Feedback, cascades & self-regulation in galactic disks

Alexei Kritsuk

University of California, San Diego

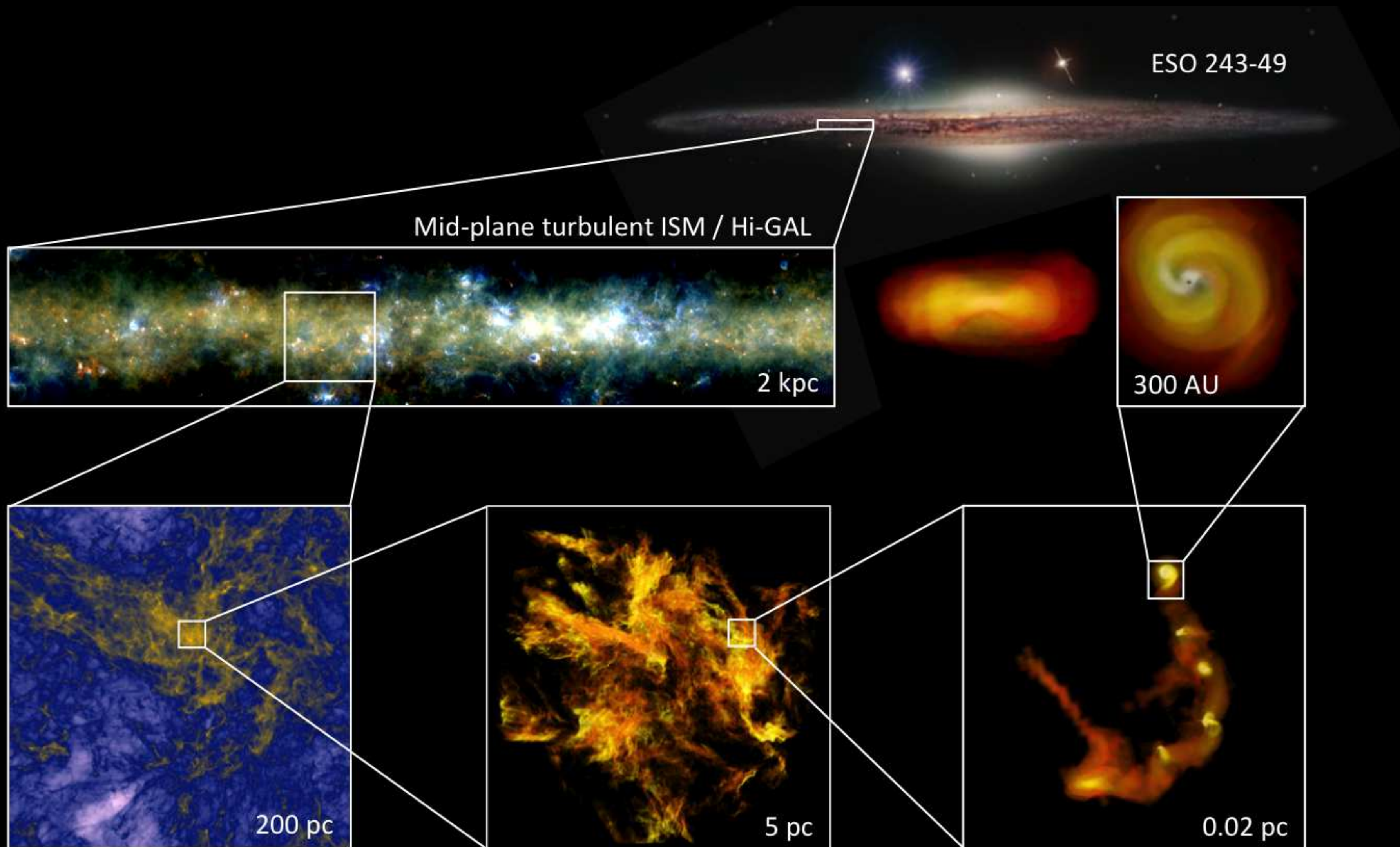
Collaborators: Rick Wagner, Mike Norman (SDSC)

Outline

- 3D direct energy cascade in supersonic turbulence
  - New exact scaling relations
  - Verification in numerical experiments
- Quasi-2D inverse energy cascade in galactic disks (?)
  - Theoretical concepts, effects of stratification and rotation
  - Predictions: self-regulation of GMC structures (?)

How could we simulate star formation ab initio?

Zooming-in from the disk scale height ($h \sim 100$ pc) down to rotationally supported disks (~ 100 AU)



Outline

I. Supersonic Turbulence in 3D

Density field in a thin slice

(Loading movie...)

Mach 6, grid resolution 2048^3, 5 flow crossing times [Kritsuk et al. 2009]

Projected density

(Loading movie...)

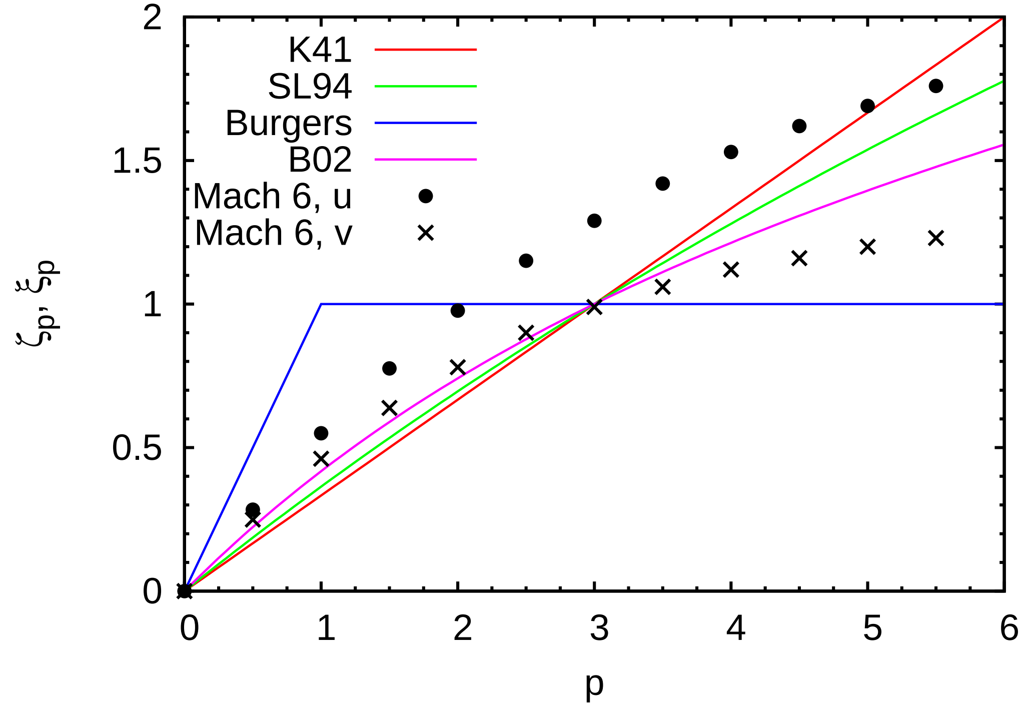
Mach 6, grid resolution 2048^3, 5 flow crossing times [Kritsuk et al. 2009]

What is supersonic turbulence about: div() or curl()?

