Brief Introduction

Background

Based on **DNS** data, it is well established that in **compressible** turbulent flows:

- ☞ **velocity power spectra** tend to get steeper as the Mach number increases, reaching the Burgers **slope of** -2 asymptotically (Boldyrev 2002, Biskamp 2003)
- ☞ **density power spectra** instead get shallower at high Mach numbers, approaching a **slope of** -1 (Kritsuk et al 2004, 2006; Kim & Ryu 2005)
- ☞ **density PDF** in isothermal turbulent flows is well represented by a **lognormal** distribution (Vázquez-Semadeni 1994; Padoan et al 1997; Passot & Vázquez-Semadeni 1998; Nordlund & Padoan 1999)
- ☞ **fractal dimension** of singular **velocity structures** increases from $D_{s,u} \sim 1$ in a subsonic regime to $D_{s,u} \sim 2$ in highly supersonic (Padoan et al 2004)
- ☞ **mass dimension** of the turbulent structures decreases from $D_m = 3$ in weakly compressible flows to $D_m \sim 2.5$ in highly compressible (Kritsuk et al 2007)

Compressible Cascade Model

The kinetic energy is transferred through a hierarchy of scales by nonlinear interactions. In a compressible fluid, the mean *volume* energy transfer rate $\rho u^2 u/\ell$ is constant in a statistical steady state [e.g., Lighthill 1955], therefore

$$u \sim (\ell/\rho)^{1/3}. \tag{1}$$

Let's consider scaling relations for $\boldsymbol{v} \equiv \rho^{1/3}\boldsymbol{u}$

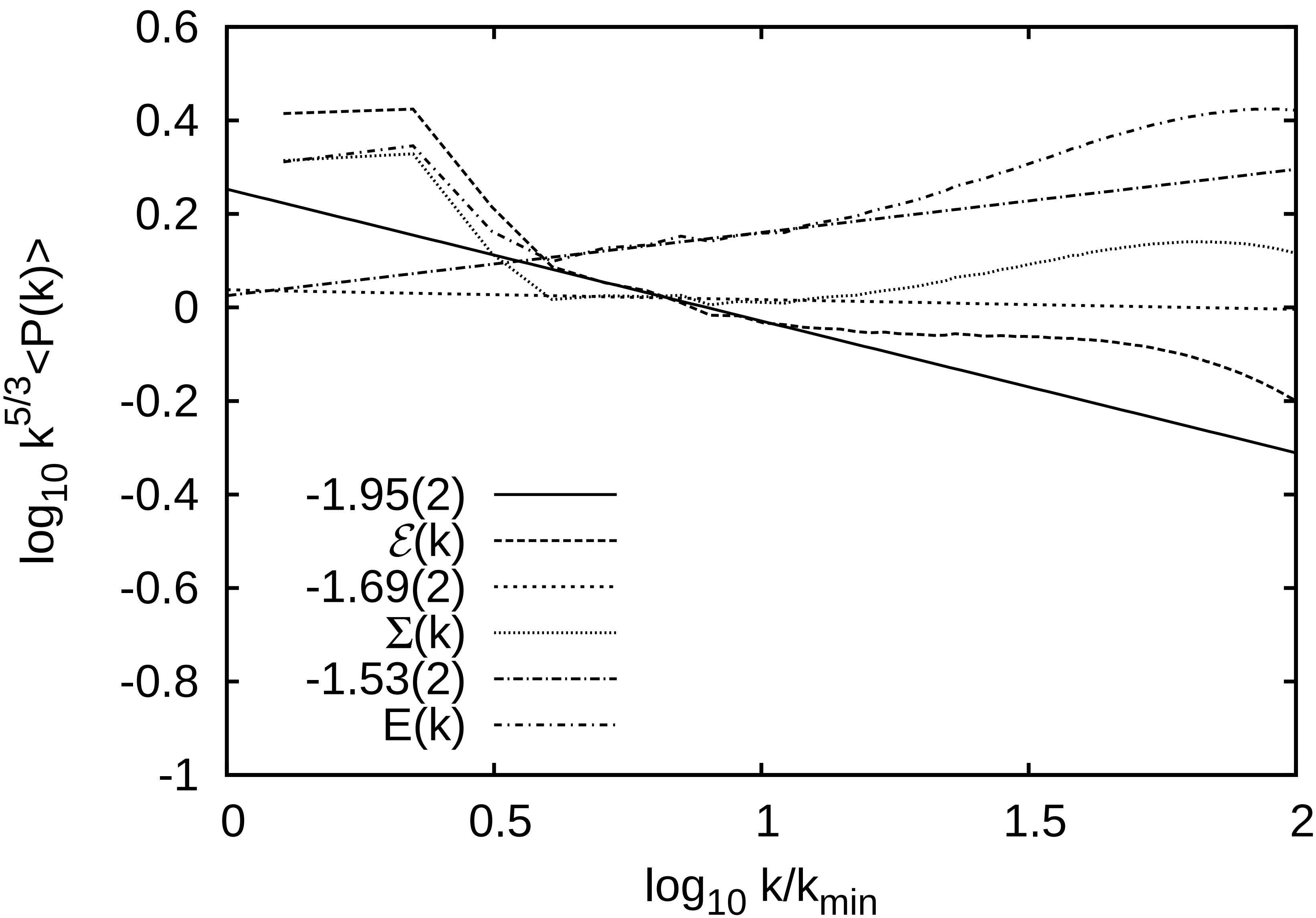
$$v^p = (\rho^{1/3}u)^p \sim \ell^{\,p/3}. \tag{2}$$

