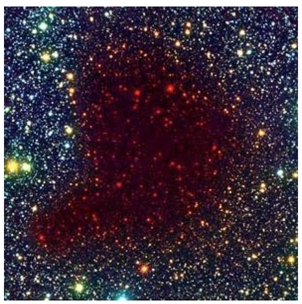
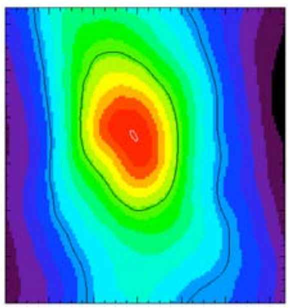
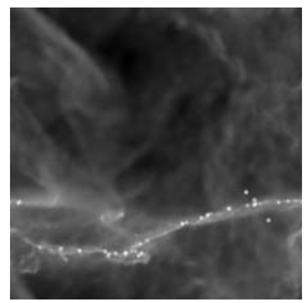
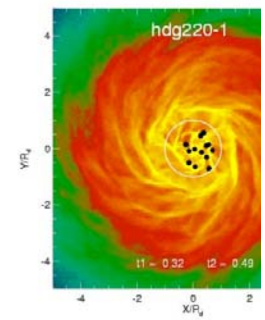
role of thermodynamics in star formation





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thermodynamics and the IMF

- IMF determined thermodynamic properties of gas (in addition to turbulence)

Richard Larson in MNRAS, 359, 211 - 222 (2005)

The thermal properties of star-forming clouds have an important influence on how they f
ment into stars, and it is suggested in this paper that the low-mass stellar initial mass func
(IMF), which appears to be almost universal, is determined largely by the thermal physic
these clouds. In particular, it is suggested that the characteristic stellar mass, a little below
solar mass, is determined by the transition from an initial cooling phase of collapse to a
phase of slowly rising temperature that occurs when the gas becomes thermally couple
the dust. Numerical simulations support the hypothesis that the Jeans mass at this transi
point plays an important role in determining the peak mass of the IMF.

(see also Y. Li et al. 2003, Jappsen et al. 2005, Bonnell et al. 2006)

agenda

- EOS and critical exponents
- on the role of thermodynamics in gravo-turbulent fragmentation
- three examples
  - solar neighborhood
  - starburst regions
  - early universe: transition Pop III to Pop II.5

hydrodynamics

$$\frac{d\rho}{dt} = \frac{\partial\rho}{\partial t} + \vec{v}\cdot\vec{\nabla}\rho = -\rho\vec{\nabla}\cdot\vec{v}$$ (continuity equation)

$$\frac{d\vec{v}}{dt} = \frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla}p - \vec{\nabla}\phi + \eta\vec{\nabla}^2\vec{v} + \left(\zeta + \frac{\eta}{3}\right)\vec{\nabla}(\vec{\nabla}\cdot\vec{v})$$

(Navier-Stokes equation)

$$\frac{d\epsilon}{dt} = \frac{\partial\epsilon}{\partial t} + \vec{v}\cdot\vec{\nabla}\epsilon = T\frac{ds}{dt} - \frac{p}{\rho}\vec{\nabla}\cdot\vec{v}$$ (energy equation)

$$\vec{\nabla}^2\phi = 4\pi G\rho$$ (Poisson's equation)

$$p = \mathcal{R}\rho T$$ (equation of state)

energy equation

- transport equation for internal energy

$$\frac{d\epsilon}{dt} = \frac{\partial \epsilon}{\partial t} + \vec{v} \cdot \vec{\nabla} \epsilon = T \frac{ds}{dt} - \frac{p}{\rho} \vec{\nabla} \cdot \vec{v}$$

$\rightarrow$ follows from the thermodynamic relation $d\epsilon = T\, ds - p\, dV$
$T\, ds + p/\rho^2 d\rho$ which described changes in ϵ due to entro
changed and to volume changes (compression, expansion)

$\rightarrow$ for adiabatic gas the first term vanishes ($s =$constant)

$\rightarrow$ heating sources, cooling processes can be incorporated in

(conservation of energy)

equation of state -- EOS

- closure equation – equation of state
  - → general form of equation of state $p = p(T, \rho, \ldots)$
  - → ideal gas: $p = \mathcal{R}\rho T$
  - → special case – isothermal gas: $p = c_\mathrm{s}^2 T$ (as $\mathcal{R}T = c_\mathrm{s}^2$)

Note:

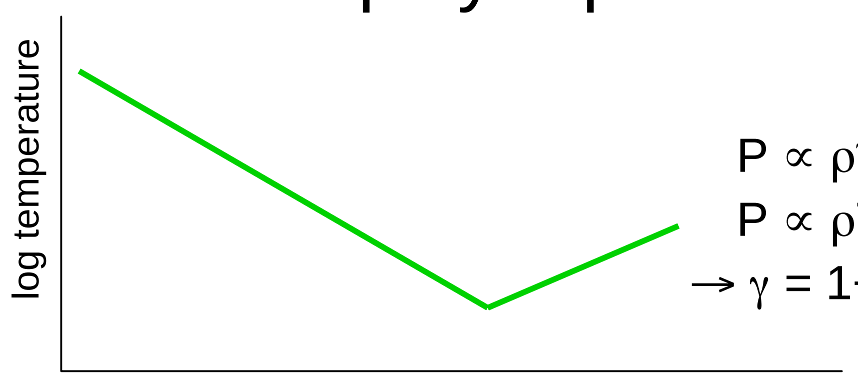
- in reality, computing the EOS is VERY complex!
- depends on detailed *balance* between *heating* and *cooling*
- these depend on *chemical composition* (which atomic and molecular species, dust)
- and on the ability to radiate away „cooling lines“ and black body radiation
  --> problem of *radiation transfer*

effective EOS

- under certain conditions: energy equation and EOS can be „combined“ to an ***effective EOS***
  - all gas particles follow an unique density - pressure - temperature curve (e.g. Spaans & Silk 2000, 2005, Omukai et al 2005)

- ideal gas EOS: $P = \rho k T / m$
- effective polytropic EOS: $P = K\rho^{\gamma}$

$\}\ \gamma = 1+\mathrm{dln}T/\mathrm{dln}$



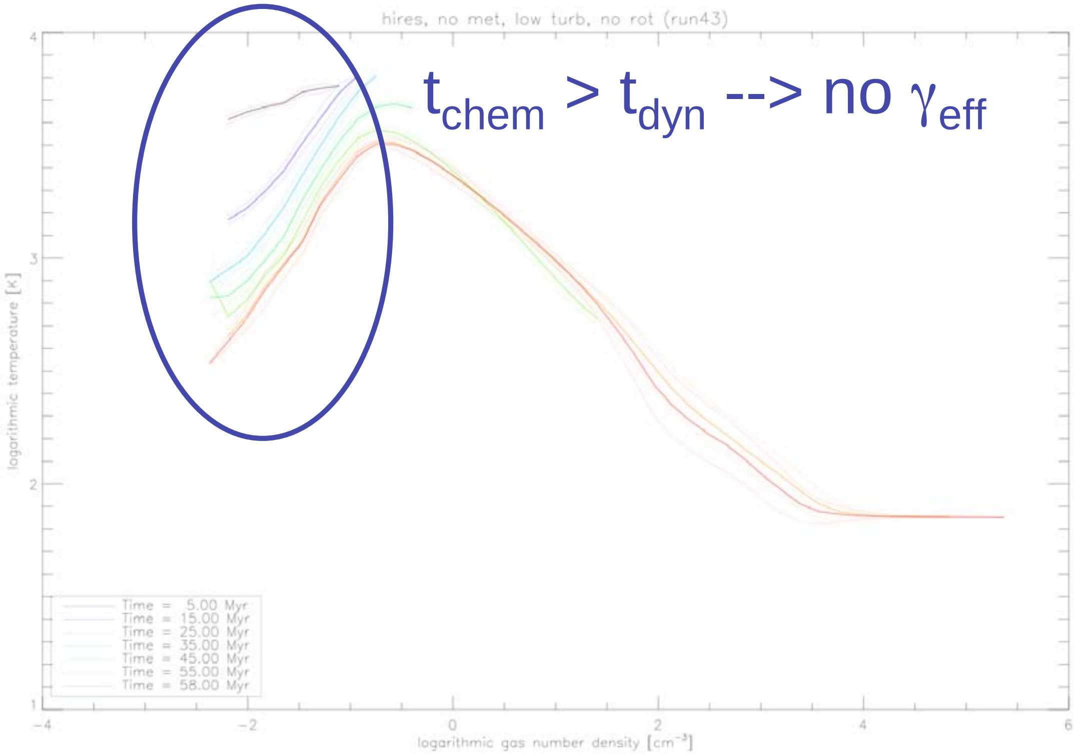
$P \propto \rho^{\gamma}$

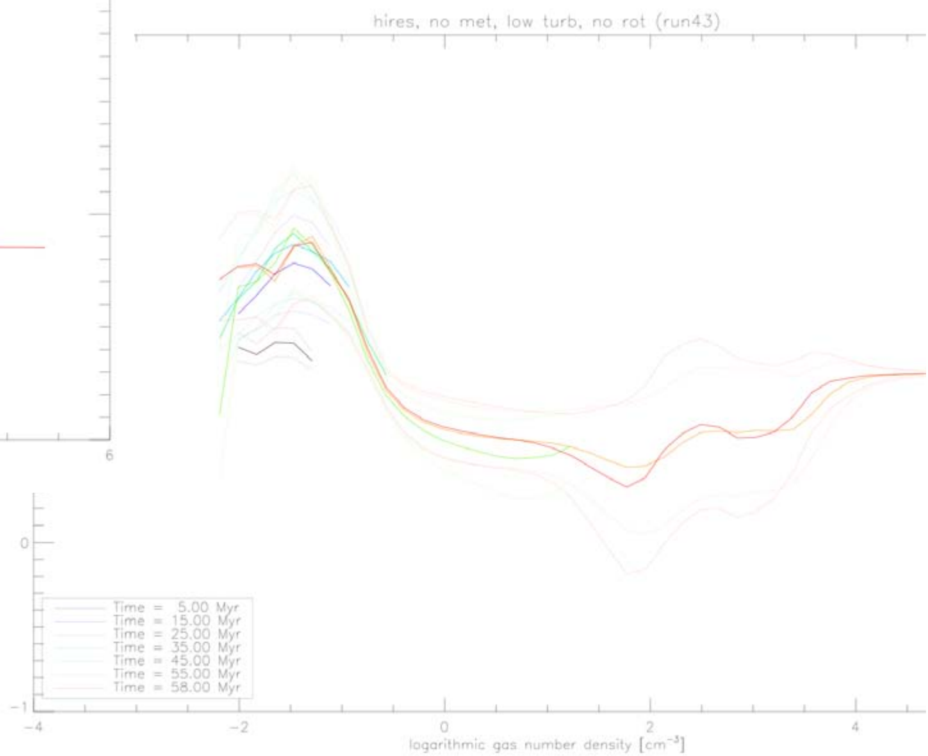
$P \propto \rho T$

$\rightarrow \gamma = 1+\mathrm{dln}T/\mathrm{dln}\rho$

P = pressure, ρ = density,
k = Boltzmann constant,
T = temperature, m = effective
particle mass, γ = polytropic
exponent, K = some constant

example





cooling in a low-mass
halo in the early universe

(from Jappsen et al. 2007,
chemistry: Glover 2007, Glover & Jappsen 2007)

critical γ from virial theorem

- virial theorem: ∃ critical exponent for stability
  - ATTENTION: depends on *dimension* of problem
    (see, e.g., Chandrasekhar & Fermi 1953)
- in 3D:
  - virial equilibrium: $3(\gamma-1)U + W = 0$ (i)
  - total energy: $E = U + W$ (ii)
  - (ii) in (i): $3(\gamma-1)E + (3\gamma-4)W = 0$ ⟶ $E = -\frac{3\gamma - 4}{3(\gamma - 1)}$
  - collapse possible for $\gamma < 4/3$