A snapshot of $\nabla \times \boldsymbol{u}$ at Mach 6, grid resolution 1024^3 [Kritsuk et al. 2007]

Third-order structure functions of velocity do not scale linearly:

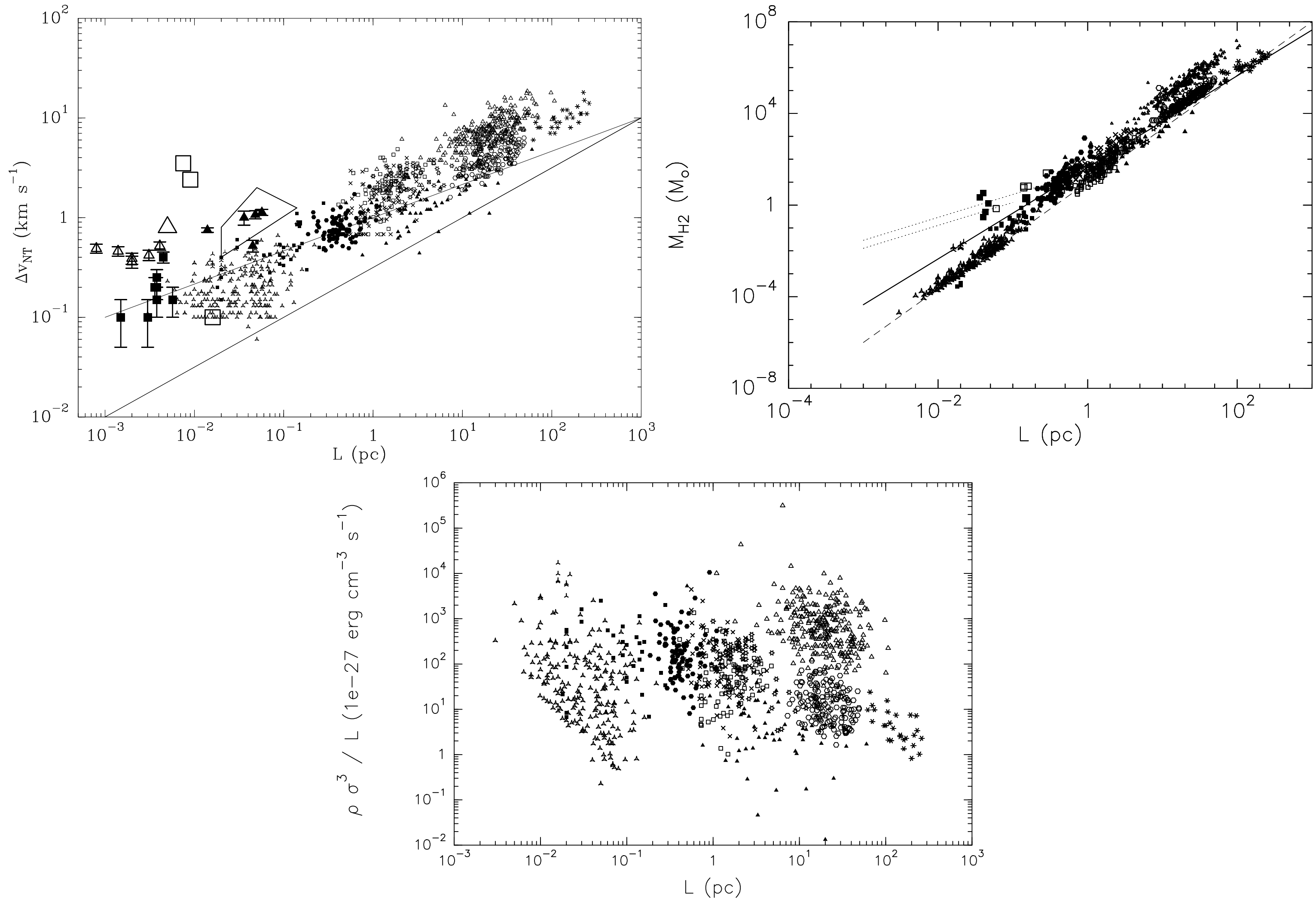
$$S_3(u,\ell) \propto \ell^{1.3}$$



- 1024^3 model of isothermal HD turbulence, Mach 6, no self-gravity [Kritsuk et al. 2007].
- Density-weighted velocity: $\boxed{v \equiv \rho^{1/3} u}$. Total energy is conserved: $E = \langle \rho u^2/2 + c_s^2 \rho \ln \rho \rangle$.
- Linear scaling: $S_3(v,\ell) \propto \ell^1$ independent of the Mach number.

$\boldsymbol{v \equiv \rho^{1/3} u}$ shows "universal" behavior?

Larson's scaling relations



^{12}CO J=1-0 data compilation for Galactic molecular clouds [Hennebelle & Falgarone 2012]

Beyond dimensional arguments

- Falkovich, Fouxon & Oz (2010, JFM)

  New relations for correlation functions in Navier-Stokes turbulence

- **Galtier & Banerjee (2011, PRL)**

  Exact relation for correlation functions in compressible isothermal turbulence

- Wagner, Falkovich, Kritsuk & Norman (2012, JFM)

  Flux correlations in supersonic isothermal turbulence

- **Aluie (2013, Phys. D)**

  Scale decomposition in compressible turbulence

- Banerjee & Galtier (2013, PRE)

  Exact relation with two-ponit correlation functions and phenomenological approach for compressible magnetohydrodynamic turbulence

- **Kritsuk, Wagner & Norman (2013, JFM Rapids)**

  Energy cascade and scaling in supersonic isothermal turbulence

- Banerjee & Galtier (2014, JFM)

  A Kolmogorov-like exact relation for compressible polytropic turbulence

Forced Navier-Stokes system

- Compressible N-S equations, isothermal EOS

$$\begin{aligned} \partial_t \rho + \nabla \cdot (\rho \boldsymbol{u}) &= 0, & (1) \\ \partial_t (\rho \boldsymbol{u}) + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) + \nabla p &= \eta \Delta \boldsymbol{u} + \frac{\eta}{3} \nabla (\nabla \cdot \boldsymbol{u}) + \boldsymbol{f} & (2) \end{aligned}$$

- $p = c_s^2 \rho$ — pressure, $\eta > 0$ — viscosity, and $\boldsymbol{f}(\boldsymbol{x}, t)$ — random force.
- Total energy density is an **ideal invariant**: $E \equiv \left\langle \rho u^2/2 + \rho e \right\rangle$, where $e = c_s^2 \ln(\rho/\rho_0)$.
- The energy balance equation: injection versus dissipation

$$\partial_t E = \langle \epsilon \rangle - \eta \left\langle \omega^2 + 4d^2/3 \right\rangle \qquad (3)$$

$\epsilon = \boldsymbol{u} \cdot \boldsymbol{f}$ – energy injection rate, $\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{u}$ – vorticity,
$d = \boldsymbol{\nabla} \cdot \boldsymbol{u}$ – dilatation, and $\langle \ldots \rangle$ – ensemble average.

