Properties of Supersonic Turbulence: Does the forcing make a difference?

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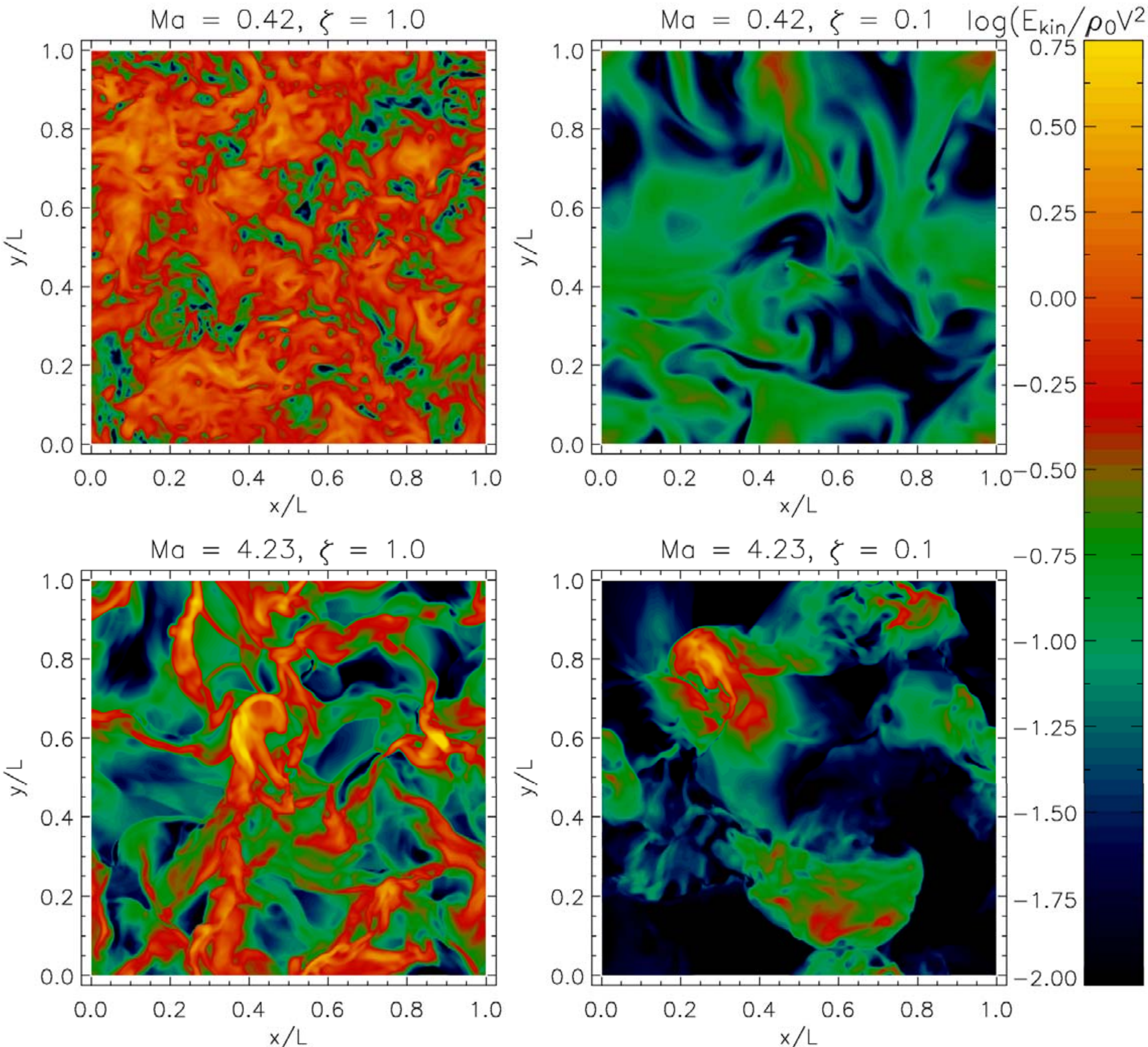
Stochastic forcing