For compressible flows, structure functions of v should be used instead of u

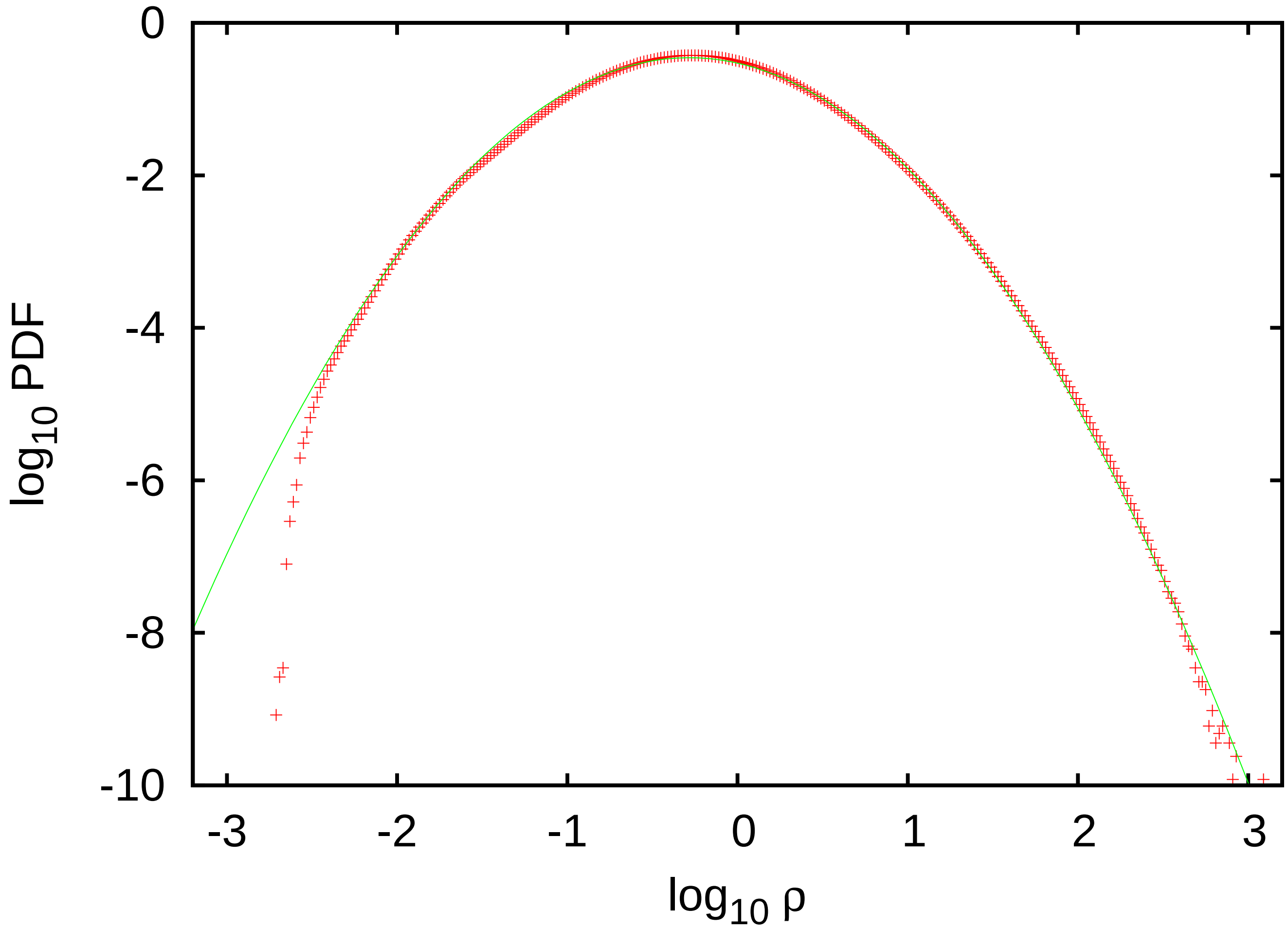
$$\mathcal{S}_p(\ell) \equiv \langle|\boldsymbol{v}(\boldsymbol{r}+\boldsymbol{\ell}) - \boldsymbol{v}(\boldsymbol{r})|^p\rangle \sim \ell^{\,p/3}, \tag{3}$$

with $\mathcal{S}_3 \sim \ell$. The scaling laws expressed by equation (3) are not necessarily exact and, as the incompressible K41 scaling, may require intermittency corrections. Using v instead of u, one properly accounts for the important density–velocity correlations in compressible flows.