- Dissipative anomaly in 3D: $\langle \epsilon \rangle = \eta \left\langle \omega^2 + 4d^2/3 \right\rangle = \mathcal{O}(1)$ even if $\eta \ll 1$

Compressible analog to Kolmogorov's 4/5 law

In a statistical steady state at $Re \gg 1$, assuming isotropy,

$$Q(r) + F_{\parallel}(r) = -\frac{4}{3}\varepsilon r,$$

where

$$F_{\parallel}(r) \equiv \boldsymbol{F}\cdot\boldsymbol{r}/r = \left\langle \left[\delta(\rho\boldsymbol{u})\cdot\delta\boldsymbol{u} + 2\delta\rho\delta e\right]\delta u_{\parallel} + \tilde{\delta} e \delta(\rho u_{\parallel})\right\rangle$$

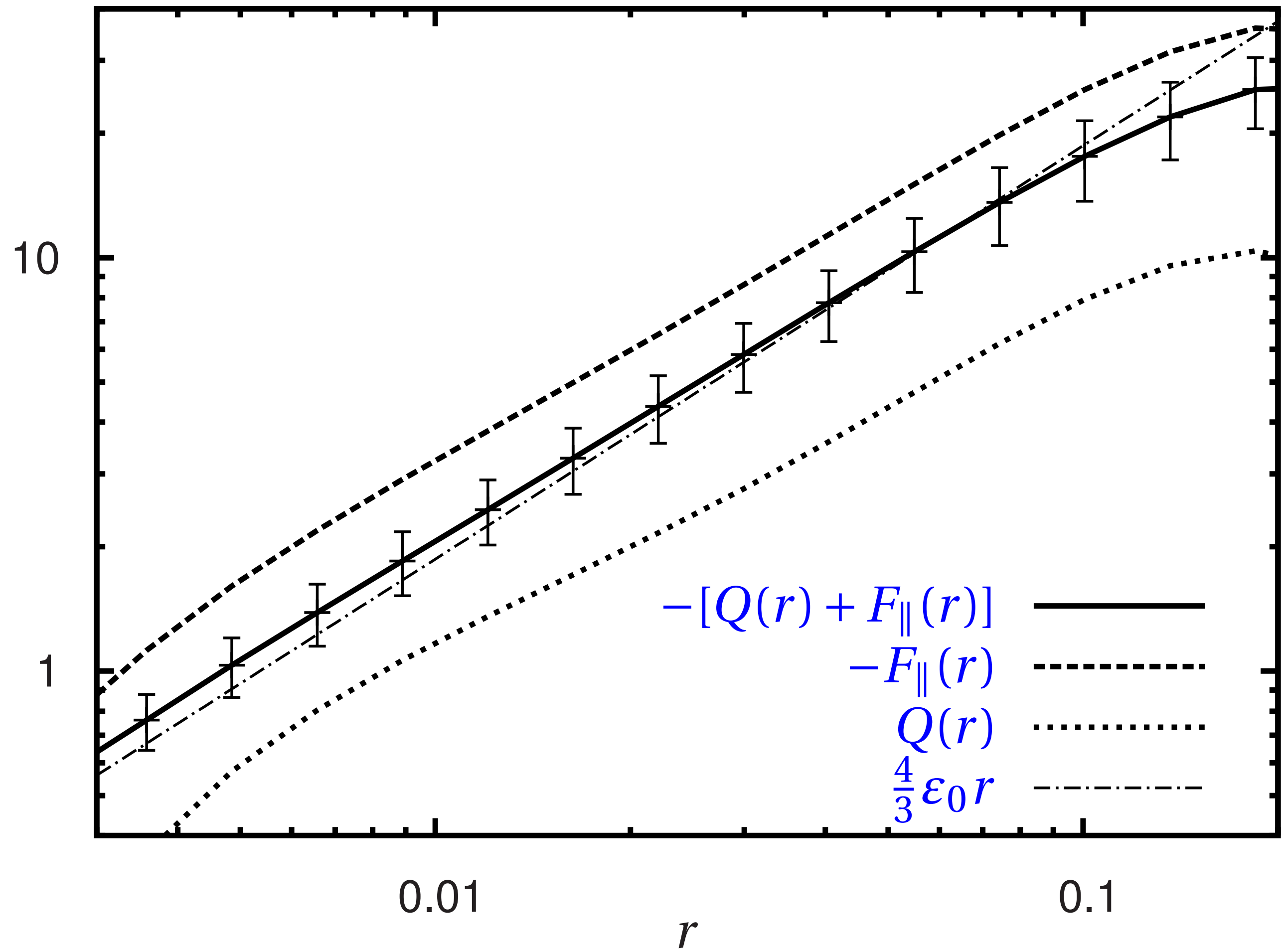
$$Q(r) \equiv \frac{1}{r^2}\int_0^r S(r) r^2 \mathrm{d}r$$

$$S(r) = \left\langle \left[\delta(d\rho\boldsymbol{u}) - \tilde{\delta} d\delta(\rho\boldsymbol{u})\right]\cdot\delta\boldsymbol{u} + 2\left[\delta(d\rho) - \tilde{\delta} d\delta\rho\right]\delta e + \delta d\delta p - 2dp\right\rangle$$

$$\varepsilon = \langle \rho\boldsymbol{u}'\cdot\boldsymbol{a} + \rho'\boldsymbol{u}'\cdot\boldsymbol{a}\rangle/2$$

[Galtier & Banerjee, 2011; Kritsuk et al. 2013a]

Evaluating flux/source contributions at Mach 6



The new relation holds reasonably well; $\mathrm{sign}(F_{\parallel}) = -\mathrm{sign}(S)$; $|F_{\parallel}|/S \approx 3.2$

Direct energy cascade with an effective sink due to compressibility.

Both $F_{\parallel}(r)$ and $Q(r)$ scale ~linearly with r

Compressible analog to Kolmogorov's 4/5 law

In a statistical steady state at $Re \gg 1$, assuming isotropy,

$$Q(r) + F_{\parallel}(r) = -\frac{4}{3}\varepsilon r,$$

where

$$F_{\parallel}(r) \equiv \boldsymbol{F}\cdot\boldsymbol{r}/r = \left\langle \left[\delta(\rho\boldsymbol{u})\cdot\delta\boldsymbol{u} + 2\delta\rho\delta e\right]\delta u_{\parallel} + \tilde{\delta} e\delta(\rho u_{\parallel})\right\rangle$$

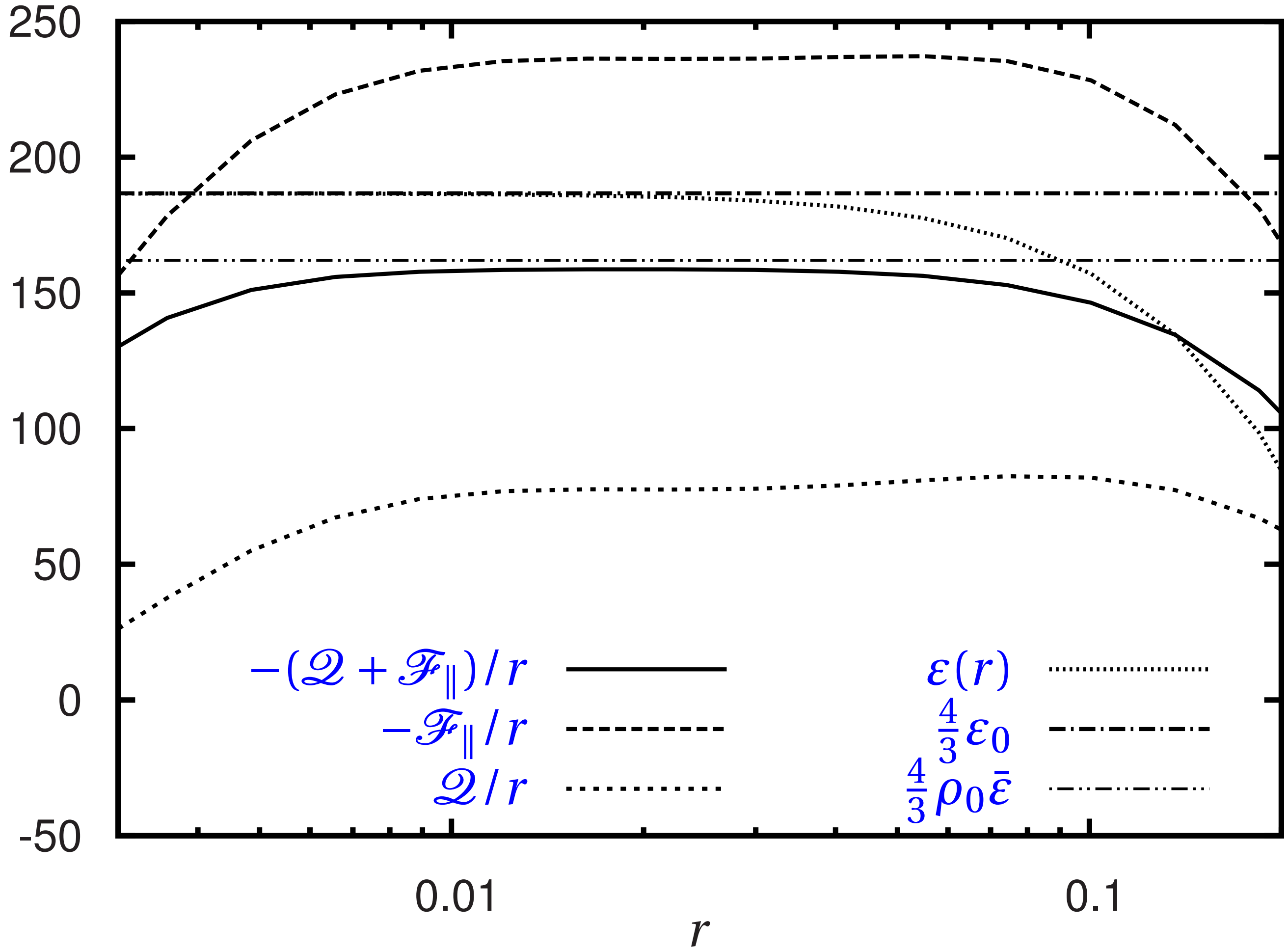
$$Q(r) \equiv \frac{1}{r^2}\int_0^r S(r) r^2 \mathrm{d}r$$

