Part 1: A few “problems” with MRI simulations and viscous disk models

Part 2: Minimalist model of Cloud Fragmentation and Feedback

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A few “Concerns” with MRI simulations and alpha disks

$$\alpha_{\rm mag} \equiv -\frac{2f(\Gamma)\langle b_y b_x \rangle}{3c_s^2} = \frac{C_{\rm mag}(\Gamma, \beta)}{\beta},$$

$$\alpha_{\rm kin} \equiv \frac{2f(\Gamma)\langle v_y v_x \rangle}{3c_s^2} \equiv \frac{C_{\rm kin}(\Gamma, \beta)}{\beta},$$

$\alpha_{tot}=\alpha_{mag}+ \alpha_{kin}$

(Blackman, Penna, Varniere 07)

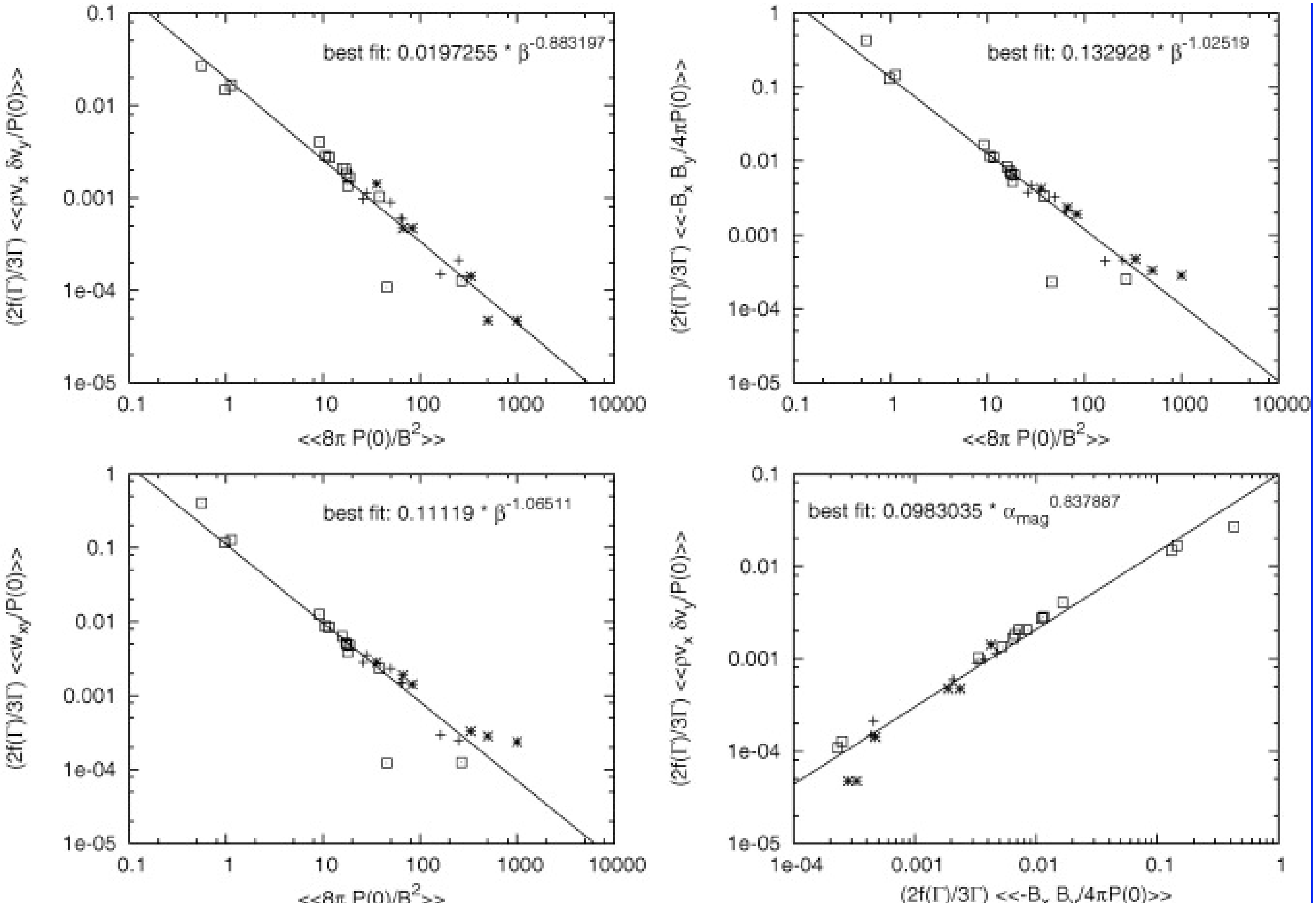

Fig. 1. Isothermal data of Table 8, Table 9 and Table 10 and best fit lines. Top row: Log $\alpha_{kin}(P(0))$ and Log $\alpha_{mag}(P(0))$ vs. Log $\beta(P(0))$, respectively. Bottom row: Log $\alpha_{tot}(P(0))$ vs. Log $\beta(P(0))$ and Log $\alpha_{kin}(P(0))$ vs. Log $\alpha_{mag}(P(0))$. The double brackets indicate that the data represent a combination of spatial average and late time average (e.g. after 15 orbits). The symbols indicate the following specific data sets, respectively: ✳ – Fleming and Stone (2003); □ – Miller and Stone (2000); + – Stone et al. (1996). Values of the magnetic field energy, Maxwell stress, and Reynolds stress are all normalized with respect to the midplane gas pressure, $P(0)$, and for all of these runs, $P(0) = 5 \times 10^{-7}$. For the most part, despite the different initial and boundary conditions and wide ranges of α_{mag}, α_{kin}, and α_{tot}, products of form $\alpha\beta$ lie close to the best fit lines shown. The line equations are at the top of each panel.

$$\frac{\bar{T}_{r\phi}}{\bar{P}} = q\alpha \simeq 0.75 \left(\frac{L}{H}\right)^{5/3} \times \begin{cases} \Delta/L & \text{if} \quad \lambda_{\rm MRI} \leq \Delta \\ \lambda_{\rm MRI}/L & \text{if} \quad \Delta < \lambda_{\rm MRI} \leq L \\ 0 & \text{if} \quad \lambda_{\rm MRI} > L \end{cases}$$

