MHD and dynamos

(i) Using the vector potential

(ii) Nearly potential turbulence

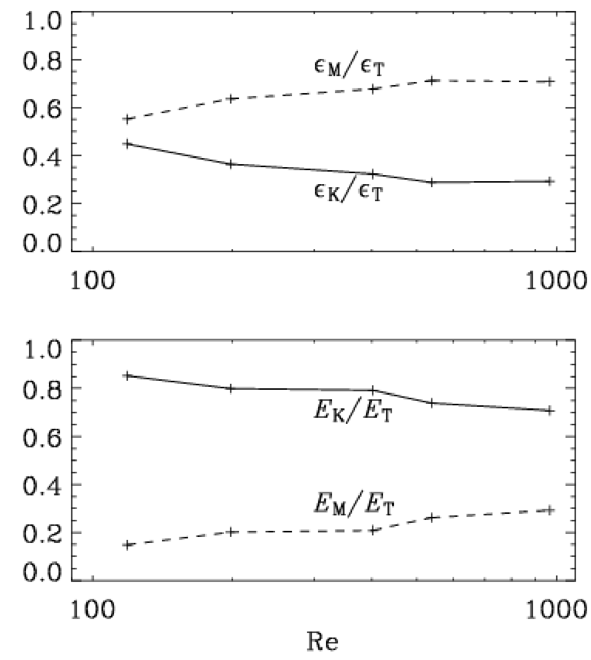
(iii) Transonic small-scale dynamo

(iv) Bottleneck and low Pr_M dynamos

Axel Brandenburg (*Nordita, Stockholm*)

Small scale vs large scale

- Conversion by $-\mathbf{u}.(\mathbf{J}\times\mathbf{B})$
- Small scale dynamo
  - if $R_m > R_{m,crit}$ $\qquad \overline{\mathbf{u}} = \overline{\mathbf{B}} = 0$
- Large scale dynamo $\quad \overline{\mathbf{B}} \neq 0$
  - if $D > D_{crit}$
  - α^2 and $\alpha\Omega$ dynamos
  - **W**x**J** dynamo (only shear)
  - Incoherent alpha-shear dynamo



MHD equations (isothermal)

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{u} \times \mathbf{B} - \eta \mathbf{J} - \nabla \phi, \qquad \frac{\partial \phi}{\partial t} = -c_\phi^2 \nabla \cdot \mathbf{A}$$

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \mathbf{u} \times \boldsymbol{\omega} - \nu \mathbf{Q} - \nabla h, \qquad \frac{\mathrm{D}h}{\mathrm{D}t} = -c_s^2 \nabla \cdot \mathbf{u}$$

Momentum and continuity eqns (usual form)

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -c_s^2 \nabla \ln \rho + \frac{1}{\rho}\left[\mathbf{J} \times \mathbf{B} + \mu\left(\nabla^2 \mathbf{u} + \tfrac{1}{3}\nabla\nabla \cdot \mathbf{u}\right)\right] + \mathbf{f}$$

$$\frac{\mathrm{D} \ln \rho}{\mathrm{D}t} = -\nabla \cdot \mathbf{u}$$

Induction Equation in suitable form for SPH

(used here for shearing box simulations)

$$\frac{\partial A_i}{\partial t} = (\mathbf{u} \times \mathbf{B})_i - \nabla_i \phi + \eta \nabla^2 A_i$$

$$= (\mathbf{u} \times \nabla \times \mathbf{A})_i - \nabla_i \phi + \ldots$$

$$= -u_j A_{i,j} + u_j A_{j,i} - \nabla_i \phi + \ldots$$

$$\frac{DA_i}{Dt} = (u_j A_j)_{,i} - A_j u_{j,i} - \nabla_i \phi + \ldots$$

$$= -A_j u_{j,i} + \ldots$$

Vector potential

- **B**=curl**A**, advantage: div**B**=0
- **J**=curl**B**=curl(curl**A**) =curl2**A**
- Not a disadvantage: consider Alfven waves

B-formulation

$$\frac{\partial u}{\partial t} = B_0 \frac{\partial b}{\partial z}, \quad \text{and} \quad \frac{\partial b}{\partial t} = B_0 \frac{\partial u}{\partial z}$$

A-formulation

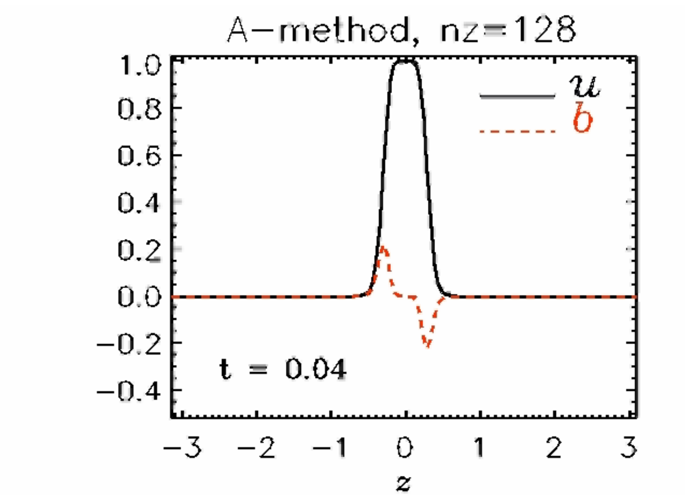
$$\frac{\partial u}{\partial t} = B_0 \frac{\partial^2 a}{\partial z^2}, \quad \text{and} \quad \frac{\partial a}{\partial t} = B_0 u$$

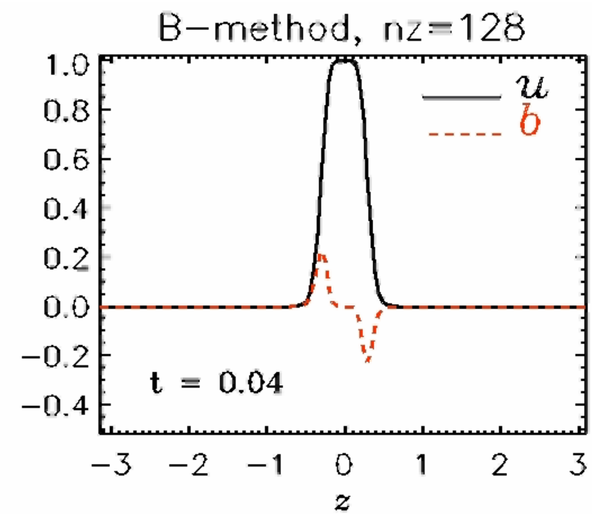
2nd der once is better than 1st der twice!

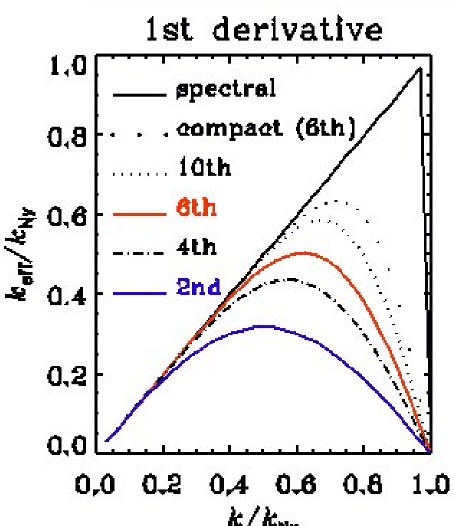
Comparison of A and B methods

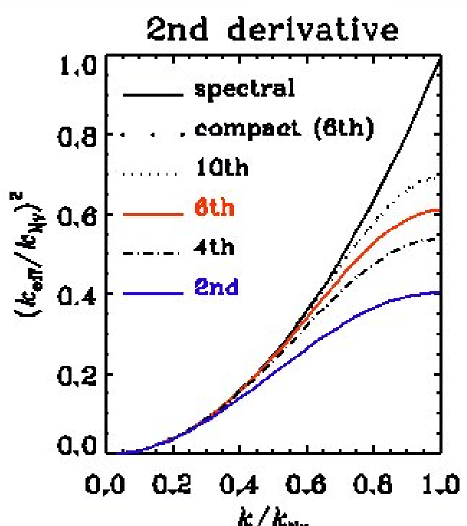
$$\frac{\partial u}{\partial t} = B_0 \frac{\partial^2 a}{\partial z^2} + \upsilon \frac{\partial^2 u}{\partial z^2}, \quad \text{and} \quad \frac{\partial a}{\partial t} = B_0 u + \eta \frac{\partial^2 a}{\partial z^2}$$