$$S(r) = \left\langle \left[\delta(d\rho\boldsymbol{u}) - \tilde{\delta} d\delta(\rho\boldsymbol{u})\right]\cdot\delta\boldsymbol{u} + 2\left[\delta(d\rho) - \tilde{\delta} d\delta\rho\right]\delta e + \delta d\delta p - 2dp\right\rangle$$

$$\varepsilon = \langle \rho\boldsymbol{u}'\cdot\boldsymbol{a} + \rho'\boldsymbol{u}'\cdot\boldsymbol{a}\rangle/2$$

[Galtier & Banerjee, 2011; Kritsuk et al. 2013a]

Evaluating flux/source contributions at Mach 6



$\mathscr{F}_{\parallel}(r) = \left\langle \left[\delta(\rho \boldsymbol{u}) \cdot \delta \boldsymbol{u}\right] \delta u_{\parallel} \right\rangle$; $\mathscr{S}(r) = \left\langle \left[\delta(d\rho \boldsymbol{u}) - \tilde{\delta} d \delta(\rho \boldsymbol{u})\right] \cdot \delta \boldsymbol{u} \right\rangle$; $\varepsilon(r) \approx \langle \rho \boldsymbol{u} \cdot \boldsymbol{a} \rangle = \varepsilon_0$.

New relation for supersonic turbulence

- Ignoring subdominant terms representing fluctuations of pressure p and compressive energy e, we get

$$\mathscr{Q}(r) + \left\langle [\delta(\rho \boldsymbol{u}) \cdot \delta \boldsymbol{u}] \delta u_{\parallel} \right\rangle = -\tfrac{4}{3} \mathscr{C} \varepsilon_0 r$$

- As $\mathscr{Q}(r) \propto r$, it can be incorporated in ϵ_{eff}

$$\boxed{\left\langle [\delta(\rho \boldsymbol{u}) \cdot \delta \boldsymbol{u}] \delta u_{\parallel} \right\rangle = -\tfrac{4}{3} \varepsilon_{\mathrm{eff}} r}$$

- Compare with a primitive version of the 4/5 law for incompressible turbulence

$$\left\langle (\delta \boldsymbol{u})^2 \delta u_{\parallel} \right\rangle = -\tfrac{4}{3} \bar{\varepsilon} r,$$

where $\bar{\varepsilon} = \varepsilon / \rho_0$

- The new relation allows to derive Larson's laws from first principles, see [Kritsuk et al. (2013b)].

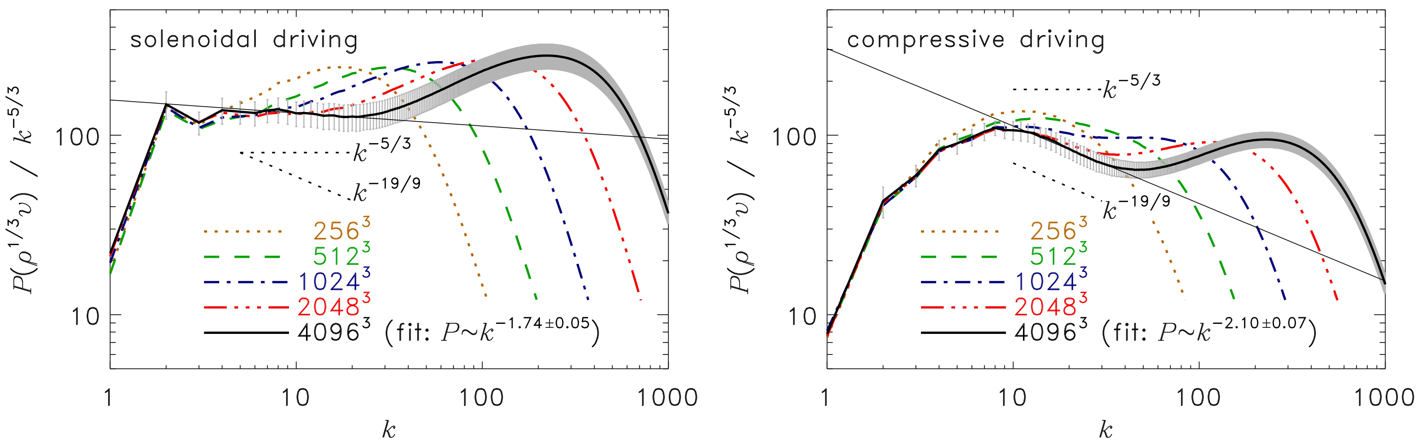
Turbulence at Mach 17

(Loading movie...)

Isothermal fluid, grid resolution 4096^3, 5 flow crossing times [Federrath 2013]

solenoidal (left) and compressive (right) random forcing

PSD confusion in Mach 17 simulations



- In fully developed turbulence at high Mach numbers, solenoidal and dilatational modes are nonlinearly coupled [Moyal (1952); Kovasznay (1953)].
- A tendency toward energy equipartition between the modes has been demonstrated 60 years ago [Kraichnan (1955)], see also [Goldreich & Kumar 1988].
- The steep spectrum of $\rho^{1/3}\boldsymbol{u}$ measured by Federrath (2013) is most likely due to a combination of nonequilibrium nature of the compressive forcing and limited grid resolution.
- Conjecture $|Q(r)| > |F(r)|$ used by Galtier & Banerjee (2011) to derive the $-19/9$ slope is **not** supported by numerical simulations.

What if we add magnetic field?