Power Spectra

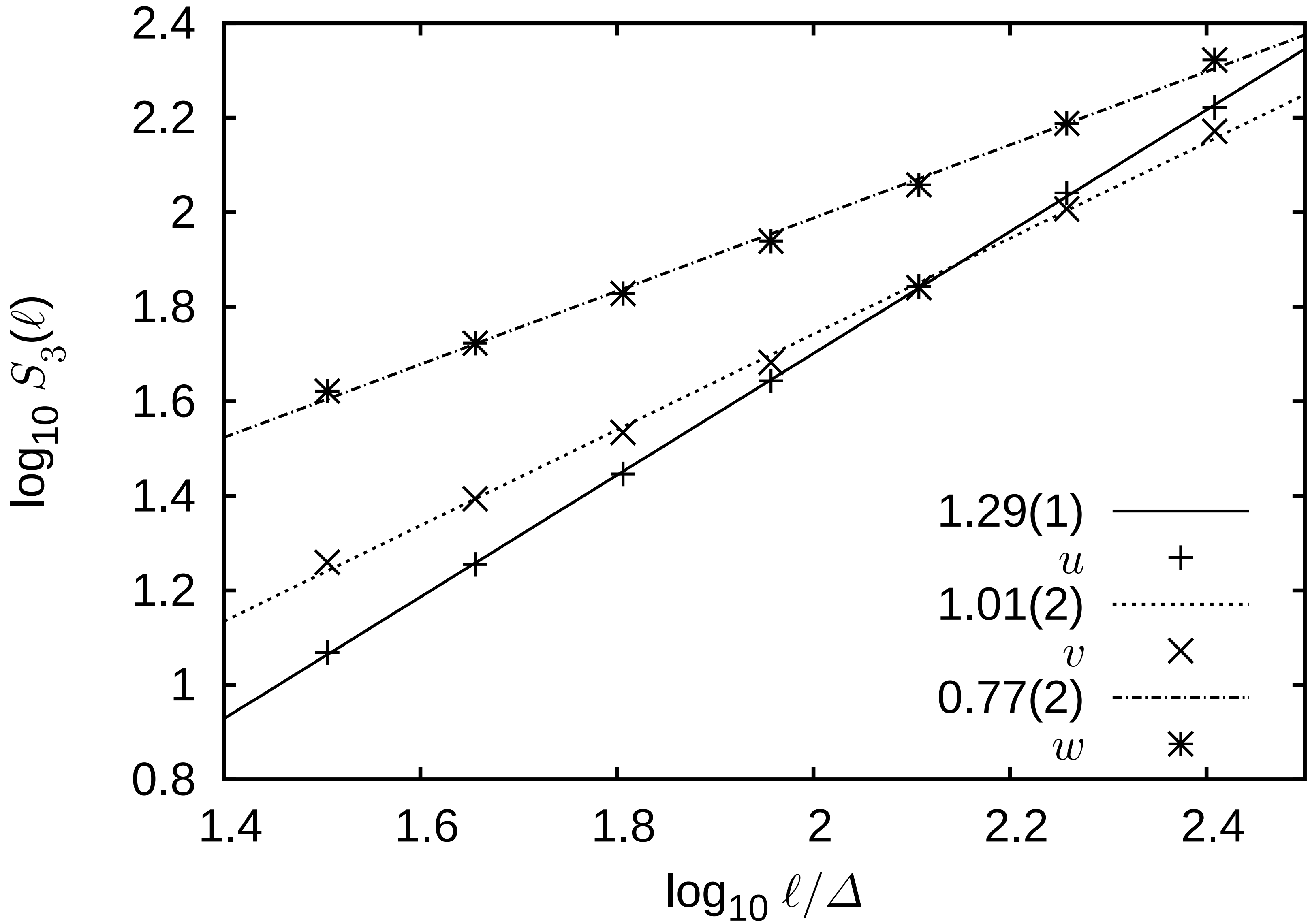


Density PDF

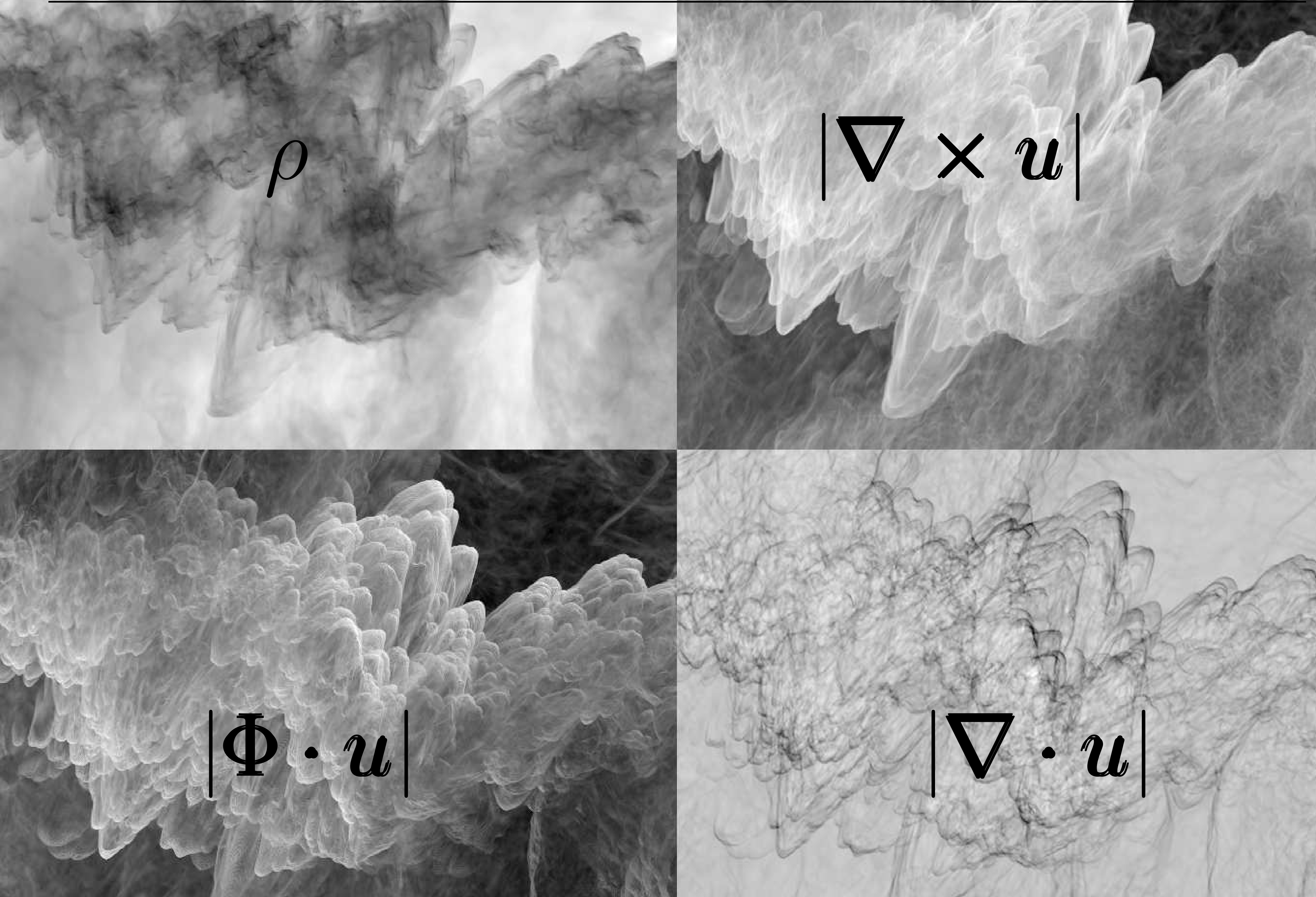


Standard deviation σ is a function of Mach number, $\sigma^2 = \ln\left(1 + b^2 \mathcal{M}^2\right)$

Structure Functions



Dissipative Structures in Mach 6 Turbulence



Dilatational vs. Solenoidal Modes(?)