- Representation in Fourier space
- Gaussian random increments
- Given magnitude and autocorrelation time
- Projection of Fourier modes
  - perpendicular to wave vector for solenoidal force
  - in longitudinal direction for dilatational force
- Inverse FFT yields physical force field

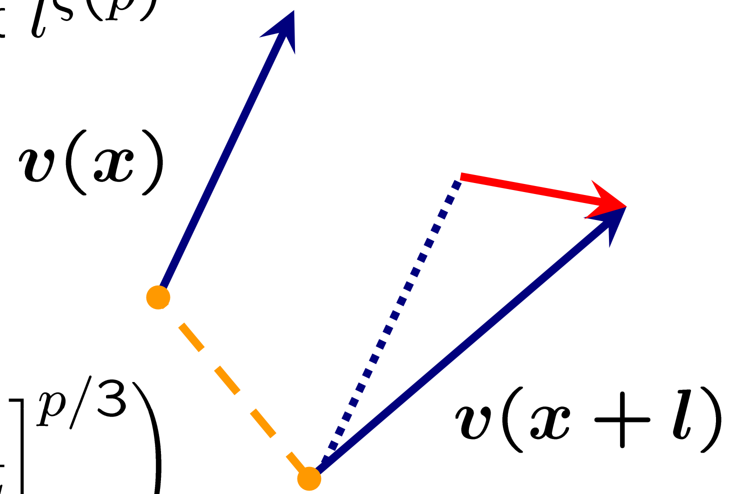
LES of isothermal turbulence, 256^3 (WS & Federrath `07)



Structure functions

- Correlation of velocties at points separated by a certain length:

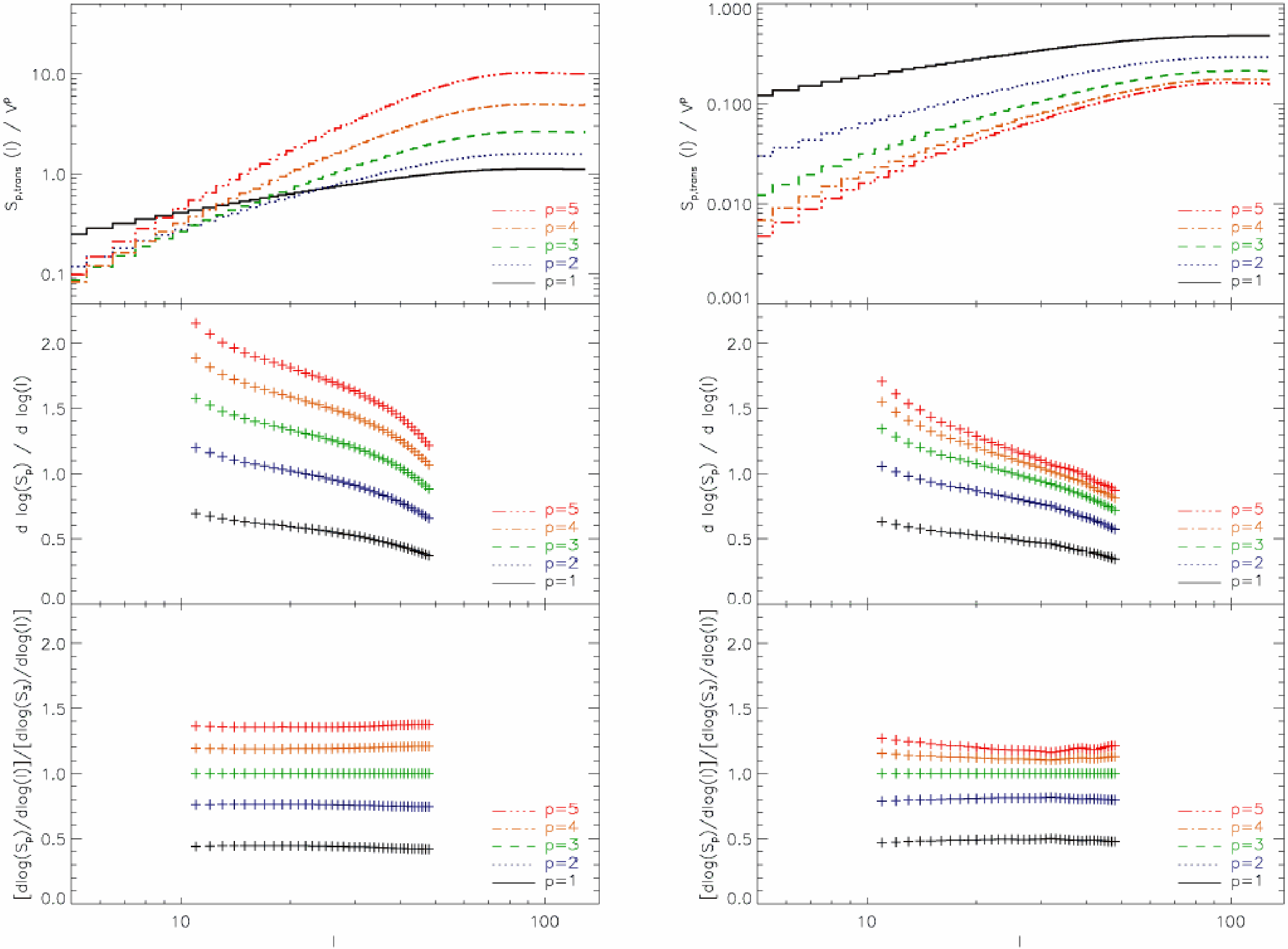
$$S_p(l) = \langle |\boldsymbol{v}(\boldsymbol{x}+\boldsymbol{l}) - \boldsymbol{v}(\boldsymbol{x})|^p \propto l^{\zeta(p)}$$



