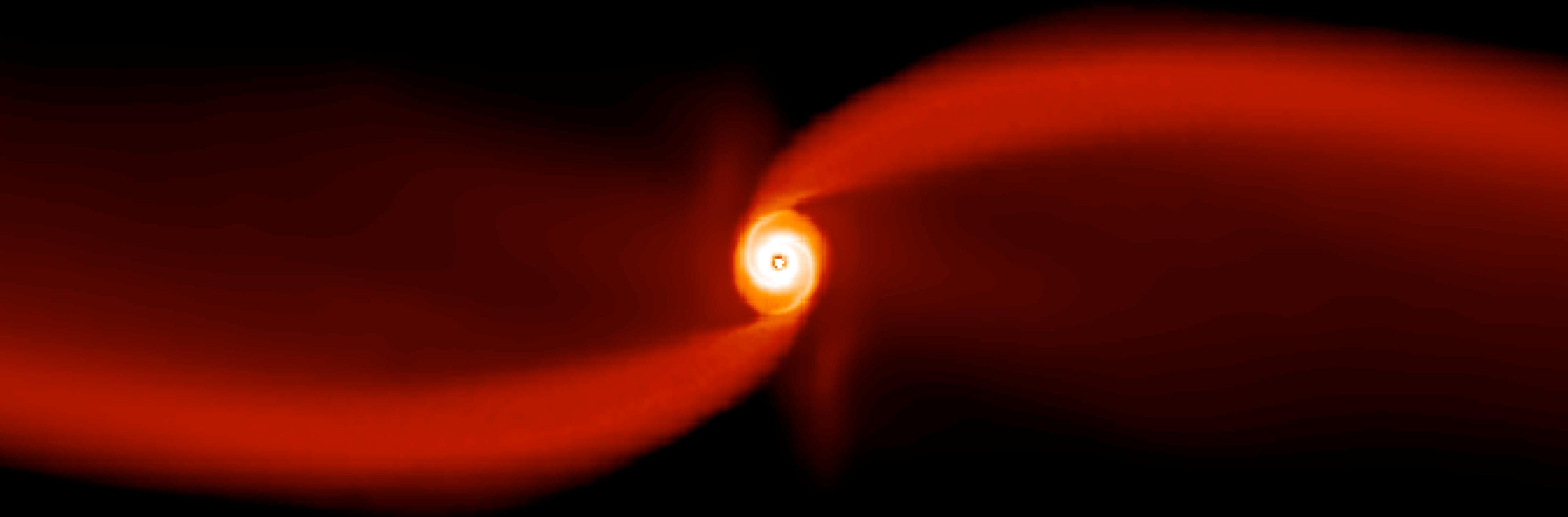
[issues with] MHD in SPH



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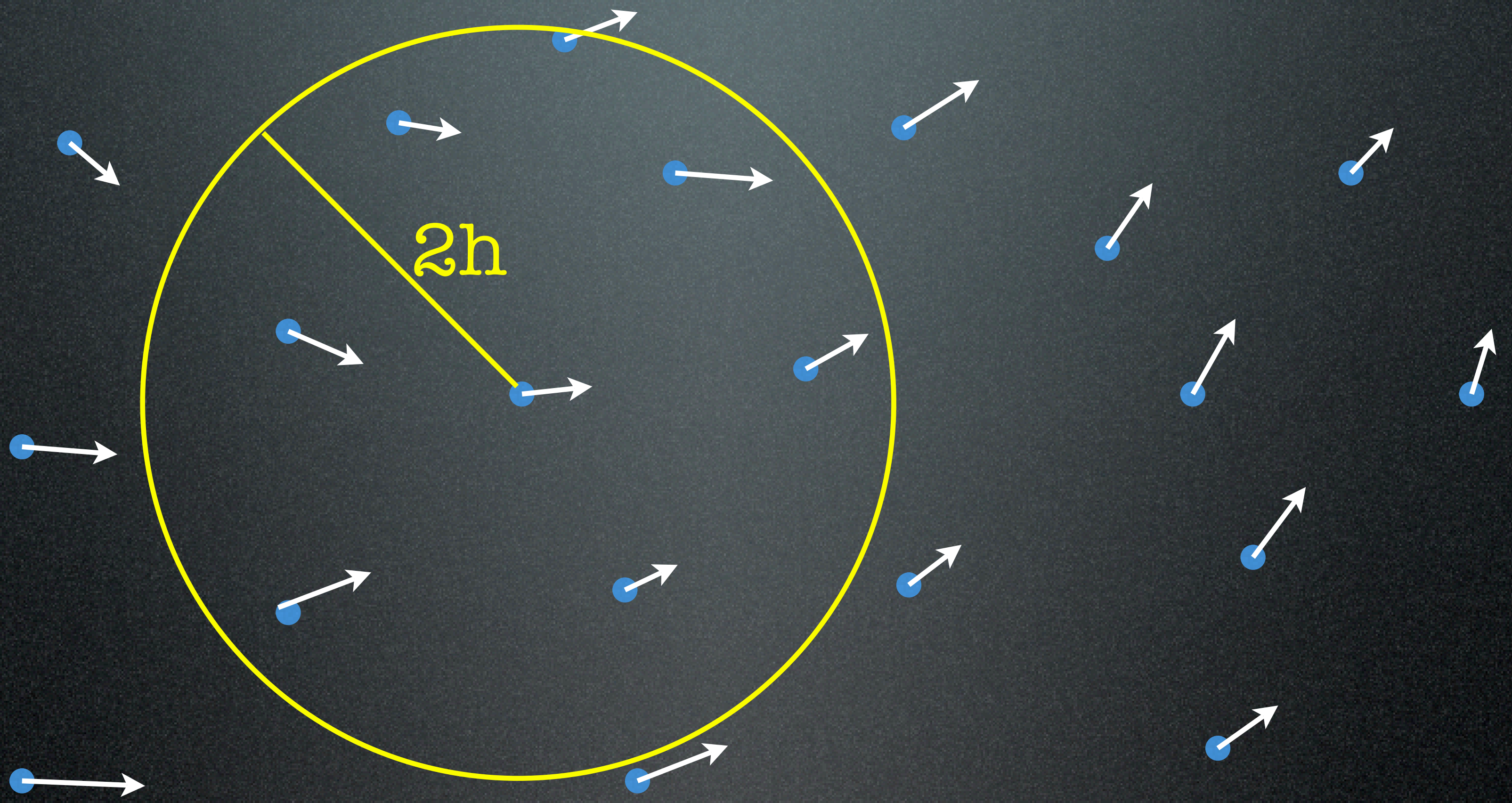
KITP 27/11/07