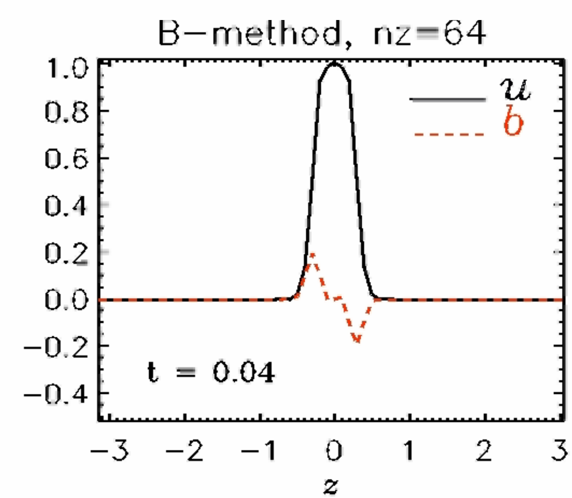
$$\frac{\partial u}{\partial t} = B_0 \frac{\partial b}{\partial z} + \upsilon \frac{\partial^2 u}{\partial z^2}, \quad \text{and} \quad \frac{\partial b}{\partial t} = B_0 \frac{\partial u}{\partial z} + \eta \frac{\partial^2 b}{\partial z^2}$$



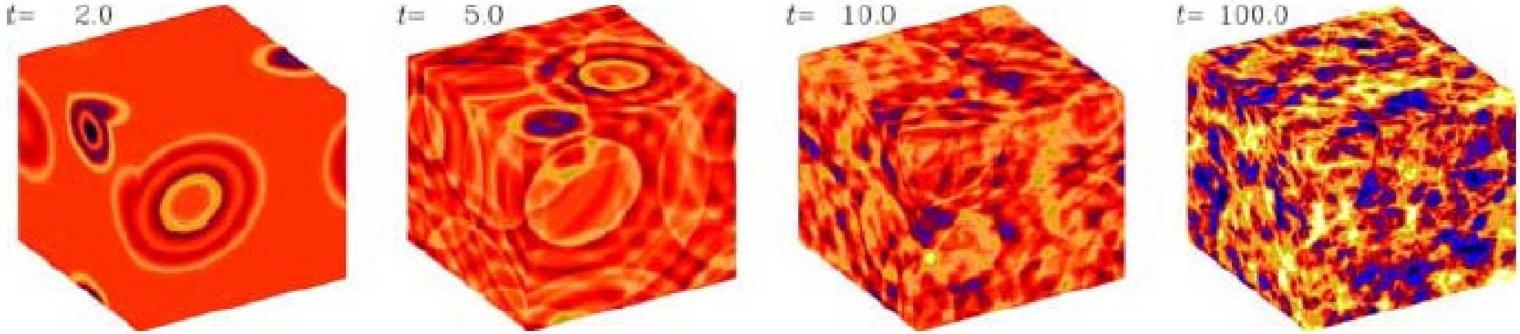


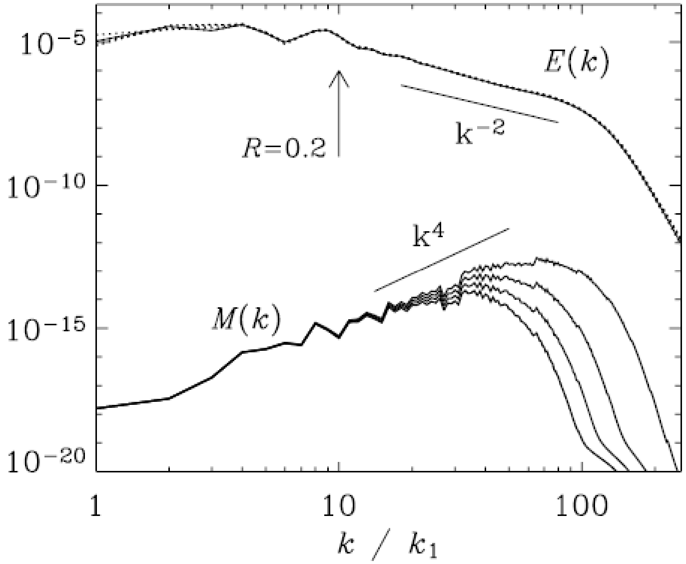






Nearly potential flows

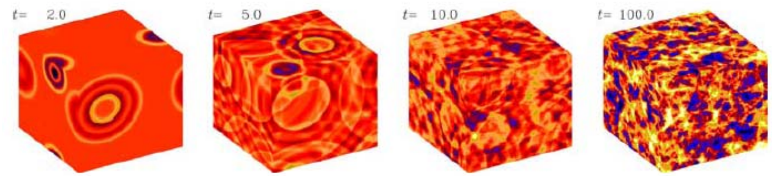




No dynamo action in nearly potential flows (at least not fo far)

Nearly potential flows

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$



$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \nabla \times (\mathbf{u} \times \boldsymbol{\omega}) + \nu \nabla^2 \boldsymbol{\omega} + \nabla \times (\mathbf{S} \cdot \nabla \ln \rho)$$

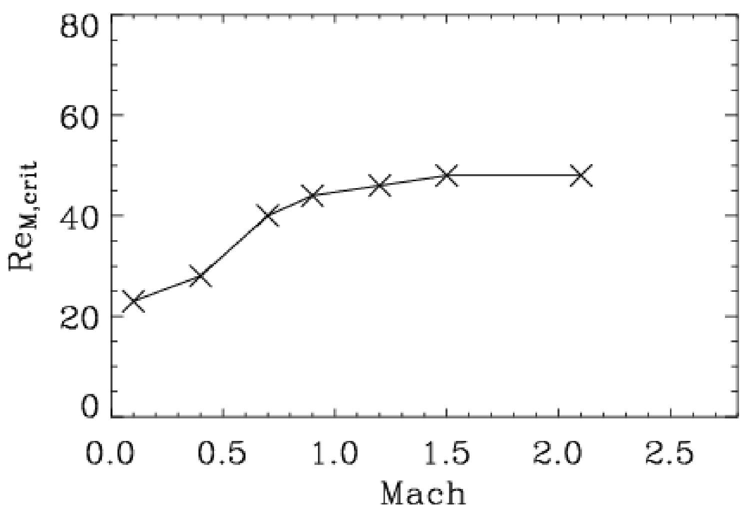
Table 1. Results for the normalized vorticity, $\omega_{\rm rms}/(u_{\rm rms}\,k_{\rm peak})$, as a function of Re and resolution. The durations of the runs vary between $N_{\rm turn} =$ 16 and 250 turnover times, and ν varies between 10^{-3} and 5×10^{-5}. For the high resolution run with 512^3 mesh points and $\nu = 5 \times 10^{-5}$, we have $N_{\rm turn} = 59$ turnover times.

Re	512^3	256^3	128^3
50	$1.4 + \times 10^{-3}$	8.7×10^{-3}	
25		1.1×10^{-2}	2.9×10^{-3}
12		1.6×10^{-2}	2.0×10^{-3}
4		7.6×10^{-3}	1.5×10^{-3}

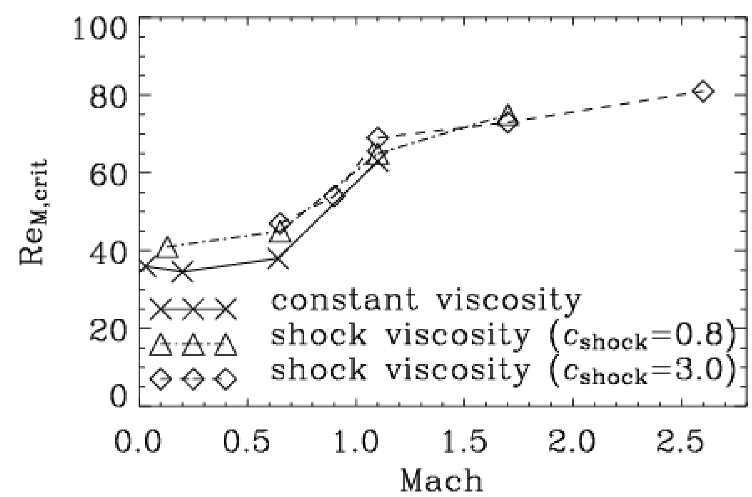
Does compressibility affect the dynamo?

Shocks sweep up all the field: dynamo harder?

-- or artifact of shock diffusion?

