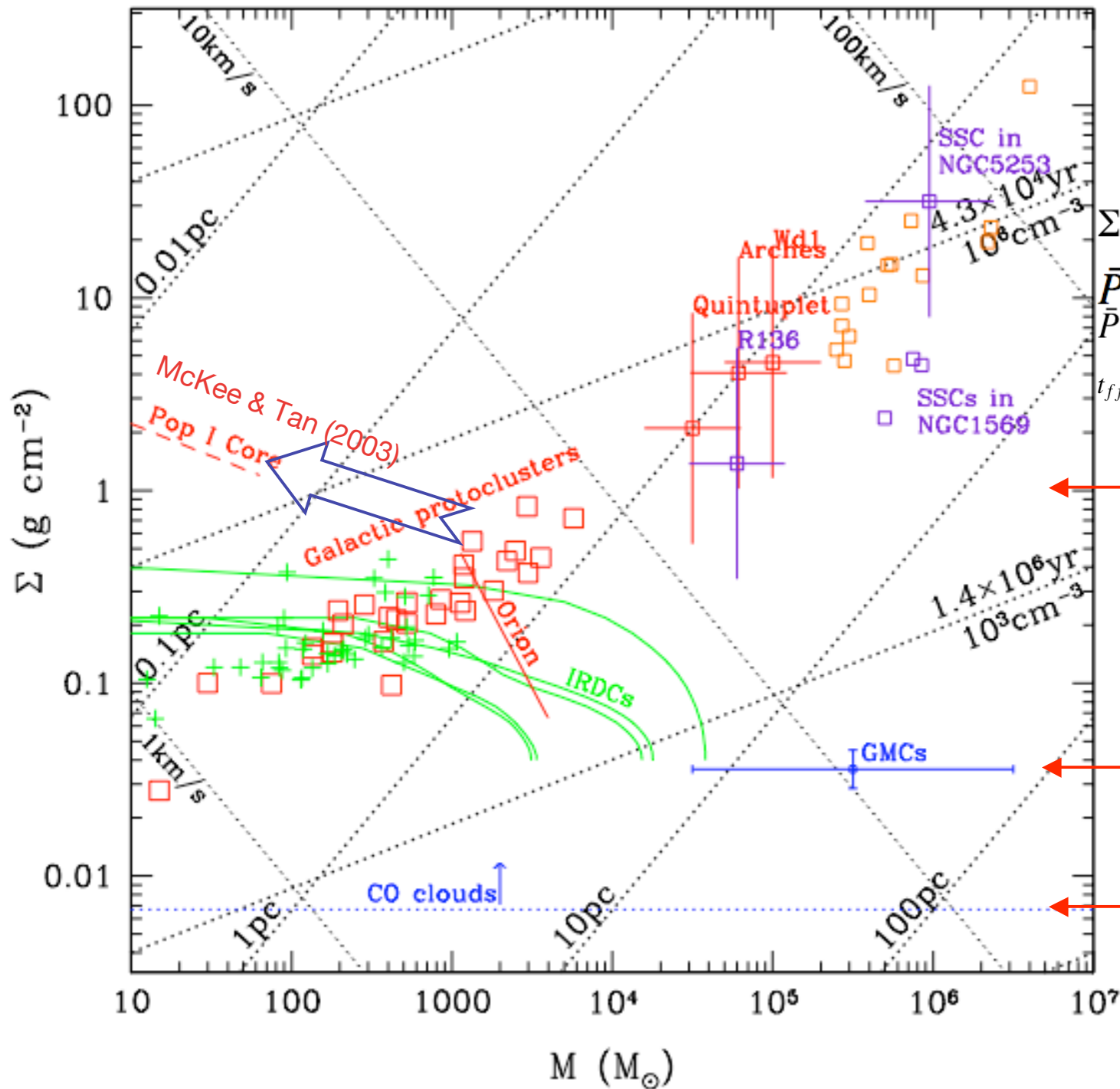
Disk structure



Support by outflows



Support by outflows
 large & small scale B-fields,
 and turbulent motions.
 Core boundaries fluctuate.