Smoothed Particle Hydrodynamics

Lucy (1977), Gingold & Monaghan (1977), Monaghan (1992), Price (2004), Monaghan (2005)



$$\rho(\mathbf{r}) = \sum_{j=1}^{N} m_j W(|\mathbf{r} - \mathbf{r}_j|, h)$$

$$L_{sph} = \sum_j m_j \left[\frac{1}{2} v_j^2 - u_j(\rho_j, s_j) \right]$$

Lagrangian

+

$$du = \frac{P}{\rho^2} d\rho$$

1st law of thermodynamics

+

$$\nabla \rho_i = \sum_j m_j \nabla W_{ij}(h)$$

density sum

+

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \mathbf{v}}\right) - \frac{\partial L}{\partial \mathbf{r}} = 0$$

Euler-Lagrange equations

=

$$\frac{d\mathbf{v}_i}{dt} = -\sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} \right) \nabla_i W_{ij}(h)$$

equations of motion!

$$\left(\frac{d\mathbf{v}}{dt} = -\frac{\nabla P}{\rho} \right)$$

Smoothed Particle Magnetohydrodynamics

Price & Monaghan (2004a,b, 2005)

$$L_{sph} = \sum_b m_b \left[\frac{1}{2} v_b^2 - u_b(\rho_b, s_b) - \frac{1}{2\mu_0} \frac{B_b^2}{\rho_b} \right]$$

$$\int \delta L \mathrm{dt} = 0$$

$$\delta \rho_b = \sum_c m_c \left(\delta \mathbf{r}_b - \delta \mathbf{r}_c \right) \cdot \nabla_b W_{bc},$$

continuity equation

$$\delta \left(\frac{\mathbf{B}_b}{\rho_b} \right) = -\sum_c m_c (\delta \mathbf{r}_b - \delta \mathbf{r}_c) \frac{\mathbf{B}_b}{\rho_b^2} \cdot \nabla_b W_{bc}$$

mag field evolution

$$\frac{dv_a^i}{dt} = \sum_b m_b \left[\left(\frac{S^{ij}}{\rho^2} \right)_a + \left(\frac{S^{ij}}{\rho^2} \right)_b \right] \nabla_a^j W_{ab},$$

equations of motion

$$S_a^{ij} = -\left(P_a + \frac{1}{2\mu_0} B_a^2 \right) \delta^{ij} + \frac{1}{\mu_0} \left(B_a^i B_a^j \right),$$

Technical issues

1) Momentum conserving force is unstable → use force which vanishes for constant stress

$$
\frac{dv^i}{dt} = -\sum_b m_b \left(\frac{P_a + \frac{1}{2}B_a^2/\mu_0}{\rho_a^2} + \frac{P_b + \frac{1}{2}B_b^2/\mu_0}{\rho_b^2} \right) \frac{\partial W_{ab}}{\partial x^i} + \frac{1}{\mu_0}\sum_b m_b \frac{(B_i B_j)_b - (B_i B_j)_a}{\rho_a \rho_b} \frac{\partial W_{ab}}{\partial x_j}.
$$

(Morris 1996)

2) Shocks → formulate artificial dissipation terms (PM04a)

$$
\left(\frac{d\mathbf{v}}{dt}\right)_{diss} = -\sum_b m_b \frac{\alpha v_{sig}(\mathbf{v}_a - \mathbf{v}_b)\cdot\hat{r}}{\bar{\rho}_{ab}} \nabla_a W_{ab},
$$

$$
\left(\frac{d\mathbf{B}}{dt}\right)_{diss} = \rho_a \sum_b m_b \frac{\alpha_B v_{sig}}{\bar{\rho}_{ab}^2} (\mathbf{B}_a - \mathbf{B}_b)\,\hat{r}\cdot\nabla_a W_{ab}
$$

$$
\left(\frac{de_a}{dt}\right)_{diss} = -\sum_b m_b \frac{v_{sig}(e_a^* - e_b^*)}{\bar{\rho}_{ab}} \hat{r}\cdot\nabla_a W_{ab}
$$

3) Variable h → use Lagrangian (Price & Monaghan 2004b)

4) The $\nabla \cdot B = 0$ constraint

- prevention vs cleanup (Price & Monaghan 2005)
- Euler potentials:

Euler (1770), Stern (1976),
Phillips & Monaghan (1985)
Price & Bate (2007), Rosswog & Price (2007)

$$\mathbf{B} = \nabla\alpha \times \nabla\beta$$

use accurate SPH derivatives (Price 2004)

$$\chi_{\mu\nu}\nabla^{\mu}\alpha_i = -\sum_j m_j(\alpha_i - \alpha_j)\nabla_i^{\nu} W_{ij}(h_i)$$

$$\chi_{\mu\nu} = \sum_j m_j (r_i^{\mu} - r_j^{\mu})\nabla^{\nu} W_{ij}(h_i).$$

$$\frac{d\alpha}{dt} = 0, \frac{d\beta}{dt} = 0$$

'advection of magnetic field lines'