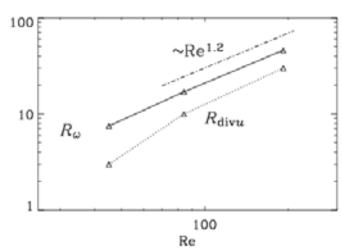


Direct simulation, $\nu/\eta=5$



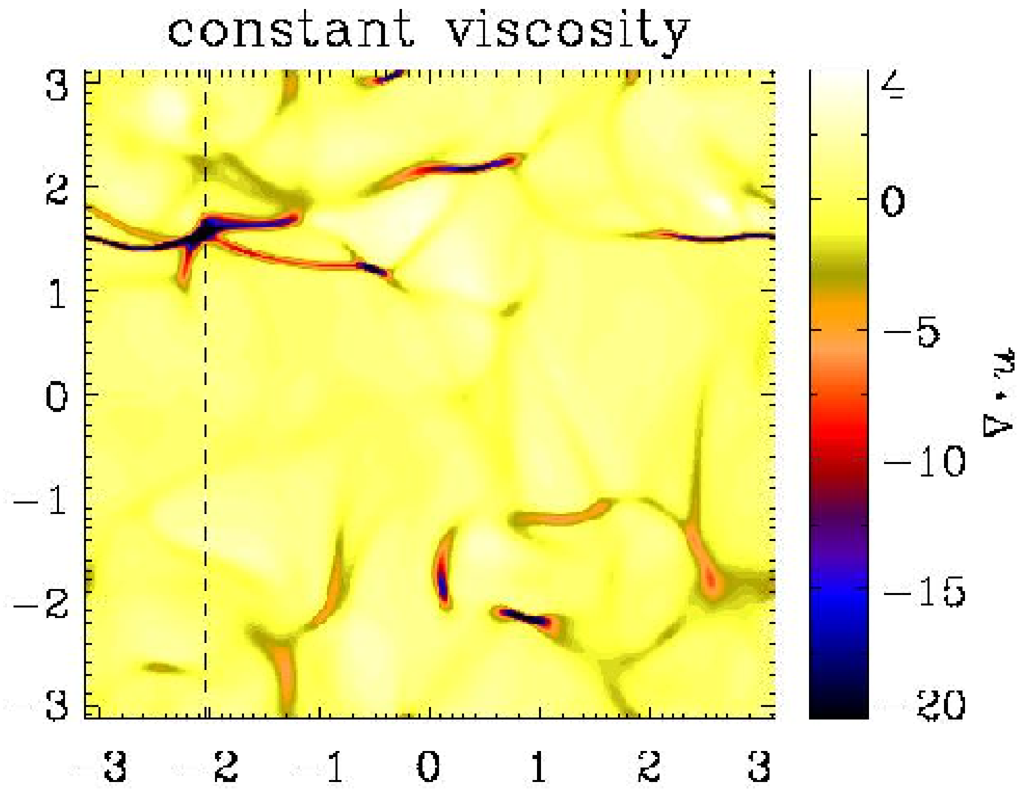
Direct and shock-capturing simulations, $\nu/\eta=1$

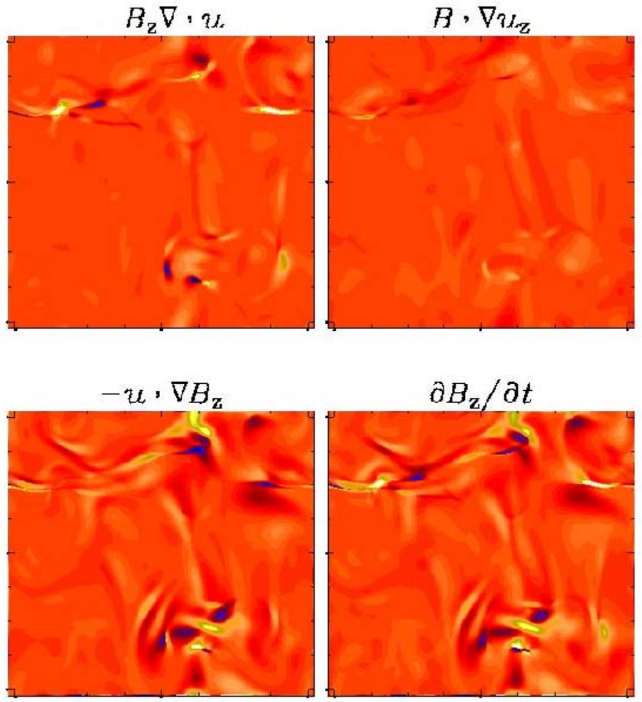
$$u = \nabla\phi + \nabla\times\boldsymbol{\psi}$$



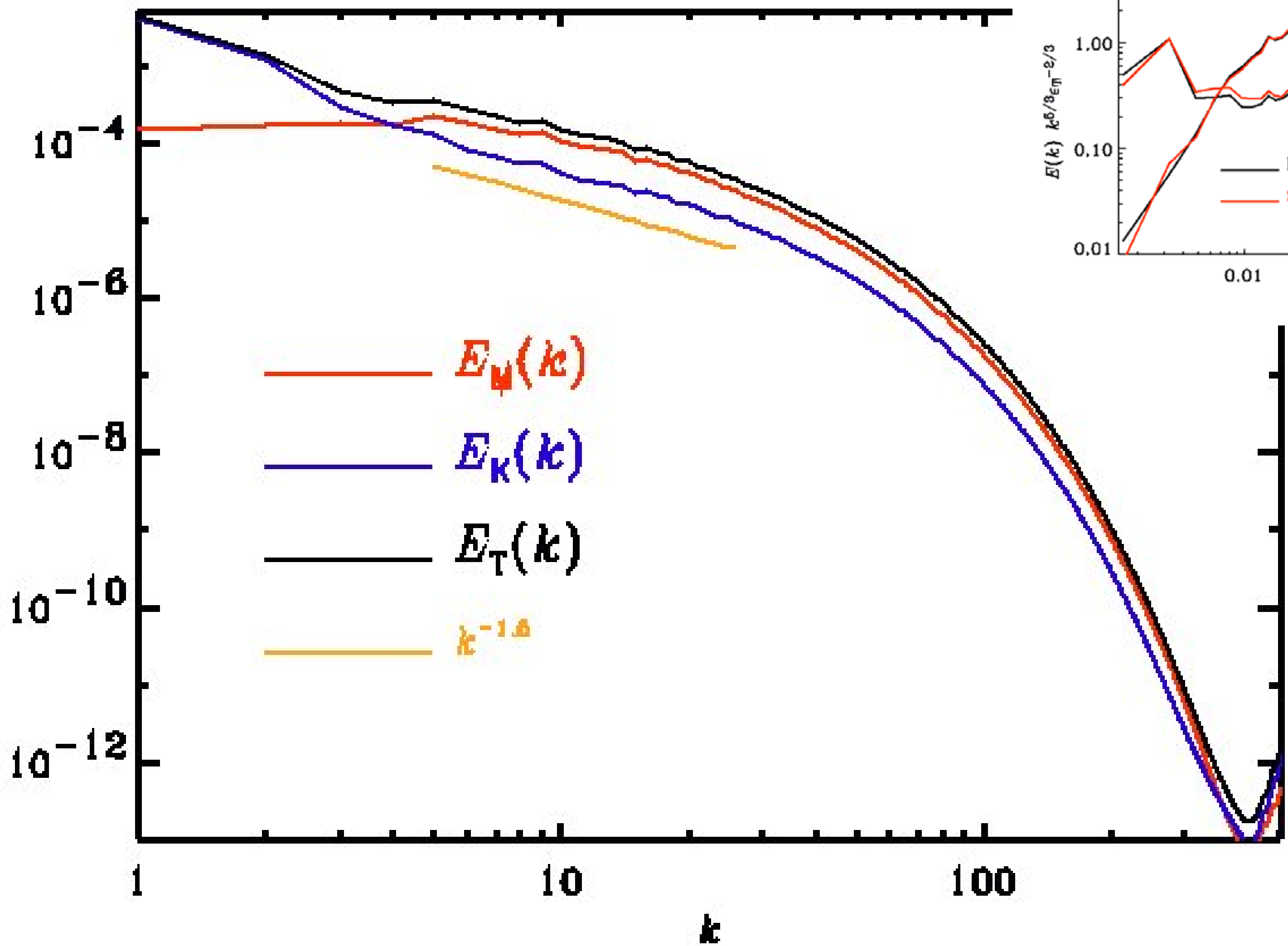
→ Bimodal behavior!