Turbulent Core Model of Massive Star Formation

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Turbulent Core Model of Massive Star Formation

Basic Model: McKee & Tan (2002; 2003)

Outflows and Hypercompact HII regions:
Tan & McKee (2003)

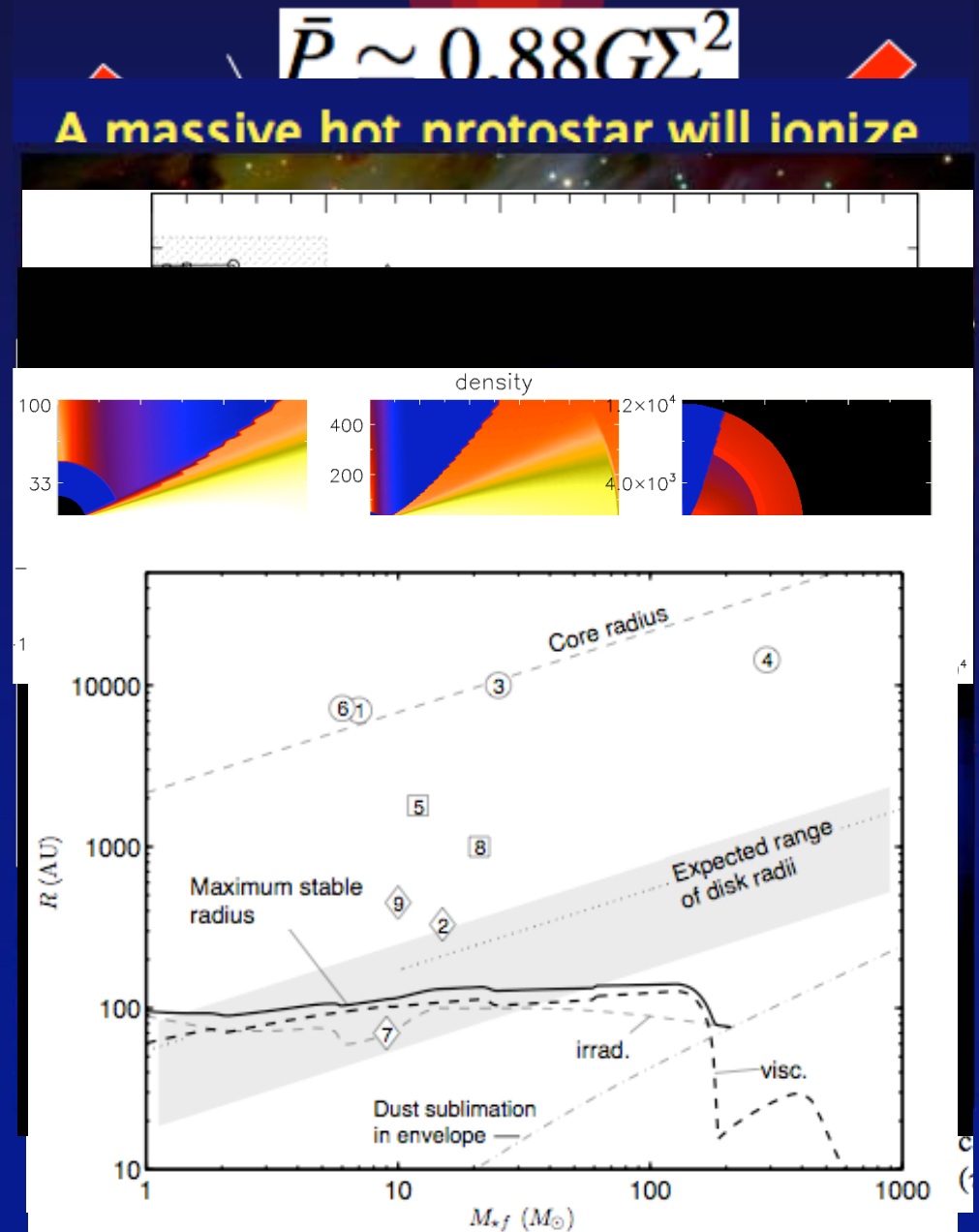
Application to Orion KL: Tan (2004)

Chemistry: Doty, van Dishoeck, Tan
(2006)

Radiation-Hydro Simulation: Krumholz,
Klein, McKee (2007); c.f. Dobbs et al.
(2005)