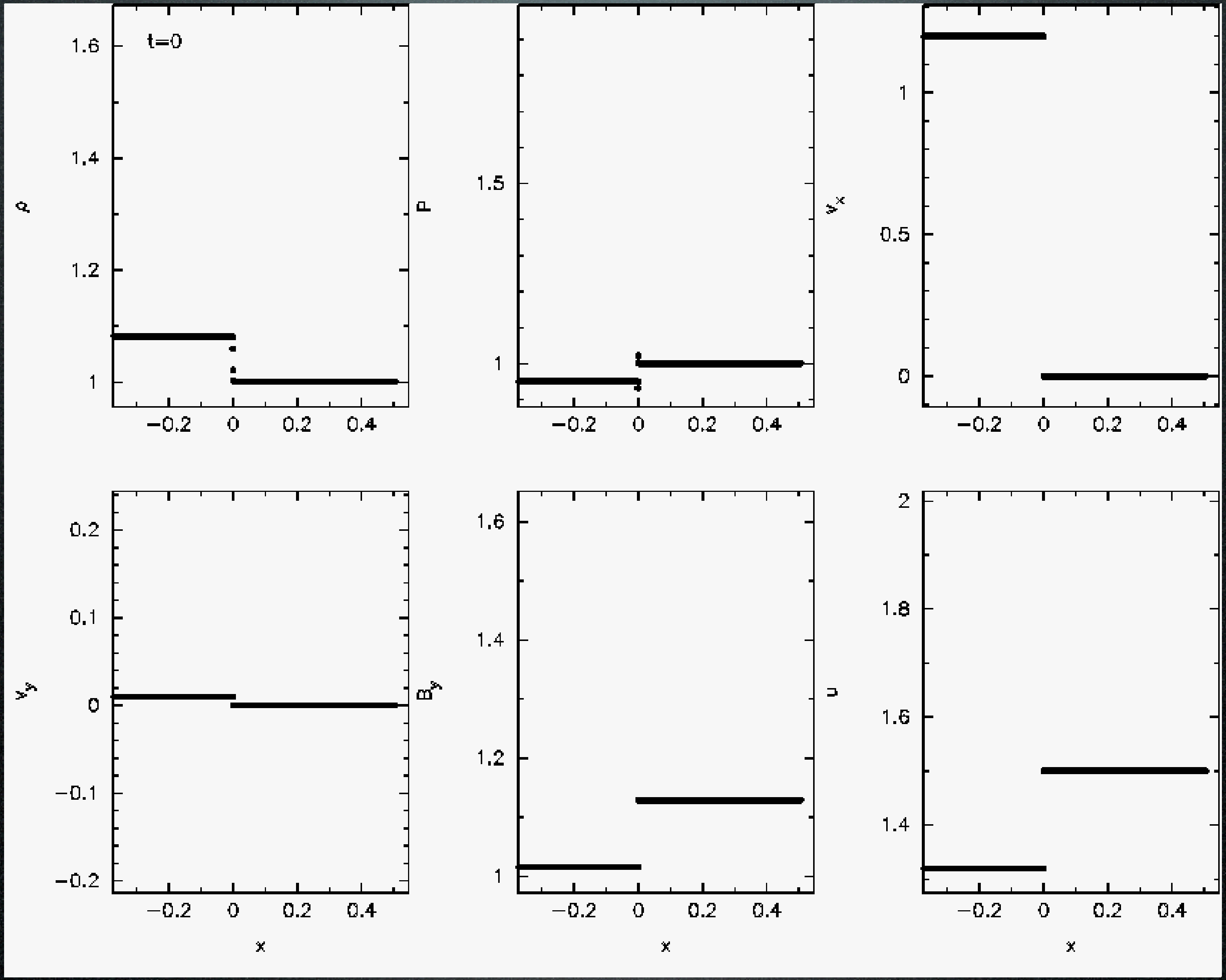
add shock dissipation

$$\frac{d\alpha}{dt} = \sum_b m_b \frac{\alpha_B v_{sig}}{\bar{\rho}_{ab}} (\alpha_a - \alpha_b)\, \hat{r}\cdot\nabla_a W_{ab}$$

$$\frac{d\beta}{dt} = \sum_b m_b \frac{\alpha_B v_{sig}}{\bar{\rho}_{ab}} (\beta_a - \beta_b)\, \hat{r}\cdot\nabla_a W_{ab}$$

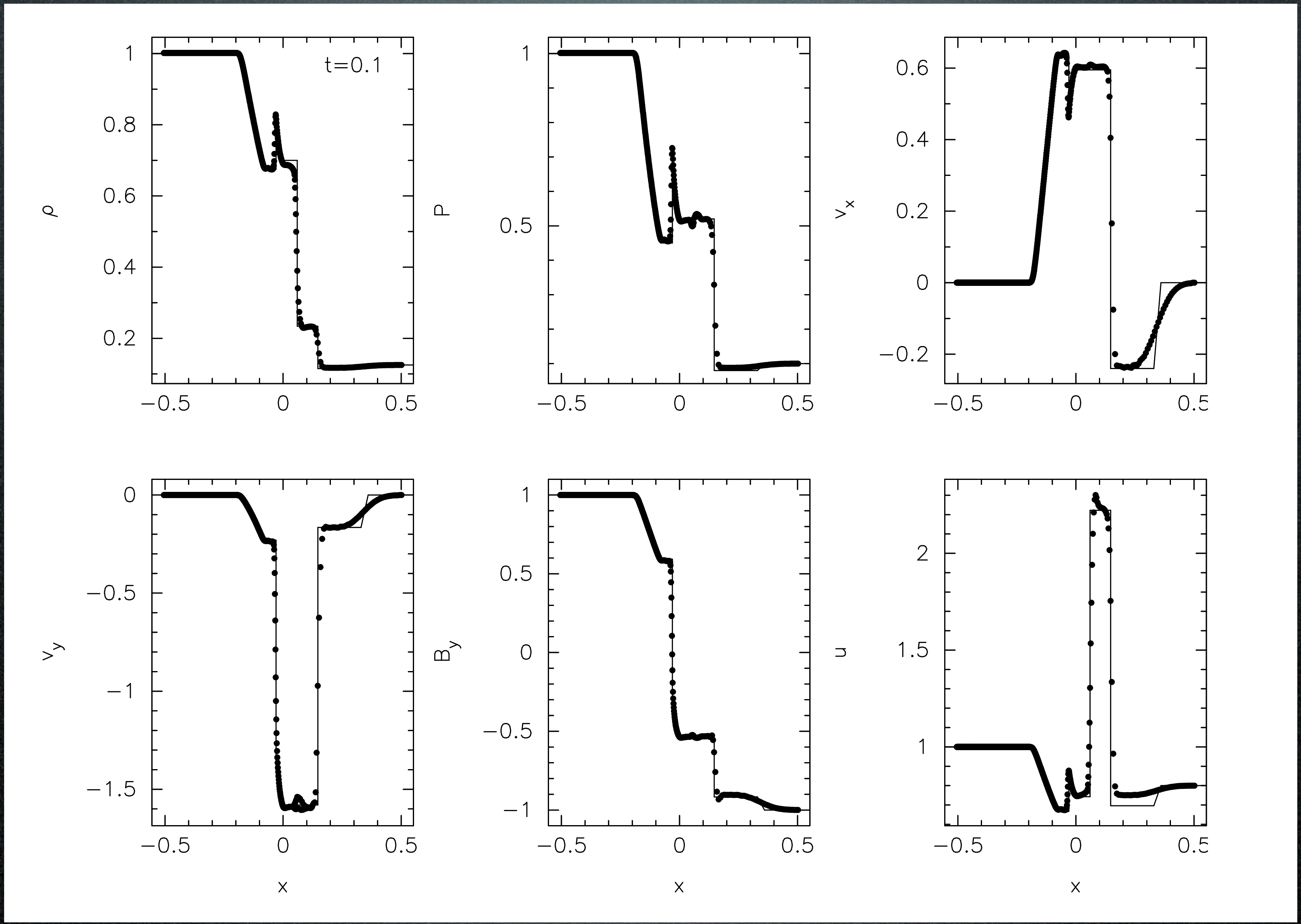
BUT: helicity constraints (A.B = const): cannot represent certain fields, no dynamo action.
Field growth suppressed once clear mapping from initial to final particle distribution is lost