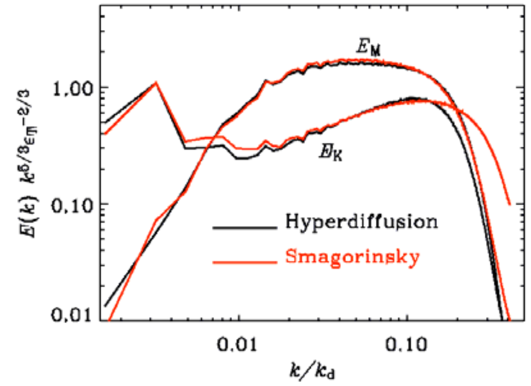
Div *u* and effect on *B*





256 processor run at 1024^3





Direct vs hyper at 512^3

Biskamp & Müller (2000, Phys Fluids 7, 4889)

D. Biskamp and W.-C. Müller

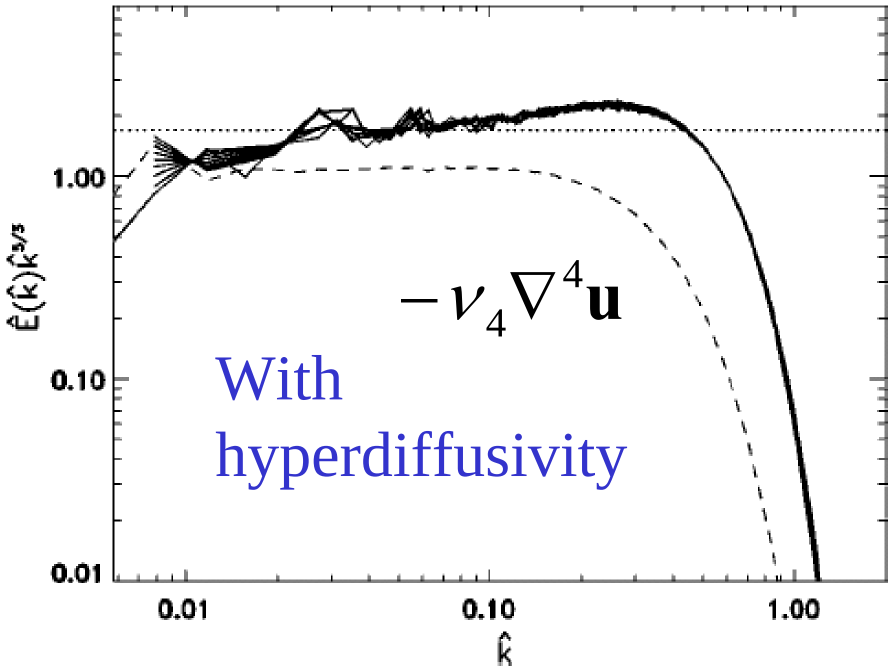


FIG. 11. Scatter plot of the normalized energy spectrum compensated by $k^{5/3}$ from the hyperdiffusive run 10. The dotted line is identical to the one in Fig. 8 for normal diffusion, indicating the same inertial range spectrum outside the bottleneck hump. The dashed line gives the one-dimensional spectrum $E(|k_z|)=E(k_z)+E(-k_z)$.

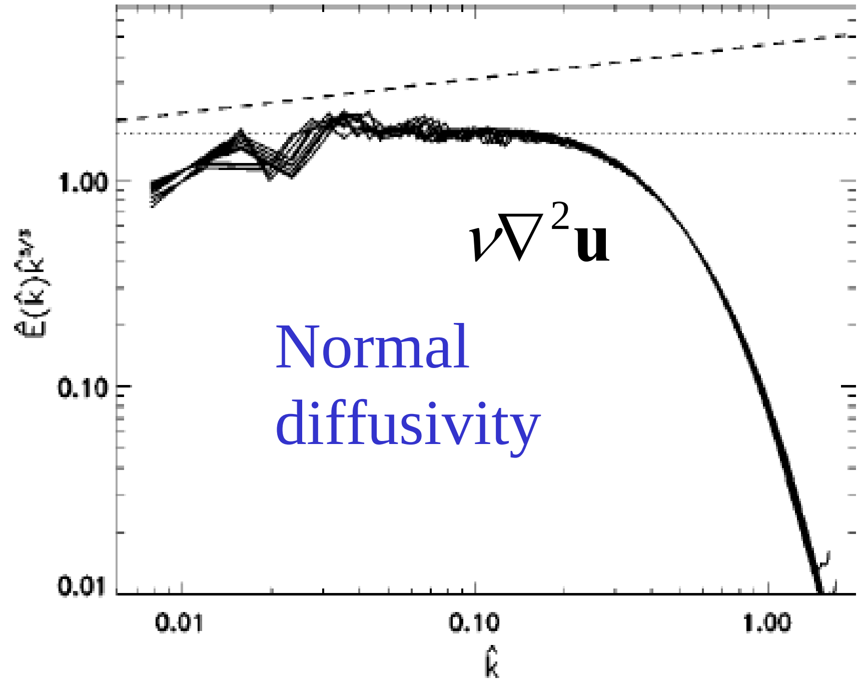
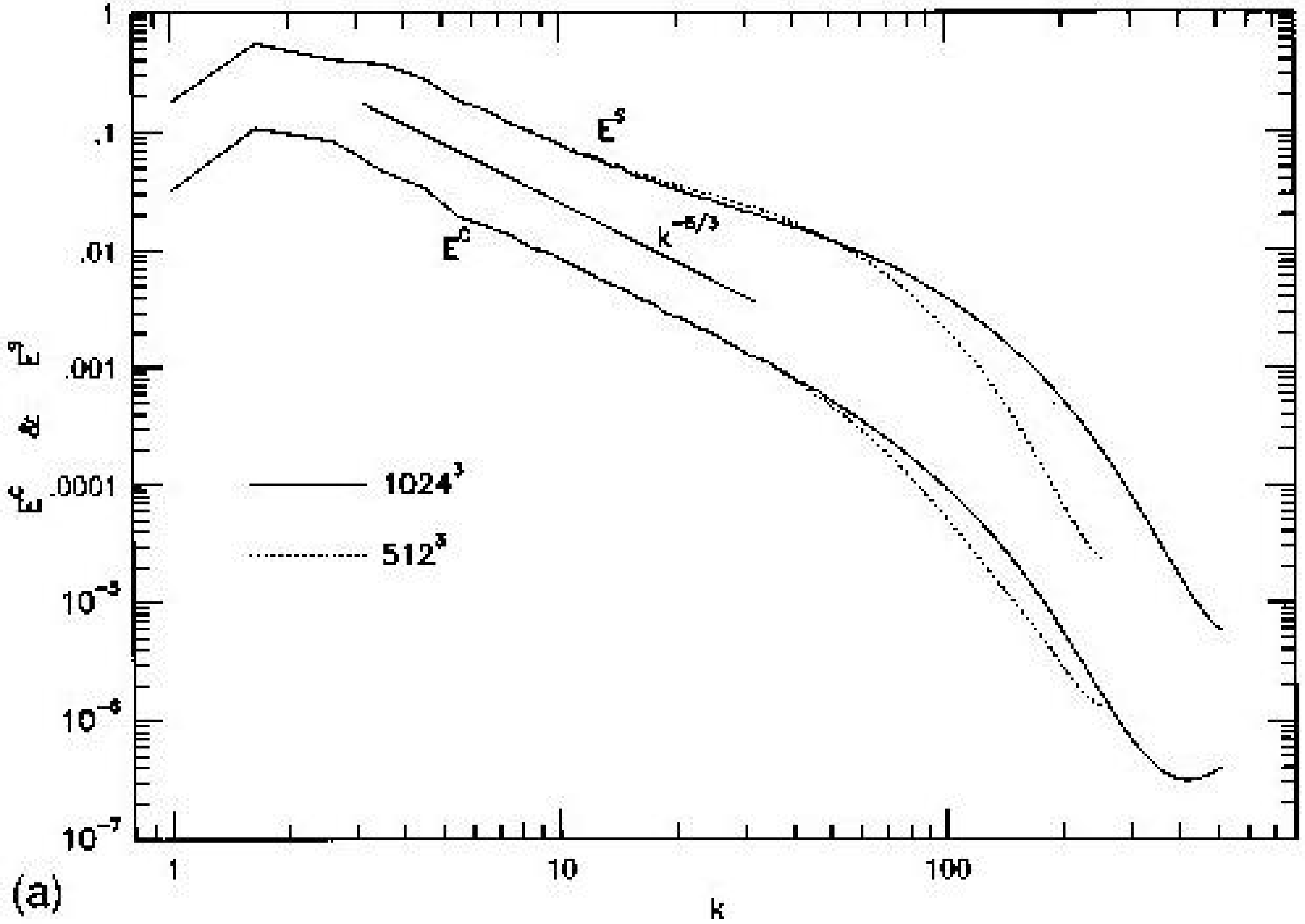


FIG. 8. Scatter plot of the normalized angle-integrated energy spectrum compensated by $k^{5/3}$ from run 6 taken during the period $t=4.5-10$. The dashed line indicates the IK-spectrum $k^{3/2}$, the dotted line the Kolmogorov spectrum with $C'=1.7$.