Lighthill 1955:

"*The interactions between shock waves actually create additional vorticity; also a single shock wave along which the entropy increase is non-uniform creates vorticity, in proportion to the gradient of this increase. Thus to some extent the shock system can generate new turbulence.* **... The author feels that rather the system has become one in which the division of motion into 'turbulence' on the one hand and 'sound' (or shock waves) on the other is almost without significance.**"

Things to discuss

- ☞ Signatures of numerical viscosity and bottleneck in different codes
- ☞ The ratio of the Kolmogorov dissipation scale, η, and the Jeans length, λ_J, is very small and remarkably independent of temperature and density,

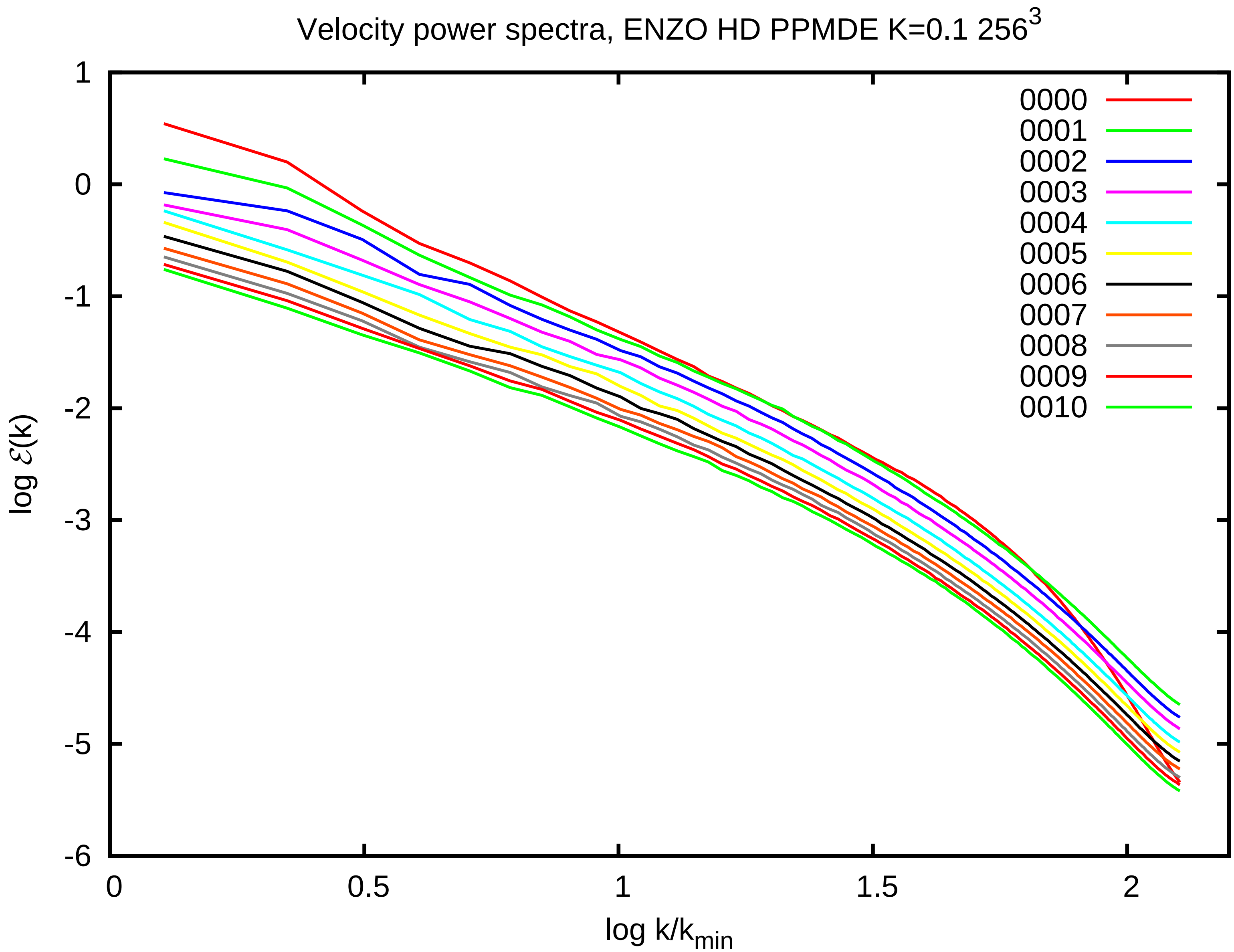
$$\eta/\lambda_J \approx 10^{-4}(T/10K)^{-1/8}(n/10^3 cm^{-3})^{-1/4}. \quad (4)$$

  Can we simulate fragmentation resolving turbulence to scales smaller than λ_J but not as small as η? This would help since $Re = 10^3$ is easier to achieve than $Re = 10^7$.