Test problems: 1D shocks



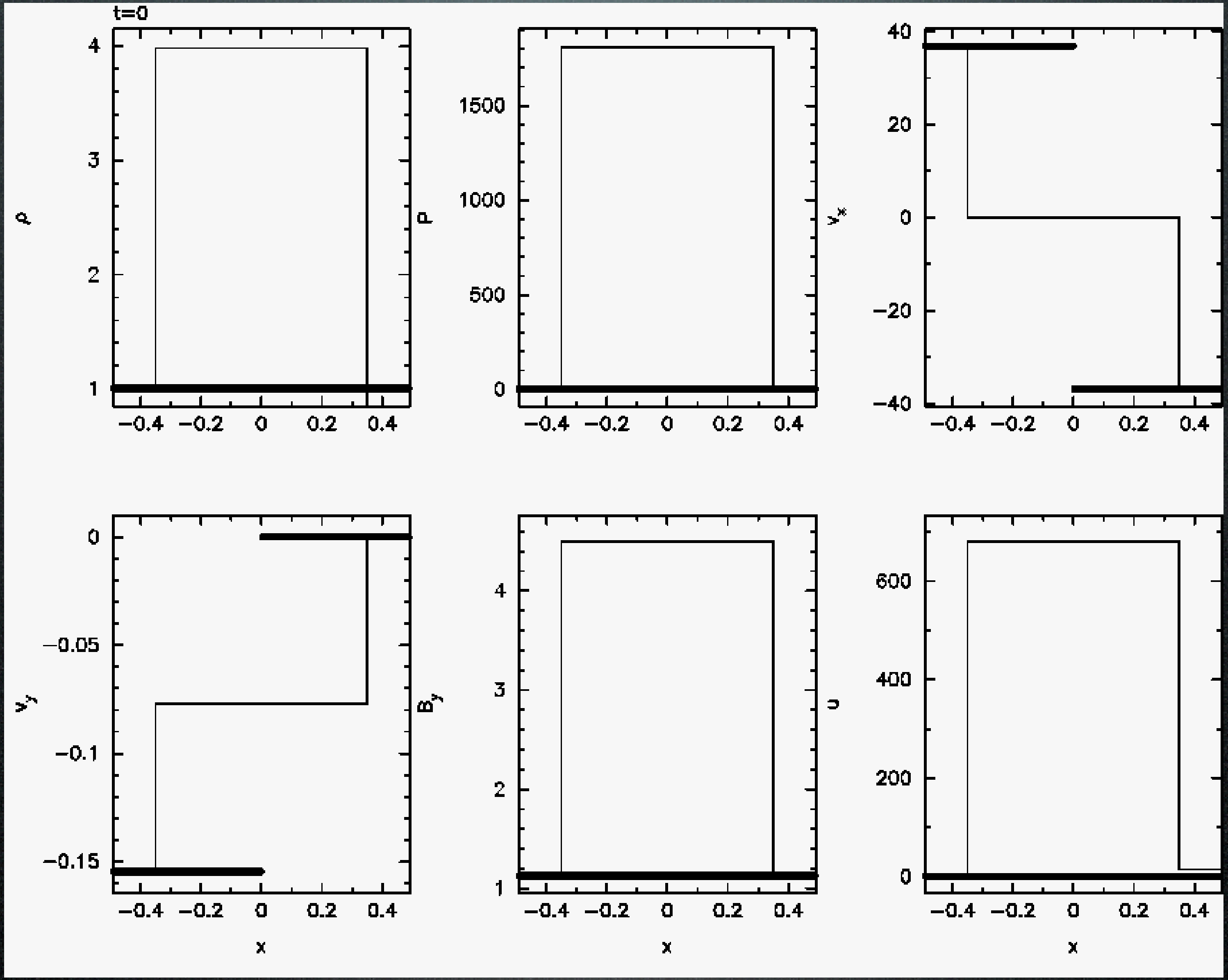
(Price & Monaghan 2004a,b, Price 2004)

Brio/Wu



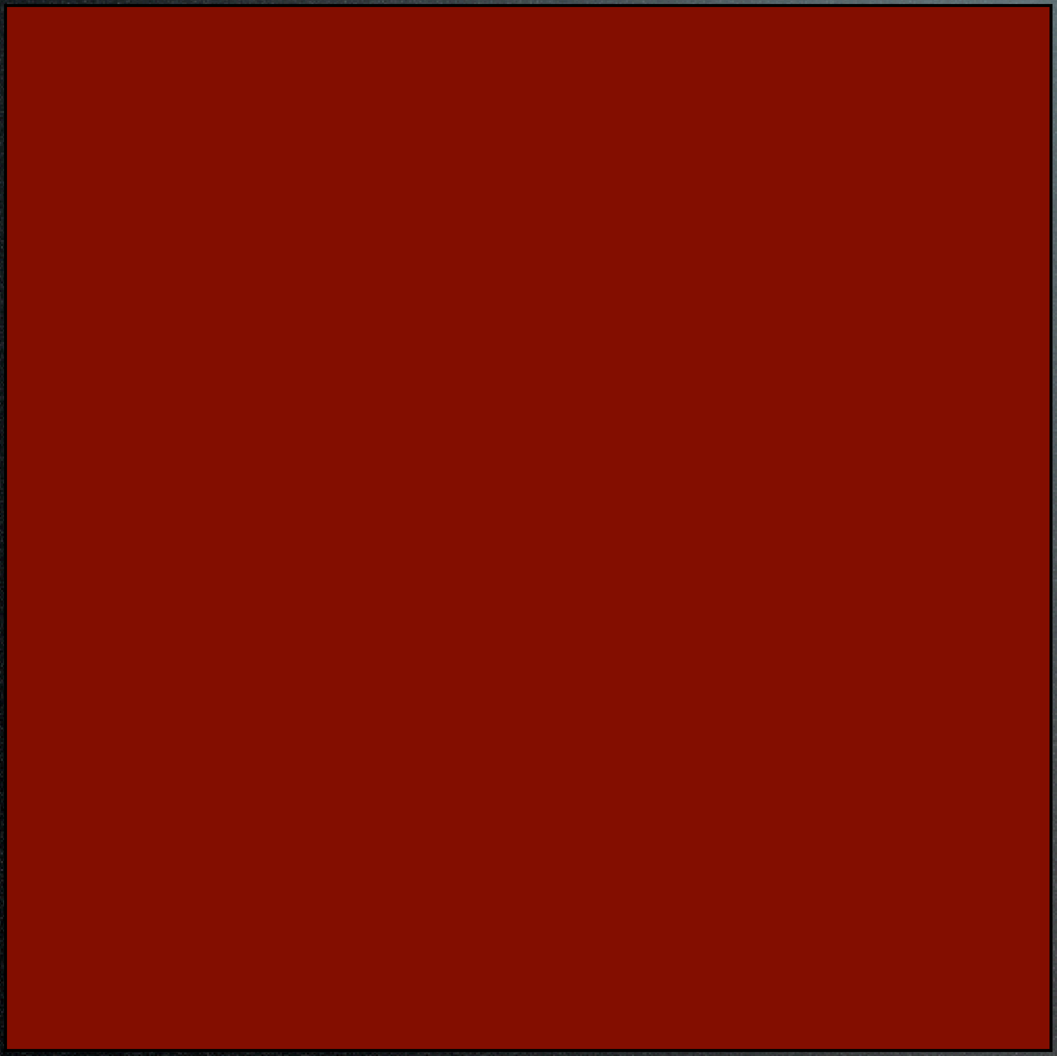
(Price & Monaghan 2004a,b, Rosswog & Price 2007)