Turbulence at 1024^3

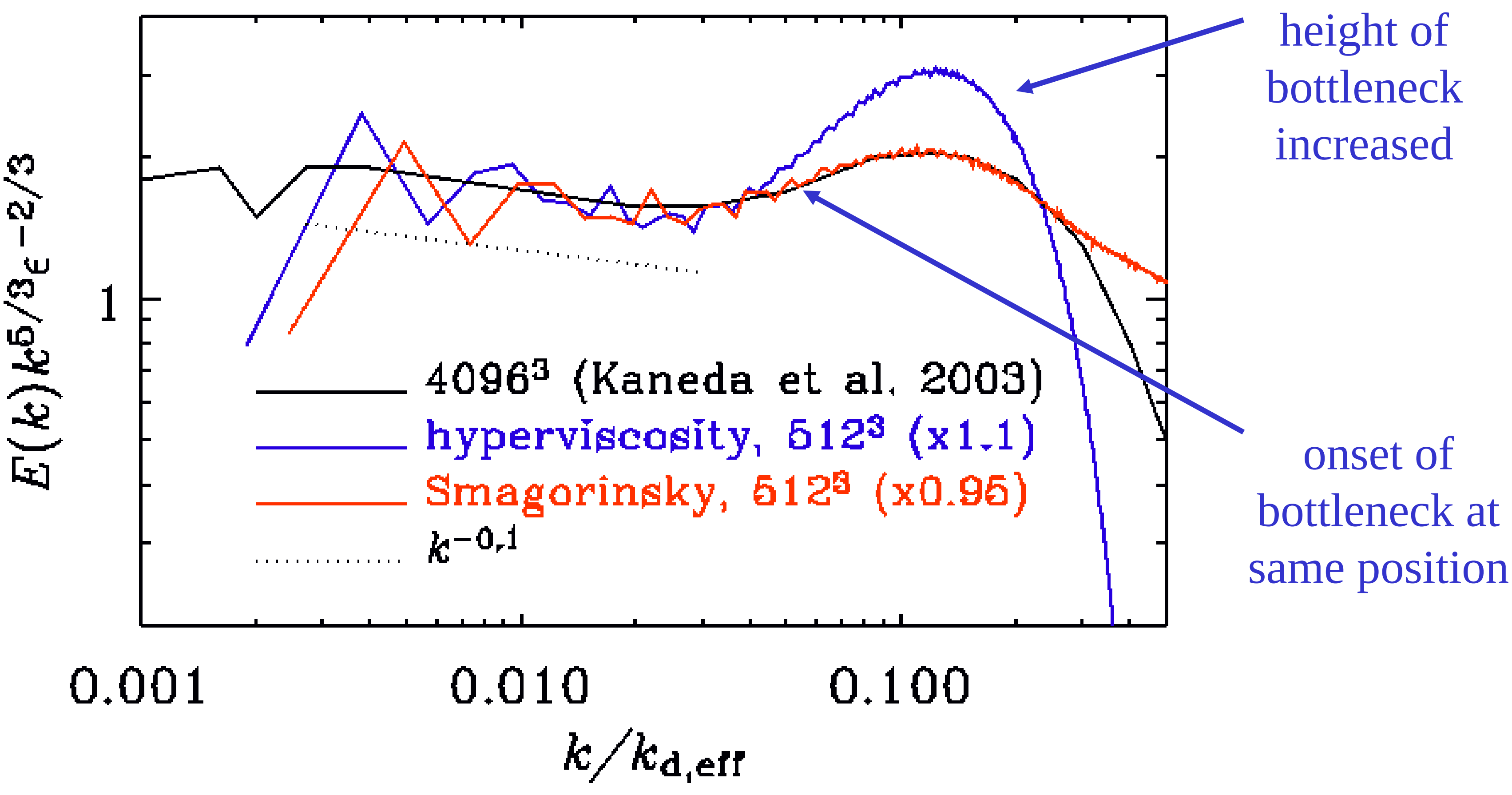
(Porter, Pouquet,& Woodward 1998)



Ideal hydro: should we be worried?

- Why this k^{-1} tail in the power spectrum?
  - – Compressibility?
  - – PPM method
  - – Or is real??
- Hyperviscosity destroys entire inertial range?
  - – Can we trust any ideal method?
- Needed to wait for 4096^3 *direct* simulations

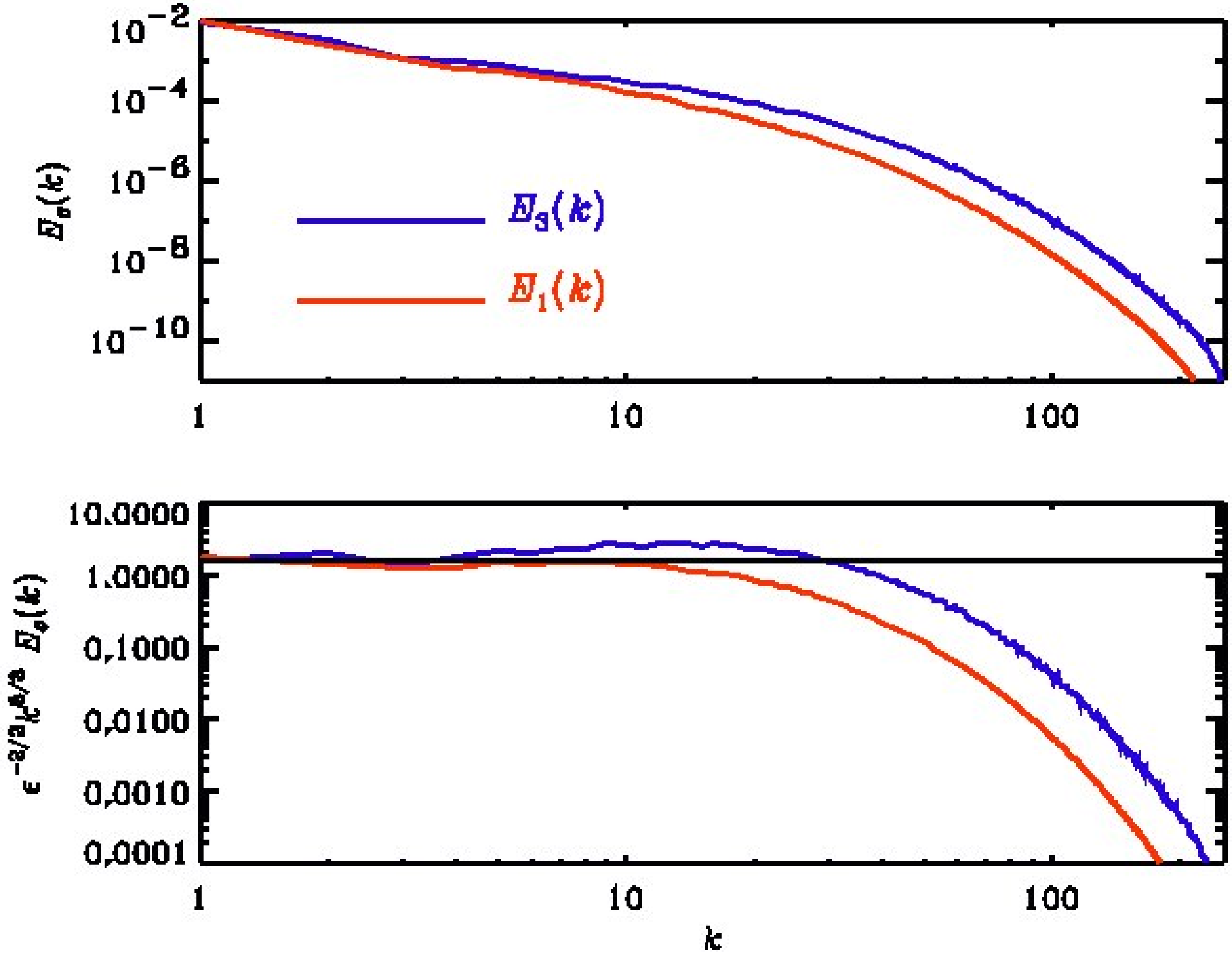
Hyperviscous, Smagorinsky, normal



Inertial range unaffected by artificial diffusion

Bottleneck effect: 1D vs 3D spectra

Why did wind tunnels not show this?