U = internal energy, W = potential energy, E = total energy we know from equation (1.105), i.e. from $\gamma\epsilon = \epsilon + P$, that not all the internal energy $\epsilon = 3/2\, nkT$ is able to do work, i.e. $P = (\gamma - 1)\epsilon$, with $\gamma = c_p/c_v$ being the ratio between the specific heat at constant pressure and constant volume; the contribution U in (1.132) from the thermal pressure therefore relates to the internal energy of the system as

$$U = \frac{3}{2}\int_V \mathrm{d}^3x\, P = \frac{3}{2}\int_V \mathrm{d}^3x\,(\gamma-1)\epsilon = \frac{3}{2}(\gamma-1)\int_V \mathrm{d}^3x\,\epsilon \quad (1.135)$$

using the total internal energy in the system, $U_{\mathrm{int}} = \int_V \mathrm{d}^3x\,\epsilon$, we get

$$U = \frac{3}{2}(\gamma - 1)U_{\mathrm{int}}\ ; \quad (1.136)$$

and consequently in the center-of-mass sytem

$$3(\gamma - 1)U_{\mathrm{int}} + W = 0 \quad (1.137)$$

the first law of thermodyna
reads. $da = T\mathrm{d}s = \mathrm{d}\epsilon + P\mathrm{d}V$

the total energy in the sytem is

$$E = (T + U_{\text{int}}) + W \ , \tag{1.138}$$

if the system is in equilibrium, then $T = 0$ and we get

$$E = U_{\text{int}} + W \ ; \tag{1.139}$$

the virial theorem (1.137) then implies

$$E = U_{\text{int}} - 3(\gamma - 1)U_{\text{int}} = -(3\gamma - 4)U_{\text{int}} \ ; \tag{1.140}$$

because $U_{\text{int}} > 0$ the equilibrium system is bound, i.e. $E < 0$, only if

$$\gamma > \frac{4}{3} \ ; \tag{1.141}$$

this defines the critical polytrope for bound self-gravitating gas spheres.

in order to see what happens in the limiting case $\gamma \to 4/3$, we write

$$\ddot{I} = 2U + W = 3(\gamma - 1)U_{\rm int} + W$$
$$= 3(\gamma - 1)(E - W) + W$$
$$= 3(\gamma - 1)E - (3\gamma - 4)W , \qquad (1.142)$$

and consequently,

$$E = \frac{\gamma - 4/3}{\gamma - 1} W + \frac{1}{3(\gamma - 1)} \ddot{I} ;$$

for $\gamma \to 4/3$, the first term on the right-hand side vanishes, and if we require that the total energy remains negative, $E < 0$, we must have

$$\ddot{I} < 0 , \qquad (1.143)$$

implying collapse;

critical γ from virial theorem

- in 1D: (e.g. Stodolkiewicz 1963, Ostriker 1964, 1965, Larson 1985, Inutsuka & Miyama 1992, 1997, Kawachi & Nagasawa 1998)
  - isotherm cylinder in hydrostatic equilibrium: line mass = $2c_s^2/G$ = const.
  - virial theorem: $2(\gamma-1)U - GM^2 = 0$ (i)
  - linear perturbation analysis: isothermal filament always unstable to radial collapse for $\gamma < 1$ (Chandrasekhar & Fermi 1953)

U = internal energy, W = potential energy, E = total energy

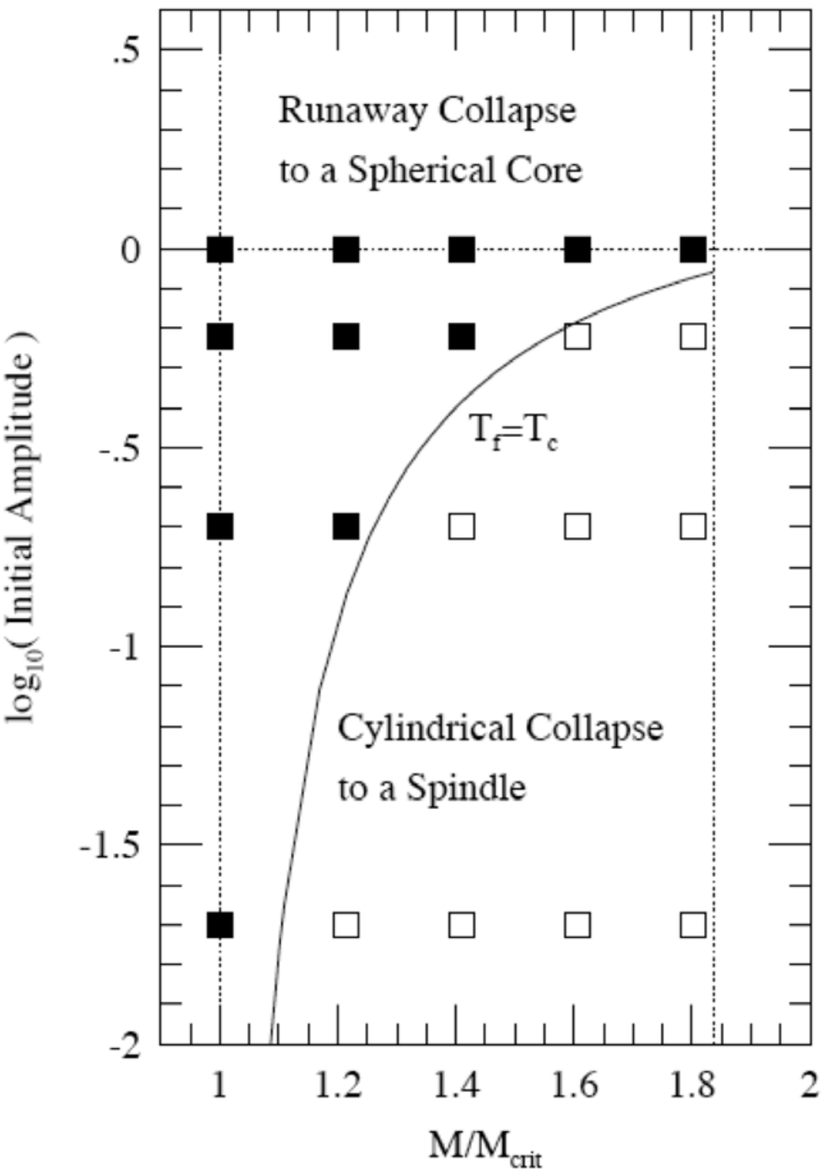


FIG. 11.—Criterion for collapse and fragmentation of a filament. The solid curve corresponds to $T_f = T_c$. The upper region corresponds to $T_f < T_c$, and the lower to $T_f > T_c$. Each filled square corresponds to a model which results in a spherical core, and each open square corresponds to a very elongated core. The solid curve clearly divides the initial parameter

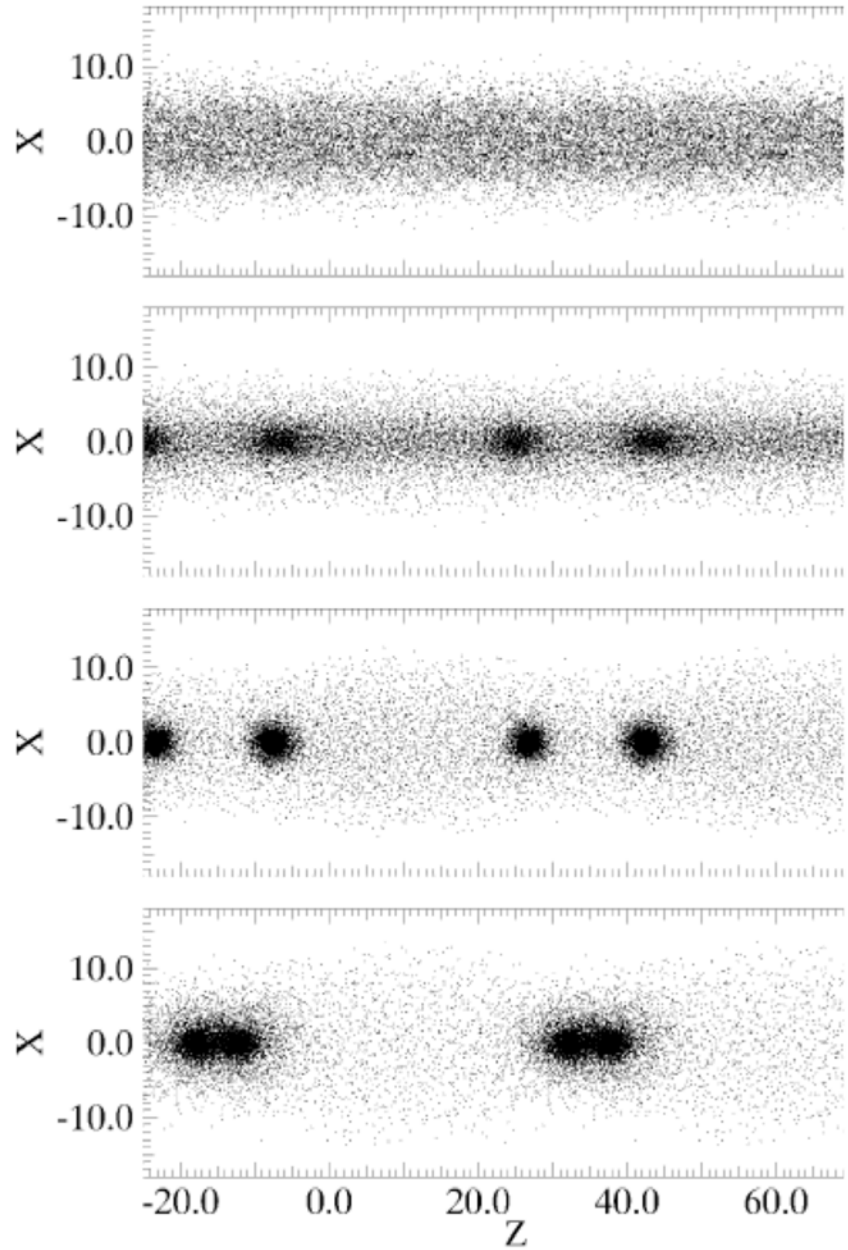


FIG. 12.—Projections of all particle positions onto the *xz*-plane. P bations are random on an equilibrium filament (model R0b).

Similarity solutions for collapsing cylinders with polytropic equations of state have been derived by Kawachi & Hanawa (1998), and these authors found that the existence of such solutions depends on the value of γ: similarity solutions exist for $\gamma < 1$, but not for $\gamma > 1$. For the solutions with $\gamma < 1$, the collapse becomes slower and slower as γ approaches unity from below, asymptotically coming to a halt when $\gamma = 1$. This result shows in a particularly clear way that $\gamma = 1$ is a critical case for the collapse of filaments. Kawachi & Hanawa (1998) suggested that the deceleration of the collapse that is predicted to occur as γ approaches unity will cause a filament to fragment into clumps because the time-scale for collapse toward the axis then becomes longer than the time-scale for fragmentation. If γ increases with increasing density, as is expected from the discussion in Sections 3 and 4, the collapse of a filament toward its axis would be expected to decelerate as γ approaches unity, favouring the breakup of the filament into clumps at this point.