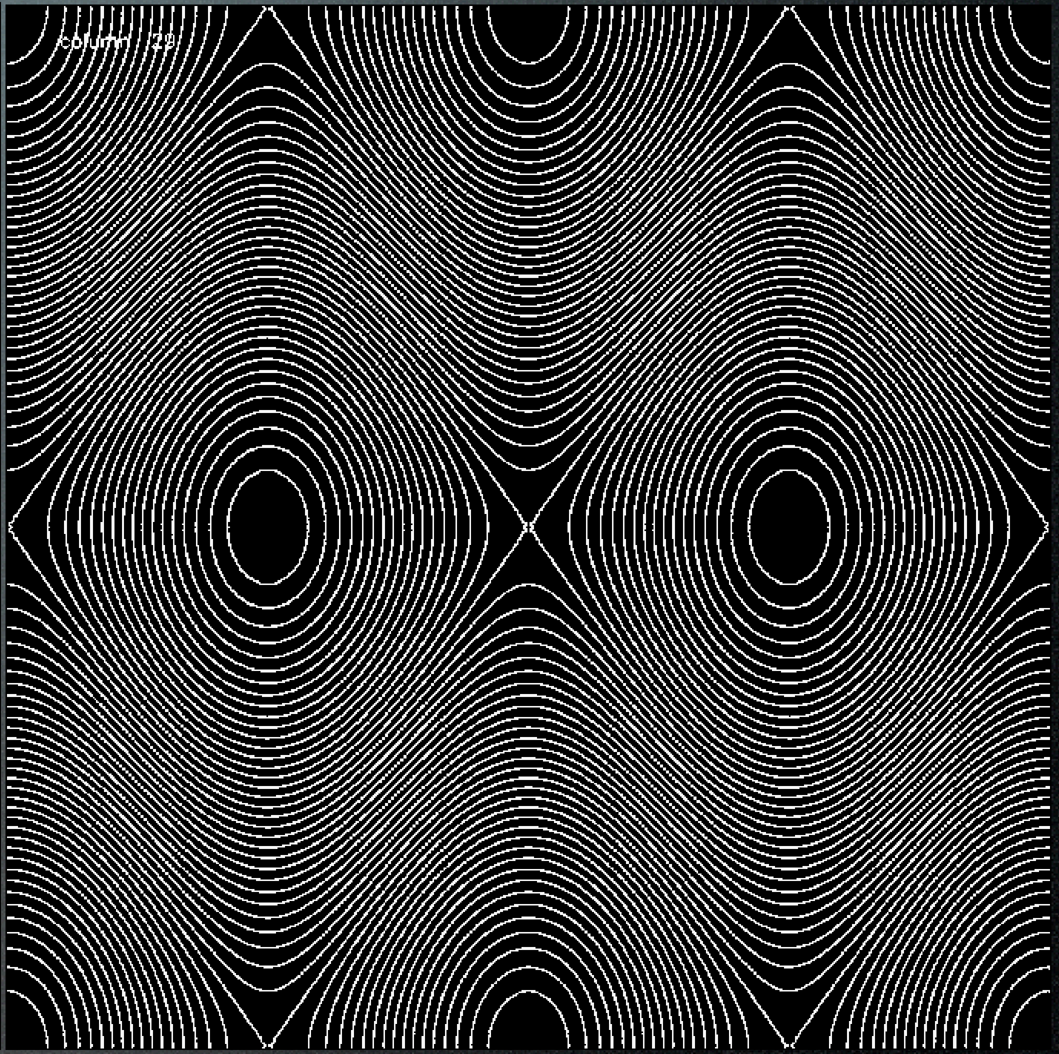
Mach 25 shock



(Price & Monaghan 2004a,b, Price 2004)