  - Kolmogorov: $\zeta(p) = \frac{p}{3}$
  - She-Lévêque:

$$\zeta(p) = \frac{p}{9} + C\left(1 - \left[1 - \frac{2}{3C}\right]^{p/3}\right)$$

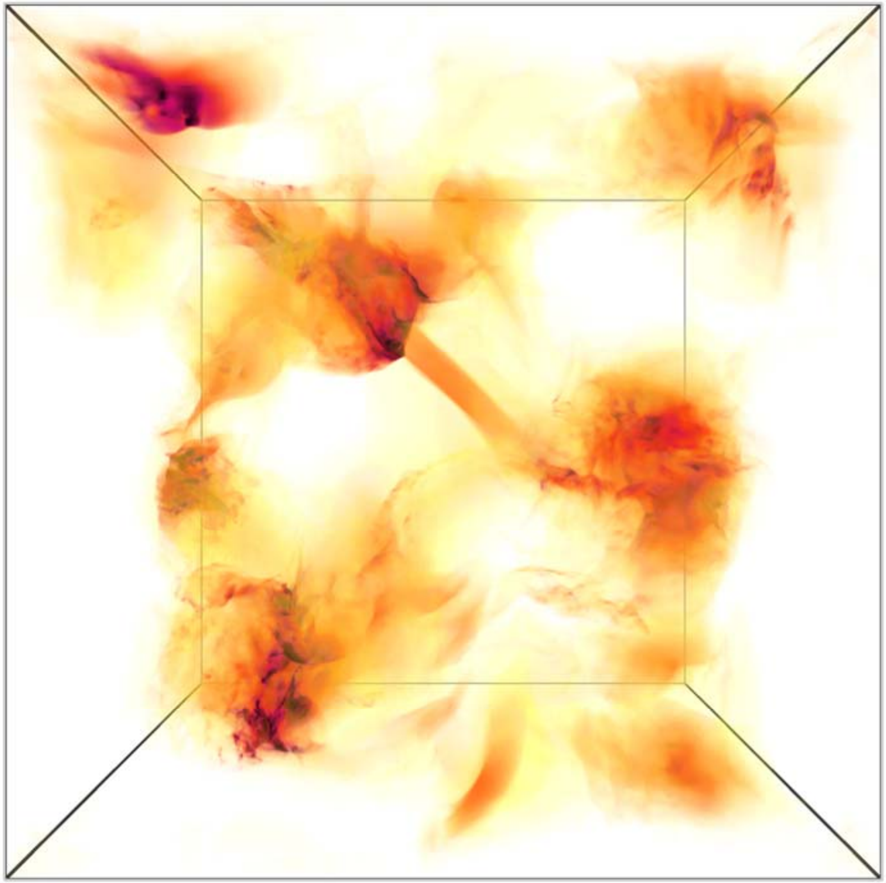
LES of isothermal turbulence, 256^3 (Federrath & WS `07)



