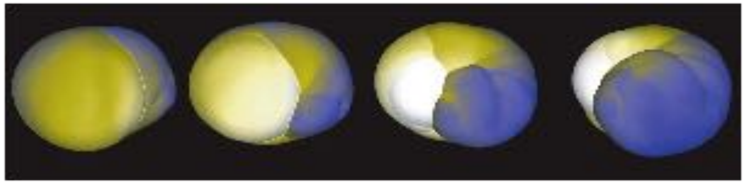
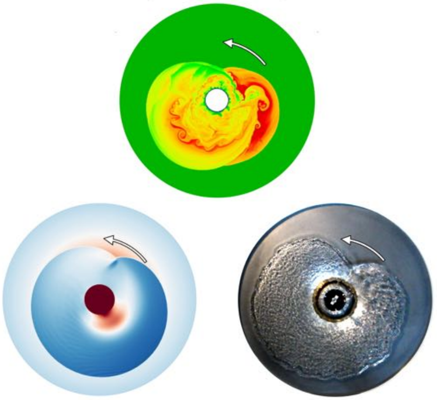
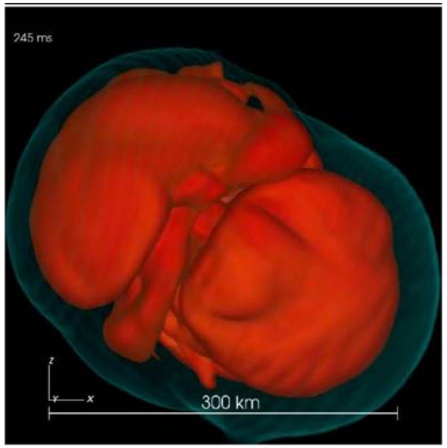
The core-collapse angular momentum budget

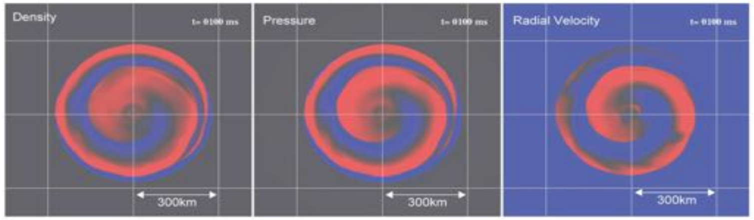


Blondin & Mezzacappa 07

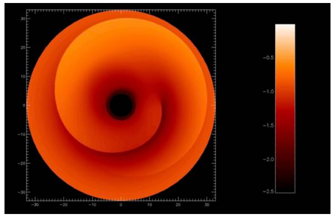


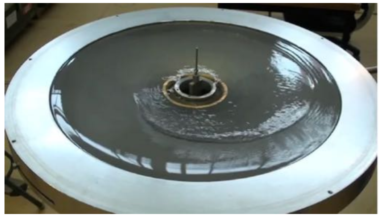


Hanke+13



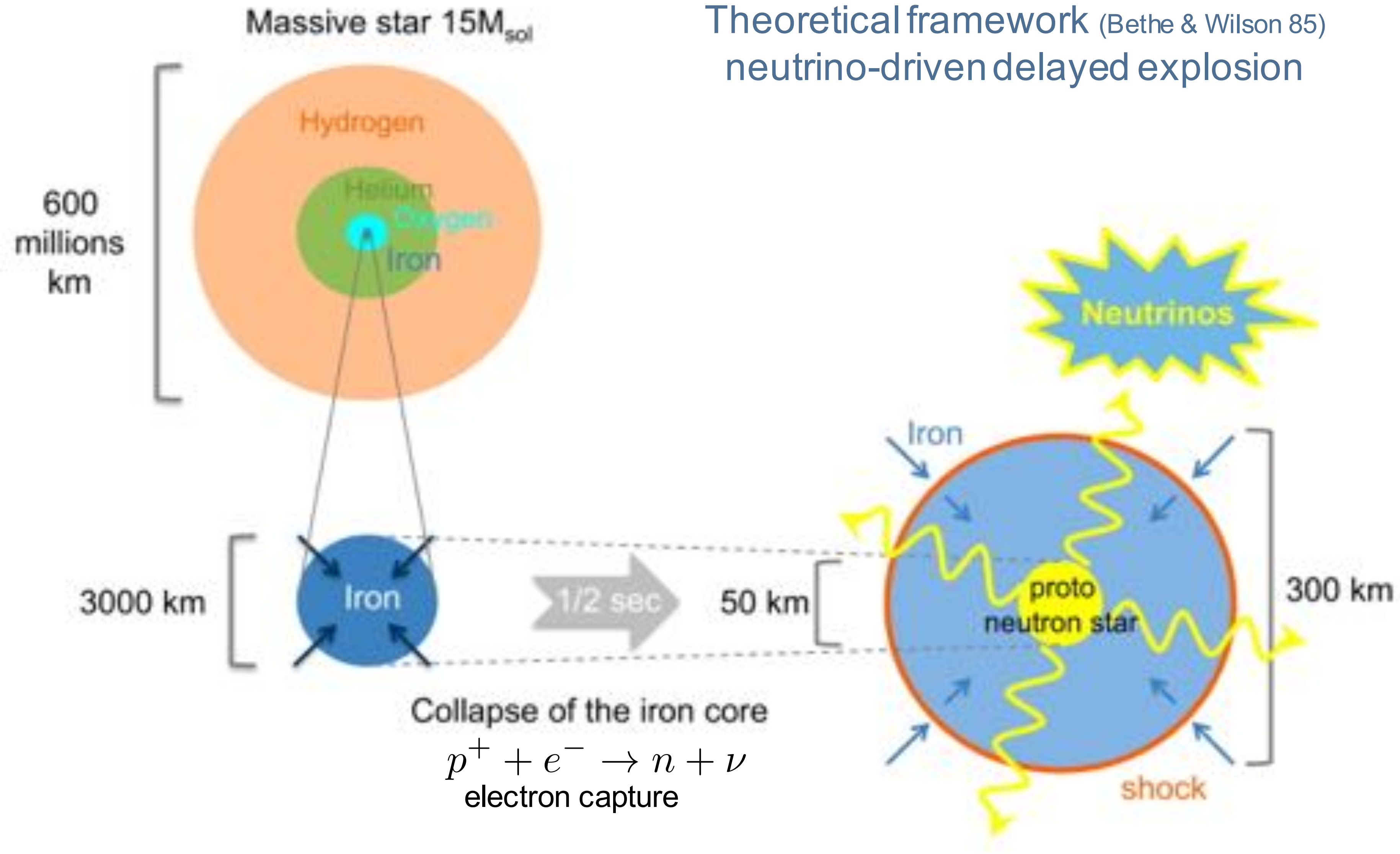
Takiwaki+16





Thierry Foglizzo, Remi Kazeroni*, Jérôme Guilet*, Matthias Gonzalez, Gilles Durand
CEA Saclay, *MPA Garching

Theoretical framework (Bethe & Wilson 85)
neutrino-driven delayed explosion



Collapse of the iron core

$$p^+ + e^- \to n + \nu$$

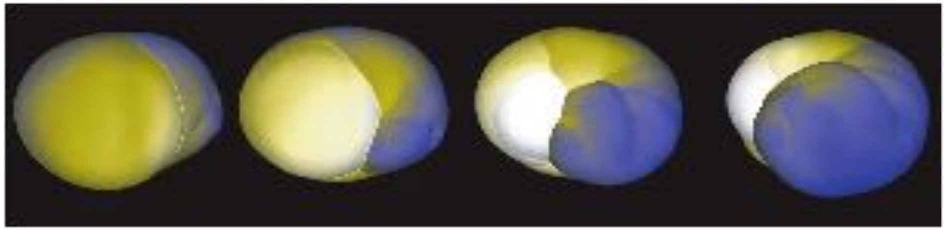
electron capture

$$\frac{GM_{\rm ns}^2}{R_{\rm ns}} \sim 2\times 10^{53}{\rm erg}\left(\frac{30{\rm km}}{R_{\rm ns}}\right)\left(\frac{M_{\rm ns}}{1.5M_{\rm sol}}\right)^2$$

modest energy in differential rotation: $E_{\rm diff} < E_{\rm rot} \sim 2.4\times 10^{50}{\rm erg}\left(\frac{M_{\rm ns}}{1.5M_{\rm sol}}\right)\left(\frac{R_{\rm ns}}{10{\rm km}}\right)^2\left(\frac{10{\rm ms}}{P_{\rm ns}}\right)^2$