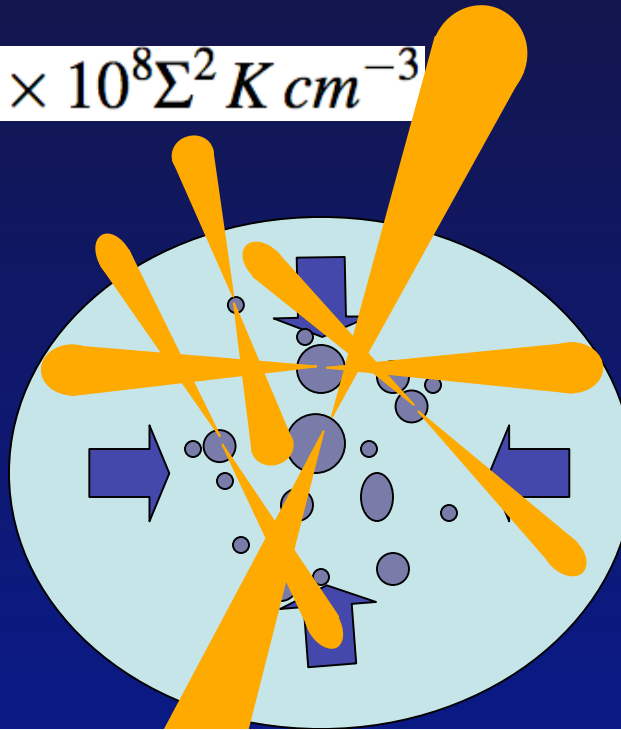
Radiative Transfer: Chakrabarti & McKee
(2005); Hernandez, Tan, Whitney, in prep.

Accretion disks (Kratter & Matzner 2007;
Kratter ea. in prep)