Pessah et al 2007

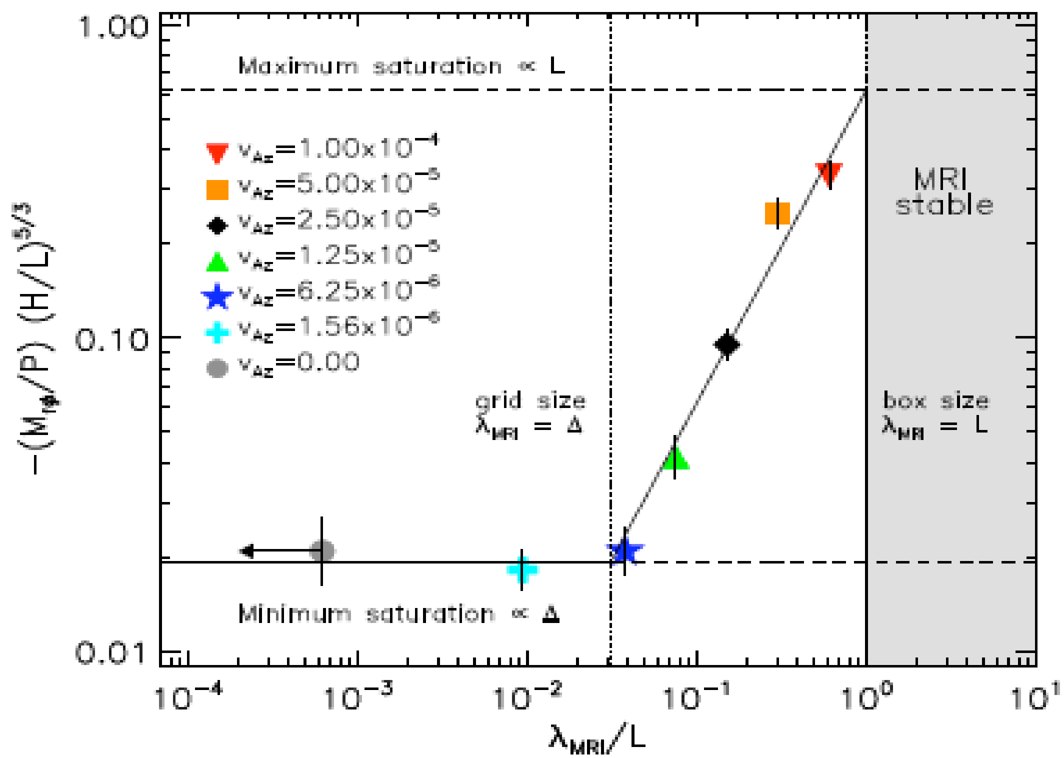

FIG. 2.— Dimensionless magnetic stress, $-\bar{M}_{r\phi}/\bar{P}$, multiplied by $(H/L)^{5/3}$, as a function of the wavelength corresponding to the most unstable MRI mode, $\lambda_{\rm MRI}$. The various types of symbols correspond to the best fit values characterizing each class of simulations according to the value of the ratio $\bar{v}_{Az}/L\Omega_0$, as inferred from Figure 1. The simulations with zero mean magnetic flux, i.e., $\bar{v}_{Az} = 0$, are displayed at some arbitrary value for visualization purposes only. The error bars quantify the scatter within each class of simulations around the corresponding mean values. Vertical *dotted lines* represent the values at which the most unstable MRI wavelength equals the grid size and the size of the box, respectively, i.e., $\lambda_{\rm MRI} = \Delta = 1/32$ and $\lambda_{\rm MRI} = L = 1$. The overall dependence of the saturation level on the ratio $\lambda_{\rm MRI}/L$ (*solid line*) is given by the saturation predictor (3). Numerical simulations for which $\lambda_{\rm MRI} > L$ are stable to the MRI (*shaded region*).