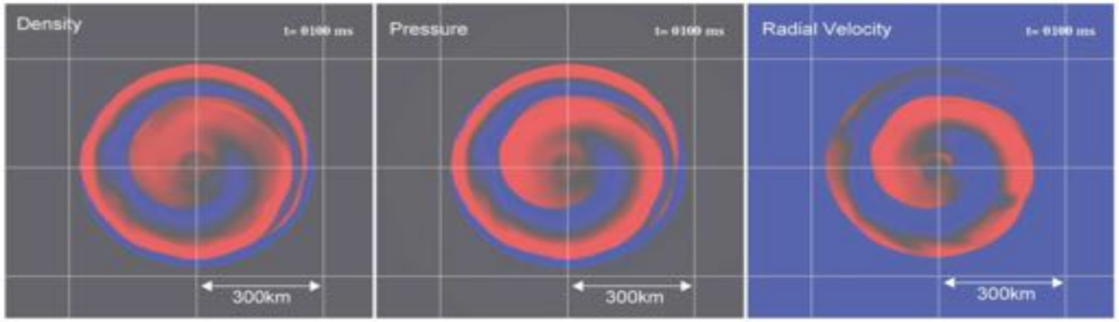
Few 3D simulations including rotation

slow rotation (j = 10^{15} cm²/s): spiral SASI

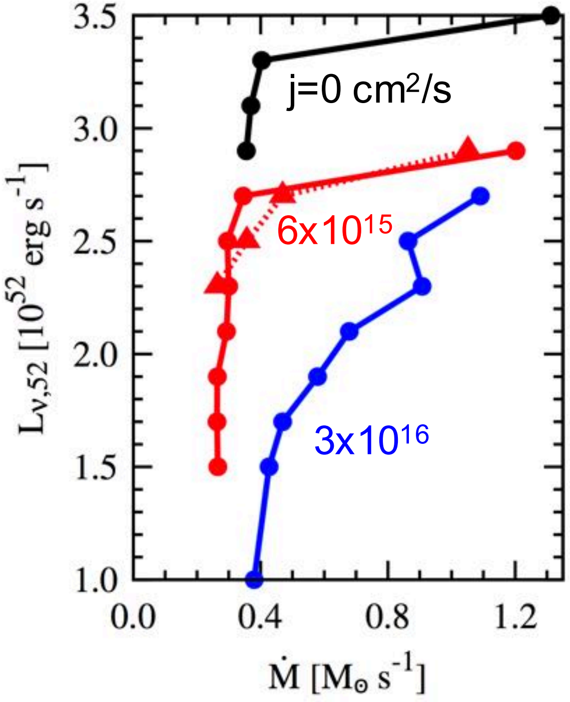


Blondin & Mezzacappa 07

Very fast rotation (j = 4x10^{16} cm²/s): low-T/|W| instability



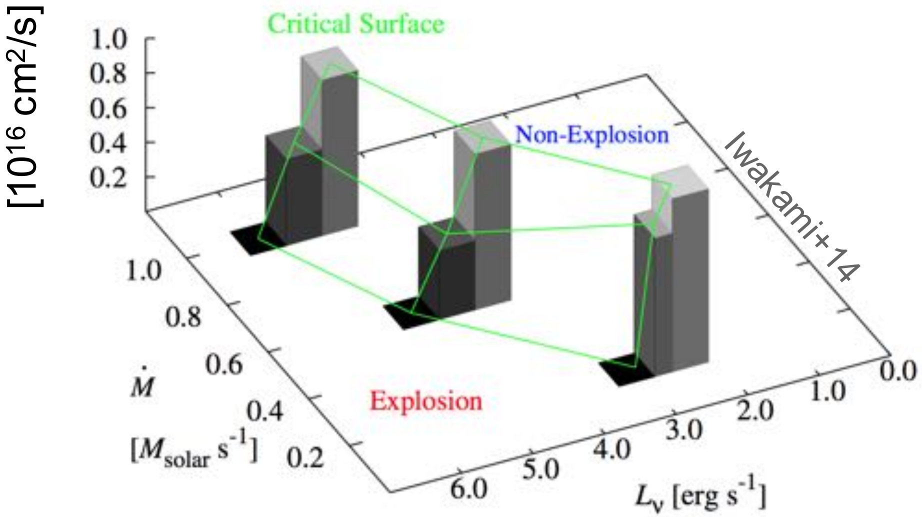
Takiwaki+16



Nakamura+14

$\Delta L_\nu/L_\nu$~10%

for j=5x10^{15}cm²/s
(~ms pulsar)
modest effect compared to the rotational kinetic energy involved



Iwakami+14

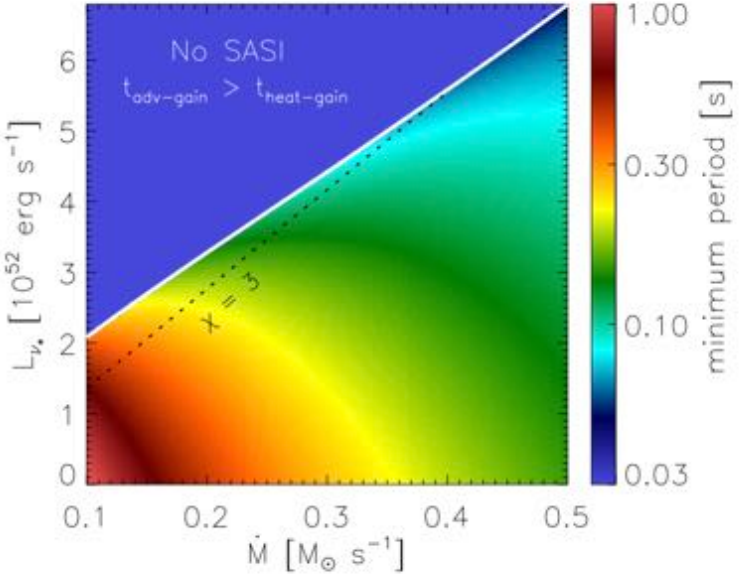
$$E_{\rm rot} \sim 1.5 \times 10^{52}{\rm erg} \left(\frac{M_{\rm ns}}{1.5 M_{\rm sol}}\right) \left(\frac{10{\rm km}}{R_{\rm ns}}\right)^2 \left(\frac{j}{5 \times 10^{15}{\rm cm}^2{\rm s}^{-1}}\right)^2$$

Angular momentum in the final stages of stellar evolution

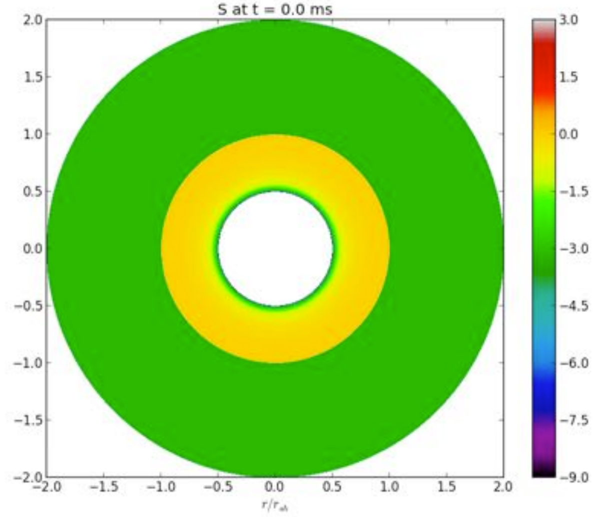
RSG/BSG	O →Si burning	Si→Fe burning	core-collapse dynamics	NS, BH
$R_{He}\sim10^{7}$km millions yrs	$R_{O}\sim3\text{-}8\times10^{3}$km months	$R_{Fe}\sim1700$km hours	$R_{sh}\sim150$km $R_{PNS}\sim50$km <sec	$R_{NS}\sim10$km >10^{3}yrs
hints of slow core rotation from RG asterosismology (Cantiello+14) possible IGW-magnetized core coupling ? (Jim's talk)	convective inhomogeneities (Müller+16)	convectively excited internal gravity waves (Fuller+15)	ν-driven convection, SASI, low $T/\lvert W\rvert$ (Kazeroni+17)	electromagnetic spindow shorter timescales? r-mode braking, ν-powered magnetic wind, fallback+propeller braking strong B>10^{13}G field ?
from "slow" to $\sim4\times10^{14}$cm^{2}/s with Tayler-Spruit dynamo to $\sim5\times10^{16}$cm^{2}/s without (Heger+05)		$\sim10^{13\text{-}14}$cm^{2}/s stochastic	$\sim10^{13\text{-}14}$cm^{2}/s stochastic	uniiform distribution $P_{NS}\sim10\text{-}100$ms at birth $j_{NS}\sim6\times10^{13\text{-}14}$ cm^{2}/s (Popov & Turolla 12) ms breakup threshold $\sim6\times10^{15}$cm^{2}/s

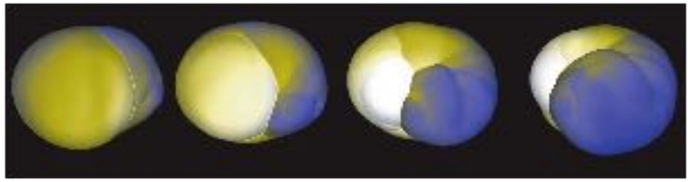
Spin up of the neutron star induced by the spiral mode of SASI