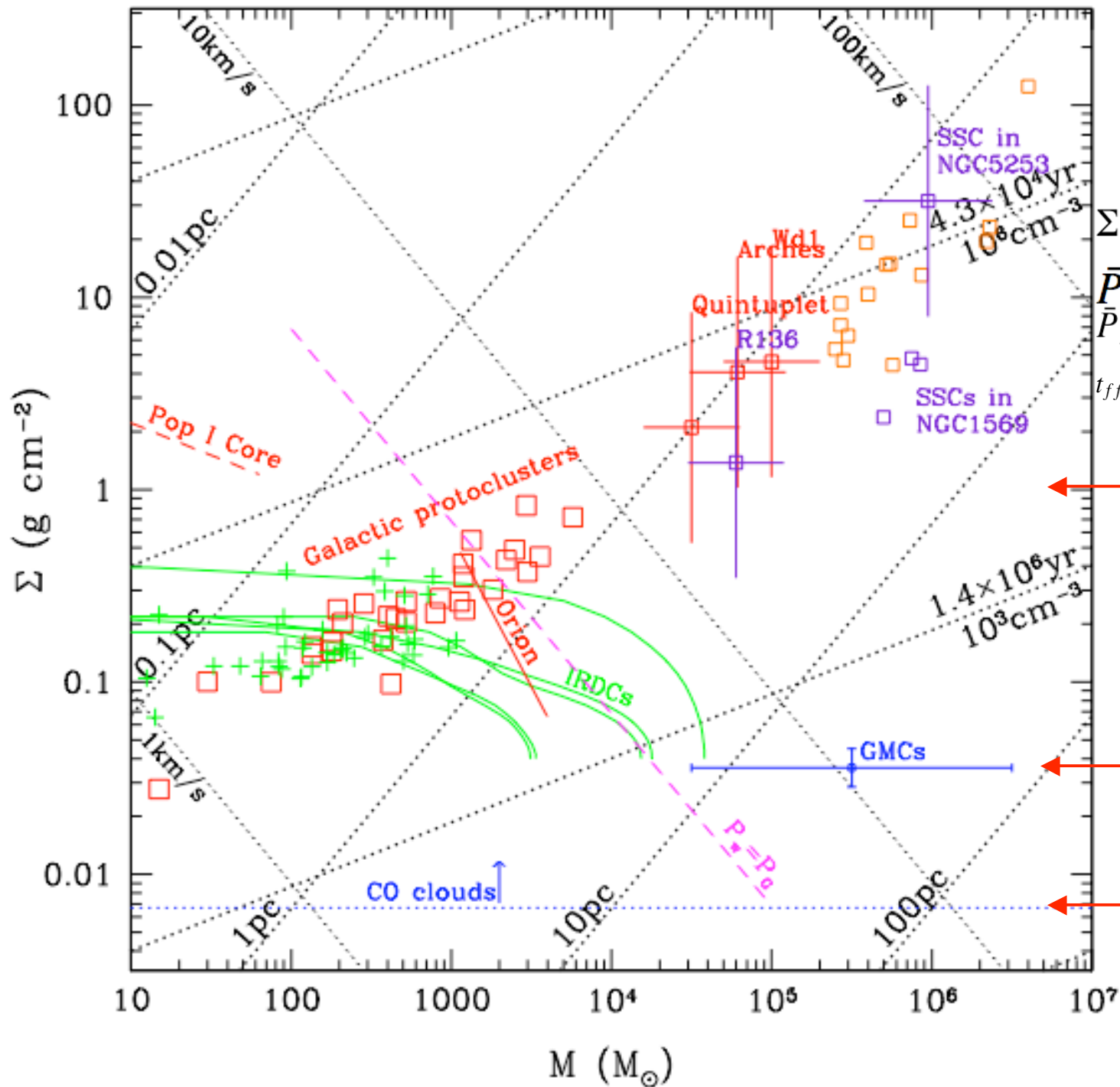
Feedback: protostellar outflows

$$\bar{P}_G \simeq 0.88 G \Sigma^2 \rightarrow 4.25 \times 10^8 \Sigma^2 \text{ K cm}^{-3}$$



$$\bar{P}_w = \frac{f_{trap} \dot{P}_w}{4\pi(0.8R)^2} \rightarrow 3.87 \times 10^8 \frac{f_{trap}}{0.5} \frac{p_*}{87 \text{ km s}^{-1}} \frac{SFR_{ff}}{0.05} \left(\frac{M}{1000 M_\odot} \right)^{-1/4} \Sigma^{7/4} \text{ K cm}^{-3}$$

$$\bar{P}_w = \bar{P}_G \rightarrow \Sigma = 0.684 \left(\frac{f_{trap}}{0.5} \frac{p_*}{87 \text{ km s}^{-1}} \frac{SFR_{ff}}{0.05} \right)^4 \left(\frac{M}{1000 M_\odot} \right)^{-1} \text{ g cm}^{-2}$$