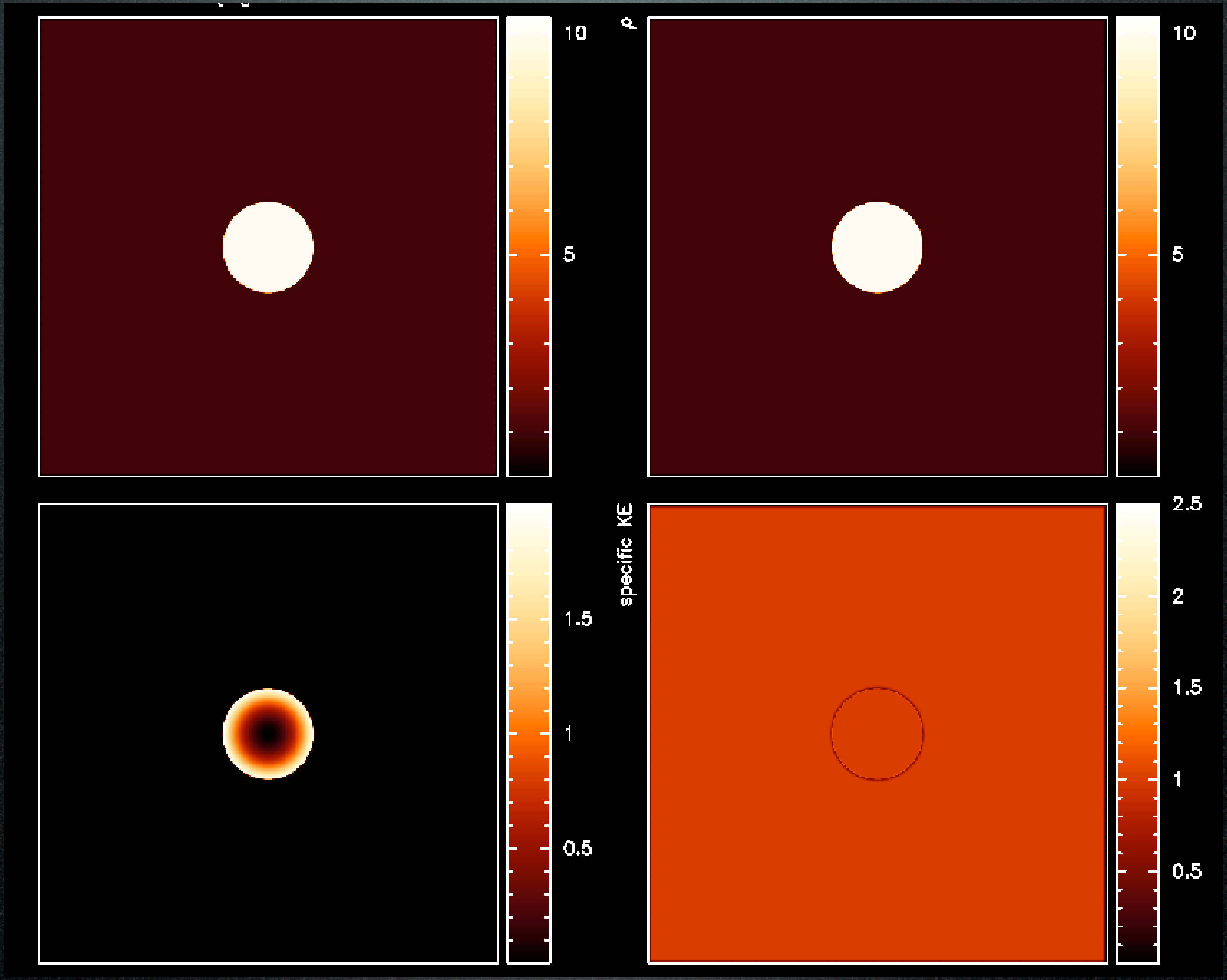
2D Orszag-Tang Vortex





(Price & Monaghan 2005, Rosswog & Price 2007)

Magnetised rotor problem



(Price & Monaghan 2005)

Current loop advection

(Gardiner & Stone 2005, JCP 205, 509)

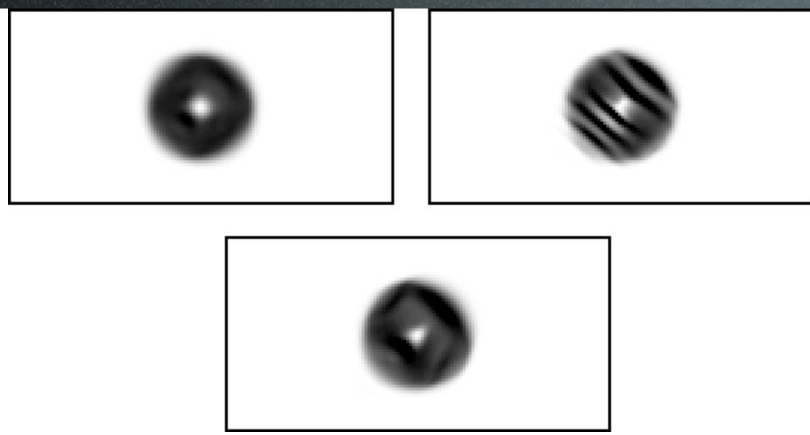


Fig. 3. Gray-scale images of the magnetic pressure $(B_x^2 + B_y^2)$ at t = 2 for an advected field loop ($v_0 = \sqrt{5}$) using the $\mathcal{E}_z^a$ (top left), $\mathcal{E}_z^{\circ}$ (top right) and $\mathcal{E}^c$ (bottom) CT algorithm.

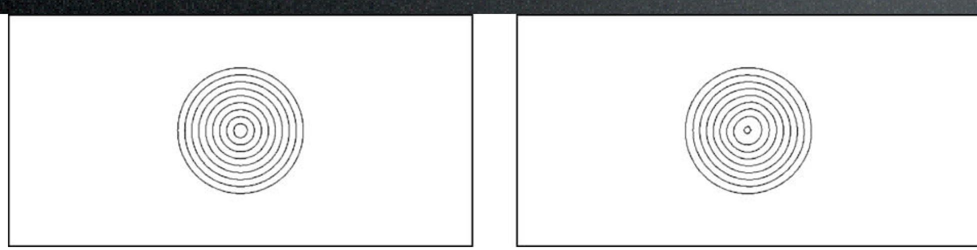
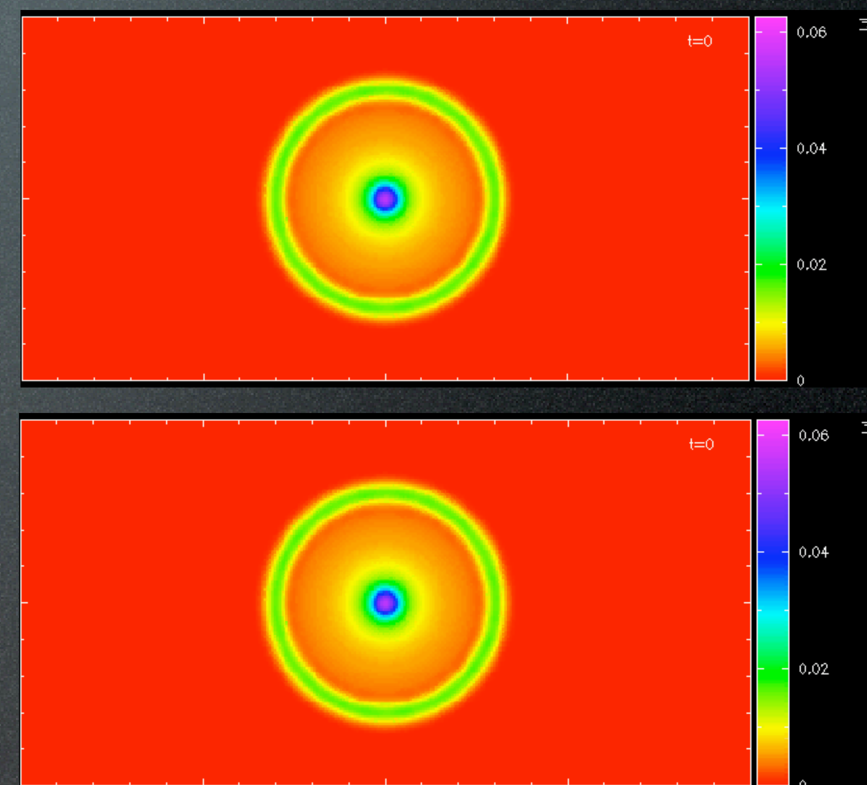