Guilet & Fernandez 14



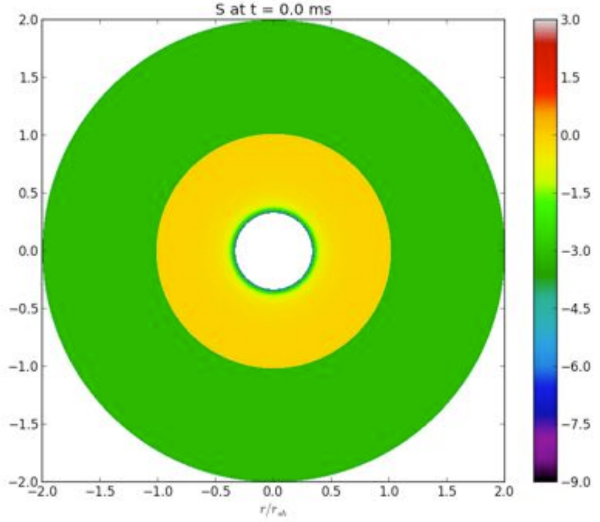
$r_{sh}/r_{NS} = 2$

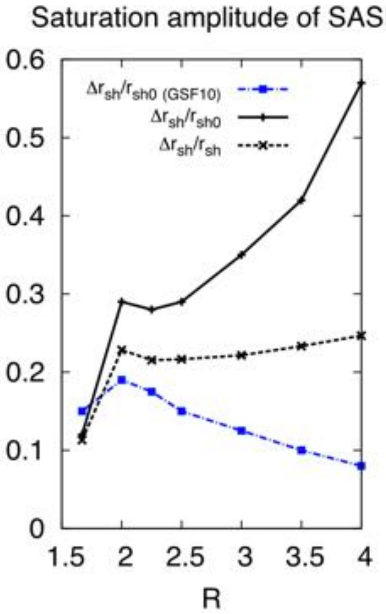


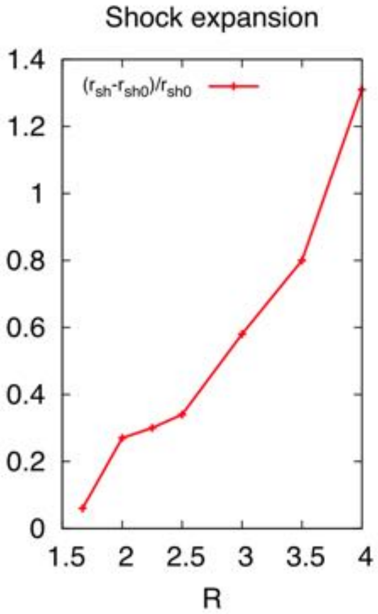


Blondin & Mezacappa 07

$r_{sh}/r_{NS} = 3$

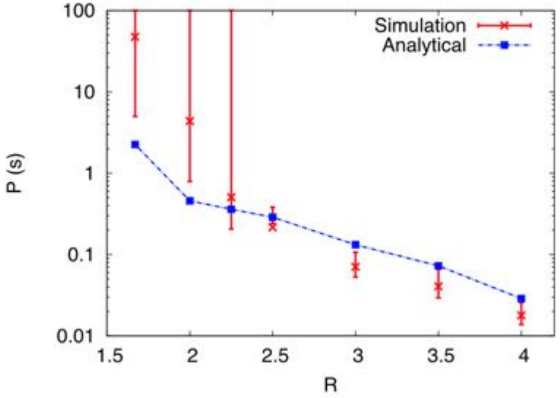






Kazeroni+16

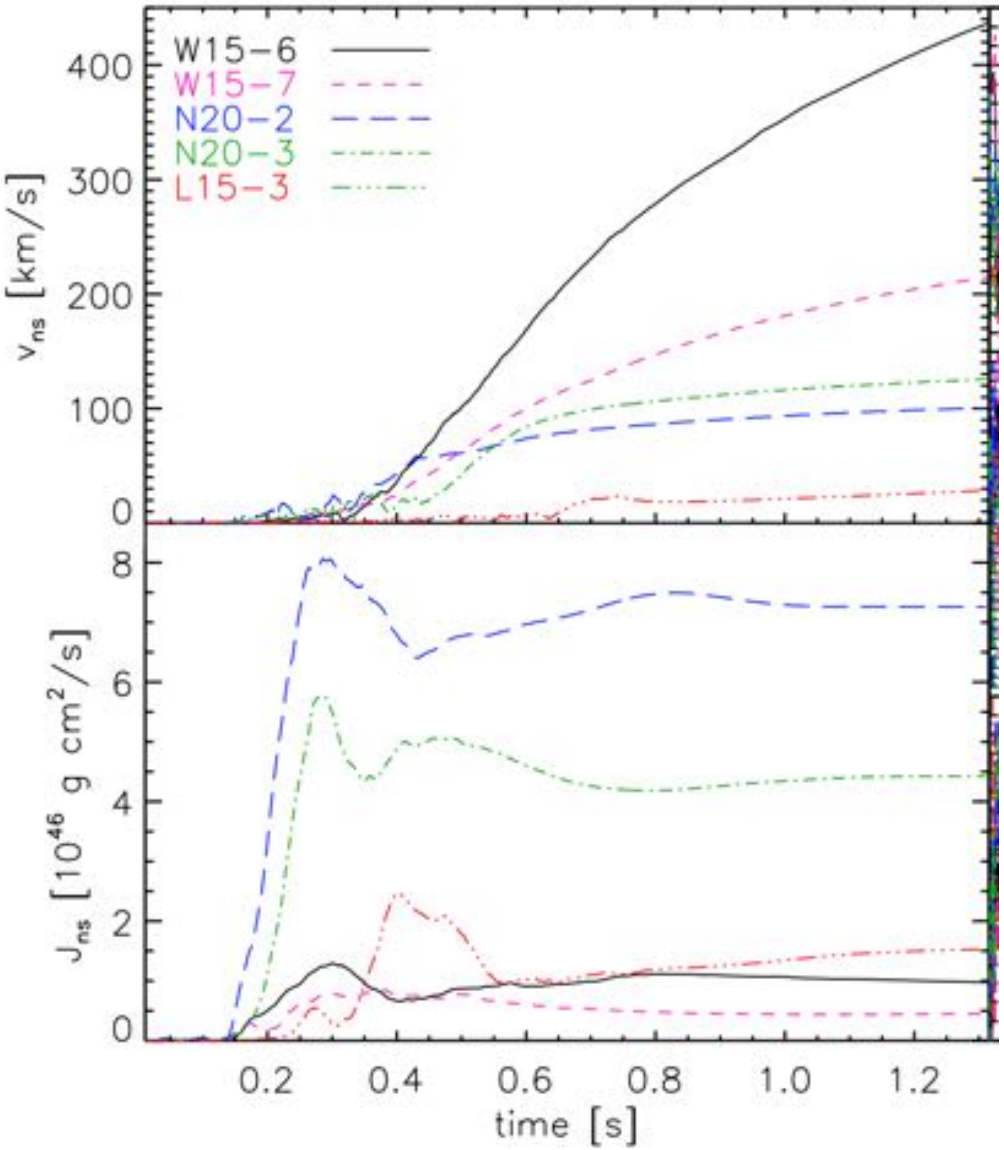
2D cylindrical
adiabatic shock
perfect gas γ=4/3
ν-cooling function
no ν-heating

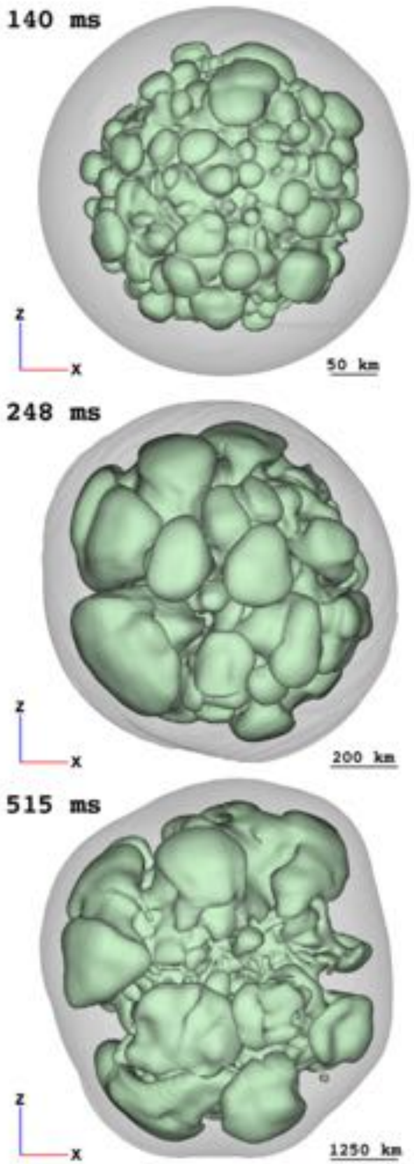


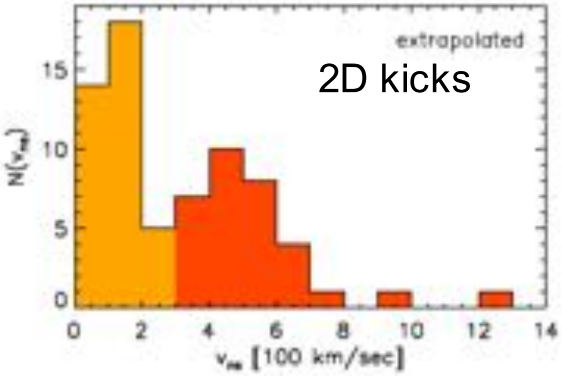
pulsar spin periods down to ~50ms

-the strength of SASI increases with the radius ratio R = r_{sh}/r_{NS}
-unexpected stochasticity

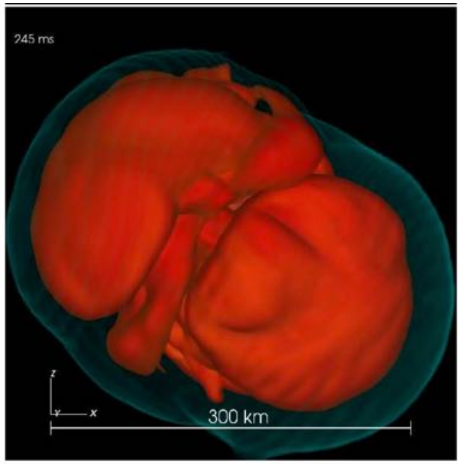
Kick and spin from multi-D simulations: different timescales