Outflow Wind Feedback

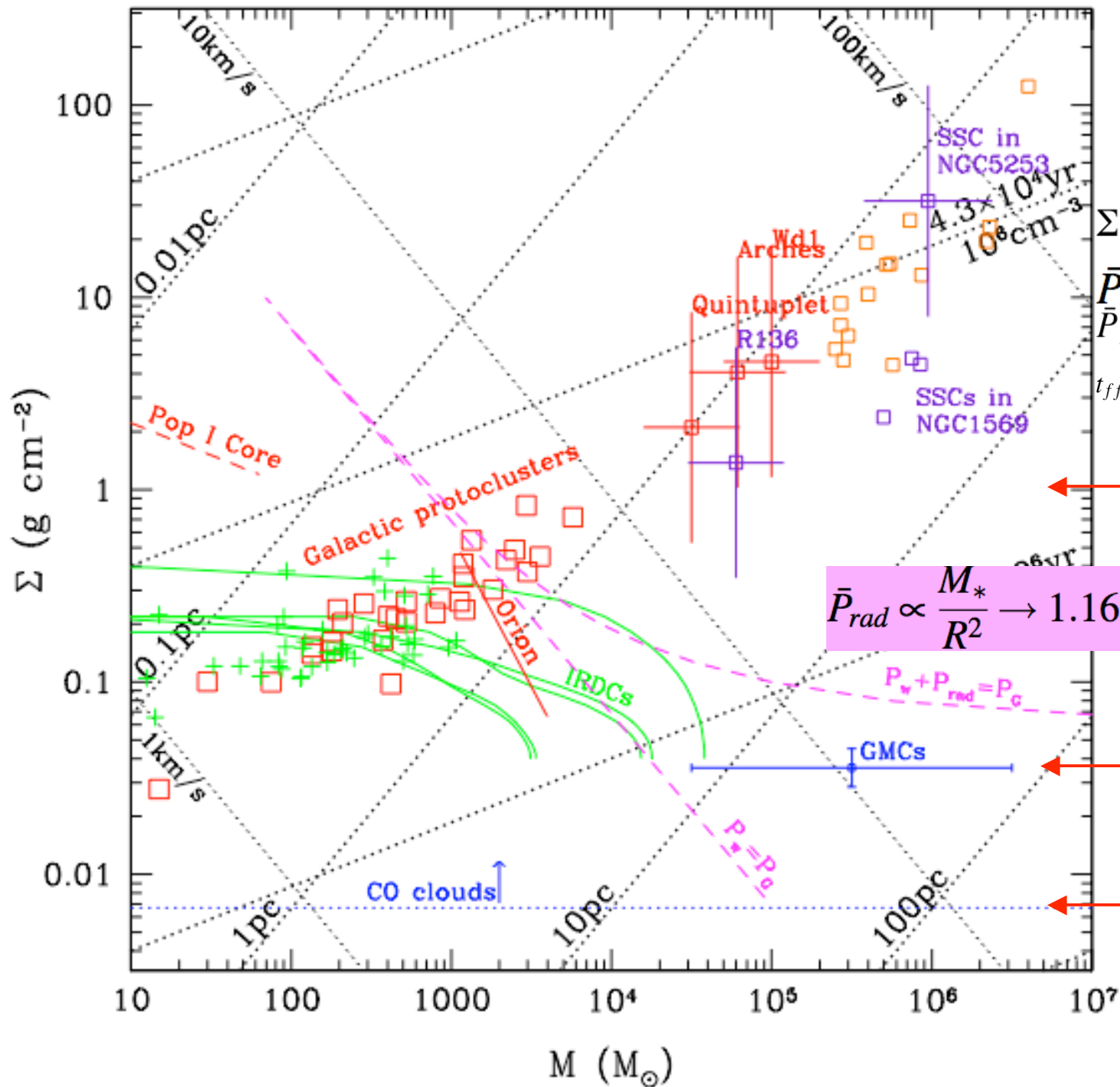
$$\Sigma \equiv \frac{M}{\pi R^2}$$

$$\bar{P} \approx G \Sigma^2$$

$$\bar{P}/k = 4.3 \times 10^8 \Sigma^2 K \text{ cm}^{-3}$$

$$t_{ff} = \left(\frac{3\pi}{32G\rho} \right)^{1/2}$$

- $A_V = 200$
 $N_H = 4.2 \times 10^{23} \text{ cm}^{-2}$
 $\Sigma = 4800 M_\odot \text{ pc}^{-2}$
- $A_V = 7.5$
 $N_H = 1.6 \times 10^{22} \text{ cm}^{-2}$
 $\Sigma = 180 M_\odot \text{ pc}^{-2}$
- $A_V = 1.4$
 $N_H = 3.0 \times 10^{21} \text{ cm}^{-2}$
 $\Sigma = 34 M_\odot \text{ pc}^{-2}$



Outflow Wind and Radiation Pressure Feedback

$$\Sigma \equiv \frac{M}{\pi R^2}$$

$$\bar{P} \simeq G \Sigma^2$$

$$\bar{P}/k = 4.3 \times 10^8 \Sigma^2 K \text{ cm}^{-3}$$

$$t_{ff} = \left(\frac{3\pi}{32G\rho} \right)^{1/2}$$

$$A_V = 200$$

$$N_H = 4.2 \times 10^{23} \text{ cm}^{-2}$$

$$\Sigma = 4800 M_\odot \text{ pc}^{-2}$$

$$\bar{P}_{rad} \propto \frac{M_*}{R^2} \rightarrow 1.16 \times 10^7 \frac{\epsilon_*}{0.33} \Sigma K \text{ cm}^{-3}$$