Pessah et al. 2006

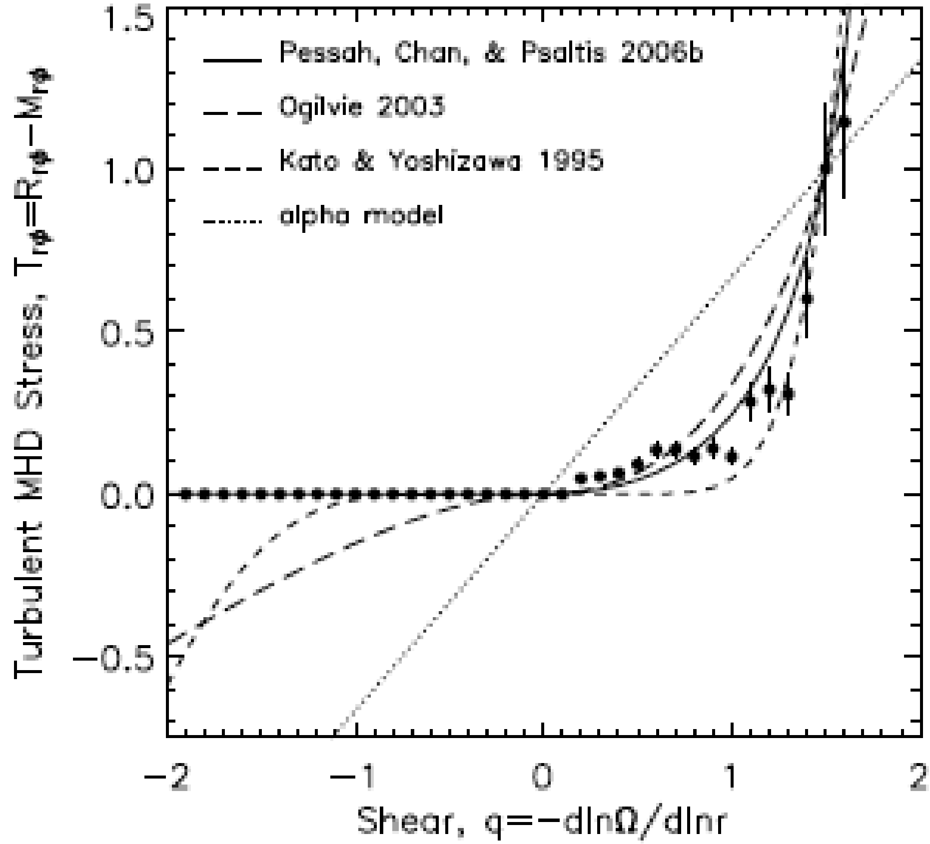

Figure 1. Total turbulent stress at saturation as a function of the shear parameter $q \equiv -d\ln\Omega/d\ln r$. The various lines show the predictions corresponding to the three models for turbulent MHD angular momentum transport discussed in §3. The filled circles correspond to the volume and time averaged MHD stresses calculated from the series of shearing box simulations described in §4. Vertical bars show the typical rms spread (roughly 20%) in the stresses as calculated from ten numerical simulations for a Keplerian disc. The dotted line corresponds to a linear relationship between the stresses and the local shear, like the one assumed in the standard model for alpha viscosity. All the quantities in the figure are normalized to unity for a Keplerian profile, i.e., for $q = 3/2$.