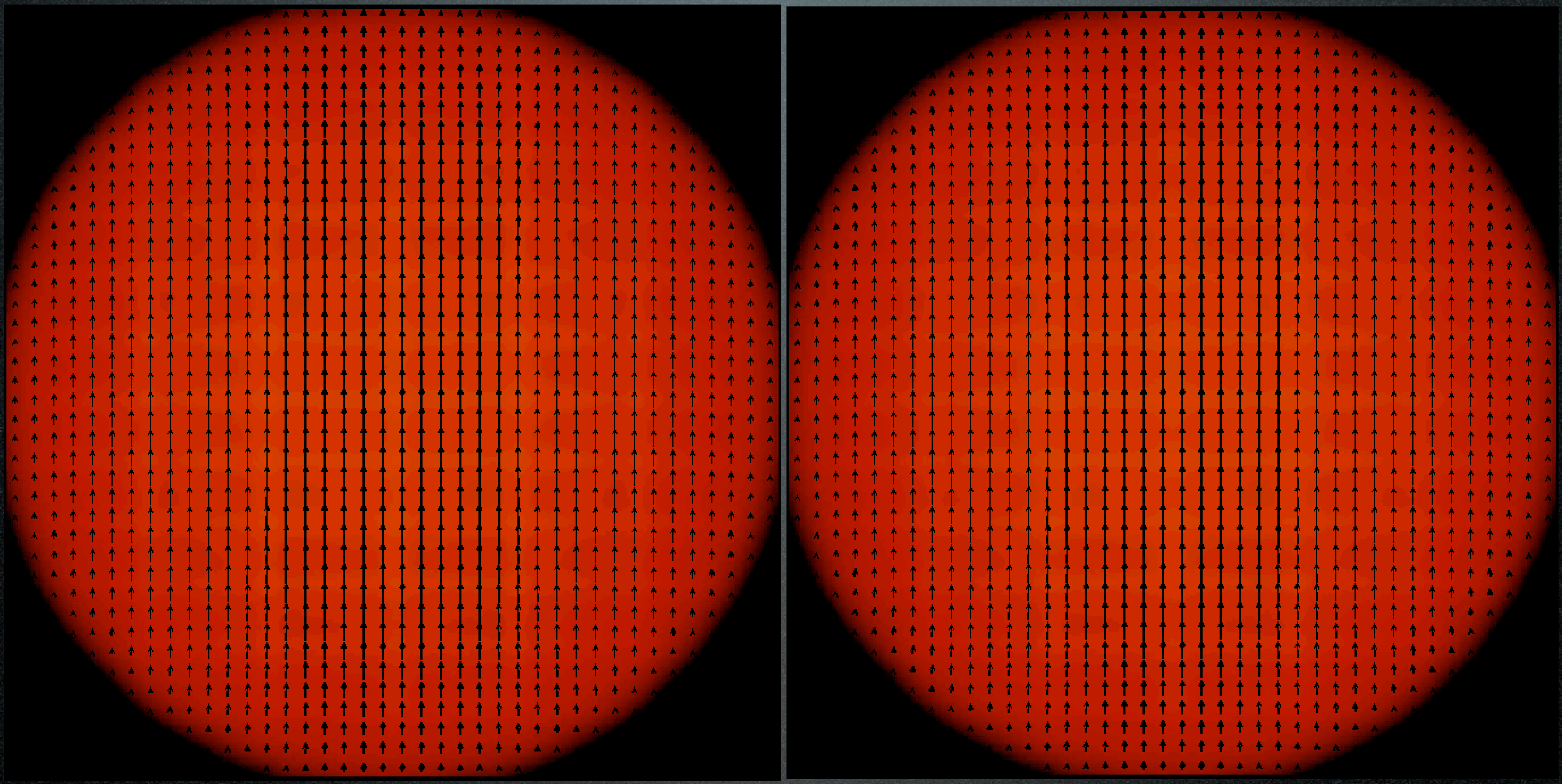
Fig. 8. Magnetic field lines at t = 0 (left) and t = 2 (right) using the CTU + CT integration algorithm.



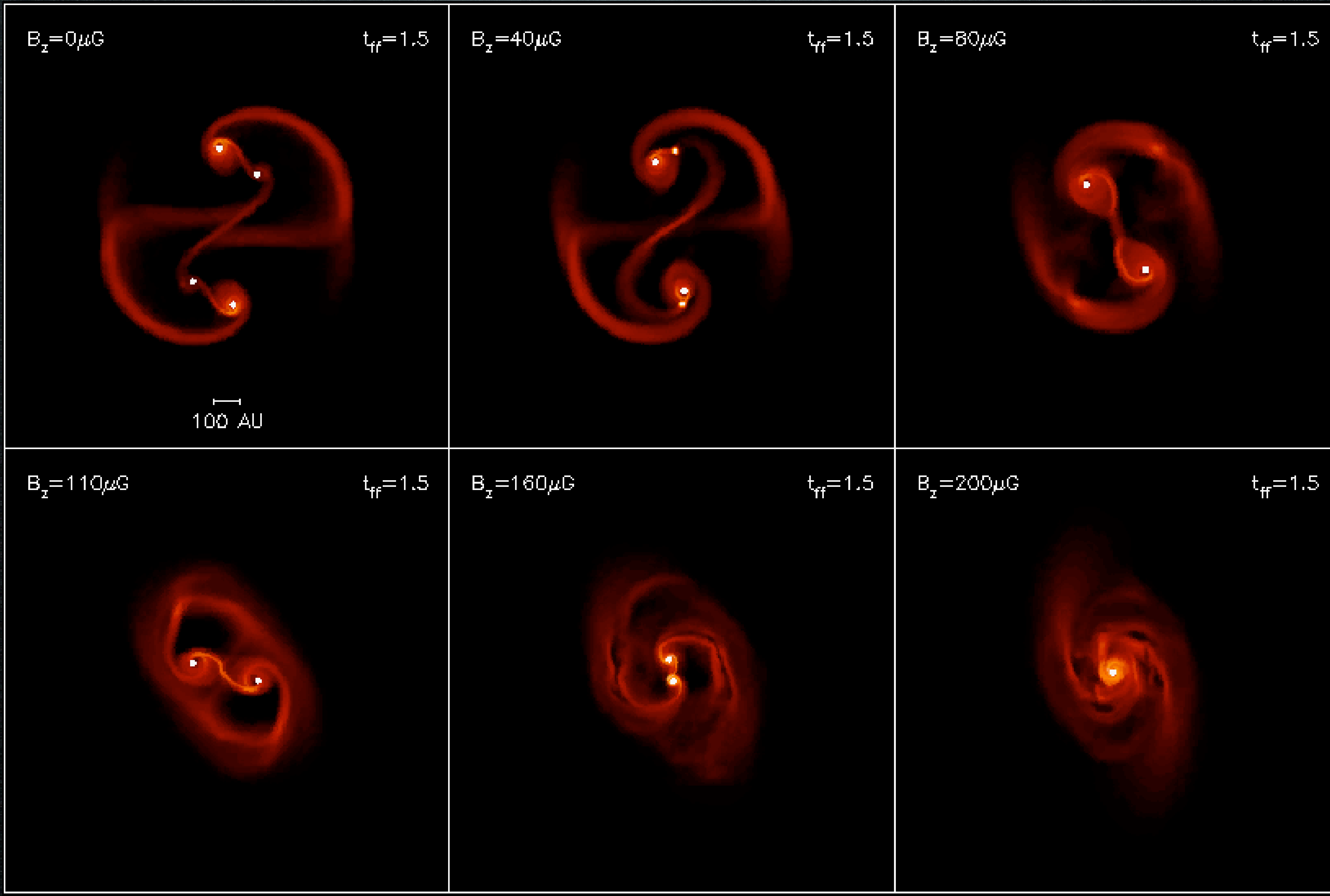
(Gardiner & Stone 2005)