Elsässer fields: $\boldsymbol{z}^{\pm} = \boldsymbol{v} \pm \boldsymbol{v}_{\mathrm{A}}$ Alfvén velocity: $\boldsymbol{v}_{\mathrm{A}} = \boldsymbol{b}/\sqrt{4\pi\rho}$

Total energy density is an ideal invariant: $E = \langle \rho(v^2 + v_{\mathrm{A}}^2)/2 + \rho e \rangle$

$$
\begin{aligned}
-2\varepsilon = {} & \frac{1}{2}\nabla_r \cdot \left\langle \left[\frac{1}{2}\delta(\rho\mathbf{z}^-)\cdot\delta\mathbf{z}^- + \delta\rho\delta e\right]\delta\mathbf{z}^+ + \left[\frac{1}{2}\delta(\rho\mathbf{z}^+)\cdot\delta\mathbf{z}^+ + \delta\rho\delta e\right]\delta\mathbf{z}^- + \bar{\delta}\left(e + \frac{v_A^2}{2}\right)\delta(\rho\mathbf{z}^- + \rho\mathbf{z}^+)\right\rangle \\
& - \frac{1}{8}\left\langle \frac{1}{\beta'}\nabla'\cdot(\rho\mathbf{z}^+ e') + \frac{1}{\beta}\nabla\cdot(\rho'\mathbf{z}'^+ e) + \frac{1}{\beta'}\nabla'\cdot(\rho\mathbf{z}^- e') + \frac{1}{\beta}\nabla\cdot(\rho'\mathbf{z}'^- e)\right\rangle \\
& + \left\langle (\nabla\cdot\mathbf{v})\left[R_E' - E' - \frac{\bar{\delta}\rho}{2}(\mathbf{v_A}'\cdot\mathbf{v_A}) + \frac{P_M' - P'}{2}\right]\right\rangle + \left\langle (\nabla'\cdot\mathbf{v}')\left[R_E - E - \frac{\bar{\delta}\rho}{2}(\mathbf{v_A}\cdot\mathbf{v_A}') + \frac{P_M - P}{2}\right]\right\rangle \\
& + \langle(\nabla\cdot\mathbf{v_A})[R_H - R_H' + H' - \bar{\delta}\rho(\mathbf{v}'\cdot\mathbf{v_A})]\rangle + \langle(\nabla'\cdot\mathbf{v_A}')[R_H' - R_H + H - \bar{\delta}\rho(\mathbf{v}\cdot\mathbf{v_A}')]\rangle\,,
\end{aligned}
$$

Compressible cross-helicity density: $H = \rho\boldsymbol{v}\cdot\boldsymbol{v}_{\mathrm{A}}$

Two-point correlations associated with the total energy and cross-helicity:

$R_E = \rho(\boldsymbol{v}\cdot\boldsymbol{v}' + \boldsymbol{v}_{\mathrm{A}}\cdot\boldsymbol{v}_{\mathrm{A}}')/2 + \rho e'$; $R_H = \rho(\boldsymbol{v}\cdot\boldsymbol{v}_{\mathrm{A}}' + \boldsymbol{v}_{\mathrm{A}}\cdot\boldsymbol{v}')/2$

[Banerjee & Galtier, 2013]

Direct cascade in 3D: summary

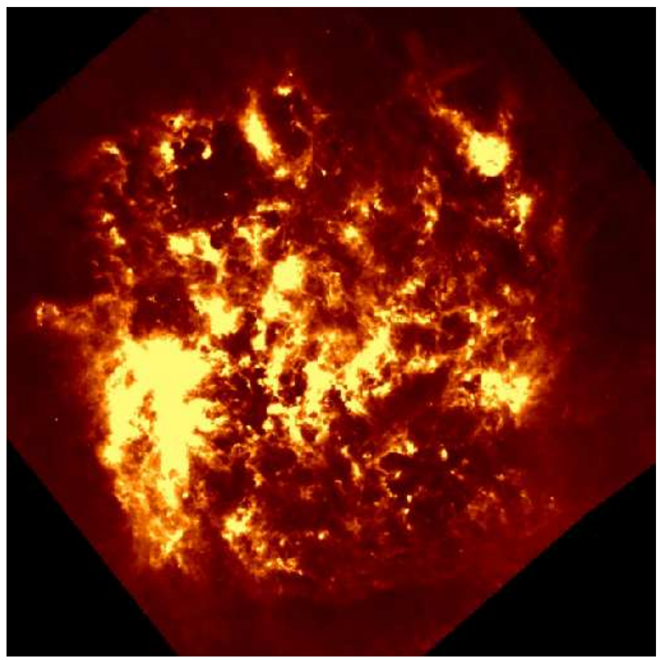
- ☞ New analytical scaling relation for supersonic isothermal turbulence.
- ☞ Linear scaling of $S_3(\mathbf{v},\mathbf{r}) \equiv \langle|\delta(\rho^{\mathbf{1/3}}\mathbf{u})|^{\mathbf{3}}\rangle$ with r previously seen in numerical experiments is dimensionally consistent with the analytical result.
- ☞ Scaling range of $\left\langle[\delta(\rho\boldsymbol{u})\cdot\delta\boldsymbol{u}]\delta u_{\|}\right\rangle \propto r$ is more extended compared to that of $S_3(\mathbf{v},\mathbf{r})$, indicating that the 'symmetric' density weighting in $S_3(\mathbf{v},\mathbf{r})$ only approximately reflects the N-S dynamics.
- ☞ A number of various statistics obtained in numerical models of supersonic turbulence (e.g. density PDF, mass–size and velocity–size correlations) agree with observational measurements in molecular clouds.
- ☞ Compressive and solenoidal modes in fully developed supersonic turbulence approach a state of equilibrium. Hence, the linear scaling of $\left\langle[\delta(\rho\boldsymbol{u})\cdot\delta\boldsymbol{u}]\delta u_{\|}\right\rangle$ is universal in the inertial range at high Mach numbers (i.e. it does not depend on how the energy is injected on large scales).

II. Quasi-2D Turbulence in Disks?

Two-component PS of FIR emission from the LMC

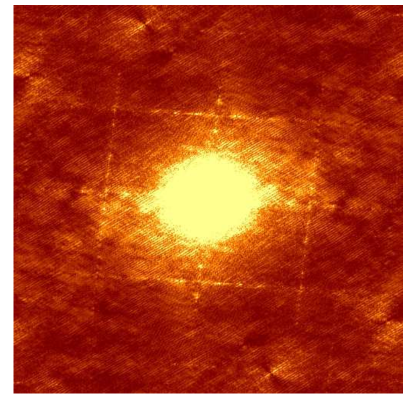
The break is at $100 - 200$ pc, interpreted as the LOS thickness of the LMC disk