(from Larson 2005)

stability of self-gravitating gas

- *Jeans (1902):* Interplay between self-gravity and thermal pressure
  - stability of homogeneous spherical density enhancements against gravitational collapse
  - dispersion relation:

$$\omega^2 = c_s^2 k^2 - 4\pi G\rho_0$$

  - instability when $\omega^2 < 0$
  - minimal mass: $M_J = \frac{1}{6}\pi^{-5/2} G^{-3/2} \rho_0^{-1/2} c_s^3 \propto \rho_0^{-1/2} T$

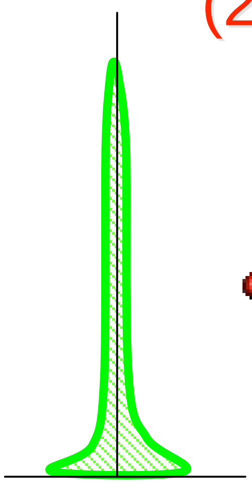
Sir James Jeans, 1877 - 1946

fragmentation depends on EOS

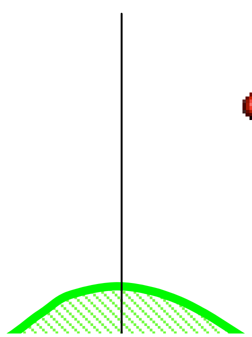
(1) $\mathbf{p \propto \rho^{\gamma}}$ → $\mathbf{\rho \propto p^{1/\gamma}}$

(2) $\mathbf{M_{jeans} \propto \gamma^{3/2} \rho^{(3\gamma-4)/2}}$

because $M_{jeans} \propto \rho^{-1/2} c_s^3$
and $c_s = (dP/d\rho)^{1/2} \propto \rho^{(\gamma-}$

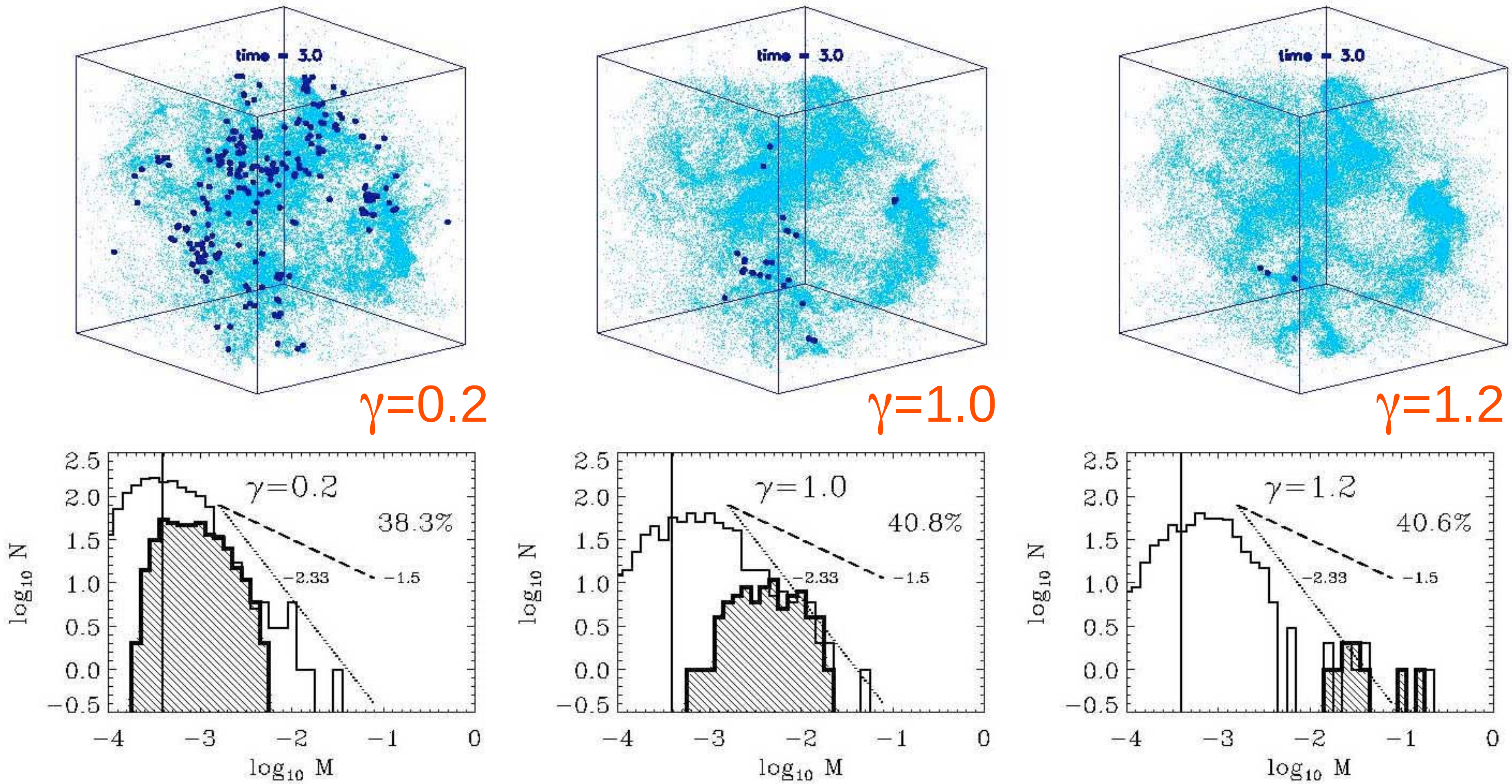


- $\gamma<1$: → *large* density excursion for given pressure
  → $\langle M_{jeans} \rangle$ becomes small
  → number of fluctuations with $M > M_{jeans}$ is large



- $\gamma>1$: → *small* density excursion for given pressure
  → $\langle M_{jeans} \rangle$ is large
  → only few and massive clumps exceed M_{jeans}

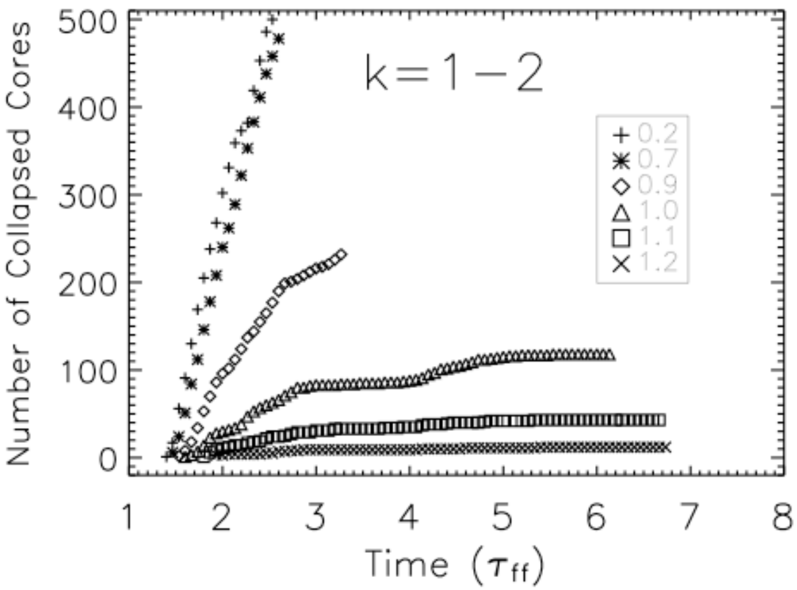
fragmentation depends on EOS

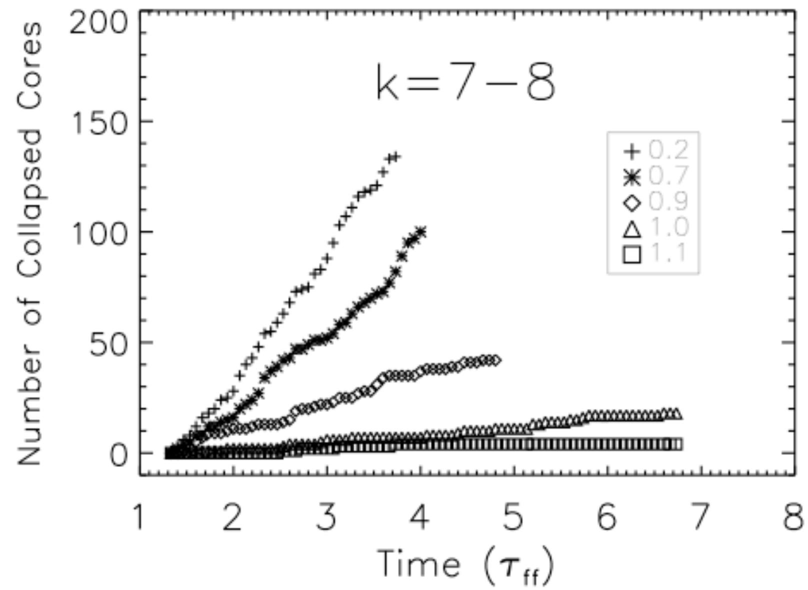


for γ<1 fragmentation is enhanced → *cluster of low-mass stars*

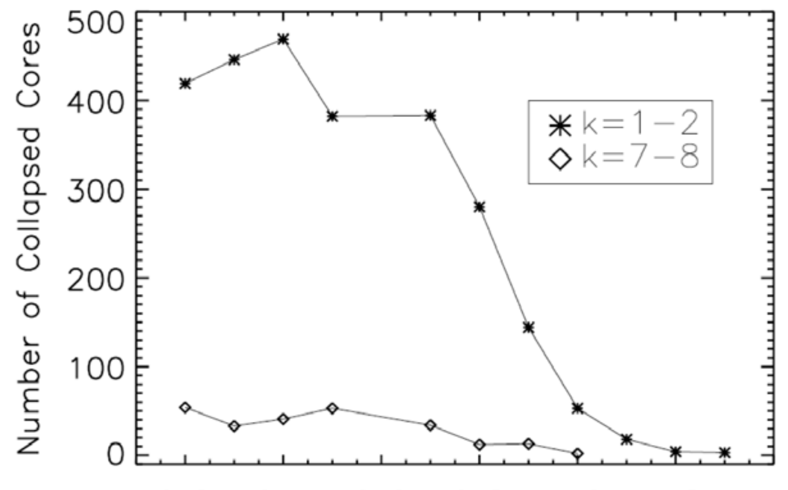
for γ>1 it is suppressed → formation of *isolated massive stars*

fragmentation depends on EOS

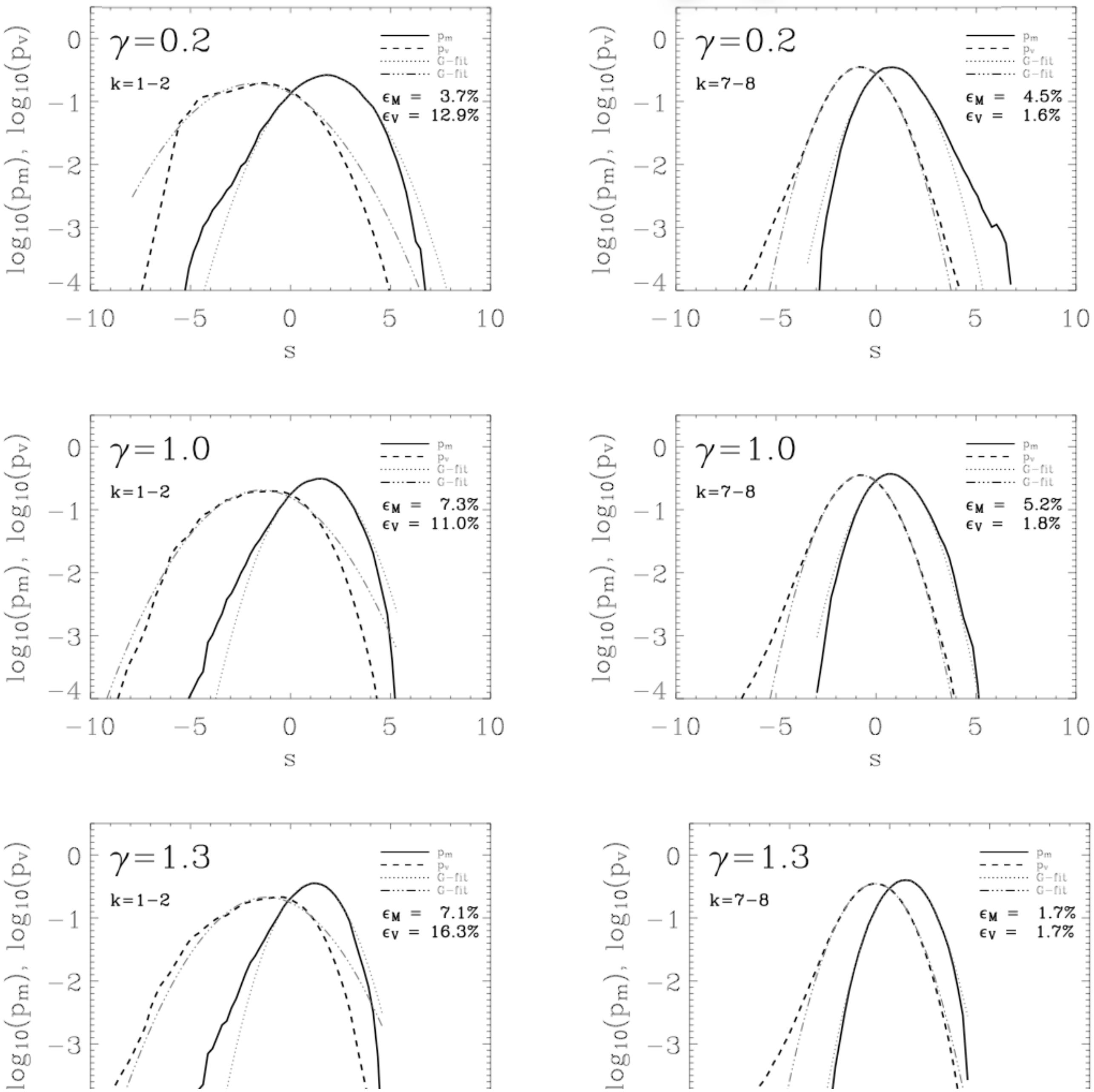




number of collapsed cores
as function of time and as
function of gamma



density pdf's



n Li, Klessen, & Mac Low 2003, ApJ, 592, 975)

possible implications

- degree of fragmentation depends on *EOS*

- polytropic EOS: $p \propto \rho^{\gamma}$
- $\gamma<1$: dense cluster of low-mass stars
- $\gamma>1$: isolated high-mass stars