Scheck+06



Hanke+13

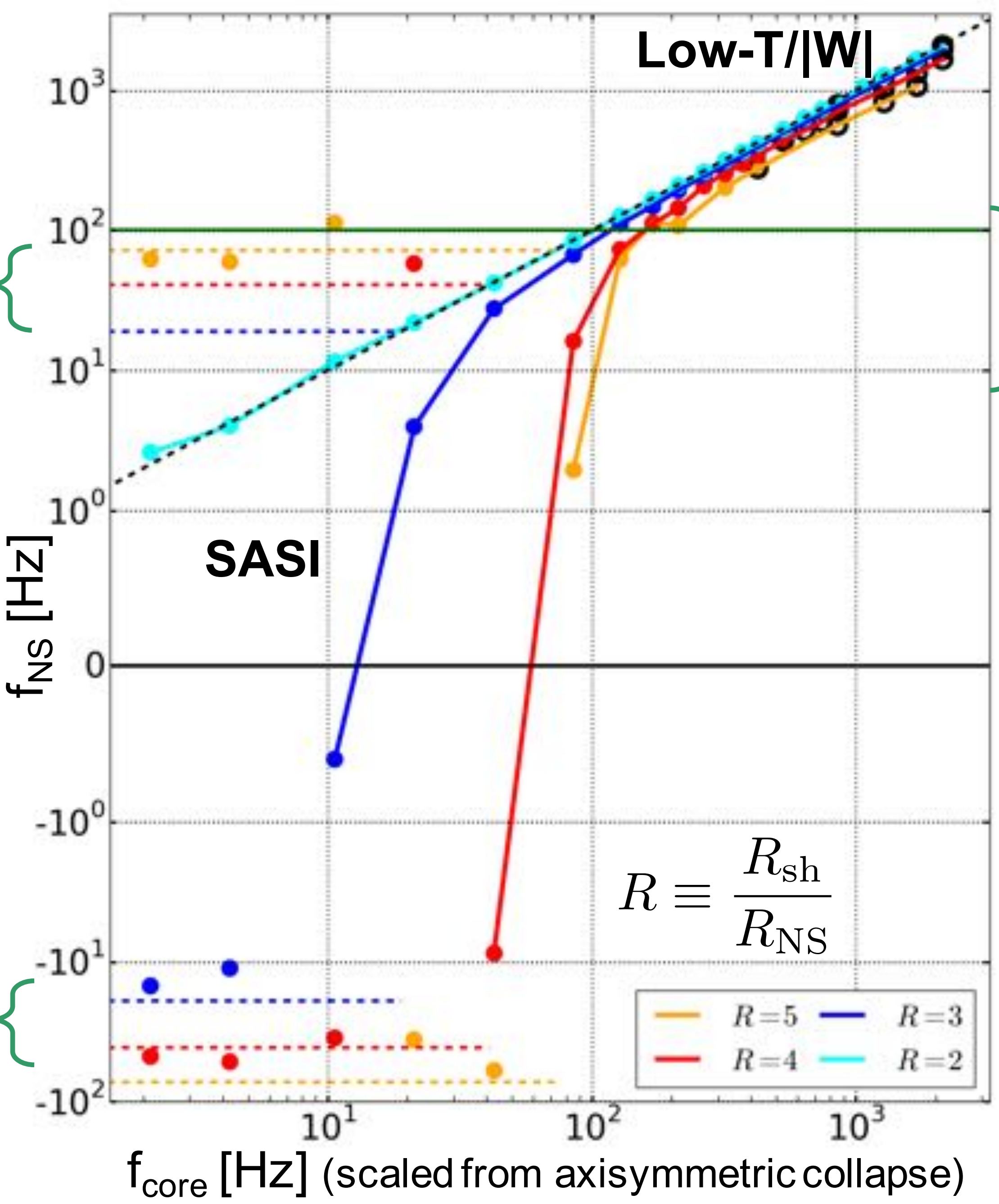
$$\chi \equiv \int_{\rm sh}^{\rm gain} \omega_{\rm BV} \frac{\mathrm{d}r}{v_r} < 3$$

P_{ns}~100ms to 8s (Wongwathanarat+13)

SASI vs convection ?

Spin-up or spin-down of the neutron star?

(Kazeroni+17)



SASI alone

range of NS spin at birth

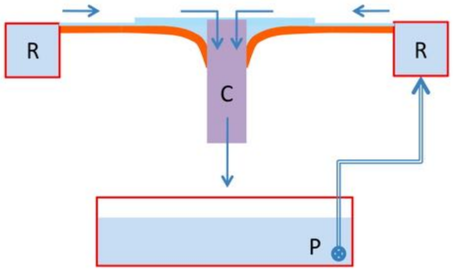
Neutron Star rotation frequency

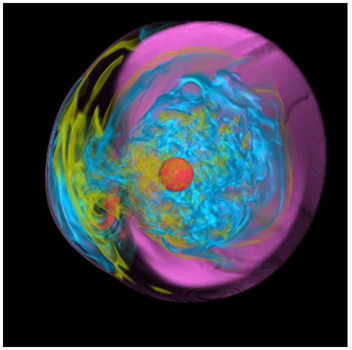
SASI alone

For a strong rotation rate, the corotation instability decelerates the neutron star by less than 30%.

SWASI: an experimental analogue of SASI

Shallow Water Analogue of a Shock Instability




Blondin & Mezzacappa 07

adiabatic gas

$$c_s^2 \equiv \frac{\gamma P}{\rho}$$

$$\Phi \equiv -\frac{GM_{ns}}{r}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

$$\frac{\partial v}{\partial t} + (\nabla \times v) \times v + \nabla \left(\frac{v^2}{2} + \frac{c_s^2}{\gamma - 1} + \Phi \right) = \frac{c_s^2}{\gamma} \nabla S$$

Inviscid shallow water is analogue to an isentropic gas $\gamma=2$

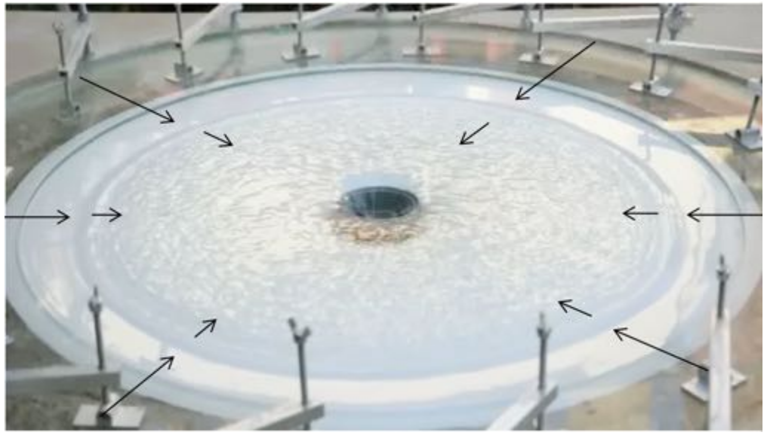
St Venant

$$c_{sw}^2 \equiv gH$$

$$\Phi \equiv gH_\Phi$$

$$\frac{\partial H}{\partial t} + \nabla \cdot (Hv) = 0$$

$$\frac{\partial v}{\partial t} + (\nabla \times v) \times v + \nabla \left(\frac{v^2}{2} + c_{sw}^2 + \Phi \right) = 0$$



$$\left.\begin{array}{l}\text{acoustic waves}\\ \text{shock wave}\\ \text{pressure}\end{array}\right\} \longleftrightarrow \left\{\begin{array}{l}\text{surface waves}\\ \text{hydraulic jump}\\ \text{depth}\end{array}\right.$$

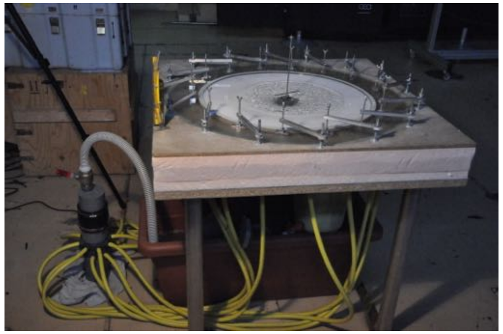
expected scaling $\frac{t_{ff}^{sh}}{t_{ff}^{jp}} \equiv \left(\frac{r_{sh}}{r_{jp}}\right)\left(\frac{r_{sh} g H_{jp}}{GM_{NS}}\right)^{\frac{1}{2}} \sim 10^{-2}$

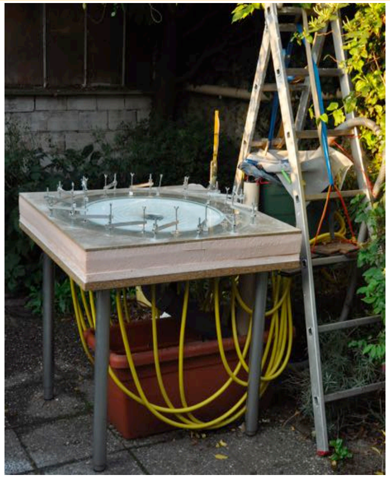
shock radius $\times 10^{-6}$ 200 km → 20 cm