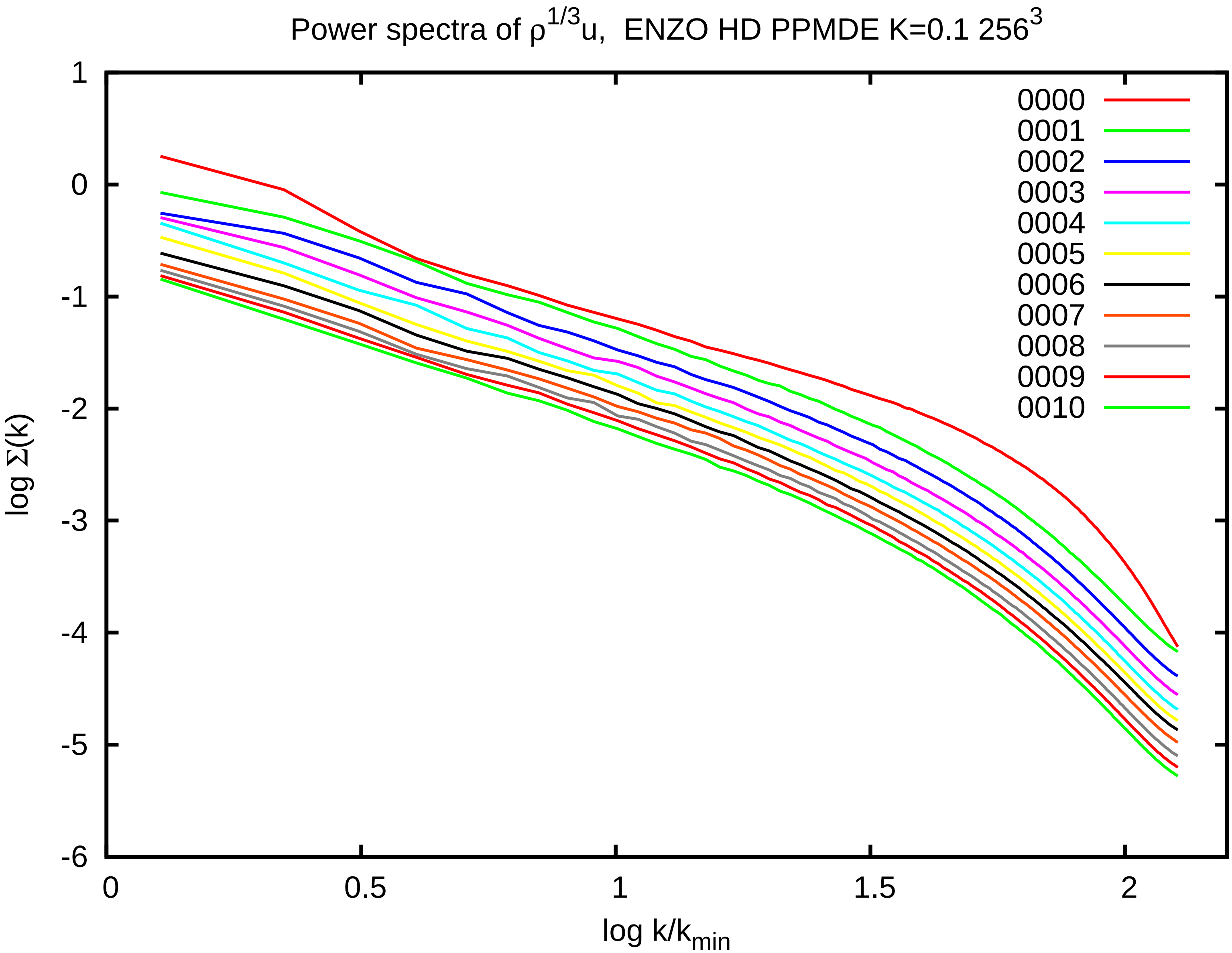
- ☞ Do we get turbulence statistics right for the high density tail with AMR/SPH?
- ☞ . . .

Sample Results

ENZO

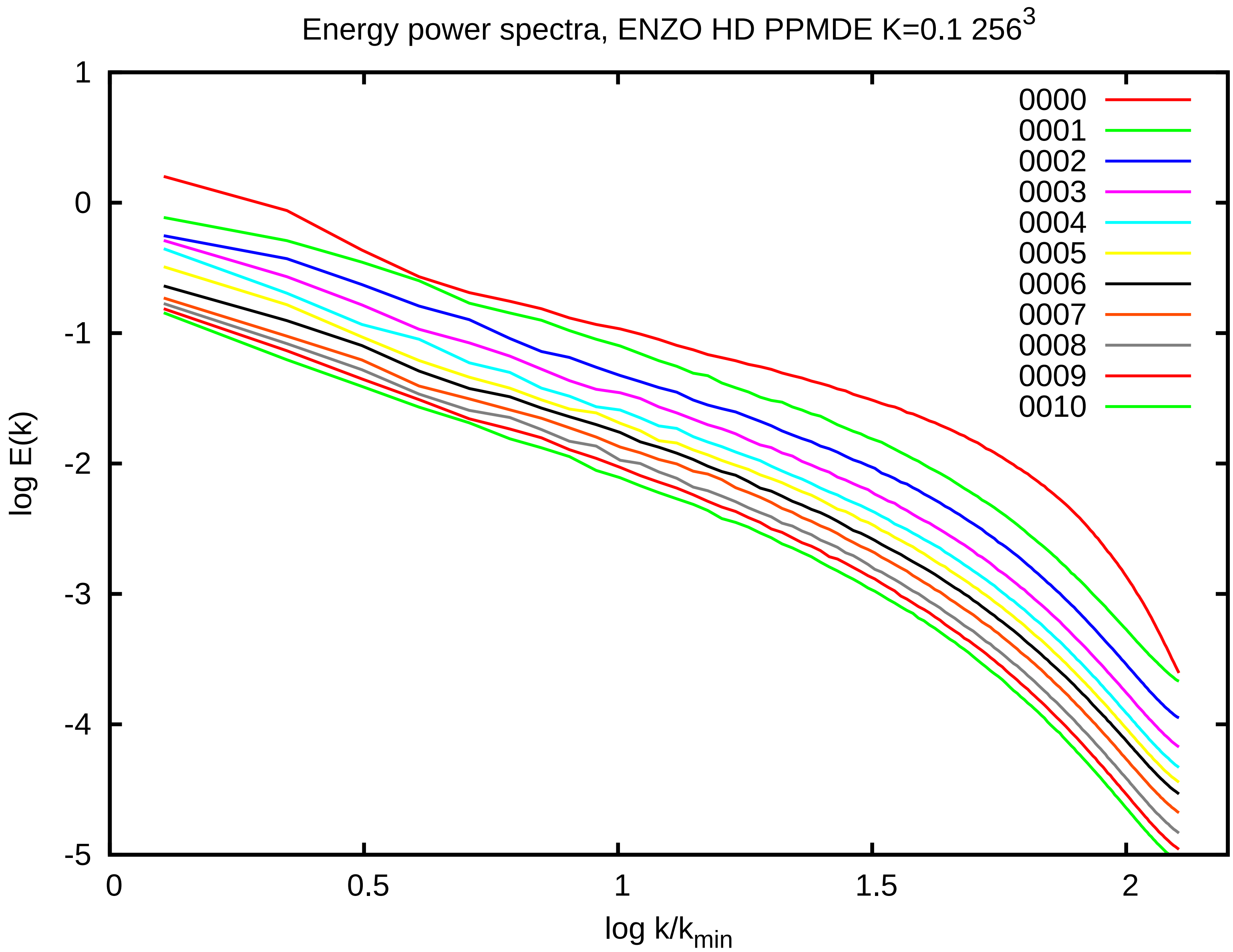

Velocity power spectra, ENZO HD PPMDE K=0.1 256^3
log $\mathcal{E}$(k)
log k/k_{min}
1
0
-1
-2
-3
-4
-5
-6
0
0.5
1
1.5
2
0000
0001
0002
0003
0004
0005
0006
0007
0008
0009
0010