  (see Li, Klessen, & Mac Low 2003, ApJ, 592, 975; Kawachi & Hanawa 1998; Larson 2003; also Jappsen, Klessen, Larson, Li, Mac Low, 2005, 435, 611)

- implications for extreme environmental conditions
  - expect Pop III stars to be massive and form in isolation
    but when is the transition from Pop III to "normal" Pop II/2 *???*

    (Bromm, Larson, Coppi 2002, Smith & Sigurdsson 2007, Omukai et al 2005, Schneider et al. 2006, Clark, Glover, Klessen 2007)

  - expect IMF variations in warm & dusty starburst regions

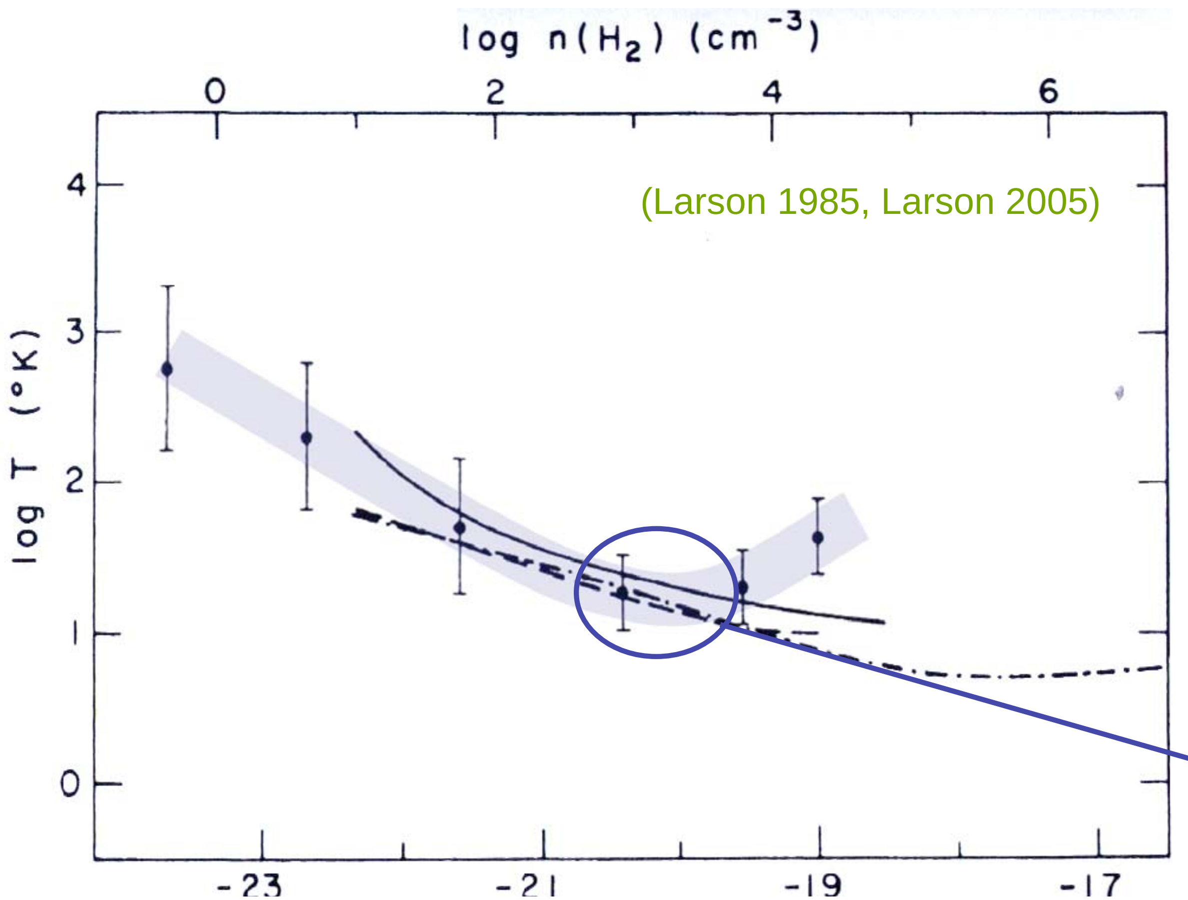
three applications

- star formation in solar neighborhood
- star formation in circum-nuclear starburst regions
- star formation in early universe: transition from Pop III to Pop II.5

EOS for solar neighborhood

below 10^{-18} gcm^{-3}: ρ ⬆ ⟹ T ⬇

above 10^{-18} gcm^{-3}: ρ ⬆ ⟹ T ⬆



(also Whitworth et a
1998, Koyama & Inu
2000, Masunaga &
Inutsuka 2000, Omu
2000, Evans et al. 2
Galli et al. 2002,