Compensated spectra (1D vs 3D)

Relation to 'laboratory' 1D spectra

$$E_{3D} = \int \left|\mathbf{u}(\mathbf{k})\right|^2 k^2 d\Omega_k = 4\pi k^2 \left\langle \left|\mathbf{u}(\mathbf{k})\right|^2 \right\rangle$$

$$E_{1D}(k_z) = 2\int \left\langle \left|\mathbf{u}(x, y, k_z)\right|^2 \right\rangle \mathrm{d}x\, \mathrm{d}y$$

$k_z > 0$

$$= 4\pi \int_0^\infty \left\langle \left|\mathbf{u}(k_\varpi, k_z)\right|^2 \right\rangle k_\varpi \,\mathrm{d}k_\varpi = 4\pi \int_{k_z}^\infty \left\langle \left|\mathbf{u}(k)\right|^2 \right\rangle k \,\mathrm{d}k$$

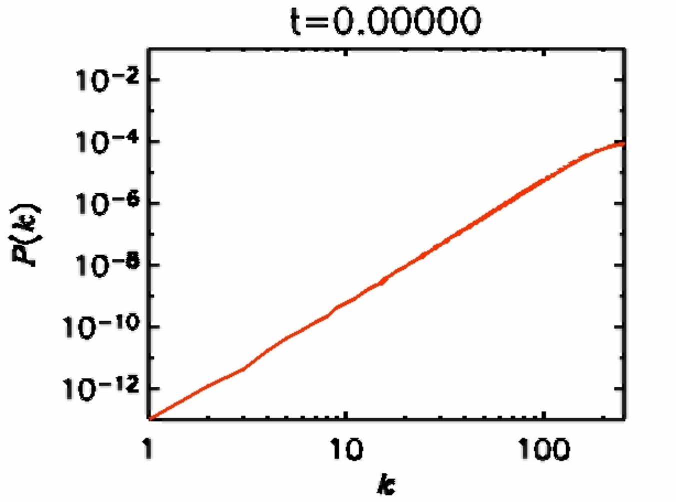
$$= \int_{k_z}^\infty \frac{E_{3D}}{k} \mathrm{d}k$$

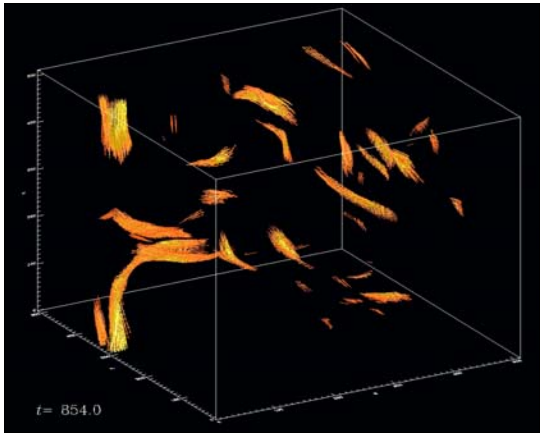
$k^2 = k_\varpi^2 + k_z^2$

Dobler, et al
(2003, PRE 68, 026304)