ENZO



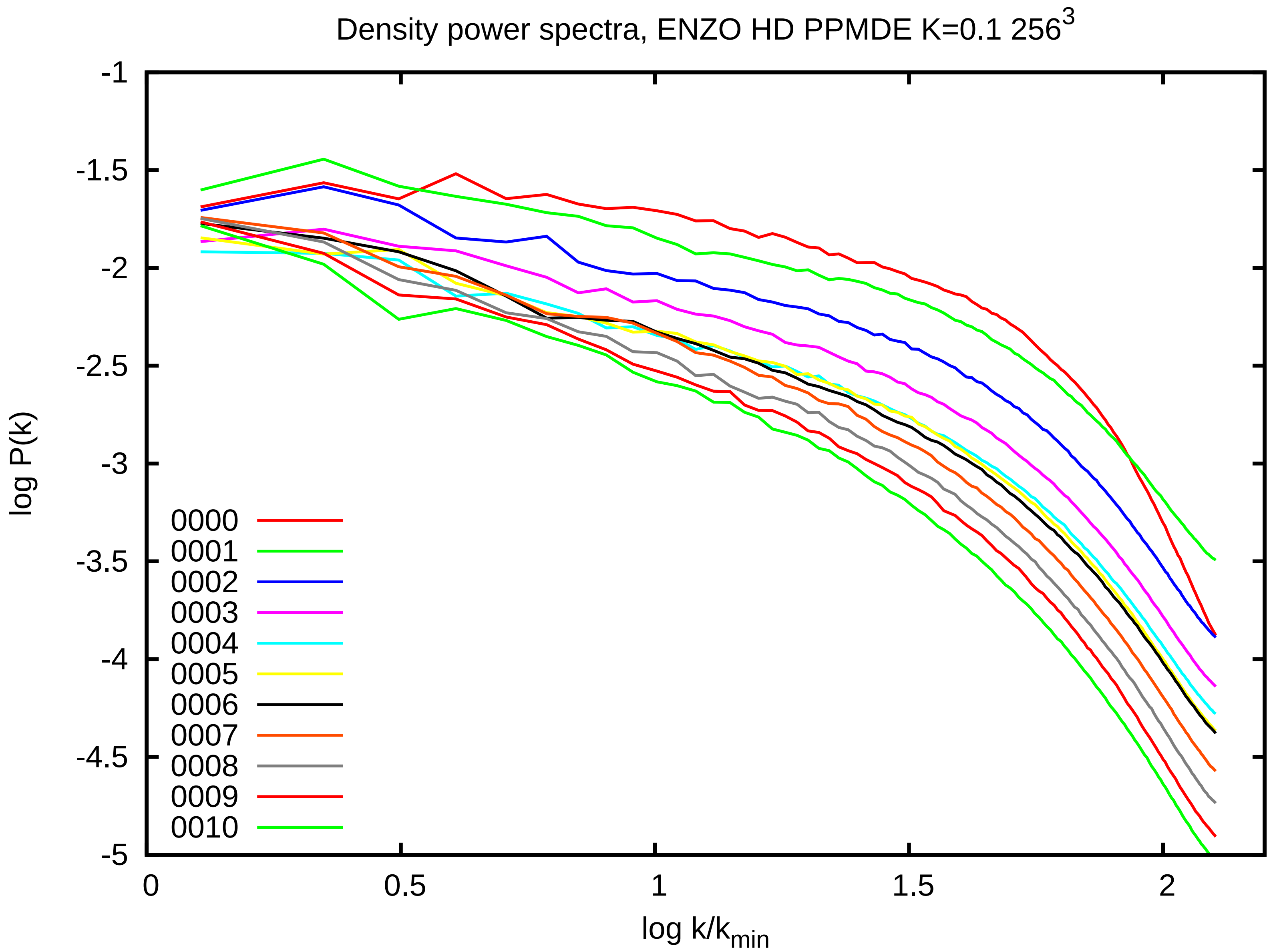
Power spectra of ρ^1/3 u, ENZO HD PPMDE K=0.1 256^3
log Σ(k)
log k/k_min
1
0
-1
-2
-3
-4
-5
-6
0
0.5
1
1.5
2
0000
0001
0002
0003
0004
0005
0006
0007
0008
0009
0010