oscillation period $\times 10^{2}$ 30 ms → 3 s

SWASI: simple as a garden experiment

November 2010





October 2010

June 2010

May 2010

February 2012

Physical Review Letters

3 February 2012

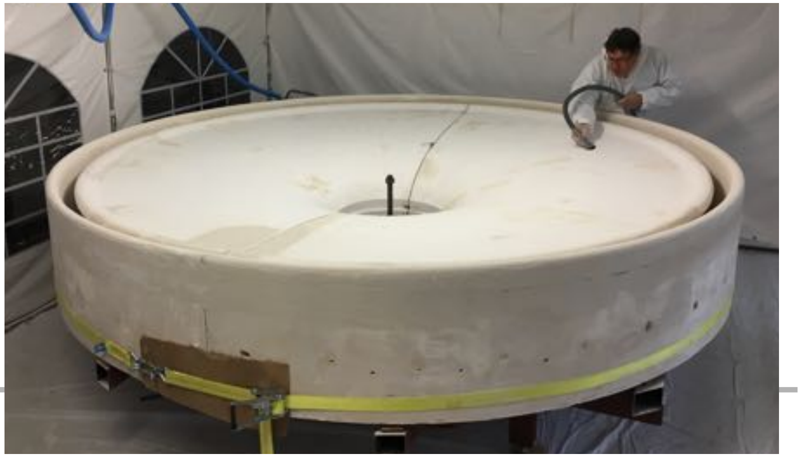
Published by American Physical Society

Volume 108, Number 5

November 2013

PRIX 2014
LE GOÛT DES SCIENCES

June 2014



February 2017

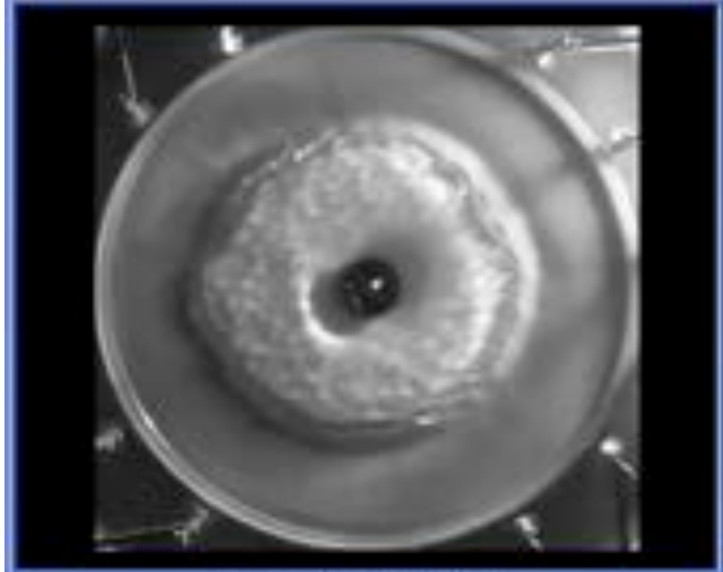
Dynamics of water in the fountain

Dynamics of the gas in the supernova core

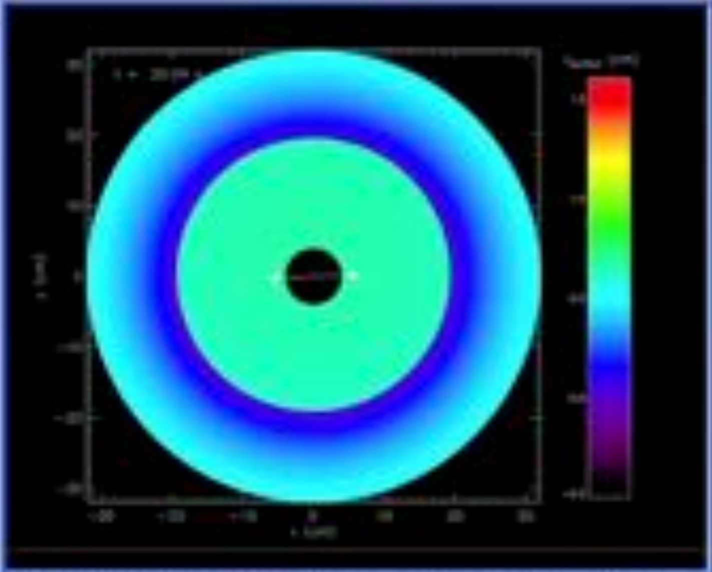
diameter 40cm
3s/oscillation

1 000 000 x bigger
100 x faster

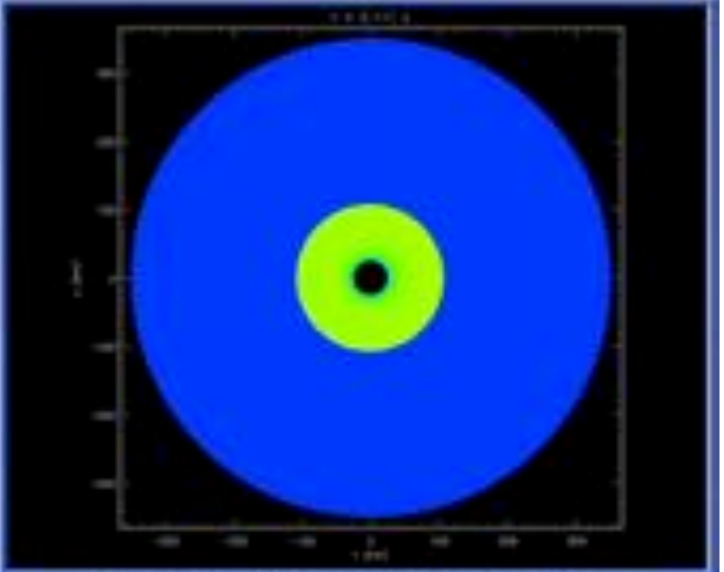
diameter 400km
0.03s/oscillation



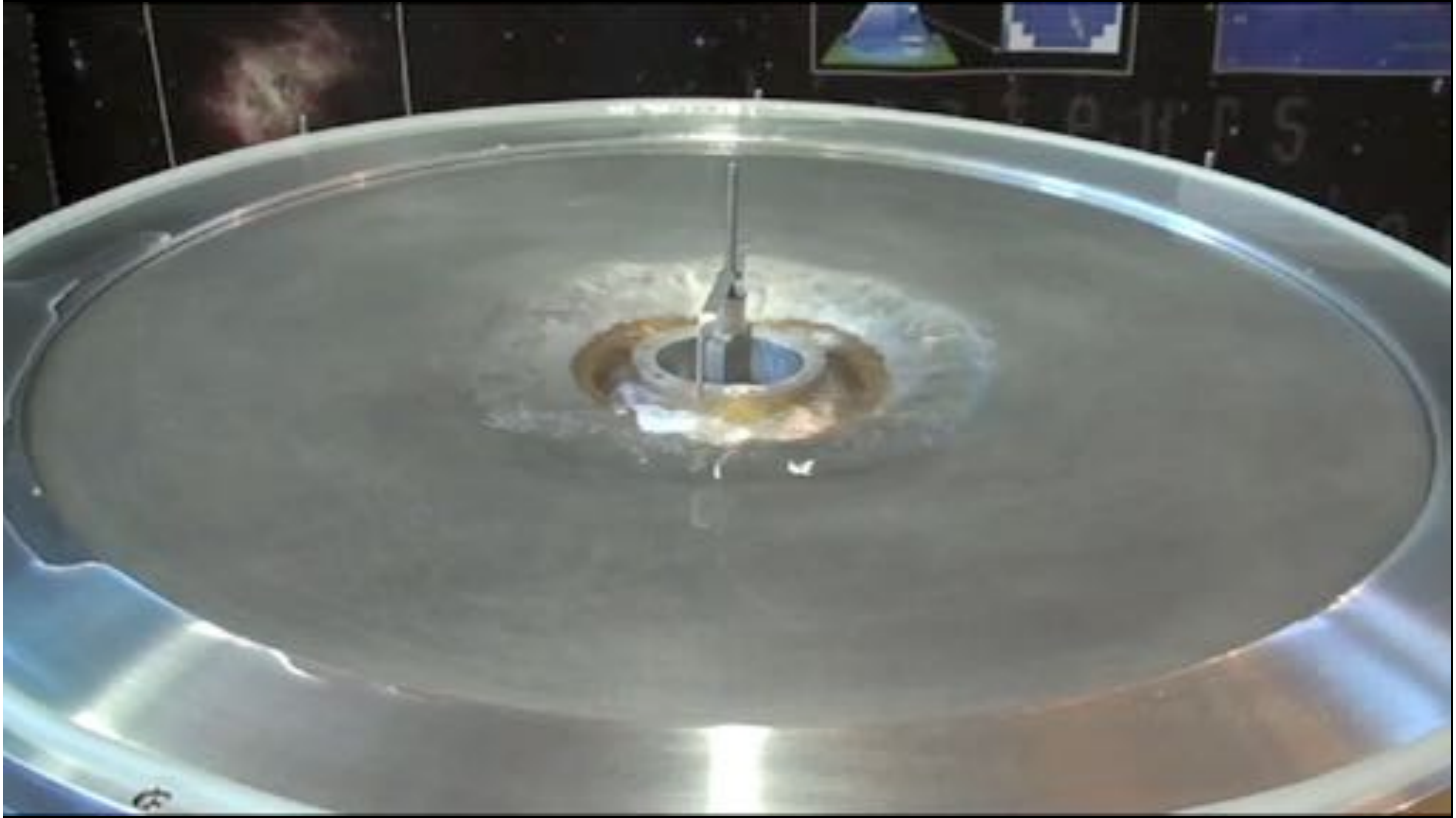
Expérience hydraulique



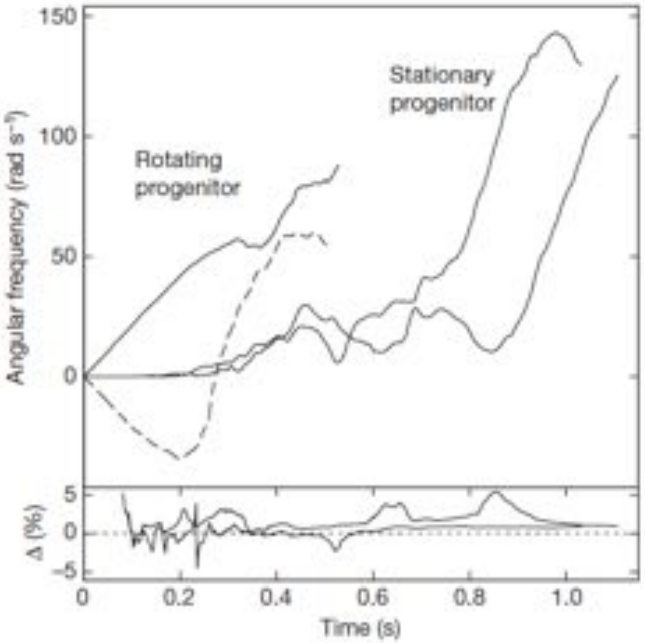
Simulation numérique de l'expérience hydraulique



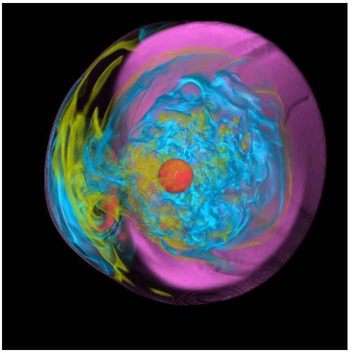
Simulation numérique de l'onde de choc dans le cœur de la supernova