Handful of "Concerns"

- Pr_M indeterminate in many simulations (other than Axel's)
- α ranges 4+ orders of magnitude but $\alpha\beta$=constant
- Box size and initial mean field strength determine saturation
- Without mean field effective alpha is too low when scaled
- Saturation in "ideal MHD" periodic boxes has linear dependence on initial vertical field seed strength. Weird.
- periodic boxes: chaos in "restricted homotopy class"
- α interpretation of MRI does not scale correctly with shear

- pure viscosity "model" of disk turbulence misses explicit transport term in surface density equation (Hubbard and Blackman in prep.).
- What are minimum properties that turbulence must have to produce outward angular momentum transfer?
- Improve closure model to allow non-viscous physics but still allow tractable analytic theory

Toward a Model of Molecular Cloud Fragmentation

collaborators: George Field, Eric Keto

(papers:Field, Blackman, Keto 2007; Blackman & Field in prep.)

Larson's Laws

$$\sigma \propto R^{p_1} \qquad \rho \propto R^{p_2}$$

- Larson (81): p_1=0.4 p_2= 1.1
- range observed but generally $0.2 < p_1 < 0.7$ (next page)

Author	Year	p_1	L(pc)	Comments
Blitz et al. 2006		0.5		Six galaxies
Brunt & Heyer 2002		0.6		Outer Galaxy
Caselli & Myers 1983		0.2	< 0.1	Massive cores
Casoli et al. 1984		0.2		
Dame et al. 1986		0.5	0–100	Large-scale survey
Fuller & Myers 1992		0.7±0.1		Dense cores
Goodman et al. 1998		0.2	0.1	Coherent cores
		0	< 0.1	"
Heithausen et al. 1996		0.5		HLC
Heyer et al. 2001		0.5	> 7	
		0	< 7	
Heyer & Brunt 2004		0.5		
Heyer et al. 2006		0.7		Rosette
Keto & Myers 1986		~0.5		HLC
Larson 1981		0.4	0.1–100	From various authors
Myers 1983		0.5	0.05–3	
Snell 1981		0.5–1	~1	
Solomon et al. 1987		0.5	0.4–40	Large-scale survey

Virial Equilibrium and Virial Parameter

- $M_v = 5\,\sigma^2\,R/G$
- $M = 4\pi\rho R^3/3$
- For $p_1=0.5$, $p_2= -1$, $M_v / M =$ constant $= \alpha\,G$
- α is virial parameter (e.g. Bertoldi & Mckee)
- $\alpha =1$ for virial equilibrium
- perhaps larger scales show $\alpha =1$ more commonly than smaller scales ⟶

Author(s)	$\alpha = M_V/M$	Mass ($M_\odot$) or L (pc)
Bertoldi & McKee 1992	$\gg 1$	$M < 100$–1000
Blitz 1987	$\cong 1$	Rosette 'large'
	$\gg 1$	Rosette 'small'
Carr 1987	> 1	$M < 30$
Dame et al. 1986	$\cong 1$	$10 < L < 100$
Heithausen 1996	$\gg 1$	HLC
Herbertz et al. 1991	$\gg 1$	$0.3 < L < 3$
Heyer et al. 2001	$\cong 1$	$M > 10000$
	> 1	$M < 1000$
Keto and Myers 1986	$\gg 1$	HLCs
Larson 1981	$\cong 1$	$0.1 < L < 100$
Leung et al. 1982	$\cong 1$	$0.3 < L < 30$
Loren 1989a,b	$\cong 1$	$M > 30$
	$\gg 1$	$M < 30$
McKee & Tan 2003	$\cong 1$	Giant molecular clouds
Myers 1983	$\cong 1$	$0.5 < L < 3$
Myers et al. 1983	$\gg 1$	$L \cong 0.3$, $M \cong 30$
Snell 1981	$\cong 1$	$L \cong 1$
Solomon et al. 1987	$\cong 1$	$0.4 < L < 40$
Strong & Mattox 1996	$\cong 1$	Giant molecular cloud
Williams et al. 1994	$\cong 1$	Rosette Nebula
	$\gg 1$	Maddelena Cloud
Williams et al. 1995	$\cong 1$	Rosette (15 per cent)
	$\gg 1$	Rosette (85 per cent)

Basic Picture

- α =1: **Large scale structures unstable to gravitational collapse** *derive observable MC cascade scalings assuming gravity is primary source of velocity dispersion*

- α >1: **Small scale 'pressure' confined structures** (Elmegreen 85; Keto Meyers 86; Bertoldi & Mckee 92): stability of HLC, and "pressure" bound clumps, consistent with p_1=0.5. (Below critical mass, both grav. unstable and pressure stable solutions are available)

- SFR mediation can be "incorporated" via scale dependent mass flux. To suppress given mass deposition rate, needed feedback power decreases with decreasing scale