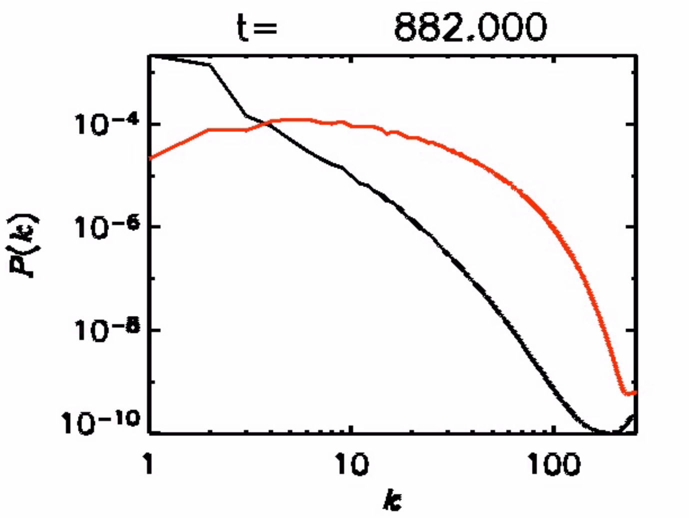
Small scale dynamo action

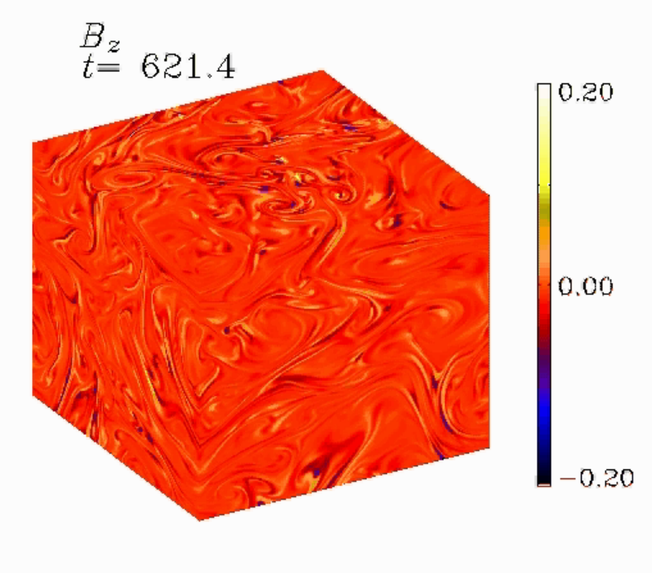
non-helically
forced turbulence





$Pr_M=\nu/\eta=1$

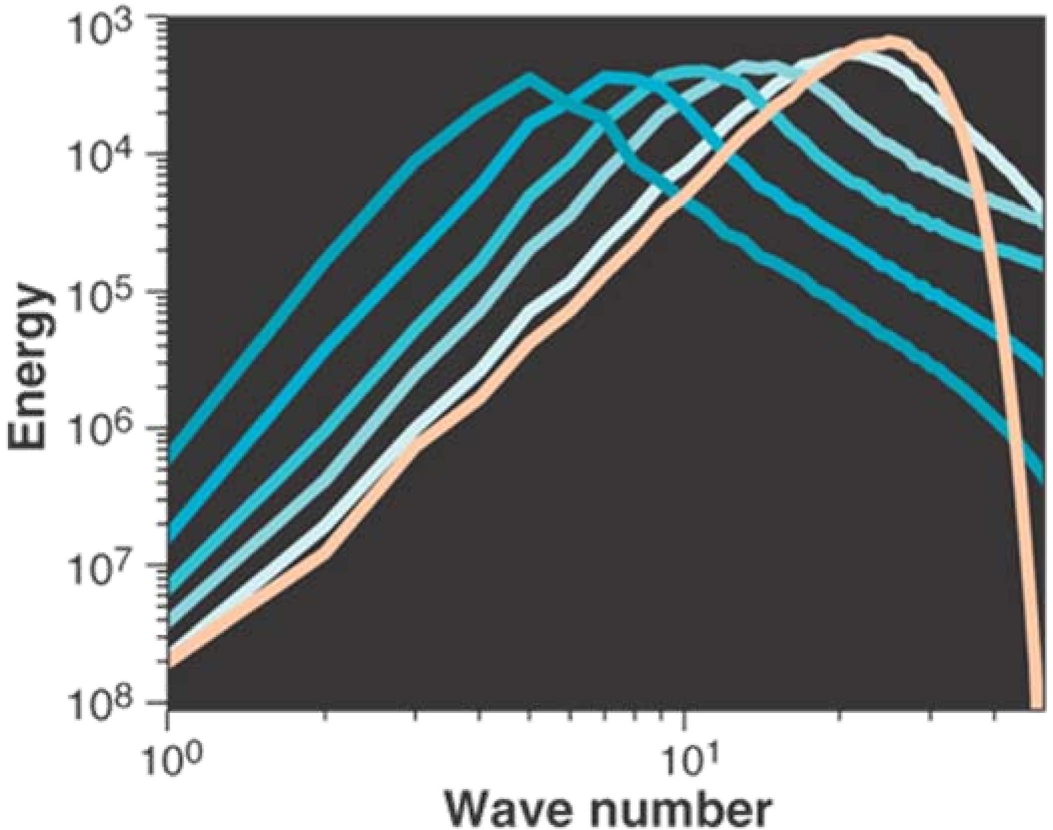




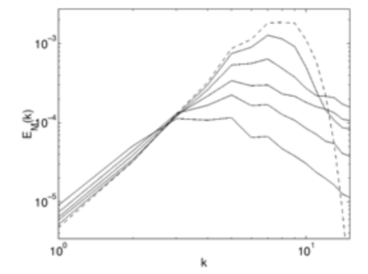
$Pr_M=\nu/\eta=50$

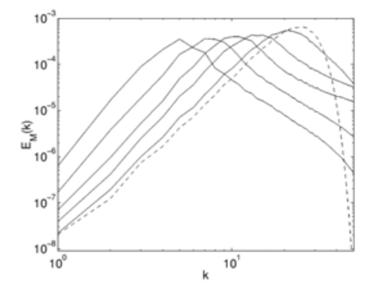
Decaying fully helical turbulence

Initial slope
$E \sim k^4$



helical vs
nonhelical

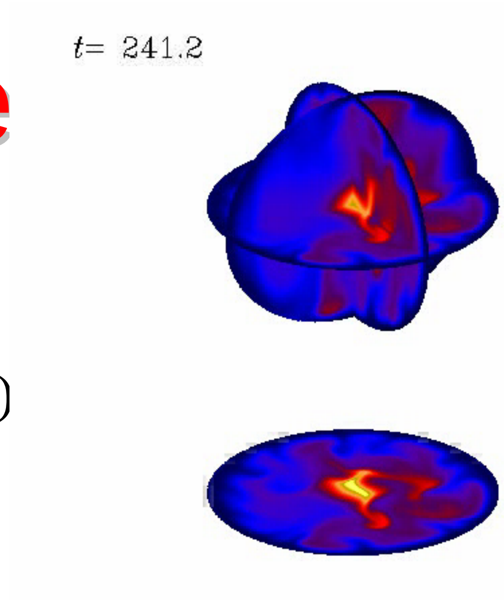




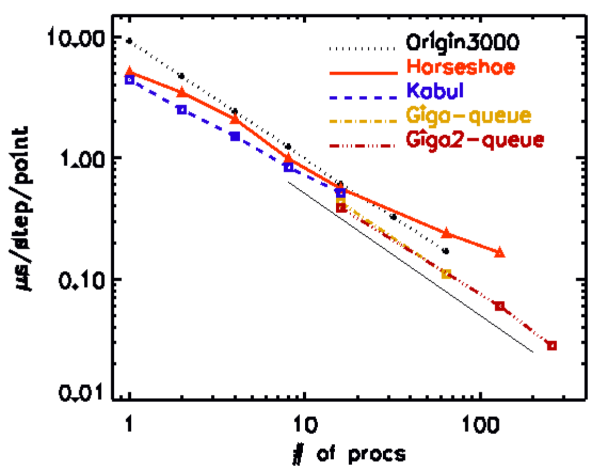
Christensson et al.
(2001, PRE 64, 056405)

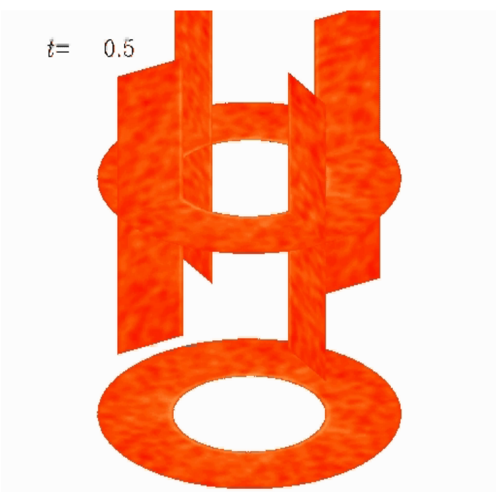
Pencil Code

- Started in Sept. 2001 with Wolfgang Dobler
- High order (6th order in space, 3rd order in time)
- Cache & memory efficient
- MPI, can run PacxMPI (across countries!)
- Maintained/developed by ~20 people (CVS!)
- Automatic validation (over night or any time)
- Max resolution so far 1024^3 , 256 procs

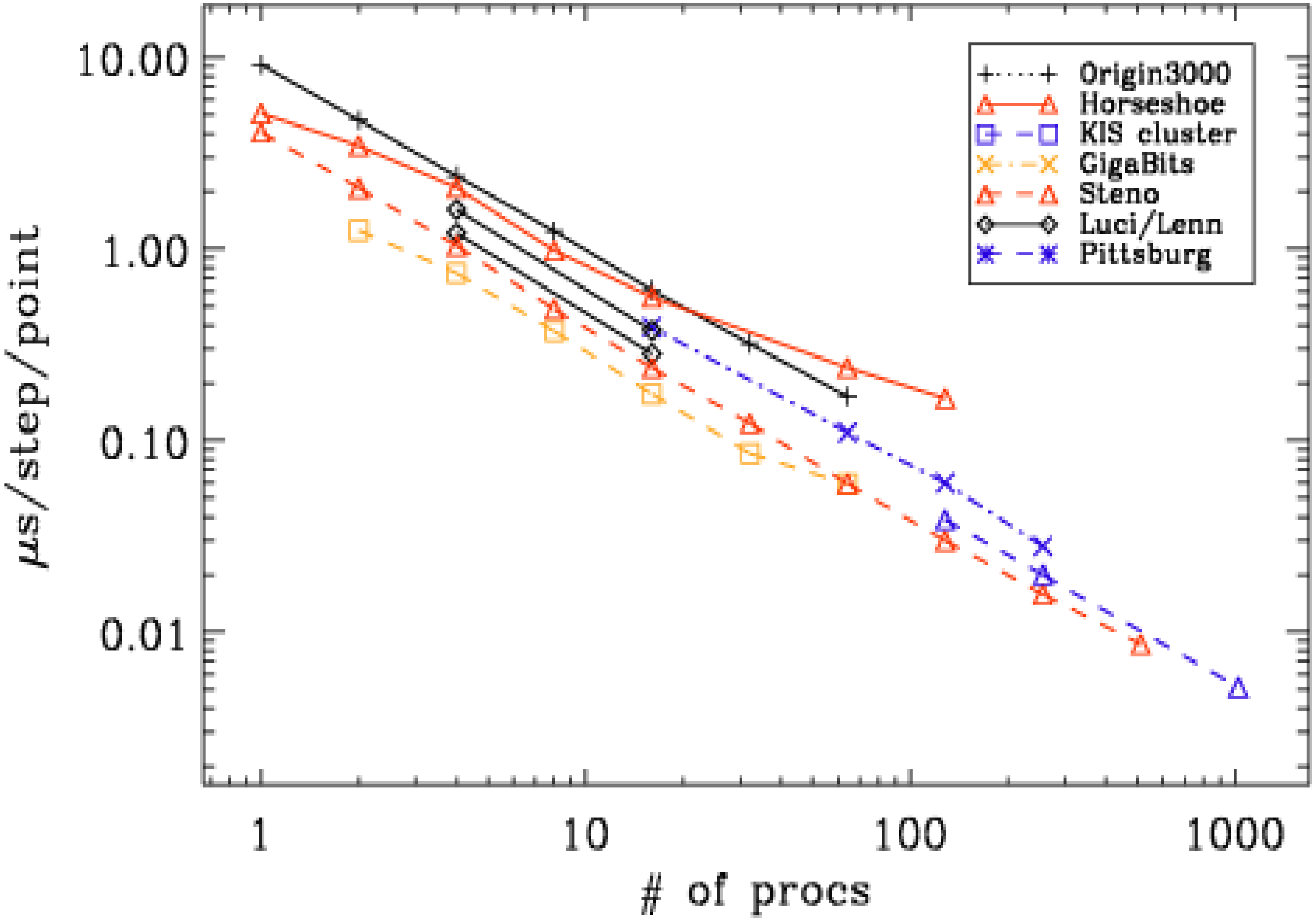


- Isotropic turbulence
  - MHD, passive scl, CR
- Stratified layers
  - Convection, radiation (gray)
- Shearing box
  - MRI, dust, interstellar
- Sphere embedded in box
  - Fully convective stars
  - geodynamo
- Other applications
  - Homochirality
  - Spherical coordinates