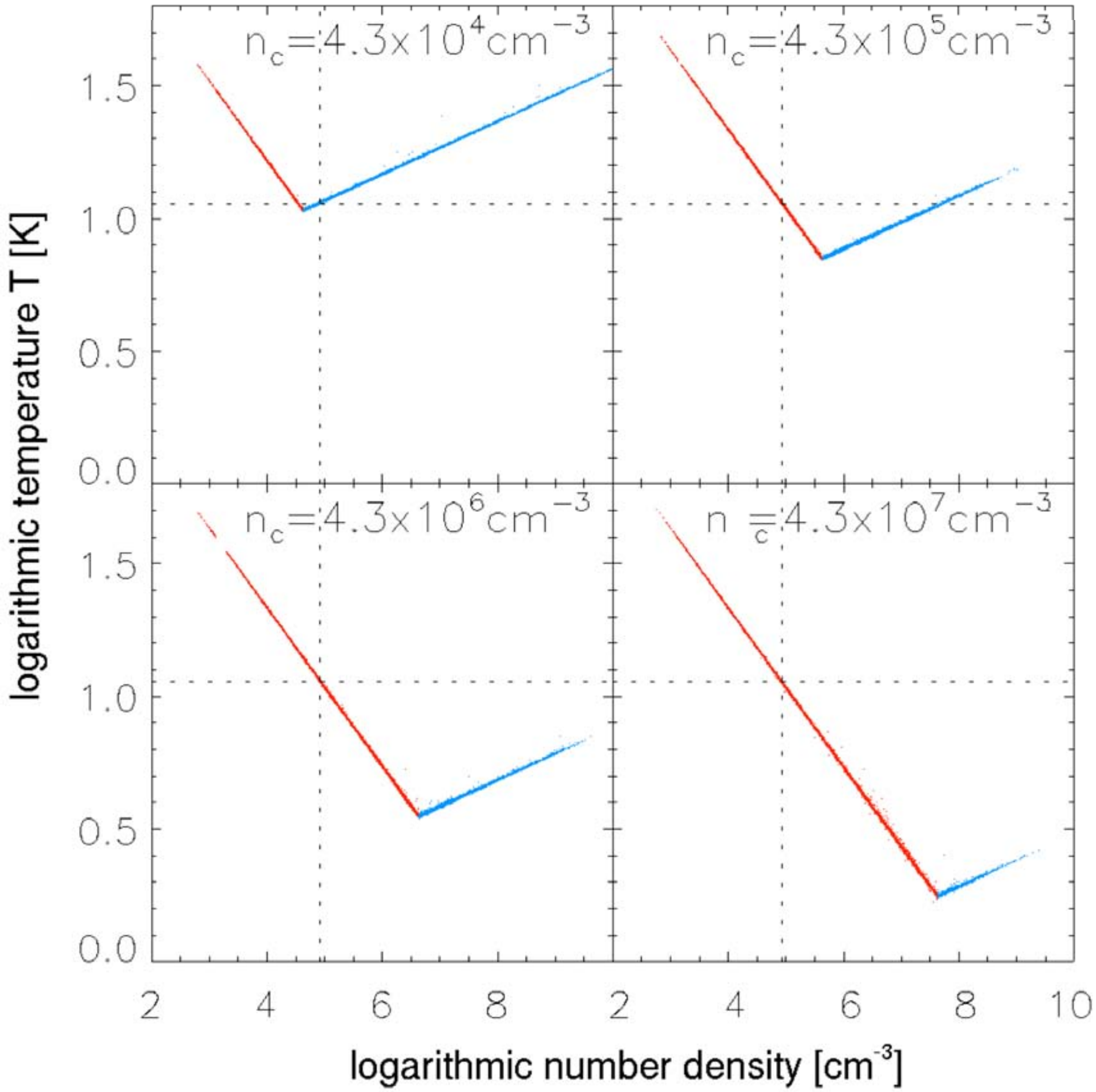
typical Jeans n
~ 0.2 - 0.3 $M_{\odot}$

IMF from simple piece-wise polytropic EOS

$\gamma_1 = 0.7$

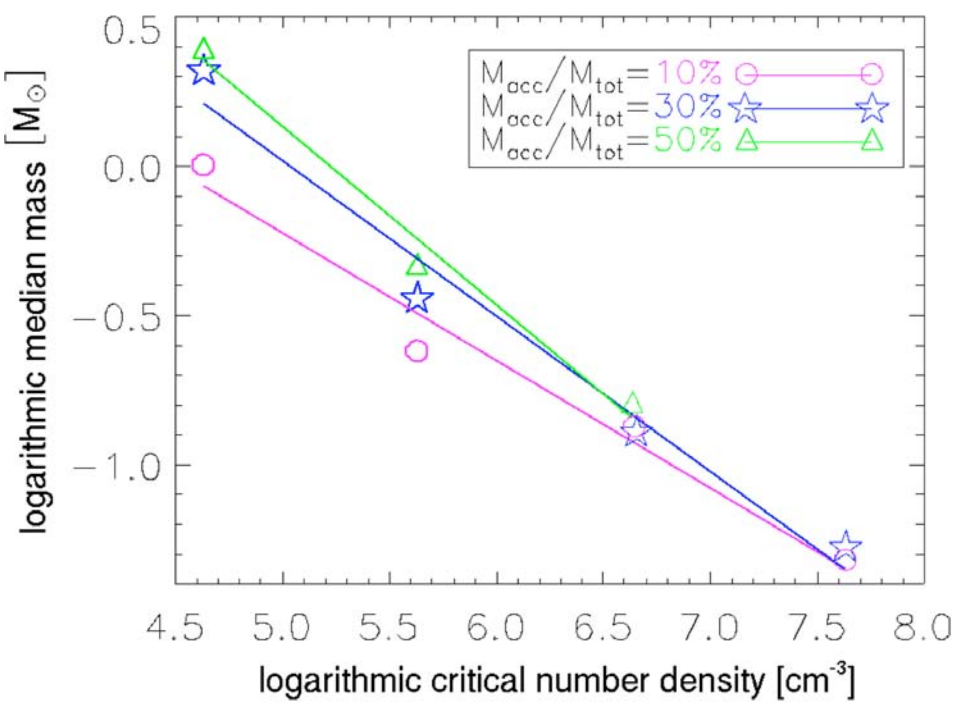
$\gamma_2 = 1.1$

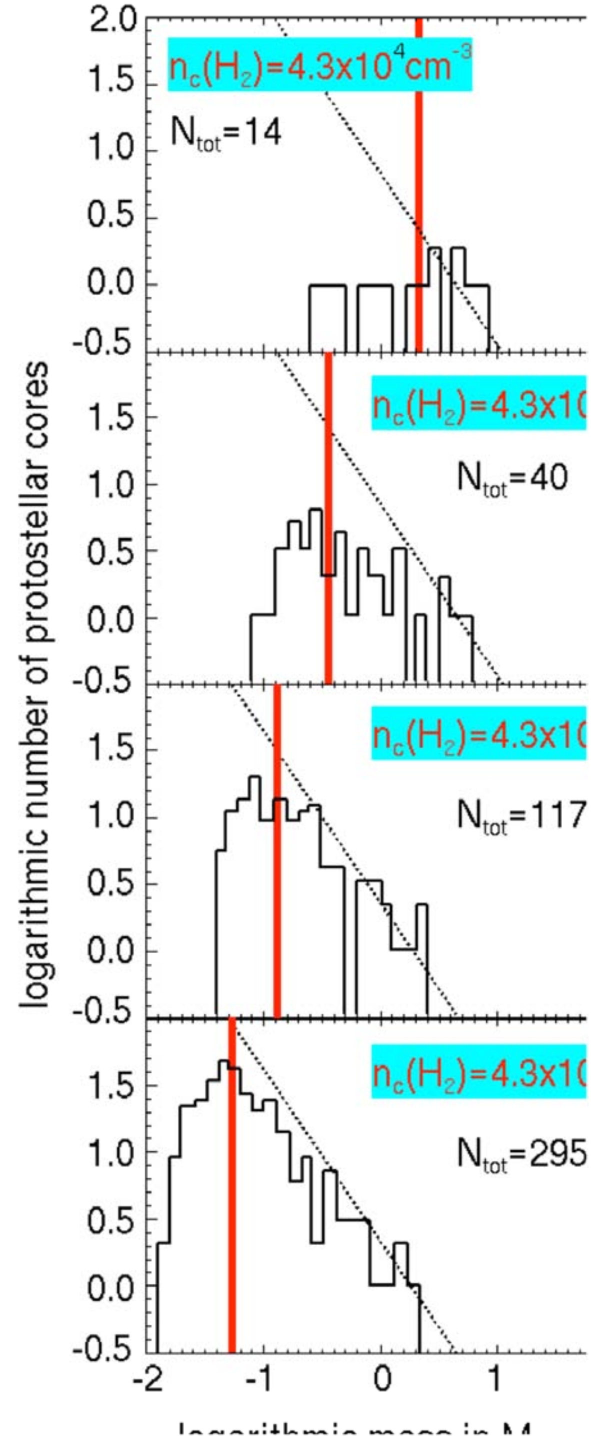
$$T \sim \rho^{\gamma-1}$$



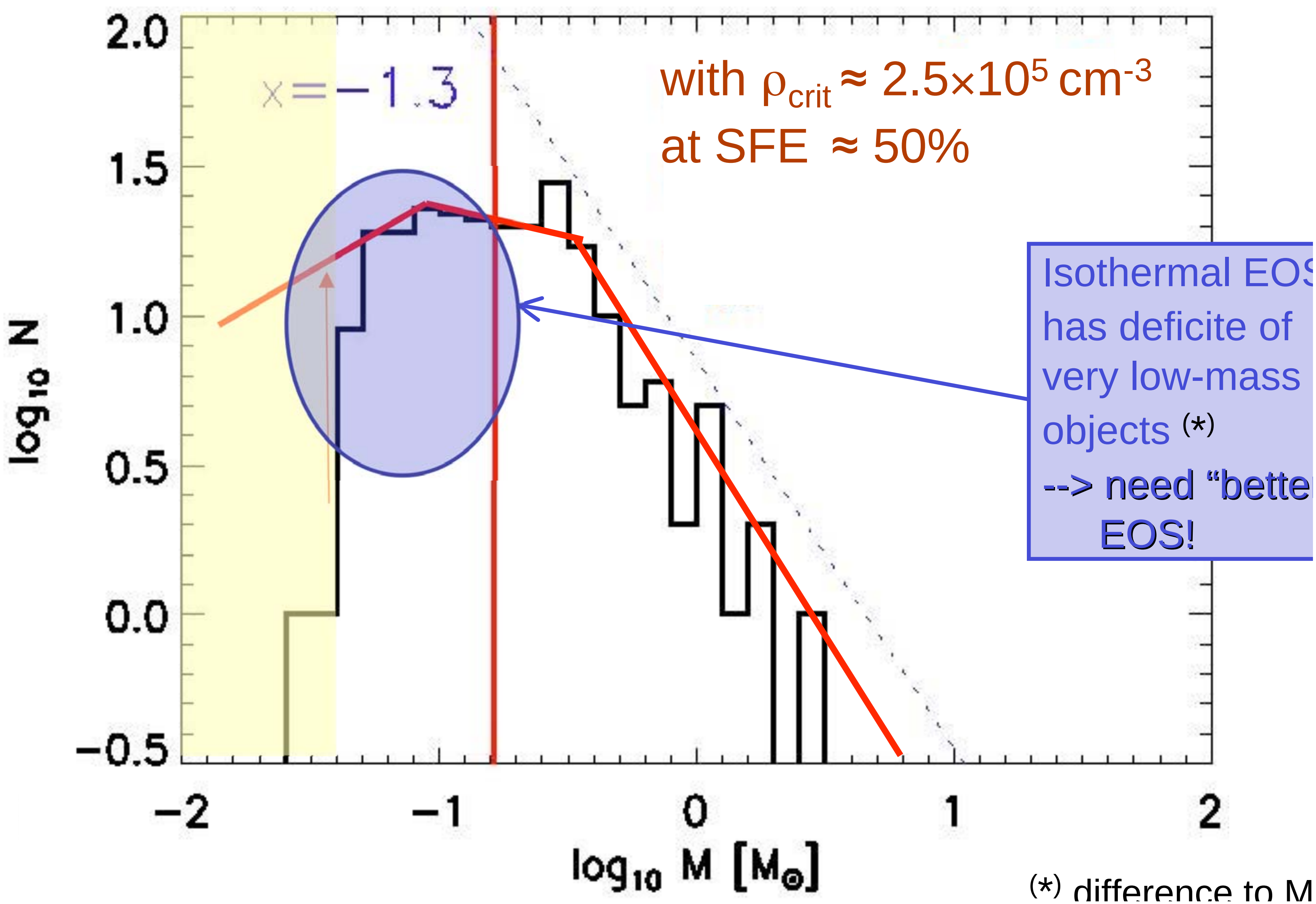
IMF from simple piece-wise polytropic EOS

critical density ⬆ ⇨ median mass ⬇





IMF in nearby molecular clouds

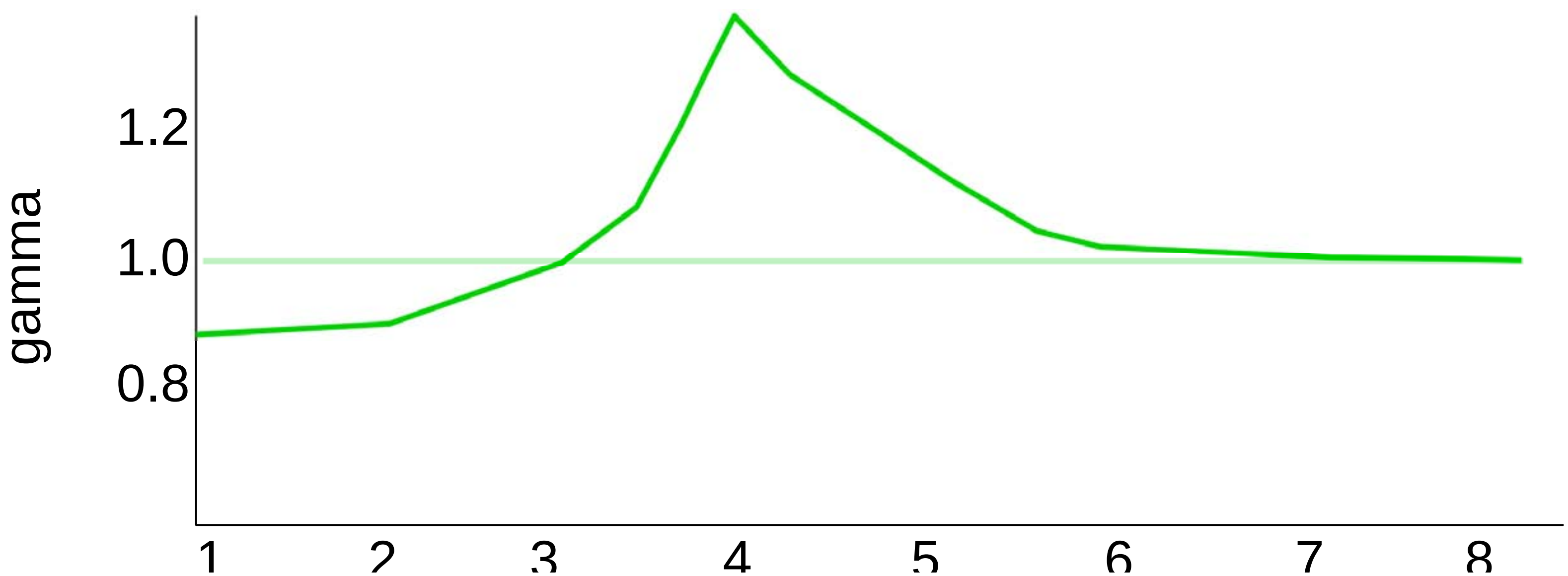


(*) difference to M

IMF in starburst galaxies

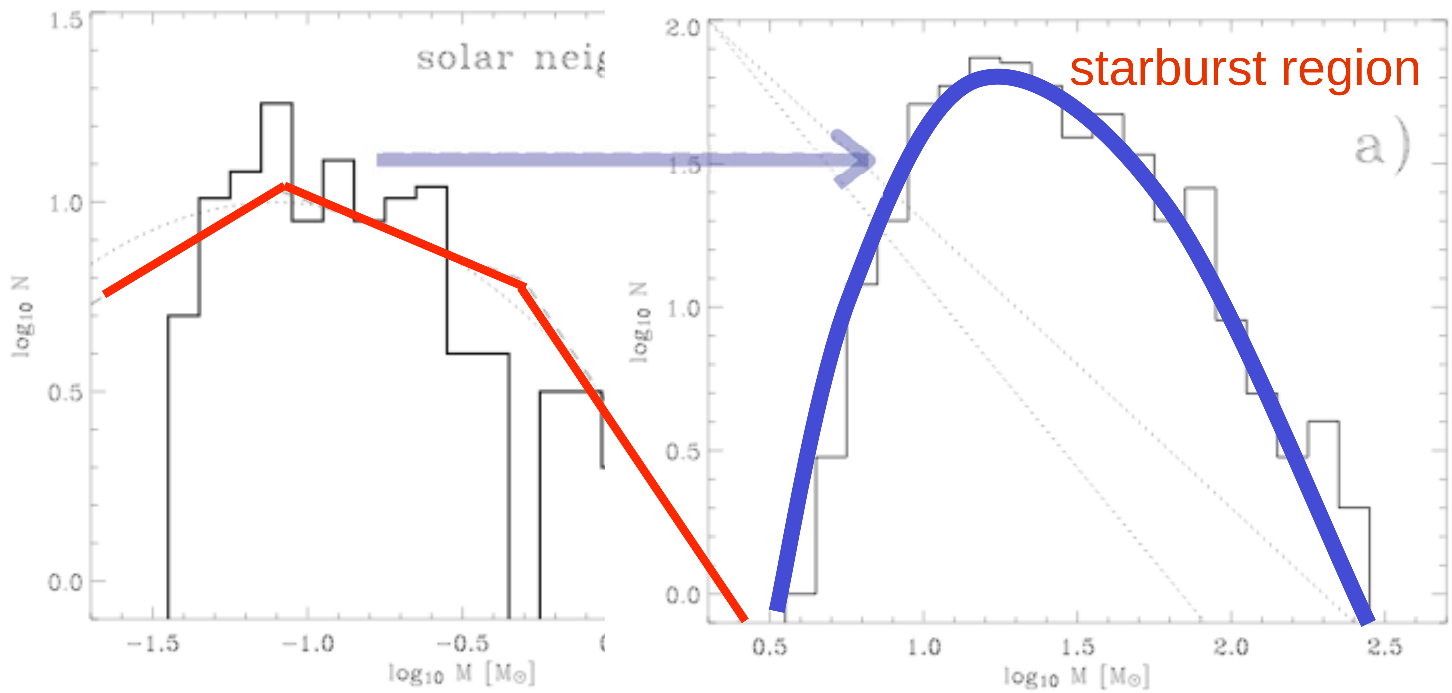
- Nuclear regions of starburst galaxies are extreme:
  - hot dust, large densities, strong radiation, etc.
- Thermodynamic properties of star-forming gas differ from Milky Way --> Different EOS!