Rotating progenitor: accreted angular momentum changes its sign as SASI grows



Blondin & Mezzacappa 07



Blondin & Mezzacappa 07

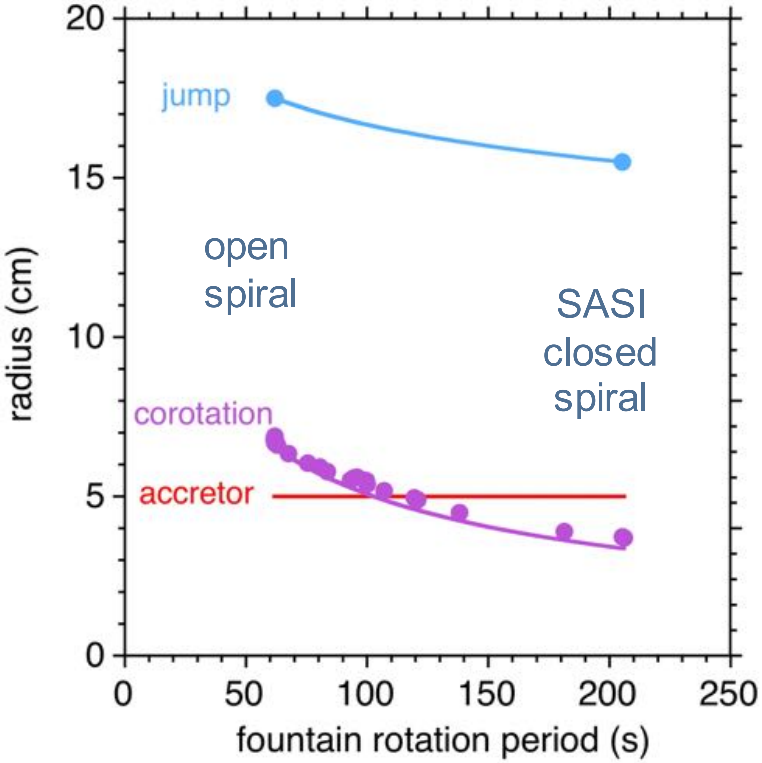
-significant shear even when the centrifugal force $\Omega^2 R$ is weak

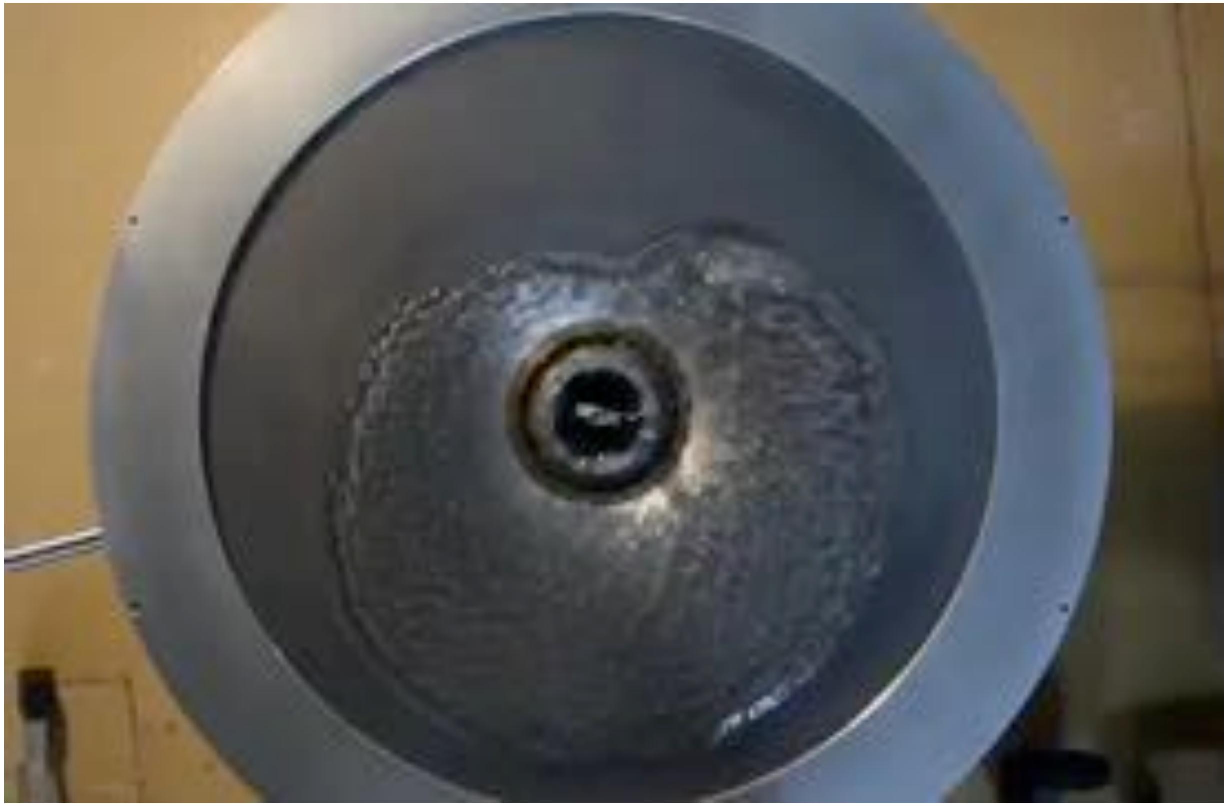
$$\frac{\Omega}{\Omega_{\rm NS}} \propto \left(\frac{R_{\rm NS}}{R}\right)^2$$

-the prograde mode is favoured by differential rotation as in shocked accretion

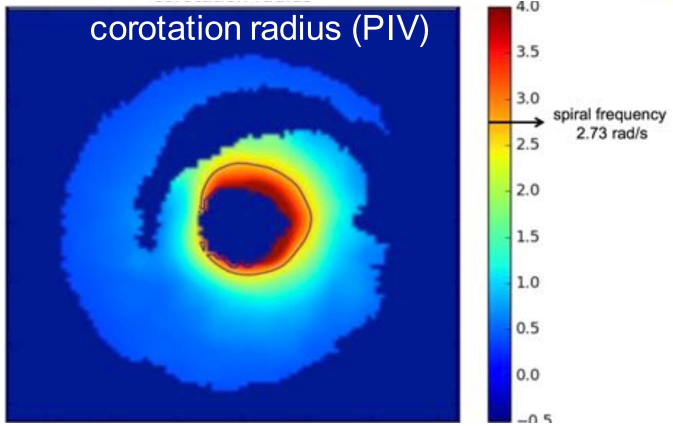
Blondin & Mezzacappa 07, Yamasaki & Foglizzo 08, Kazeroni+17

Increasing the rotation rate: continuous transition from SASI to the corotation instability

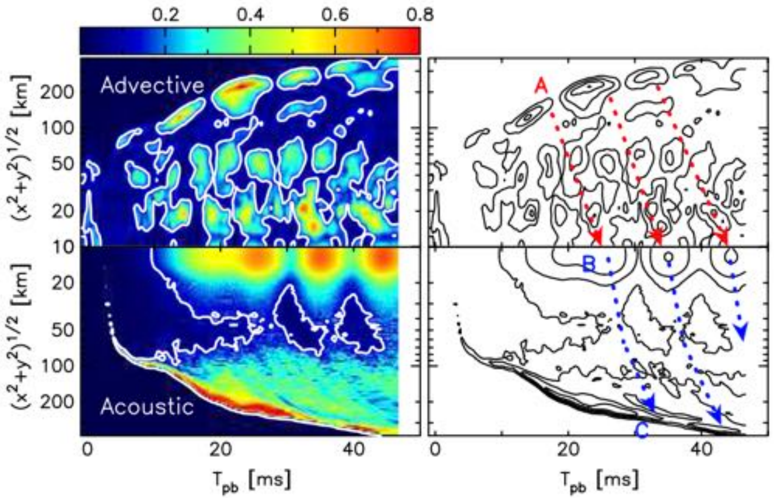




the rotation period is gradually decreased (205s → 62s)
the flow rate is gradually decreased (1.1 L/s → 0.59 L/s)



→the gravitational wave signature of the low T/|W| instability may be hard to disentangle from the SASI oscillation (Kuroda+14)



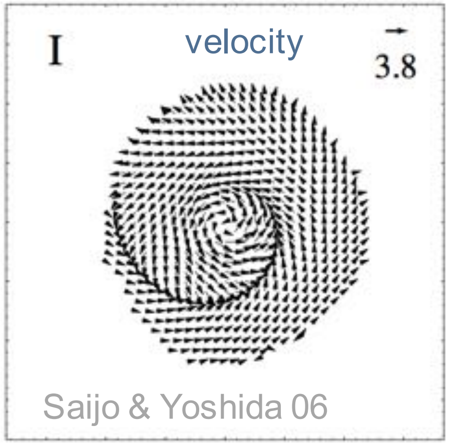
Unexpectedly robust spiral shock driven at the corotation radius when the inner rotation rate reaches 20% Kepler (low T/|W|=0.02)

Spiral instability with a weak shock

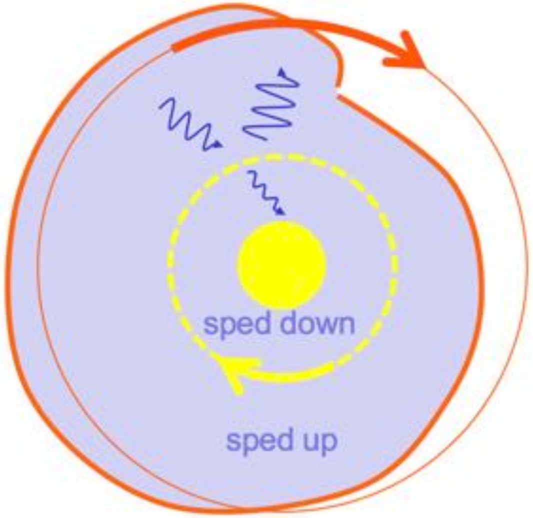
Radial accretion enforces differential rotation

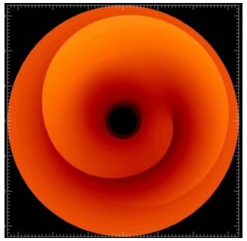
$$\frac{\Omega}{\Omega_{\rm NS}} \propto \left(\frac{R_{\rm NS}}{R}\right)^2$$