Wallclock time versus processor



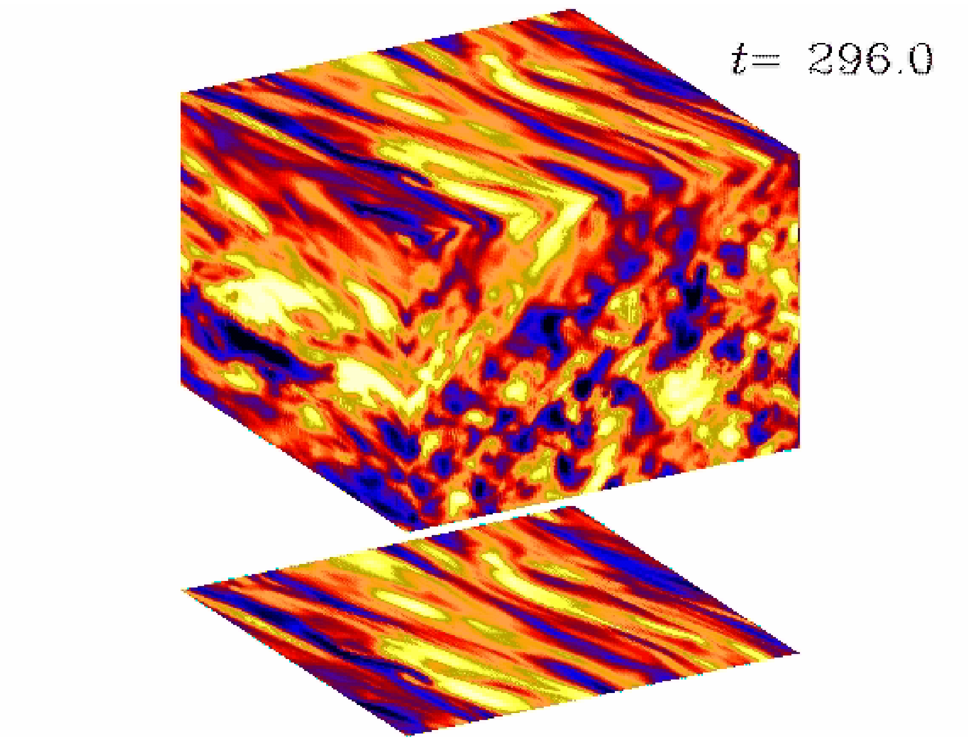
nearly linear
Scaling

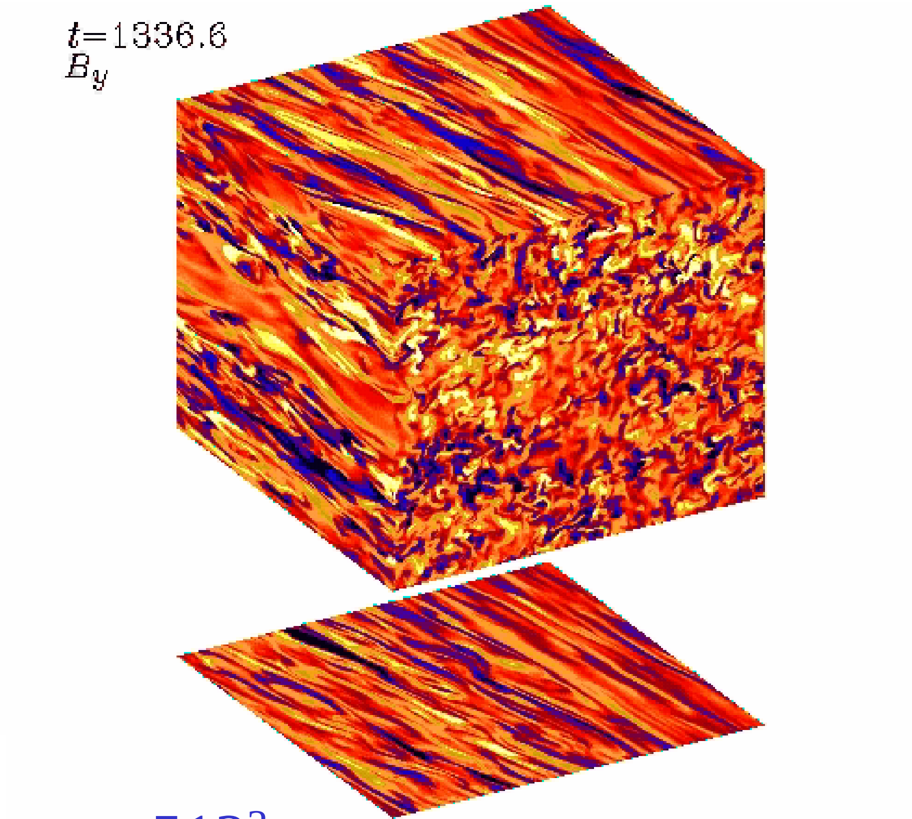
100 Mb/s shows
limitations

1 - 10 Gb/s
no limitation

MRI turbulence

MRI = magnetorotational instability

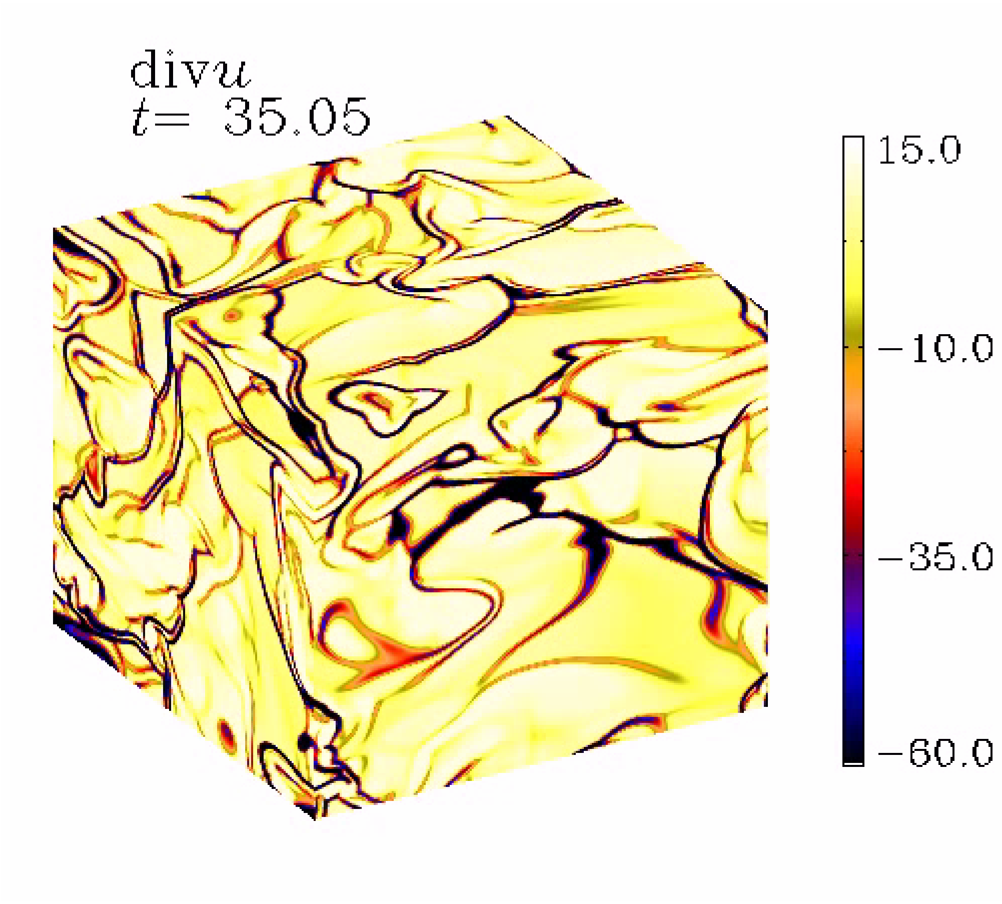




256^3
w/o hypervisc.
t = 600 = 20 orbits

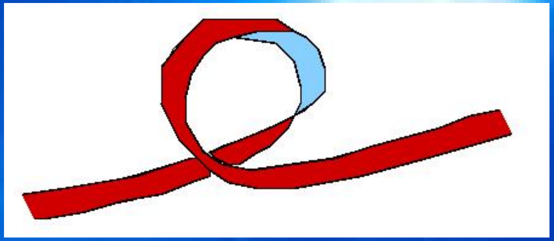
512^3
w/o hypervisc.
Δt = 60 = 2 orbits

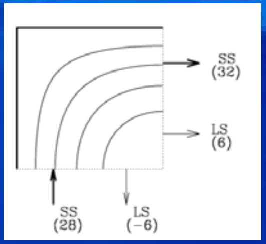
Ma=10 supersonic turbulence



Conclusions

- Vector potential useful for MHD simulations
- No dynamo action in nearly potential flow
- Small scale dynamo cares about solenoidal part of the flow, not the potential part
- Bottleneck effect is real, and it affects the small-scale dynamo





10^{46} Mx^2/cycle
(for the sun)