(see Spaans & Silk 2005)

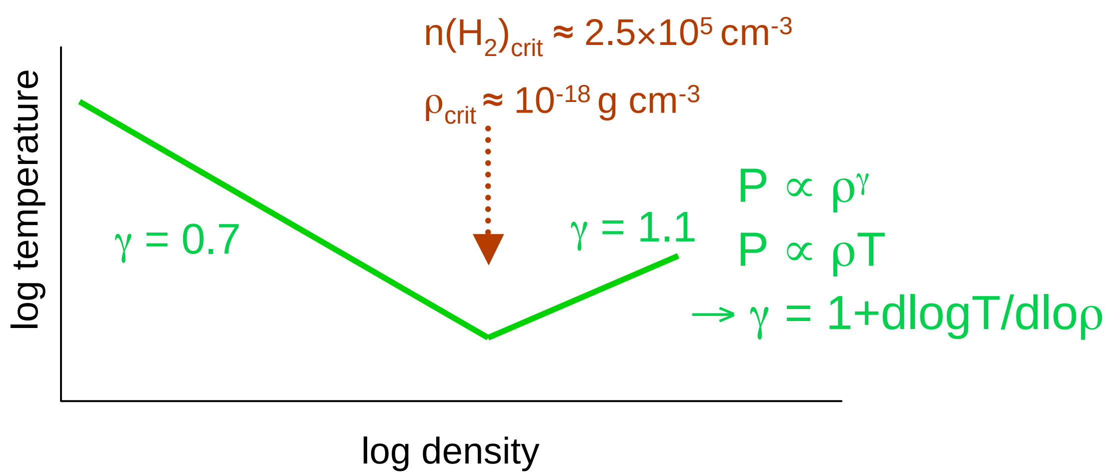


IMF in starburst galaxies

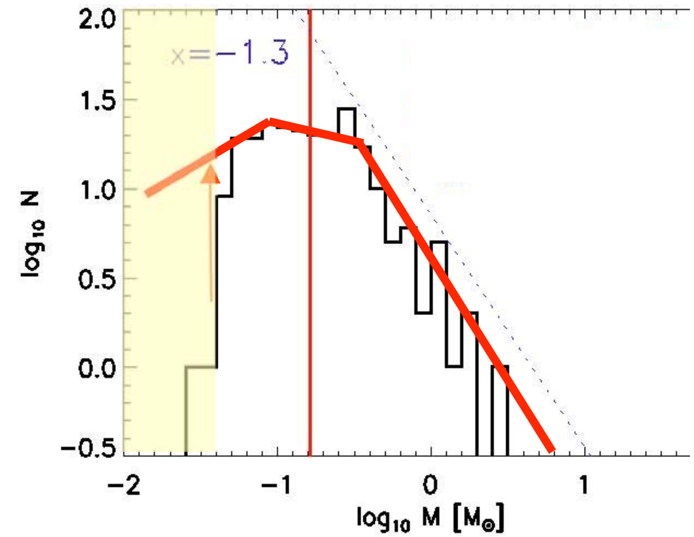
- Starburst EOS --> top-heavy IMF
  (Klessen, Spaans, Jappsen, 2007)



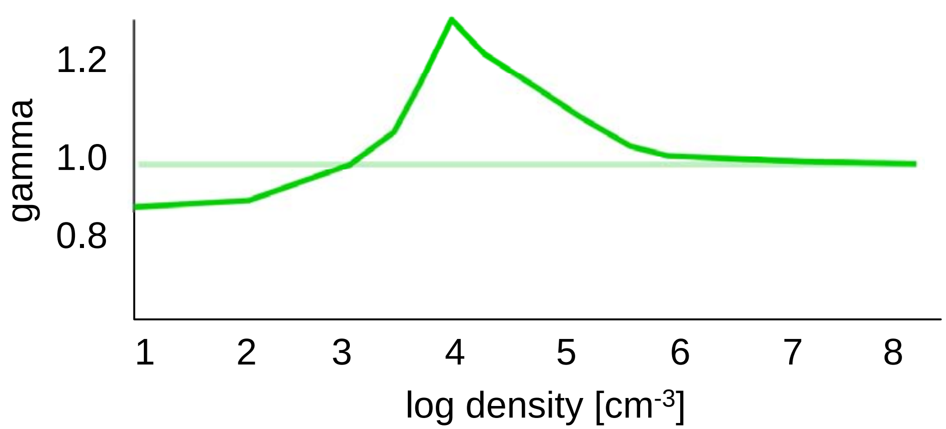
fragmentation depends on EOS

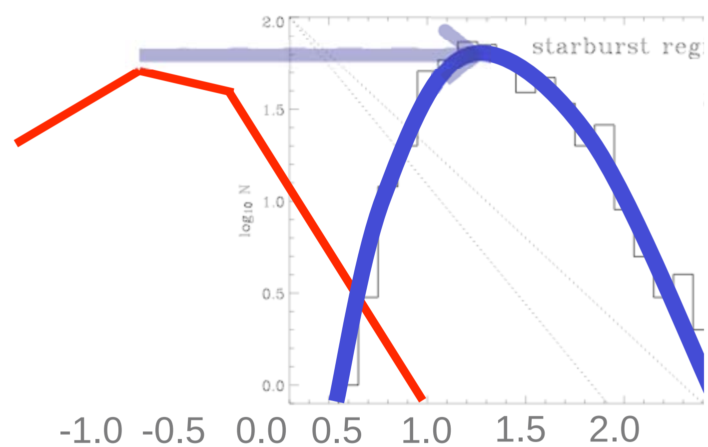


(Larson 2005)