$$A_V = 7.5$$

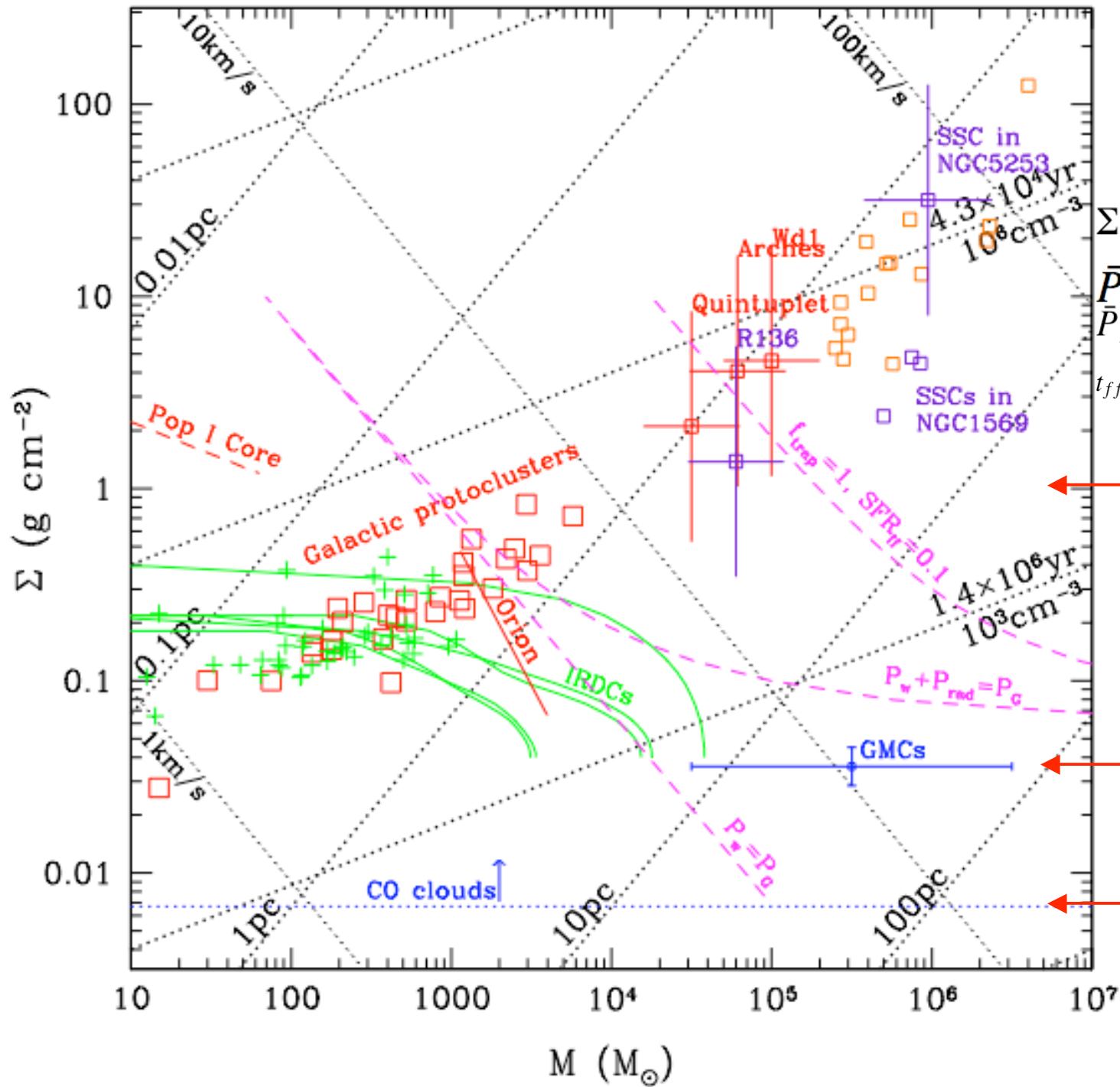
$$N_H = 1.6 \times 10^{22} \text{ cm}^{-2}$$