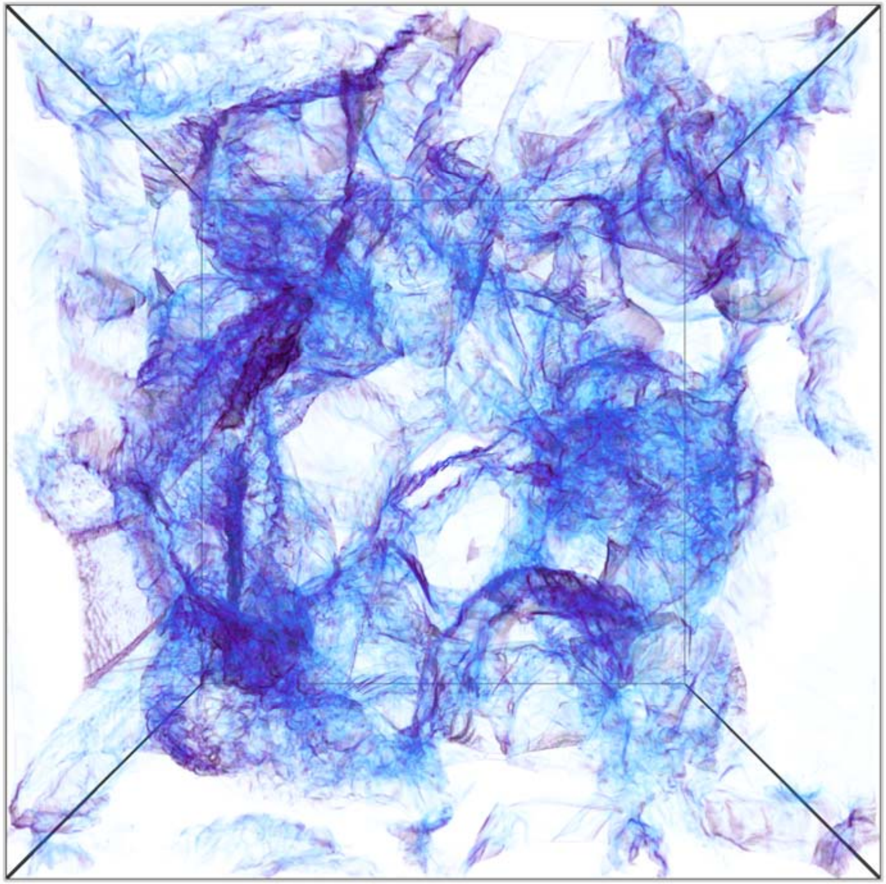
(a) $\zeta = 1.0$

(b) $\zeta = 0.1$

AMR simulation of isothermal turbulence, $\zeta = 0.1$, effective 768^3 (WS et al. `07)