ENZO



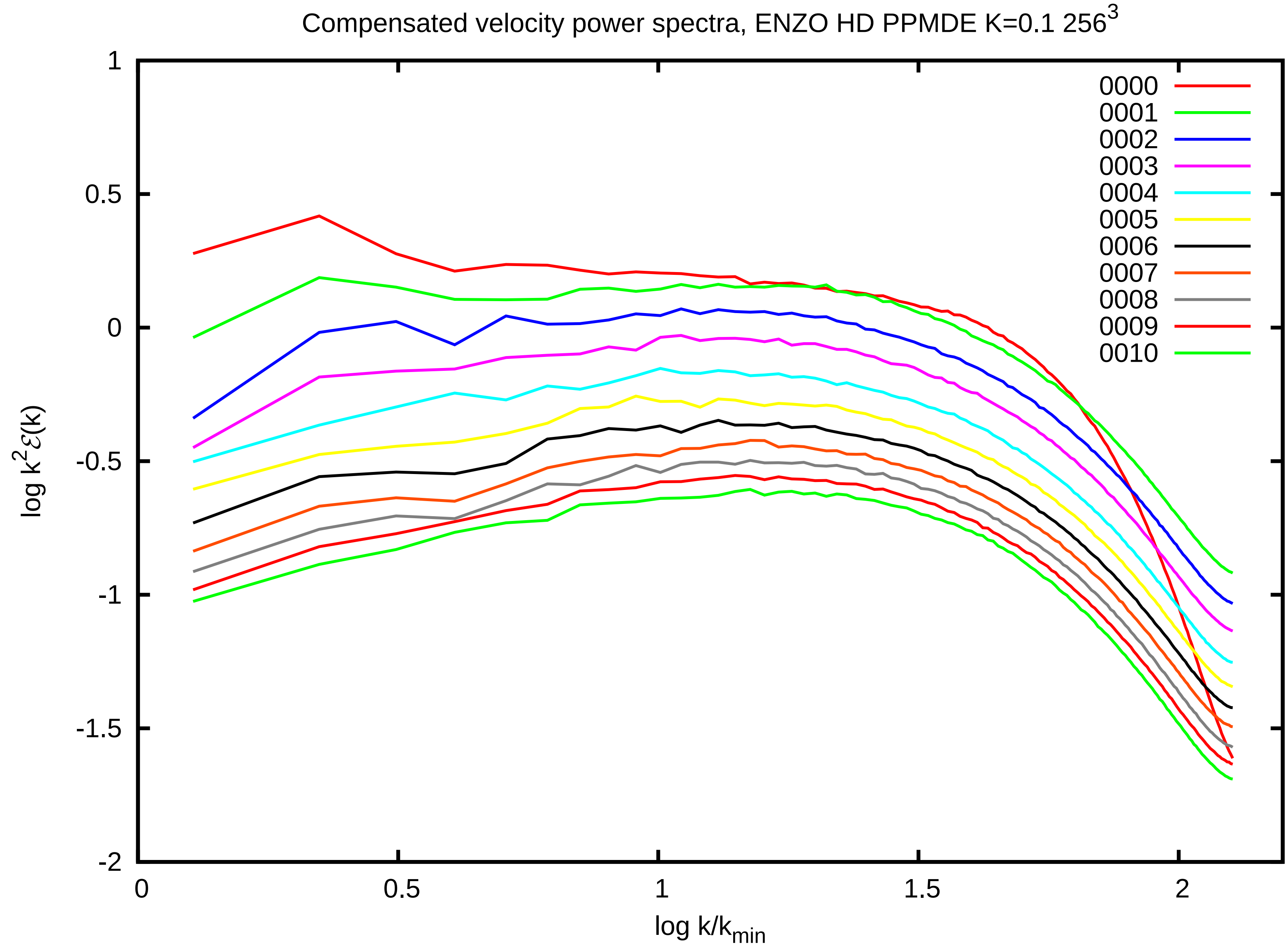
Energy power spectra, ENZO HD PPMDE K=0.1 256^3
1
0
-1
-2
-3
-4
-5
0
0.5
1
1.5
2
log E(k)
log k/k_min
0000
0001
0002
0003
0004
0005
0006
0007
0008
0009
0010