(Rosswog & Price 2007)

Star formation



Effect on binary formation (B_z field):



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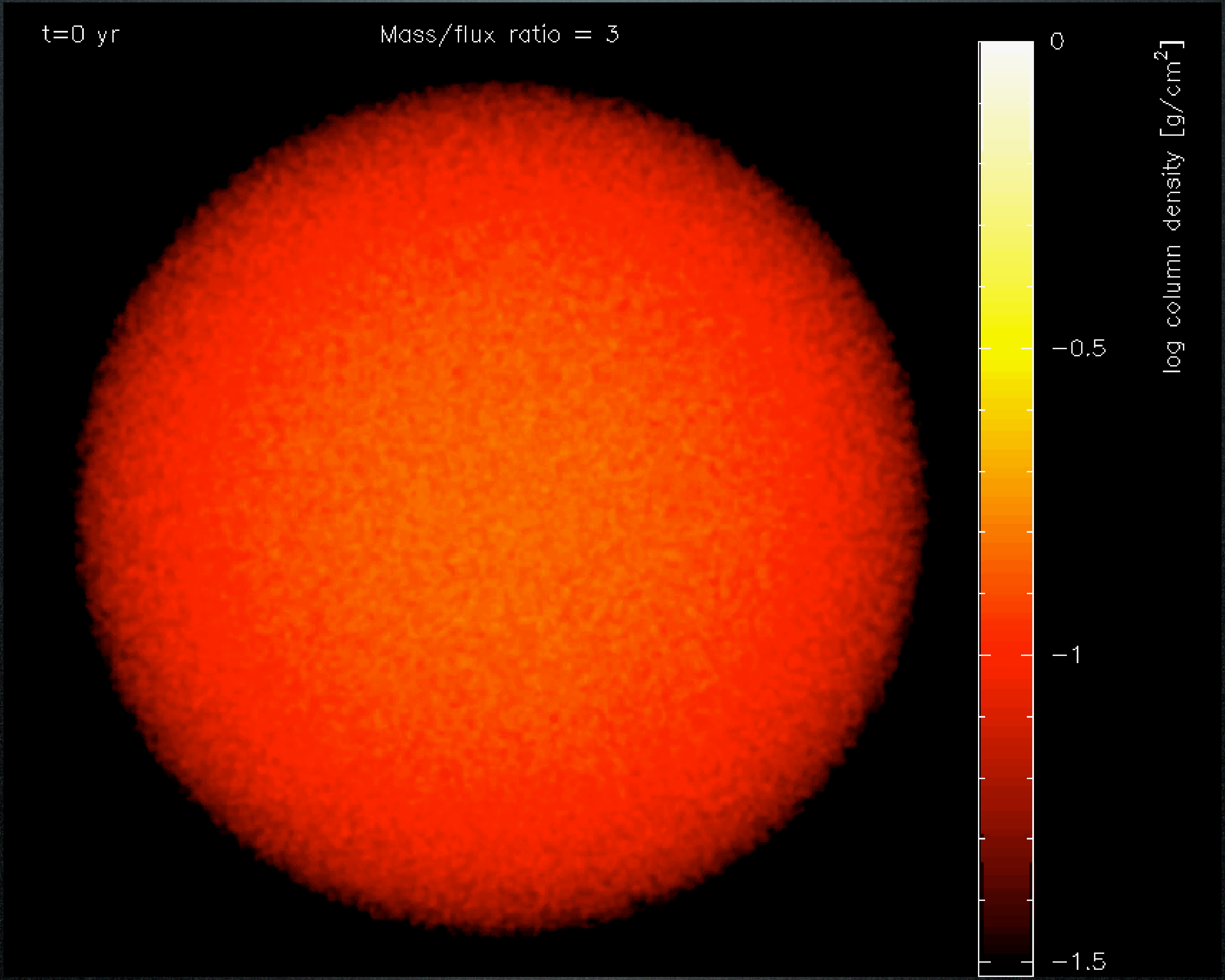
t=0 yr
Mass/flux ratio = 3
0
−0.5
−1
−1.5
log column density [g/cm^2]

Theoretical questions

1) What is the effect of magnetic fields on fragmentation?
suppress or enhance? effect on IMF?

2) Role of magnetic pressure vs magnetic tension? super/sub critical, super/sub Alfvenic, beta < 1 or beta > 1?

3) Magnetic fields -> outflow connection? what are the necessary ingredients for outflow production?

4) What are the effects of magnetic fields on the star formation rate/efficiency? support of low density regions, suppression of accretion, generation of outflows

5) What are the important numerical issues to get right?
resolution criterion for MHD?

6) Importance of non-ideal effects? ambipolar...hall...resistive. In what order?