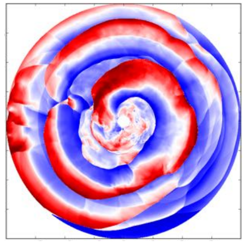
Analogue to the "low T/|W| instability" of a neutron star rotating differentially (Shibata+02,03, Saijo+03,06, Watts+05, Corvino+10, Passamonti & Andersson 15)



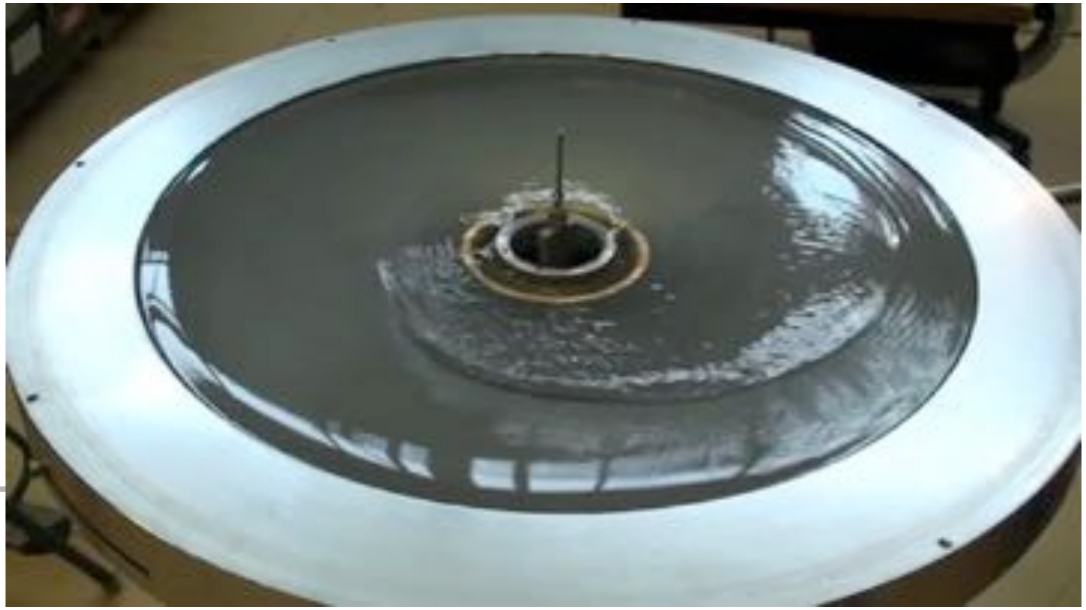
Saijo & Yoshida 06







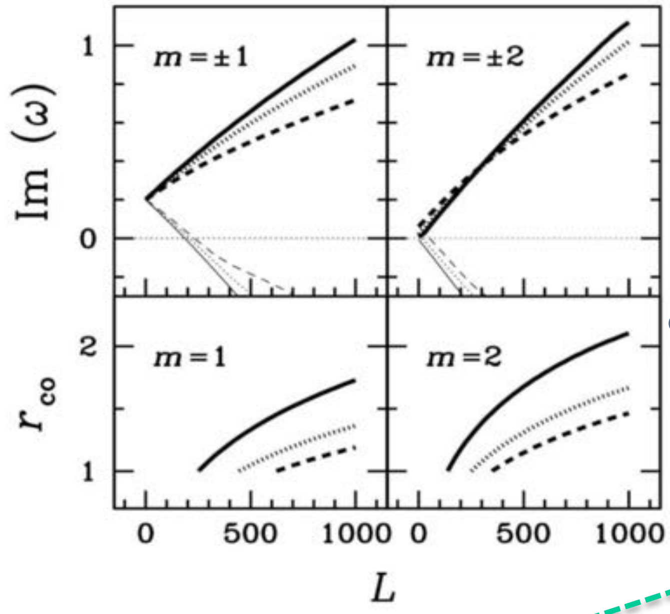
Spiral instability with subsonic accretion



Instability mechanism: interaction of a corotation radius with acoustic waves (Papaloizou & Pringle 84, Goldreich & Narayan 85)

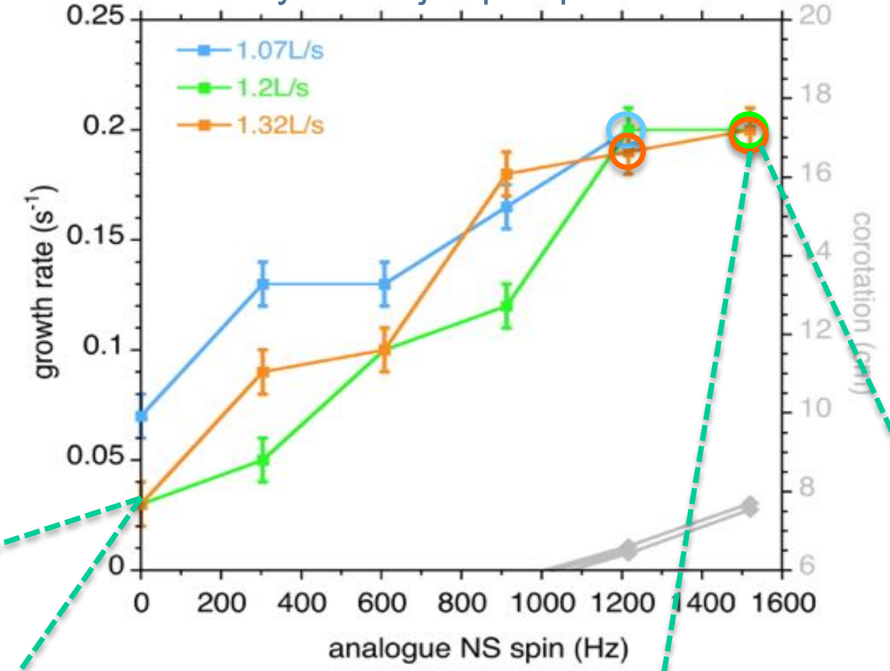
Rotation effects in the experiment compared to gas dynamics