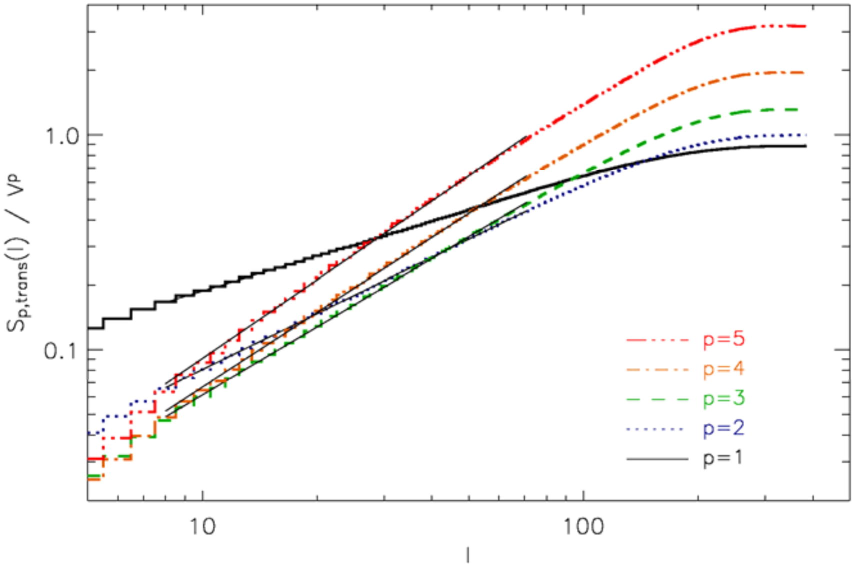


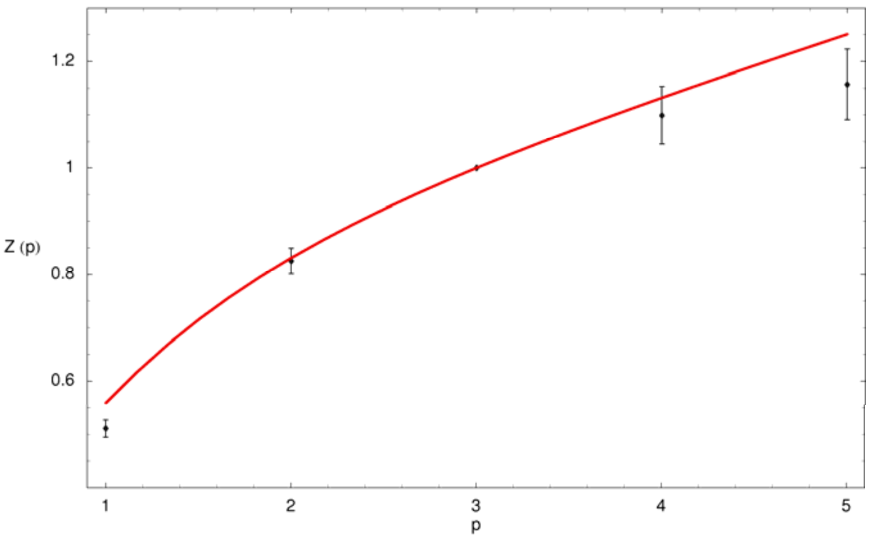
mass density



vorticity

Simulation of isothermal turbulence, ζ = 0.1, 768^3 (Federrath et al. `07)





p		SL94	B02	BNP02	B02g	this work
		(C = 2)	(C = 1)		(C = 0.7)	
1	0.33	0.36	0.42	0.42	0.56	0.51 ± 0.02
2	0.67	0.70	0.74	0.74	0.83	0.83 ± 0.02
3	1.00	1.00	1.00	1.00	1.00	1.00
4	1.33	1.28	1.21	1.20	1.13	1.10 ± 0.05
5	1.67	1.54	1.40	1.38	1.25	1.16 ± 0.07