ENZO

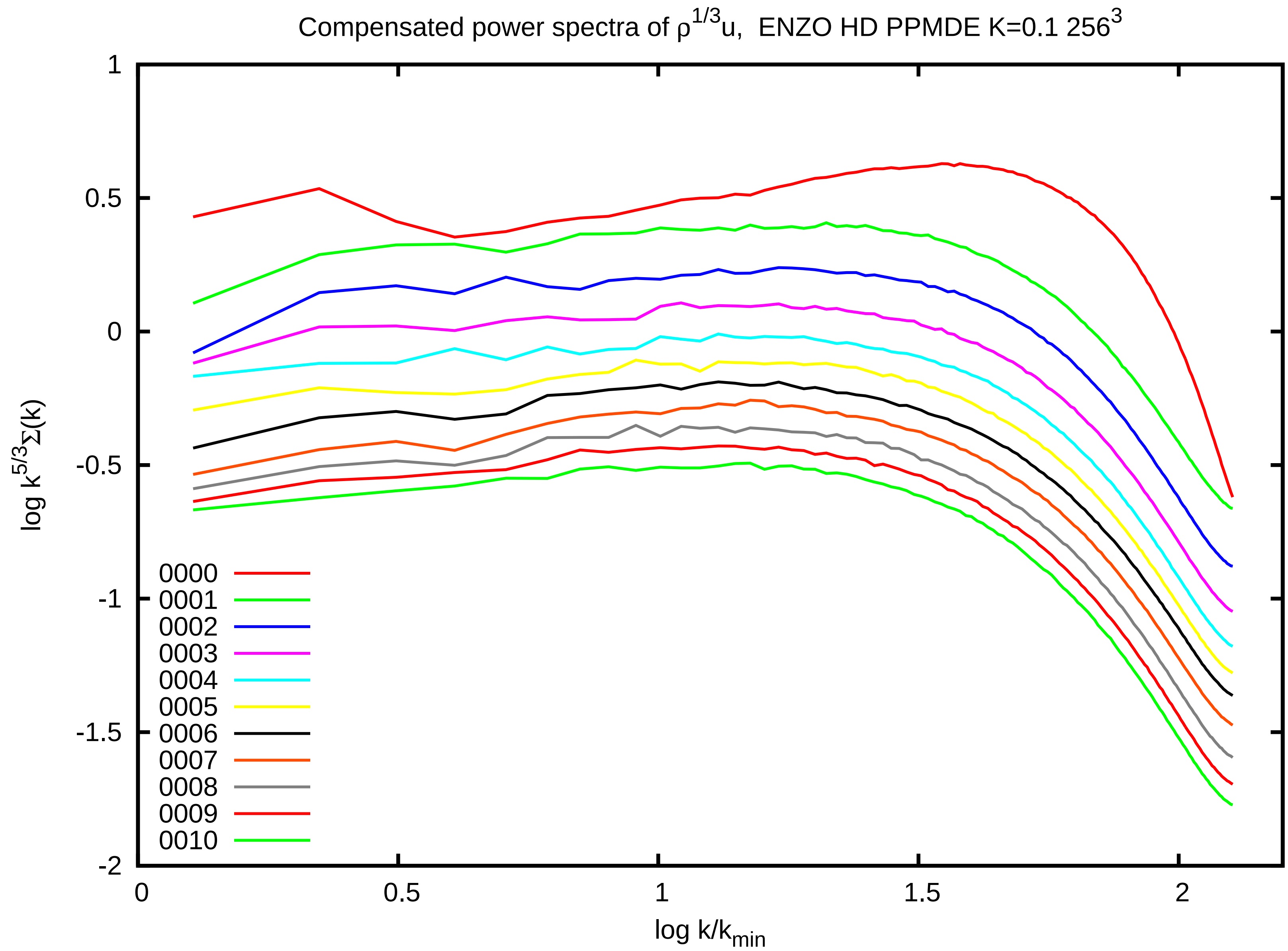


Density power spectra, ENZO HD PPMDE K=0.1 256^3
log P(k)
log k/k_min
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
0
0.5
1
1.5
2
0000
0001
0002
0003
0004
0005
0006
0007
0008
0009
0010

ENZO


Compensated velocity power spectra, ENZO HD PPMDE K=0.1 256^3
log k^2 ℰ(k)
log k/k_min
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
0000
0001
0002
0003
0004
0005
0006
0007
0008
0009
0010

ENZO



Compensated power spectra of ρ^1/3 u, ENZO HD PPMDE K=0.1 256^3
log k^5/3 Σ(k)
log k/k_min
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
0000
0001
0002
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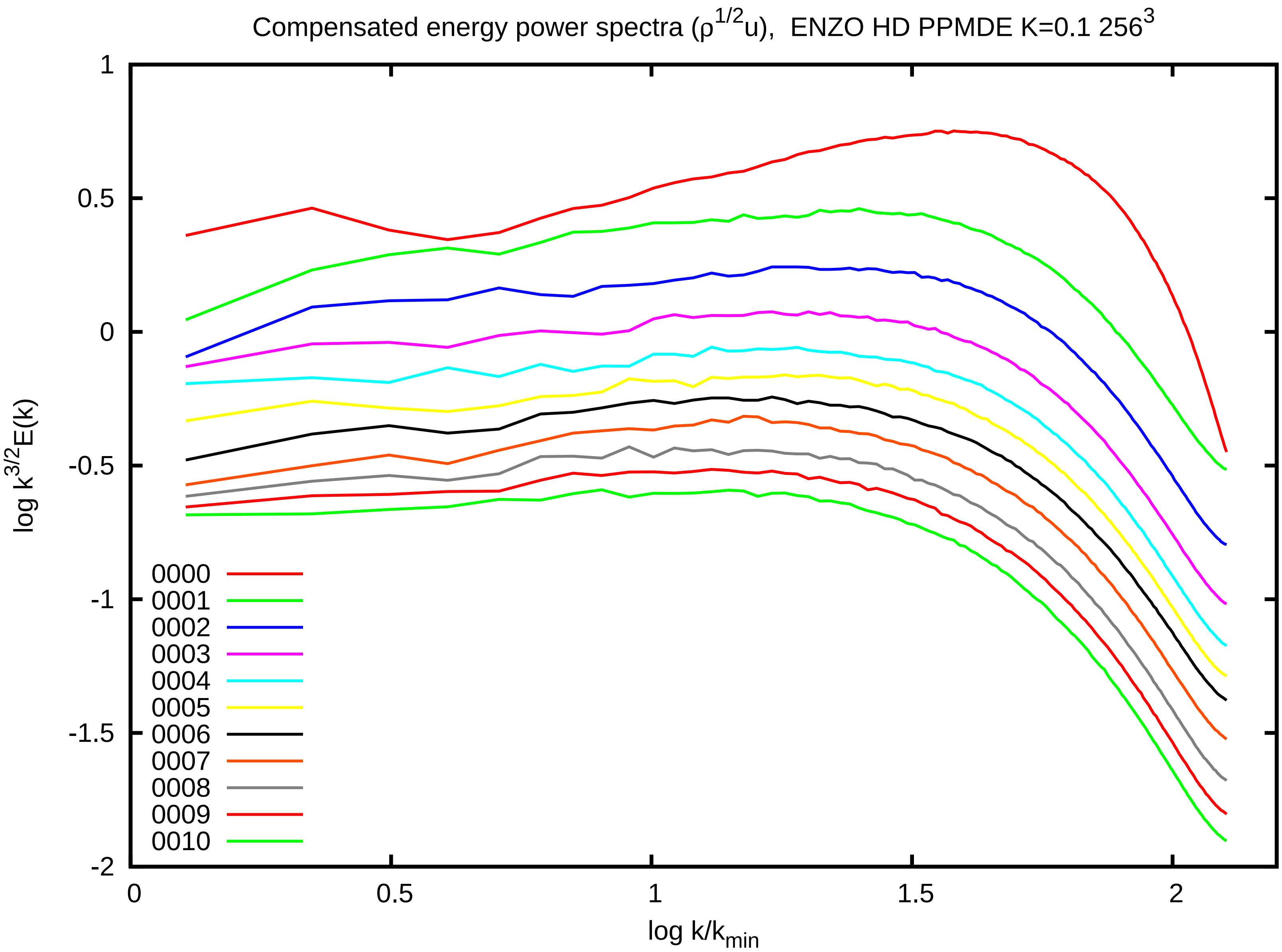
ENZO


Compensated energy power spectra ($\rho^{1/2}u$), ENZO HD PPMDE K=0.1 256^3
log $k^{3/2}E(k)$
log k/k_{min}
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
0000
0001
0002
0003
0004
0005
0006
0007
0008
0009
0010