$$\Sigma = 180 M_\odot \text{ pc}^{-2}$$

$$A_V = 1.4$$

$$N_H = 3.0 \times 10^{21} \text{ cm}^{-2}$$

$$\Sigma = 34 M_\odot \text{ pc}^{-2}$$



Outflow Wind and Radiation Pressure Feedback

$$\Sigma \equiv \frac{M}{\pi R^2}$$

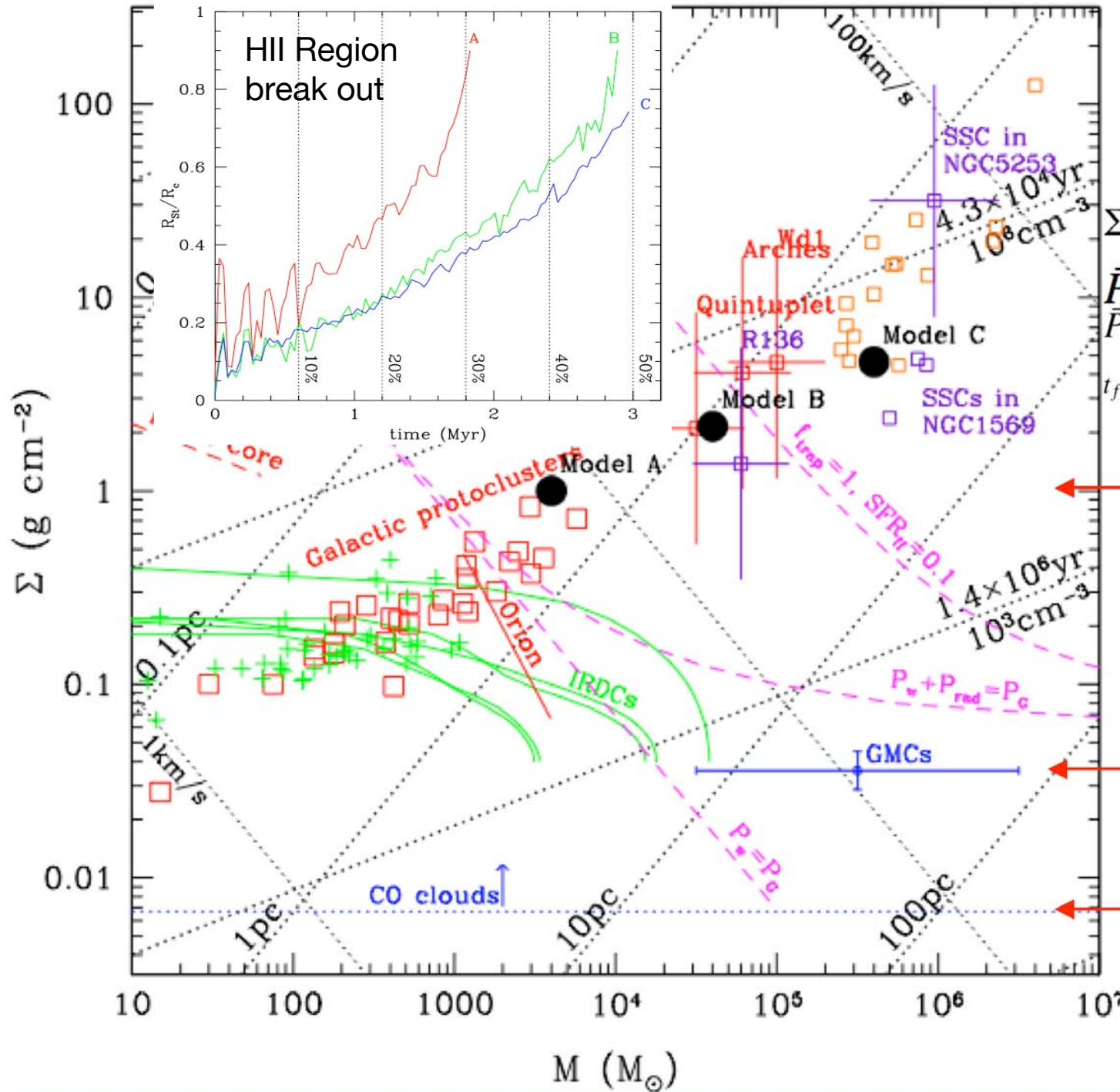
$$\bar{P} \simeq G \Sigma^2$$

$$\bar{P}/k = 4.3 \times 10^8 \Sigma^2 K \text{ cm}^{-3}$$

$$t_{ff} = \left(\frac{3\pi}{32G\rho} \right)^{1/2}$$

- $A_V = 200$
 $N_H = 4.2 \times 10^{23} \text{ cm}^{-2}$
 $\Sigma = 4800 M_\odot \text{ pc}^{-2}$
- $A_V = 7.5$
 $N_H = 1.6 \times 10^{22} \text{ cm}^{-2}$
 $\Sigma = 180 M_\odot \text{ pc}^{-2}$
- $A_V = 1.4$
 $N_H = 3.0 \times 10^{21} \text{ cm}^{-2}$
 $\Sigma = 34 M_\odot \text{ pc}^{-2}$