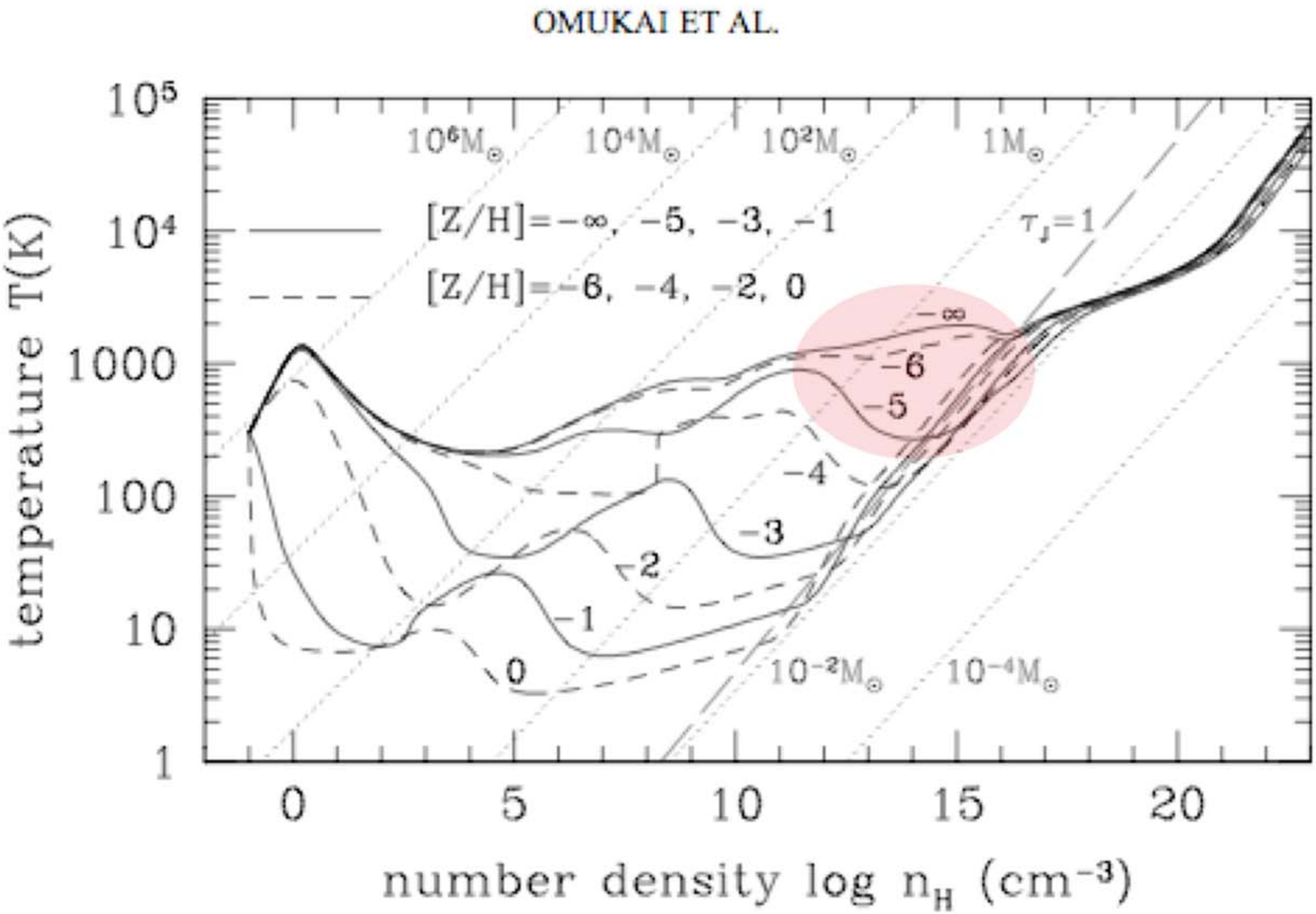


(Jappsen et al. 2

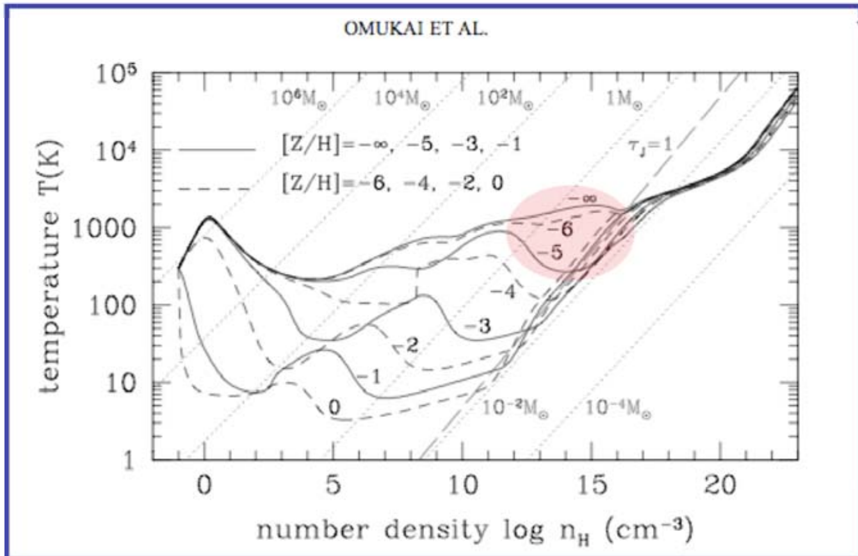


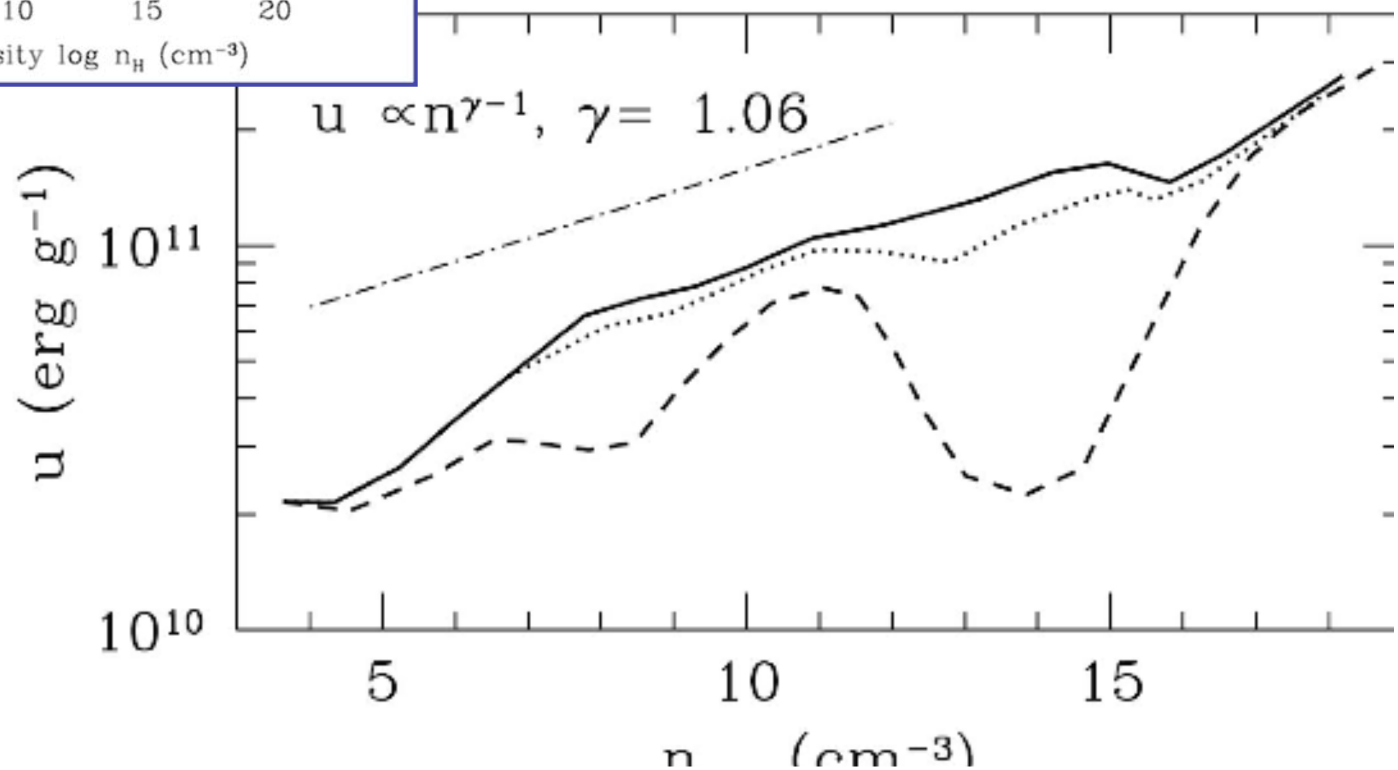


transition: Pop III to Pop II.5

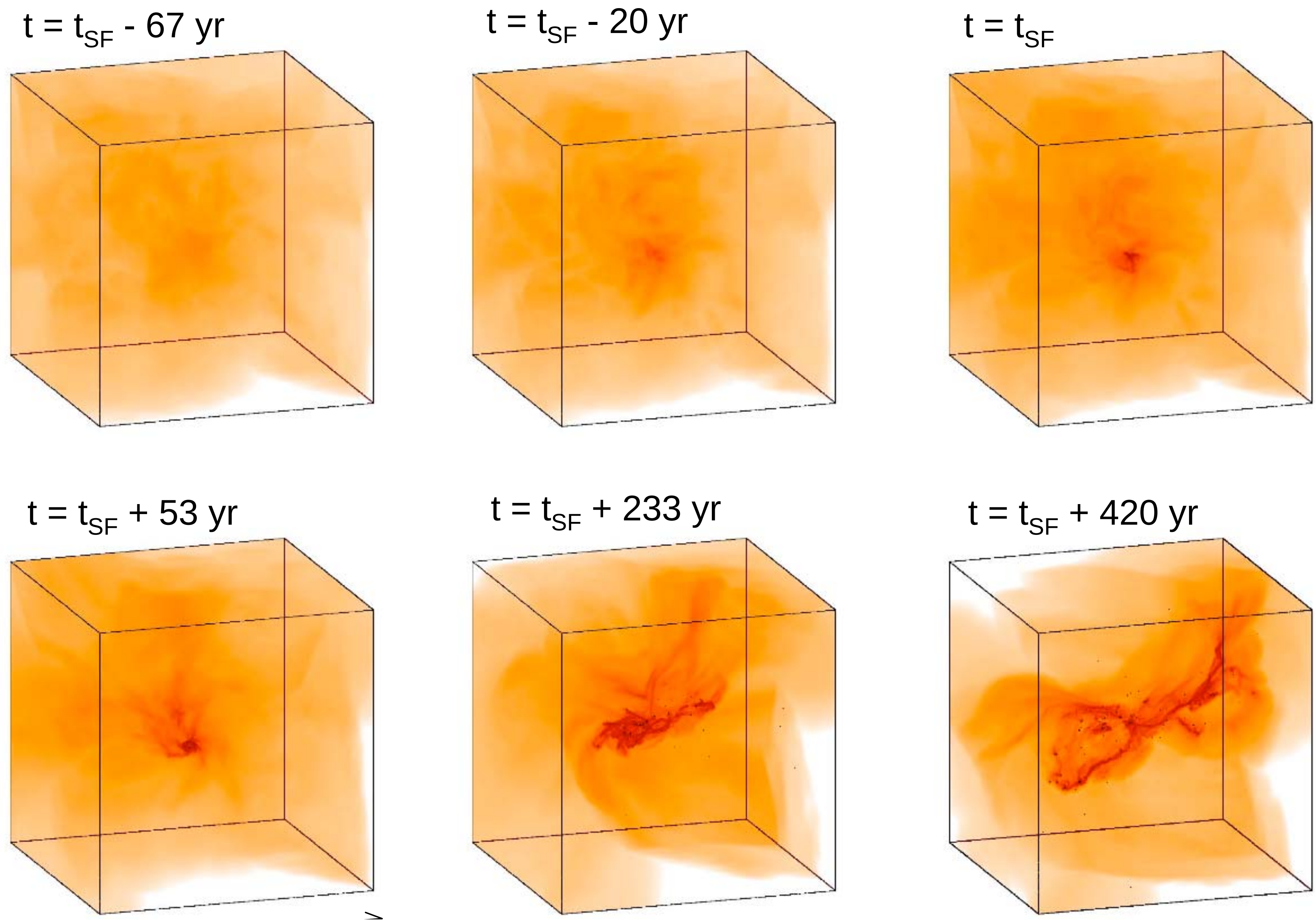


transition: Pop III to Pop II.5



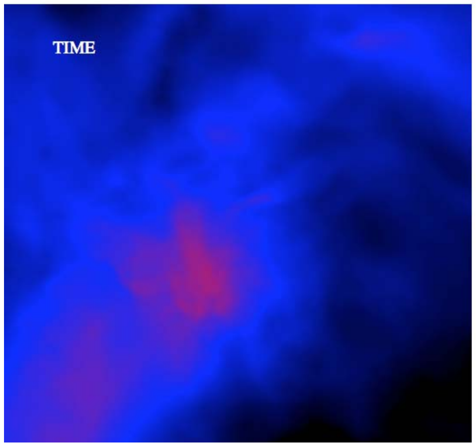


dust induced fragmentation at Z=1(

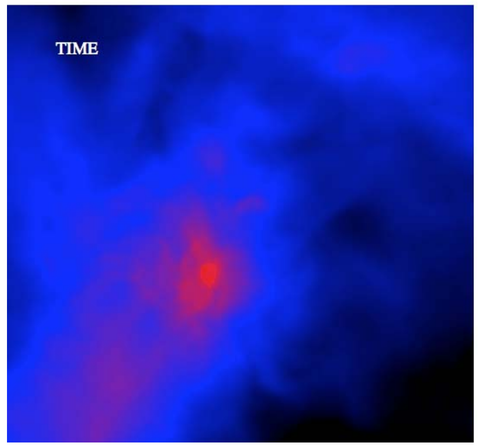


dust induced fragmentation at Z=1

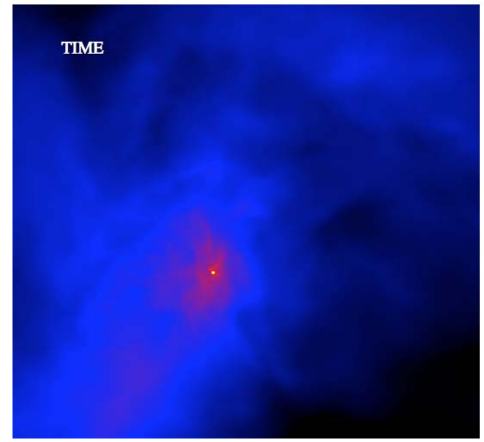
$t = t_{SF}$ - 67 yr



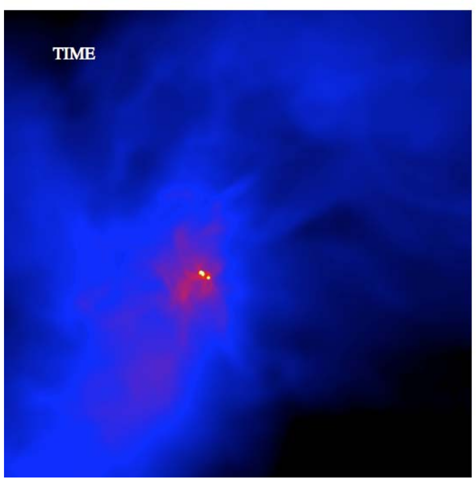
$t = t_{SF}$ - 20 yr



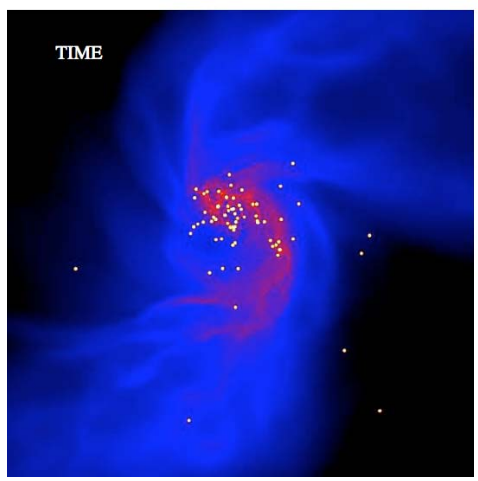
$t = t_{SF}$



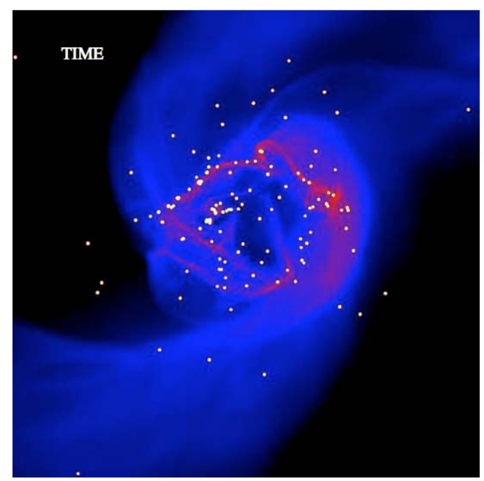
$t = t_{SF}$ + 53 yr



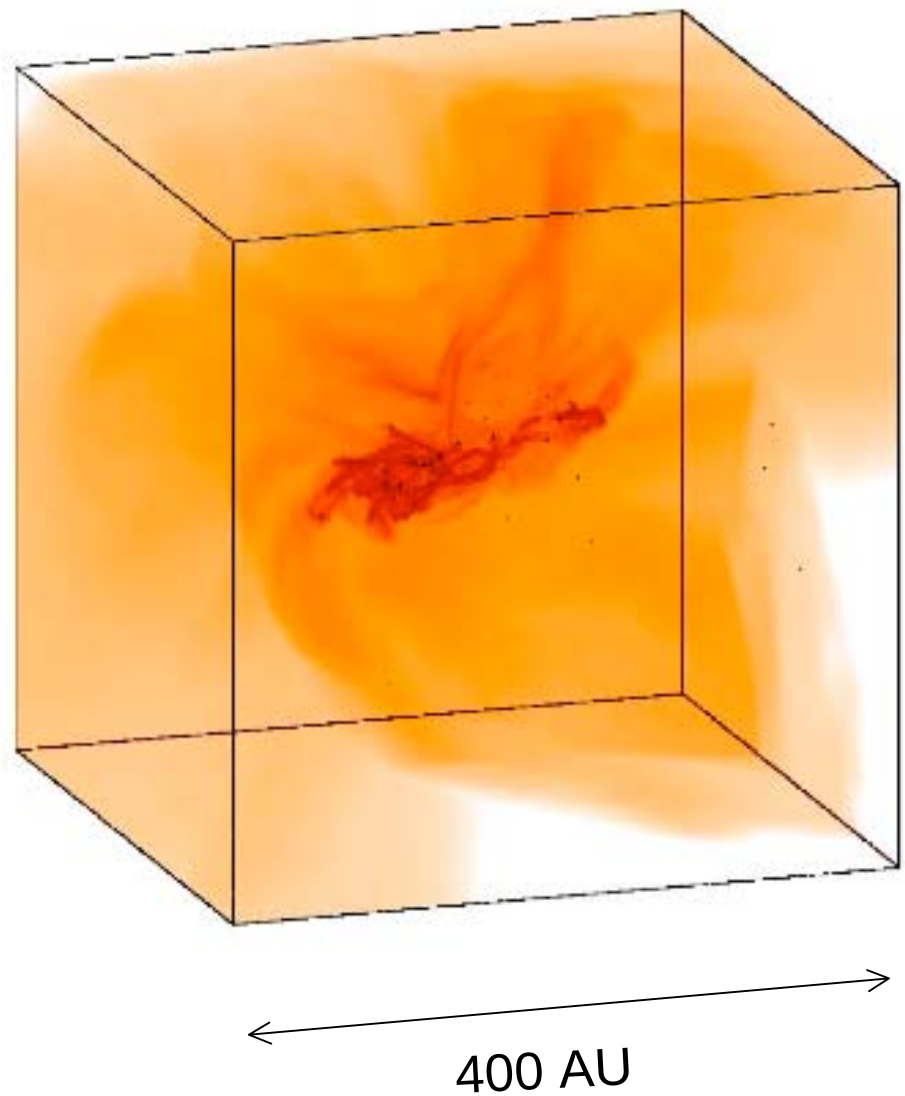
$t = t_{SF}$ + 233 yr



$t = t_{SF}$ + 420 yr



dust induced fragmentation at Z=1(



dense cluster of lov
mass protostars
builds up:

- mass spectrum
  peaks below 1 M_s
- cluster VERY den
  $n_{stars} = 2.5 \times 10^9$ p
- fragmentation
  at density
  $n_{gas} = 10^{12} - 10^{13}$ (

(Clark et al. 2007)

cluster build-up

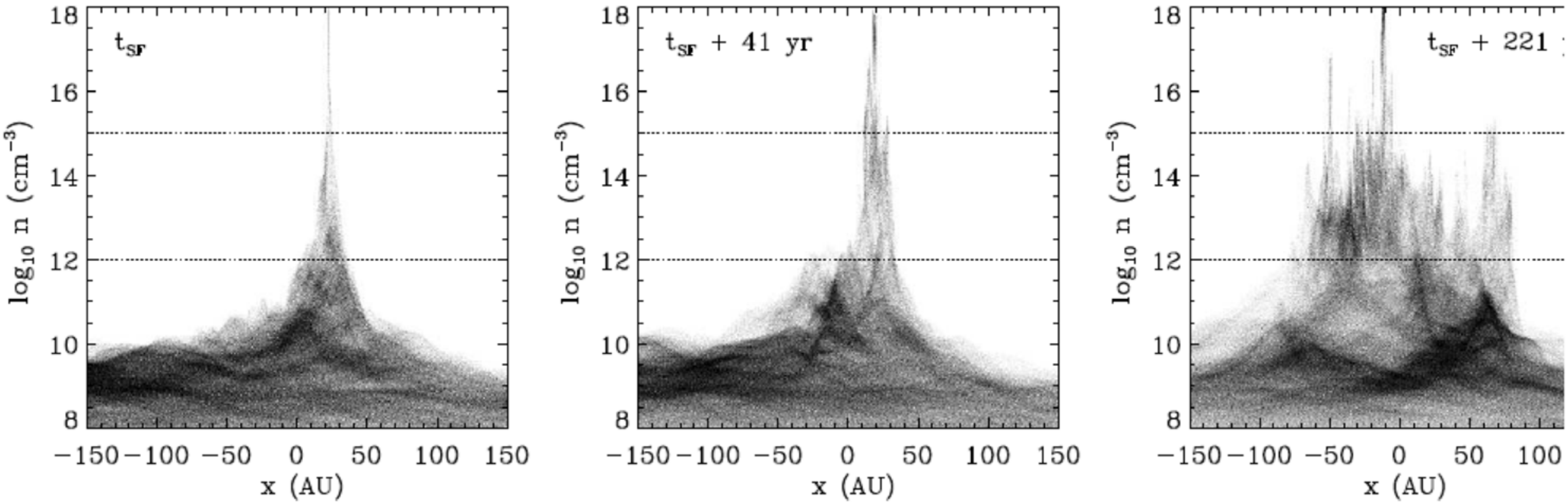
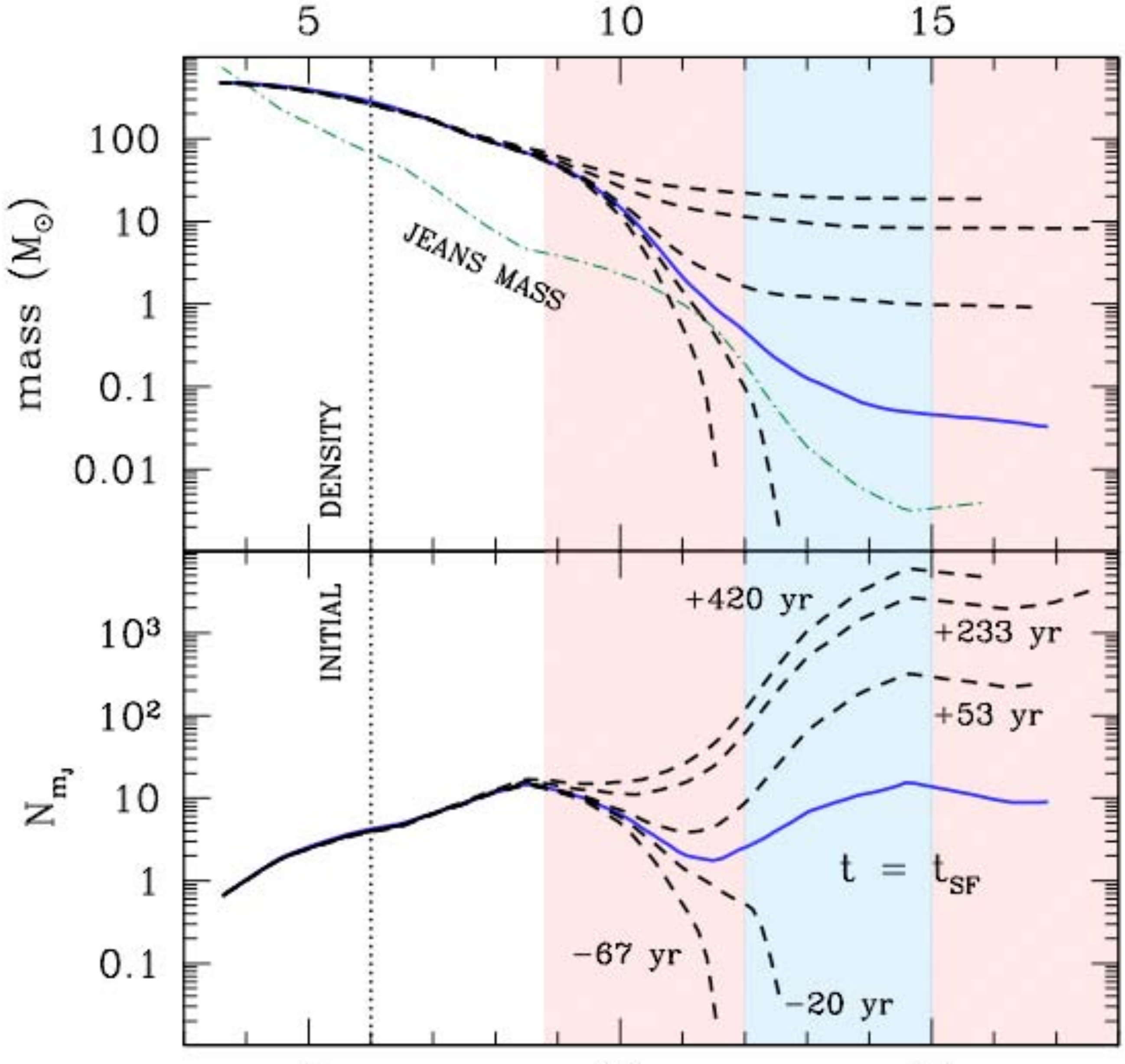


FIG. 3.— We illustrate the onset of the fragmentation process in the high resolution $Z = 10^{-5}\,Z_{\odot}$ simulation. The graphs sh densities of the particles, plotted as a function of their x-position. Note that for each plot, the particle data has been centered on the of interest. We show here results at three different output times, ranging from the time that the first star forms (t_{sf}) to 221 years afte The densities lying between the two horizontal dashed lines denote the range over which dust cooling lowers the gas temperature.

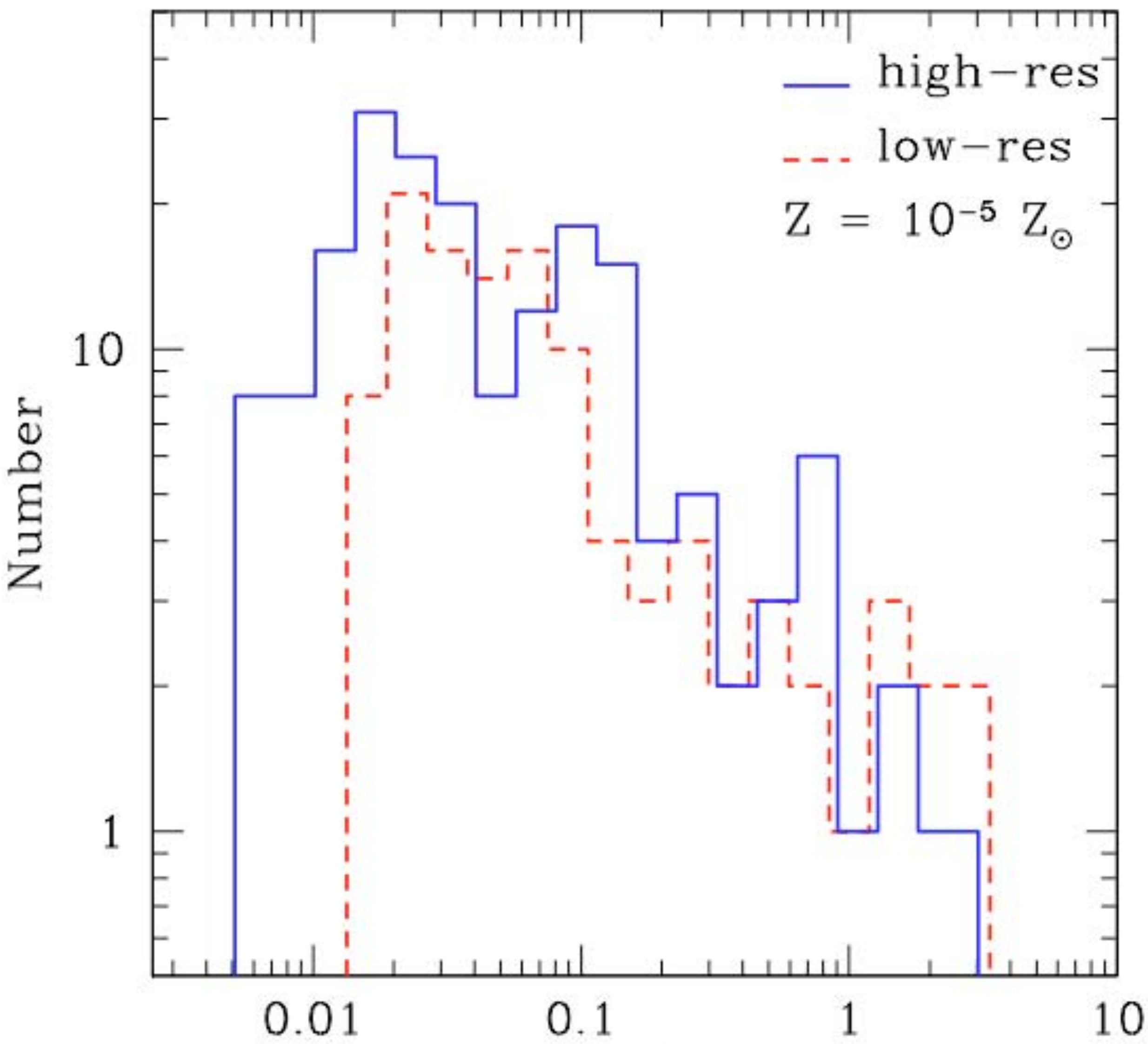
cluster build-up



γ >
(heati

γ <
(cooli

dust induced fragmentation at Z=1(



dense cluster of lov
mass protostars
builds up:

- mass spectrum
  peaks below 1 M_s
- cluster VERY den
  n_{stars} = 2.5 x 10^9 p
- fragmentation
  at density
  n_{gas} = 10^{12} - 10^{13} (

(Clark et al. 2007)

summary

- basic parameters of IMF can be explained by interplay between gravity and turbulence

- thermodynamic properties of gas are very important (balance between heating and cooling determines compressibility of gas and, hence, probability for gravitational collapse)
  - universality in solar neighborhood (can be based on atomic and molecular parameters)
  - dependency on metallicity (transition Pop III -> Pop II/2)