CASE 1: α=1

- "Inertial range" gravity driven
- $\rho R^2 / \sigma^2$ = constant: $p_2 = 2p_1 - 2$
- Nested structure: $x=R/R_1$

$$M_1 = -\int_0^1 M(x)\frac{dN}{dx}dx$$

$$M(x) = M_1 x^{p_4} = M_1 x^{2p_1+1}$$

$$\frac{dN}{dx} \propto x^{p_3}$$

- mass flux $$F_M = -M(x)\frac{dN}{dx}\frac{dx}{dt}$$

•Constant F_M $\longrightarrow$ $p_3 = -3p_1 - 1$
•Constant dF_M/dR $\longrightarrow$ $p_3 = -3p_1$

Resulting Scaling Relations, (F_M constant)

$$\sigma \propto x^{p_1} \sim x^{0.5}$$

$$\rho \propto x^{2p_1-2} \sim x^{-1}$$

$$M(x) \propto x^{2p_1+1} \sim x^2$$

$$\frac{dN}{dx} \propto x^{-3p_1-1} \sim x^{-2.5}$$

$$\frac{dN}{dM} = \frac{dN/dx}{dM/dx} = x^{-5p_1-1} \propto M^{\frac{-5p_1-1}{2p_1+1}} \sim M^{-1.75}$$

- Not horrible $\longrightarrow$

Resulting Scaling Relations, (dF_M/dR =constant)

$$\sigma \propto x^{p_1} \sim x^{0.5}$$

$$\downarrow$$

$$\rho \propto x^{2p_1-2} \sim x^{-1}$$

$$M(x) \propto x^{2p_1+1} \sim x^2$$

$$\frac{dN}{dx} \propto x^{-3p_1} \propto x^{-1.5}$$

$$\frac{dN}{dM} = \frac{dN/dx}{dM/dx} = x^{-5p_1} \propto M^{\frac{-5p_1}{2p_1+1}} \sim M^{-1.25}$$

- Not horrible either ⟶

$$\frac{dN}{dM} \propto M^{-p_5}$$

Author Year	p_5
Blitz et al. 2006	−1.7
Casoli et al. 1984	−1.4 to −1.6
Loren 1989a	−1.1
Myers et al. 1983	−1 to −1.5
Snell et al. 2002	−1.9
Solomon et al. 1987	−1.5
Williams & Blitz 1995	−1.3

Comment on p_1 and conservation laws

- Would like to derive p_1 from conservation relations
- conservation of momentum/mass $\longleftrightarrow$ $p_1 = 0.5$
- Maybe physically motivated for supersonic grav driven flow (note alternate context of outflow e.g. Matzner 07)

CASE 2: $\alpha > 1$

- Viral theorem with ext pressure

$$\frac{3GM^2}{20\pi R^4} - \frac{3M\sigma^2}{4\pi R^3} + P_e = 0$$

- For $p_1 = 0.5$ (σ^2 / r = constant) this becomes an equation for **surface density**:

$$y^2 - 2y + y_0 = 0$$

$$y = \frac{M}{\pi R^2}\frac{2\pi G R}{5\sigma^2} \qquad y_0 = \frac{P_e}{P_c};\ P_c = \frac{15\sigma^4}{16\pi G R^2}$$

$$y = 1 \pm (1 - y_0)^{1/2}$$

- No equilib for $P_e > P_{e,c}$ and two solns for $P_e < P_{e,c}$

For given P_e, M_c is the M for which P_e=$P_{e,c}$

- $GM_c^2/RV \approx M_c \sigma^2/V^{4/3}$ at critical point
- $V \approx M_c\sigma^2/P_e$ for a given P_e
- $M_c \approx \sigma^4/(GP_e^{1/2})$
- For $M > M_c$ unstable collapse
- For $M < M_c$ pressure confined solutions, collapse not inevitable for pressure branch (grav branch are unstable Keto Field 05)
- Requires source of P_e

Application to Observations

Pressure bound HLC (Magnani et al 85, Keto & Meyers 86; Heithhause 1990)

– p_1=0.5 is measured, σ^2/R measured, then compute $P_{e,c}$=1.2 x 10^{-10} dyn/cm^2

– Infer $P_e/P_{e,c}$ = 10^{-3} observationally

– Solve for surface density in above formalism: =1.3 x 10^{-4} g/cm^2 and A_v=0.2

– Agrees with Keto & Meyers 86

Pressure Bound clumps

- Orphiucus (Loren 89) M> 30Msun, L>0.8pc a=1; L< 0.8pc a >> 1
- Bertoldi & Mckee (92): α >> 1; P_e inferred (10^5 k_b)