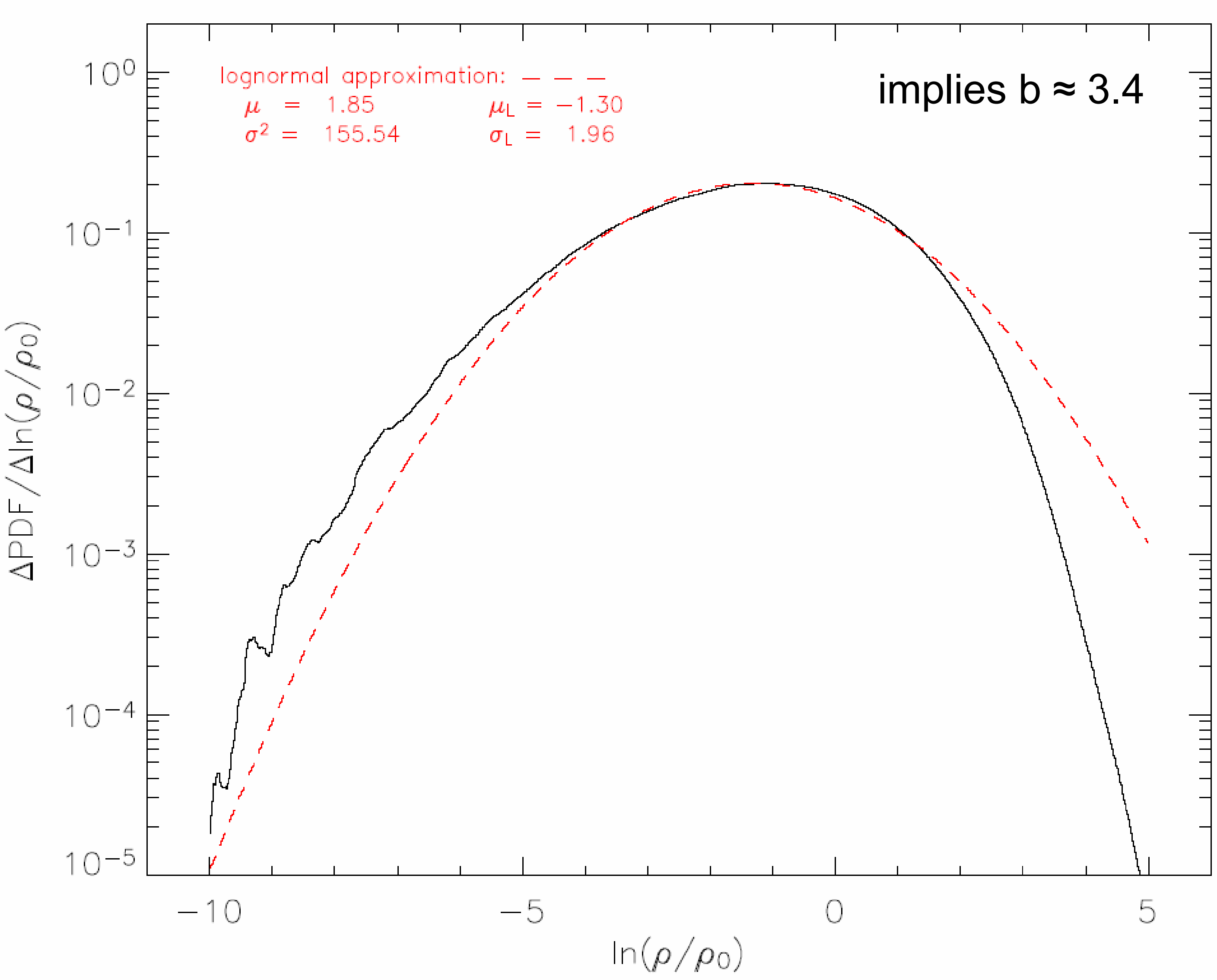
Probability density functions

- Probability of finding a random variable in a certain bin per width of the bin
- For isothermal turbulence, the pdf of mass density was found to be lognormal (most recently, Kritsuk et al. `07):

$$p(x)\mathrm{d}x = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2\right]\mathrm{d}x, \quad x = \ln\rho$$