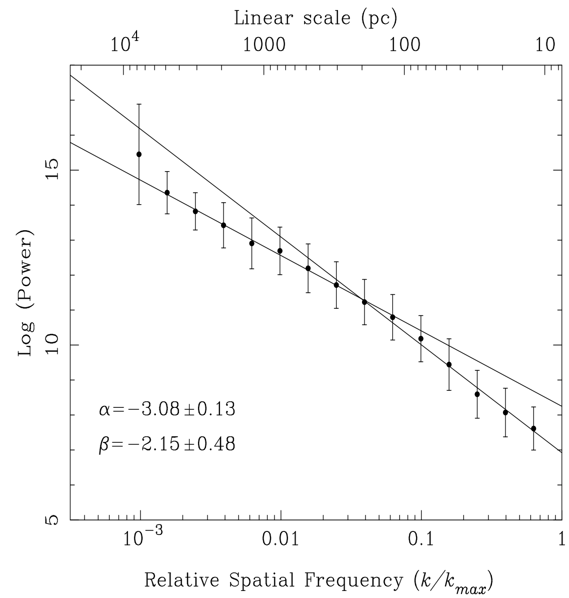
LMC − 160 microns



8.16 degrees

Fourier Transform



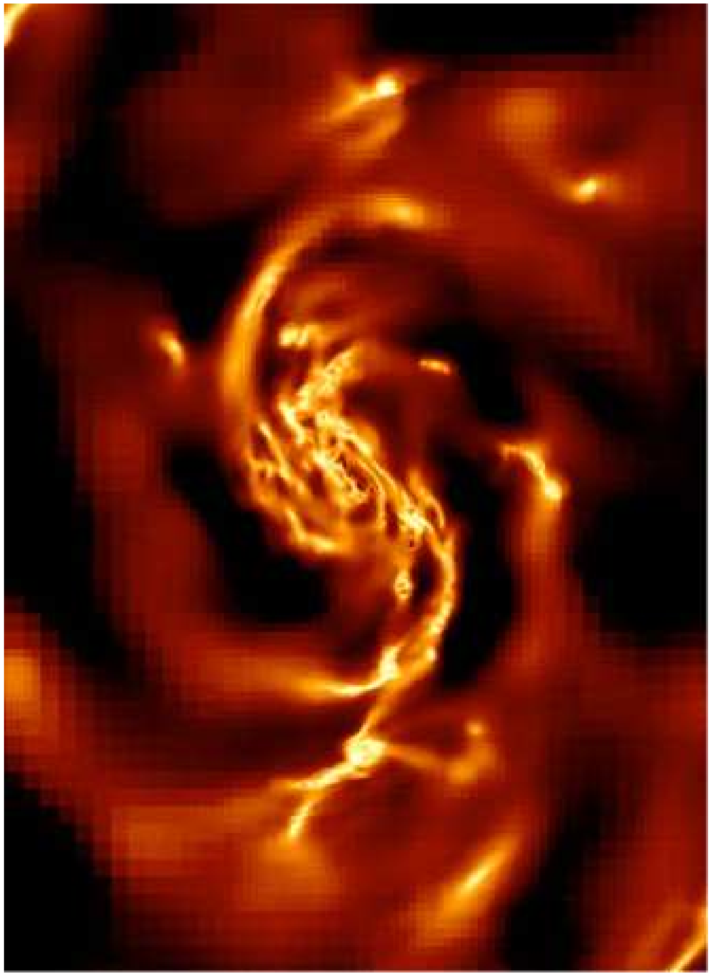


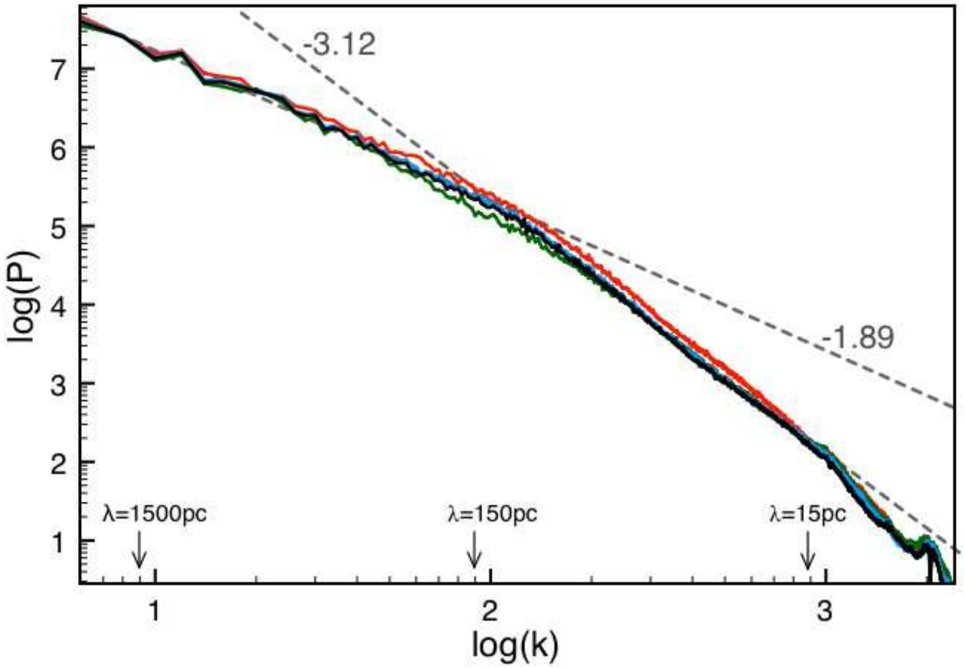
LMC with *Spitzer* at 160 μm, see also similar breaks in M33 [Combes et al. 2012]

[Block et al. 2014; also Elmegreen et al. 2001, Padoan et al. 2001]