shocked gas dynamics

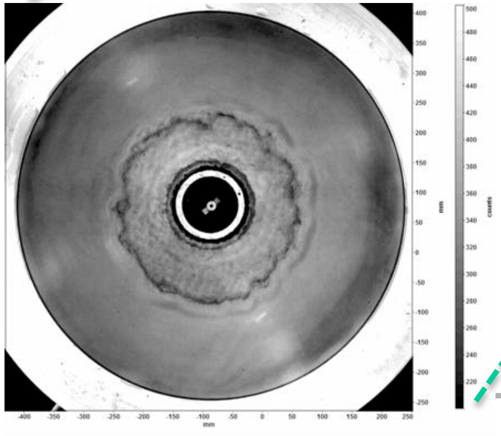


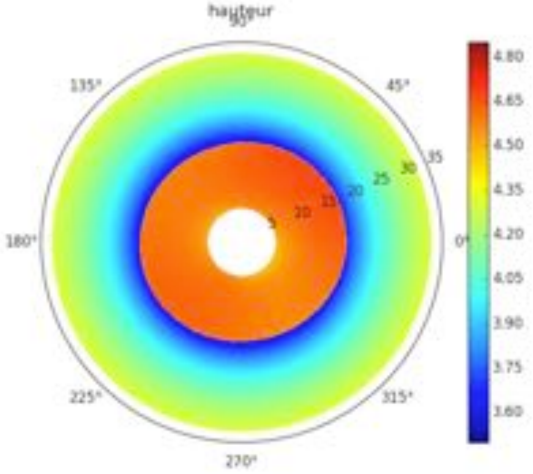
Yamazaki & Foglizzo 08

hydraulic jump experiment

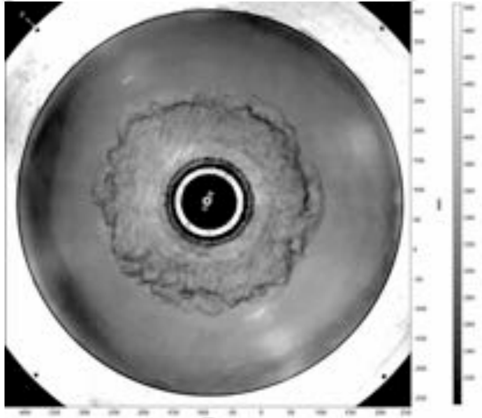


SASI
m=±1
sloshing→spiral



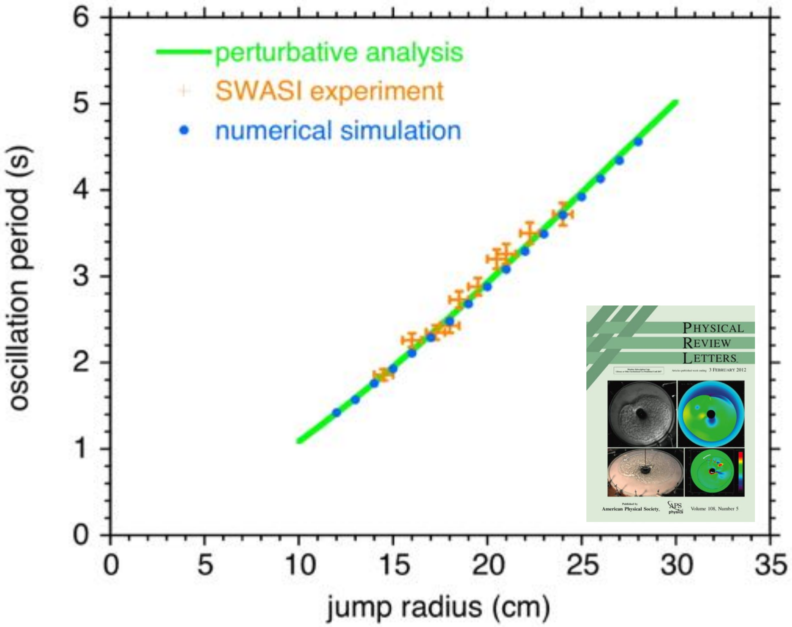


m=1,2 spiral
with corotation

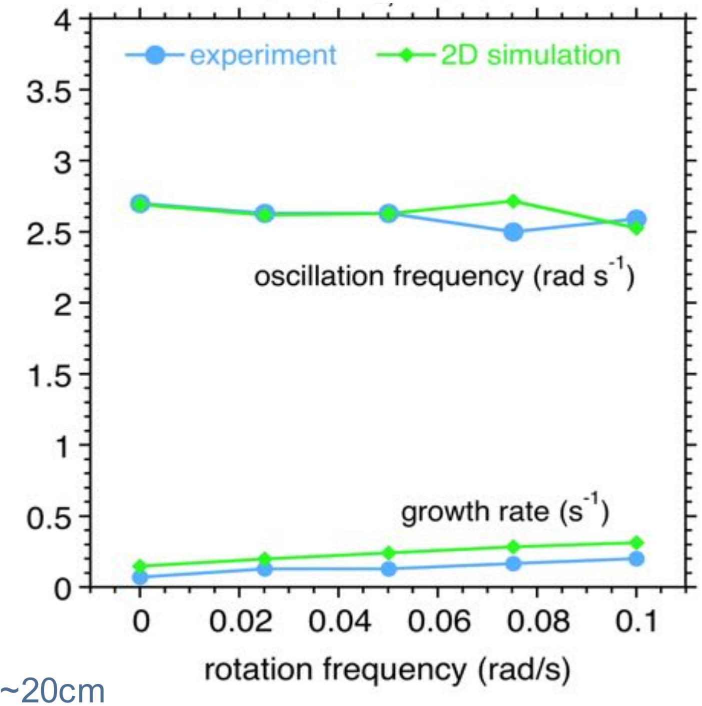


same increase of the growth rate with the core rotation as in gas accretion
& same trend towards a m=2 linear instability

Experimental growth rate and oscillation period compared to shallow water modelling



Foglizzo+12



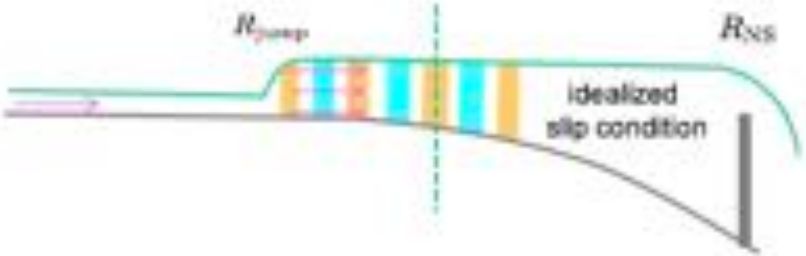
R_{jp}~20cm

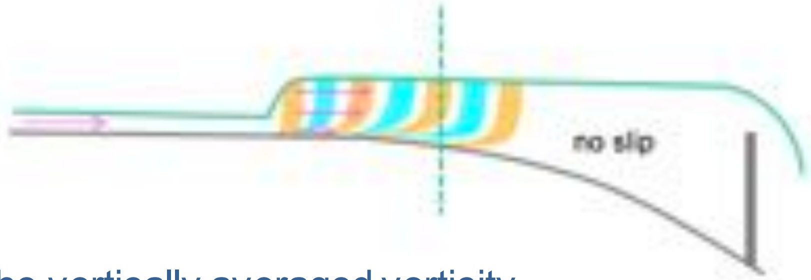
-excellent modelling of the oscillation frequency

limited by the measured radial width of the hydraulic jump

-systematic offset of the experimental growth rate

expected phase mixing of the dragged vorticity





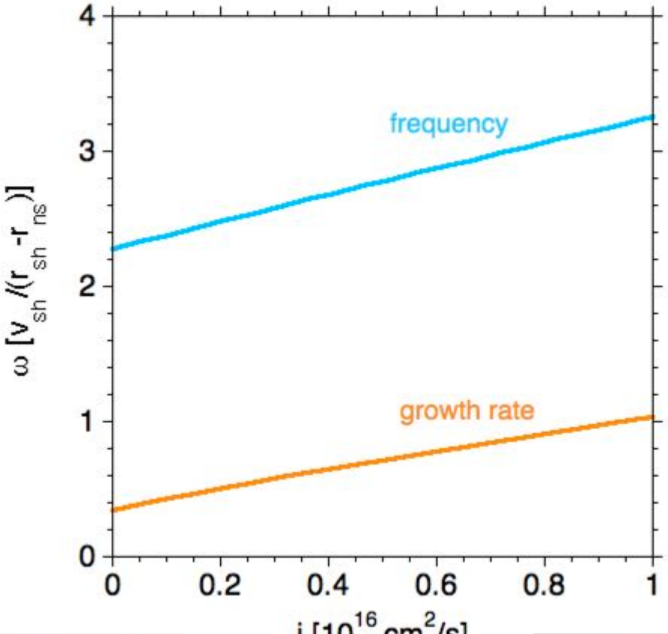
The vertically averaged vorticity is damped by a factor Q

$$Q \sim \int_0^H \frac{\mathrm{d}z}{H} \cos\left[\frac{\omega_{\rm SASI}\Delta R}{v(z)}\right] \sim 0.27\ (\text{laminar})$$

$$\sim 0.52\ (\text{turbulent})$$

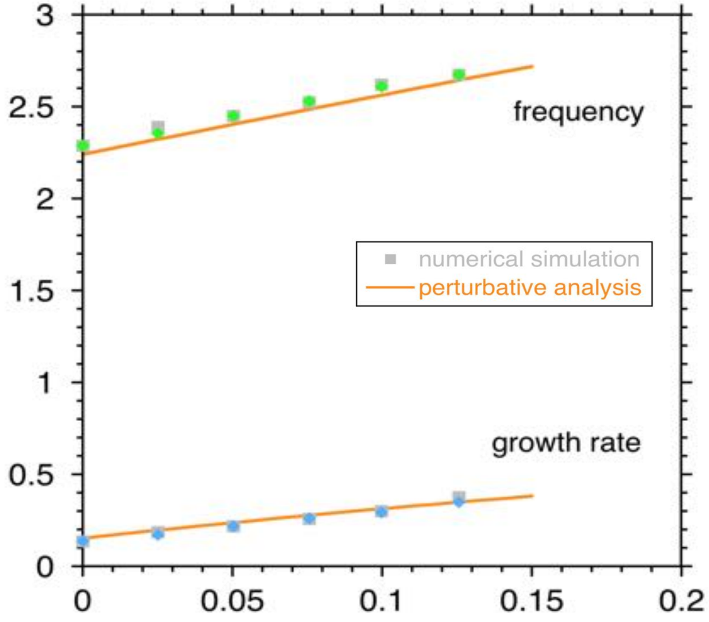
Rotation effect in shallow water equations compared to gas dynamics

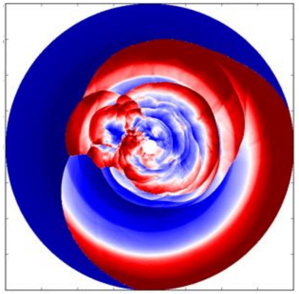
shocked gas dynamics

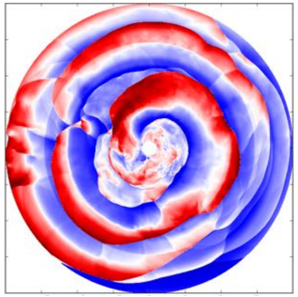


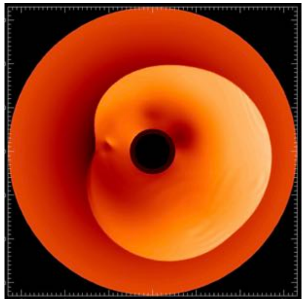
Yamazaki & Foglizzo 08

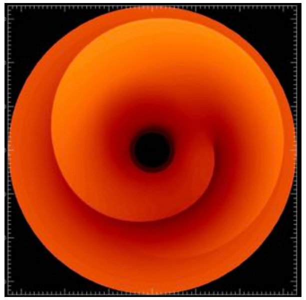
shallow water equations











Why is the prograde mode of SASI destabilized by rotation?
Why is the transition to the corotation instability so smooth?

The St Venant system of shallow water equations is a simpler framework to understand the coupling of SASI with rotation and the transition to the low T/|W| instability:

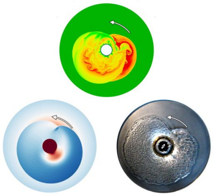
- adiabatic equations (no neutrino cooling)
- no buoyancy effects

Conclusions

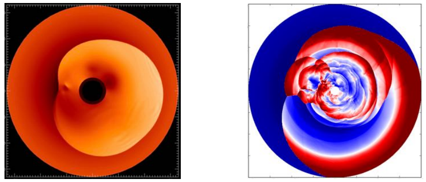
2D Cylindrical gas dynamics suggests that (Kazeroni+17)

-SASI can account for pulsar rotation periods down to ~50ms

-for rotation rates >100Hz the corotation instability decreases the pulsar spin by <30%



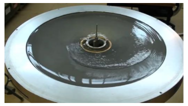
Two core collapse instabilities captured in a hydraulic experiment



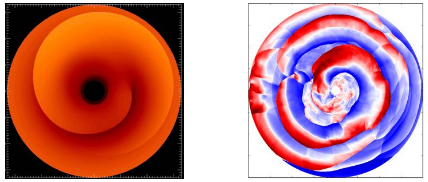
-an intuitive approach to multi-D processes that produce GW

-experimental results confirmed by a shallow water numerical model

-**first** experimental confirmation that spiral SASI can produce a counter-spinning neutron star



-**first** experimental demonstration of the 'low T/|W|' instability



-the corotation instability 'low T/|W|' connects smoothly to SASI
→GW signatures?

-the theory of the corotation instability in a postshock accretion flow is stIll missing