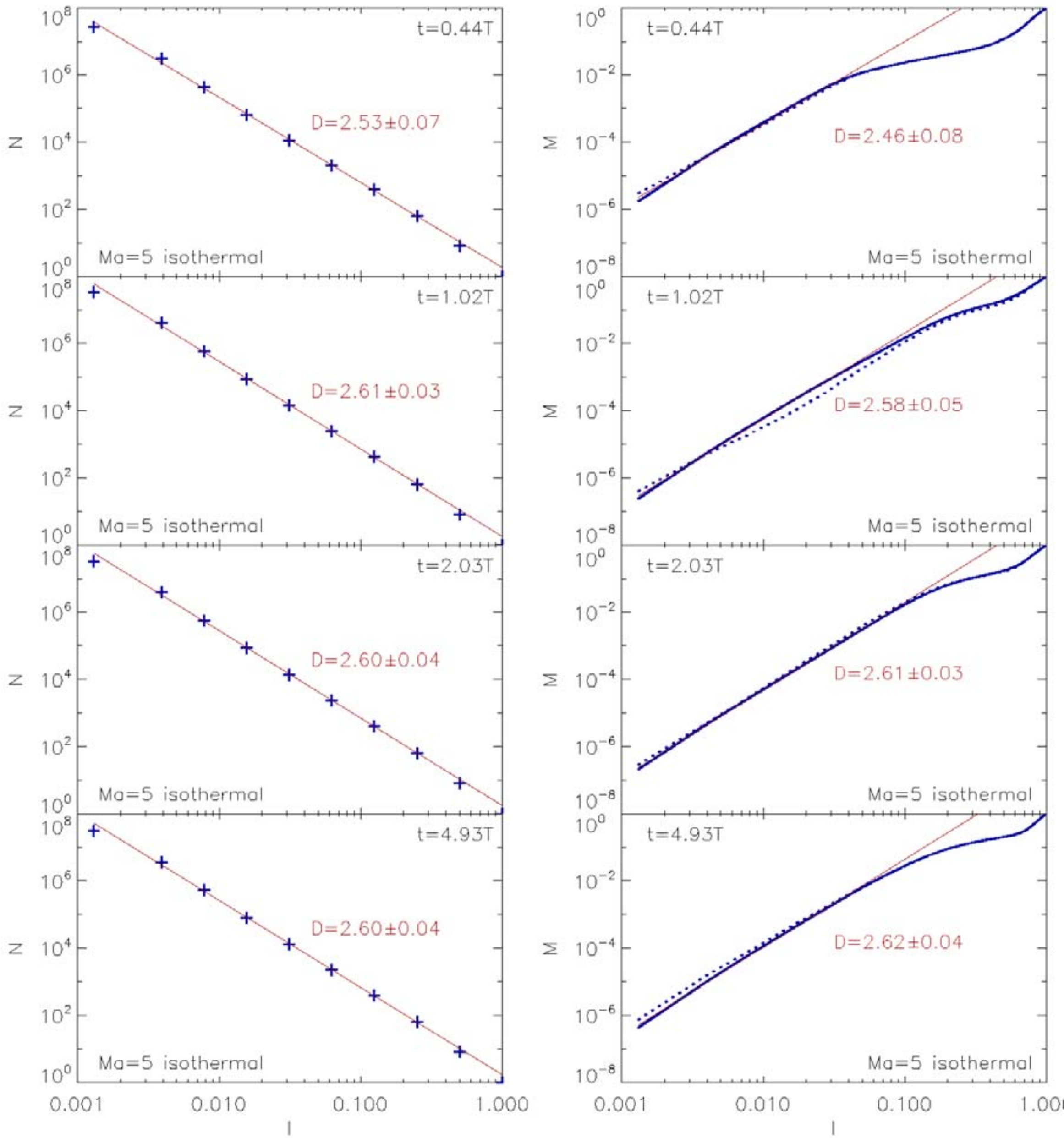
Padoan & Nordlund (`02): $\sigma = \left[\ln\left(1+\frac{3}{4}\mathcal{M}^2\right)\right]^{1/2}$

Simulation of isothermal turbulence, ζ = 0.1, 768^3 (Hupp et al. `07)



Simulation of isothermal turbulence, $\zeta = 0.1$, 768^3 (Federrath et al. `07)

box counting dimension | fractal mass dimension



Kritsuk et al. `07: $D_M \approx 2.4$