Column density PS of LMC-sized model w. feedback

The break is at ~ 150 pc; the scale height of the disk ~ 200 pc



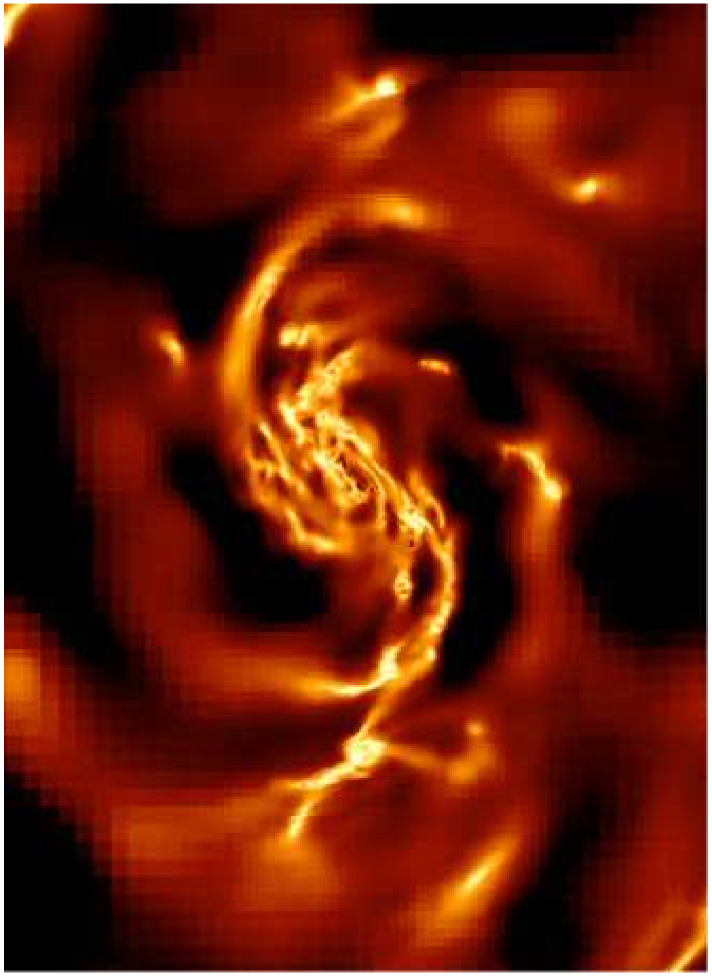


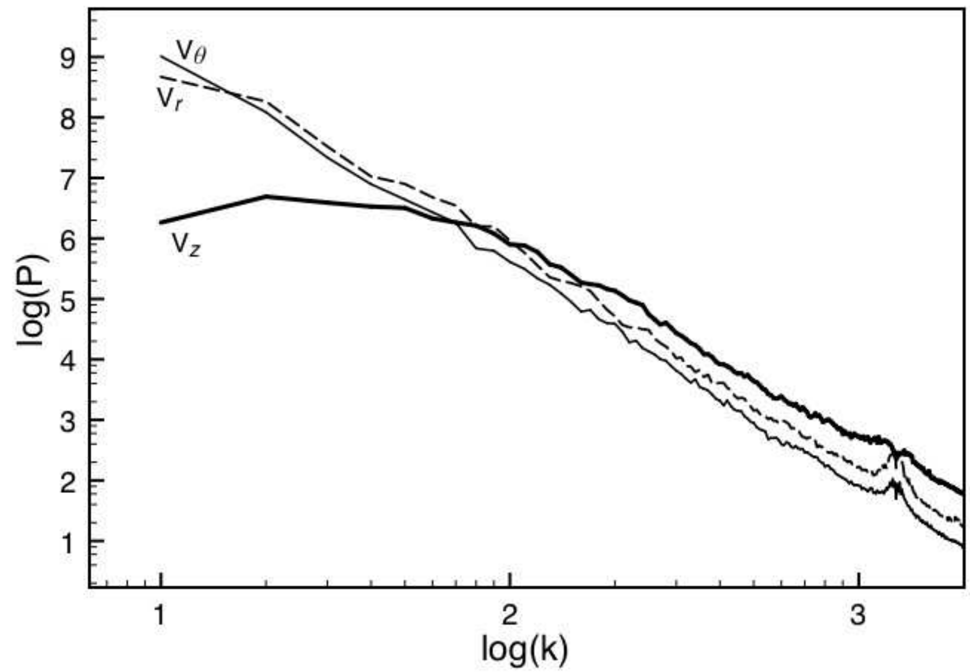
4×7 kpc snapshot from AMR simulation with Ramses ($\Delta_{\rm eff} = 0.8$ pc)

[Bournaud et al. 2010]

LMC-sized model with stellar feedback

Power spectra for the three velocity components; $P(V_z)$ flattens at scales $\gtrsim 150$ pc



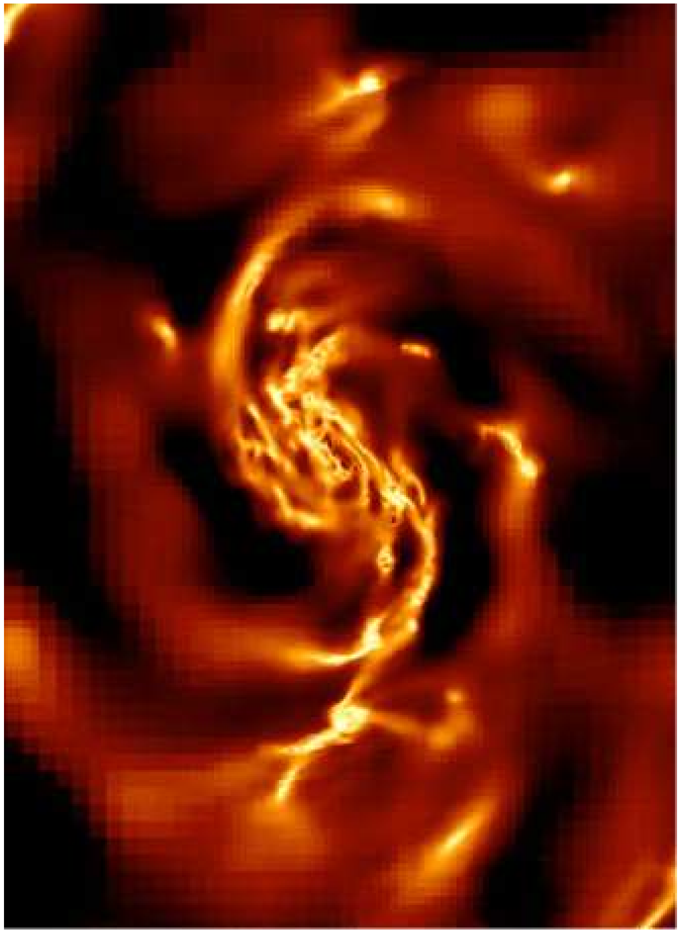


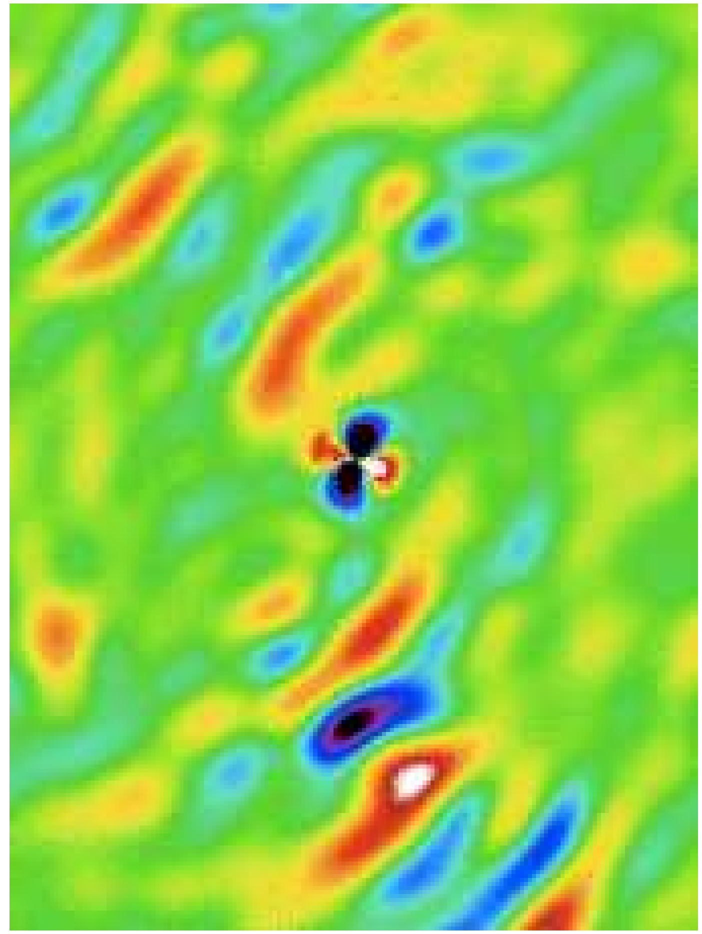
The large-scale gas motions in the disk are quasi-2D; small-scale – globally isotropic

[Bournaud et al. 2010]

LMC-sized model with stellar feedback

Vortices of various sizes, intensity, and spin directions (global rotation subtracted)





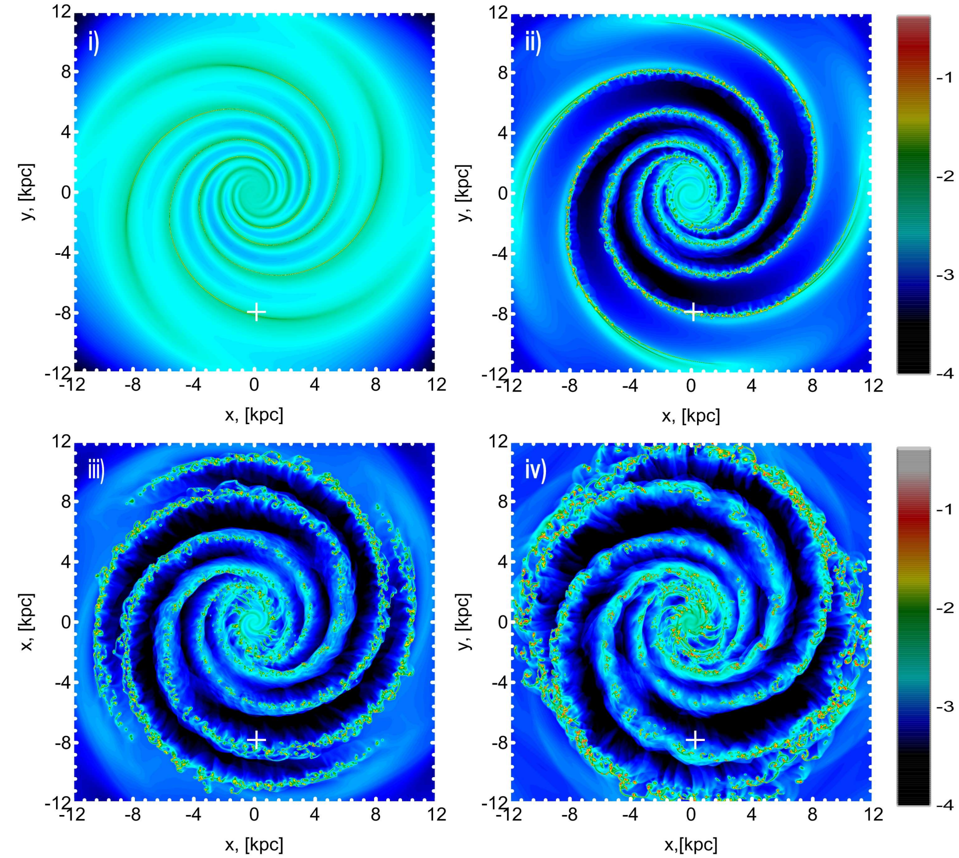
Right: Vorticity map for large-scale in-plane velocity components