Ionization Feedback



$$\Sigma \equiv \frac{M}{\pi R^2}$$

$$\bar{P} \simeq G \Sigma^2$$

$$\bar{P}/k = 4.3 \times 10^8 \Sigma^2 K\ cm^{-3}$$

$$t_{ff} = \left(\frac{3\pi}{32G\rho} \right)^{1/2}$$

$$A_V = 200$$

$$N_H = 4.2 \times 10^{23}\ cm^{-2}$$

$$\Sigma = 4800\ M_{\odot}\ pc^{-2}$$

$$A_V = 7.5$$

$$N_H = 1.6 \times 10^{22}\ cm^{-2}$$

$$\Sigma = 180\ M_{\odot}\ pc^{-2}$$

$$A_V = 1.4$$

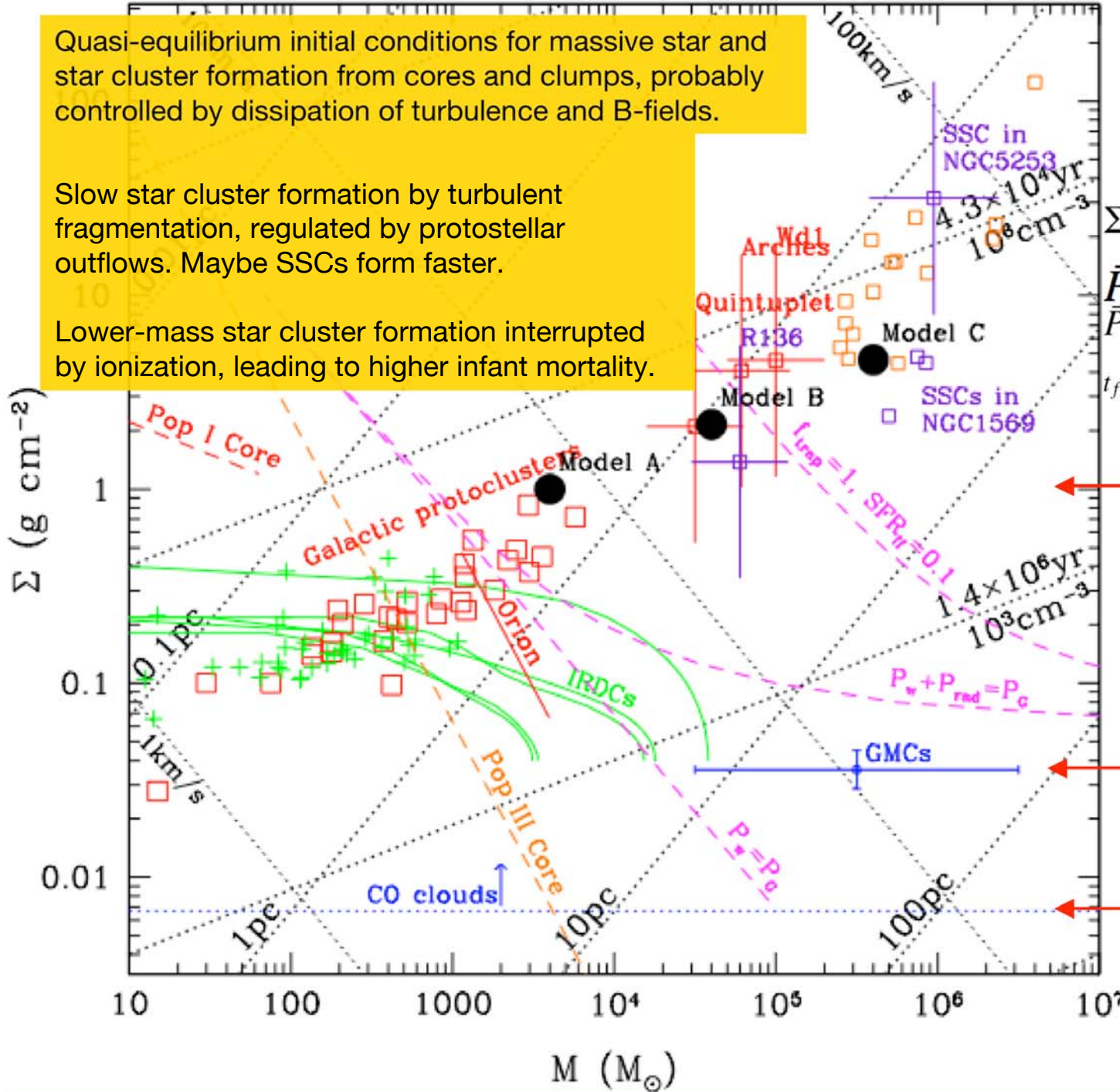
$$N_H = 3.0 \times 10^{21}\ cm^{-2}$$

$$\Sigma = 34\ M_{\odot}\ pc^{-2}$$

Quasi-equilibrium initial conditions for massive star and star cluster formation from cores and clumps, probably controlled by dissipation of turbulence and B-fields.

Slow star cluster formation by turbulent fragmentation, regulated by protostellar outflows. Maybe SSCs form faster.

Lower-mass star cluster formation interrupted by ionization, leading to higher infant mortality.



Conclusions

$$\Sigma \equiv \frac{M}{\pi R^2}$$

$$\bar{P} \simeq G \Sigma^2$$

$$\bar{P}/k = 4.3 \times 10^8 \Sigma^2 K \text{ cm}^{-3}$$

$$t_{ff} = \left(\frac{3\pi}{32G\rho} \right)^{1/2}$$

$$A_V = 200$$

$$N_H = 4.2 \times 10^{23} \text{ cm}^{-2}$$

$$\Sigma = 4800 M_\odot \text{ pc}^{-2}$$

$$A_V = 7.5$$

$$N_H = 1.6 \times 10^{22} \text{ cm}^{-2}$$

$$\Sigma = 180 M_\odot \text{ pc}^{-2}$$

$$A_V = 1.4$$

$$N_H = 3.0 \times 10^{21} \text{ cm}^{-2}$$

$$\Sigma = 34 M_\odot \text{ pc}^{-2}$$