Disk scale height should be resolved with at least $(100-200)\Delta$!

[Bournaud et al. 2010]

Fluid/gravitational instabilities in galactic disks

Spiral potential, gas self-gravity, heating and cooling, chemistry

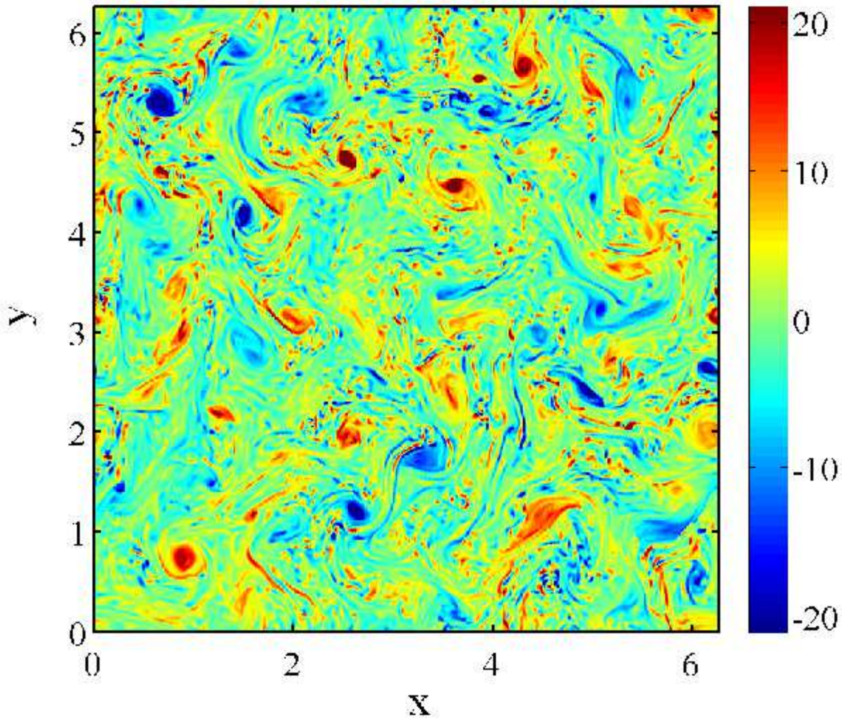


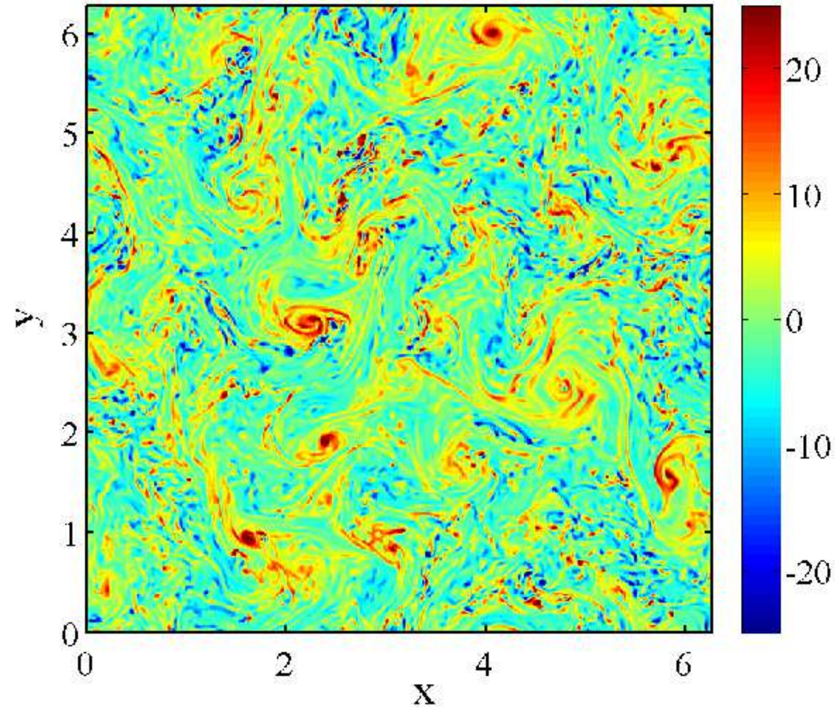
(Loading movie...)

TVD MUSCL, Cartesian grid 20×4096^2 [Khoperskov et al. 2013]

Dual cascades: Effects of rotation and confinement

Horizontal slices of vertical vorticity ω_z for two flow configurations





Left: $R = 0$, $S = 3/16$, $\epsilon_\nu/\epsilon_I = 0.71$ and **Right:** $R = 1.5$, $S = 4$, $\epsilon_\nu/\epsilon_I = 0.66$

Control parameters: $R = \frac{\Omega}{(k_f^2 \epsilon_I)^{1/3}} \sim (Ro)^{-1}$ and $S = L_z/\ell_f$

- Symmetry breaking induced by rotation
- Strong rotation and tight vertical confinement promote the inverse cascade

[Deusebio et al. 2014], see also [Pouquet & Marino 2013]

GMC rotation in models with strong feedback

Spiral potential, gas self-gravity, heating and cooling, feedback

(Loading movie...)

(Loading movie...)

SPH, 8M particles [Dobbs & Pringle 2013]

- 39%–44% of GMCs show retrograde rotation; most massive GMCs tend to be prograde
- Consistent with observations [Blitz 1993; Phillips 1999; Rosolowsky et al. 2003; Imara & Blitz 2011; Imara et al. 2011]

Questions...

- ☞ Is the inverse energy cascade (coupled with gravity) at work in ISM of disk-like galaxies?
- ☞ What determines rotational properties of GMCs?
- ☞ How star formation self-regulates accross scales?
- ☞ What feeds interstellar turbulence in galactic disks?
- ☞ At what scale most of the energy is injected?
- ☞ Where does this energy go?
- ☞ What if $\epsilon_I \neq \epsilon_\nu$?
- ☞ Why could not I come to KITP in May?

Acknowledgments

This research was supported in part by the National Science Foundation through grants AST-0808184, AST-0908740, and AST-1109570 as well as through XSEDE resources provided by NICS and SDSC (MCA07S014) and through DOE Office of Science INCITE-2009 and DD-2010,2014 awards allocated at NCCS (ast015/ast021/ast104).