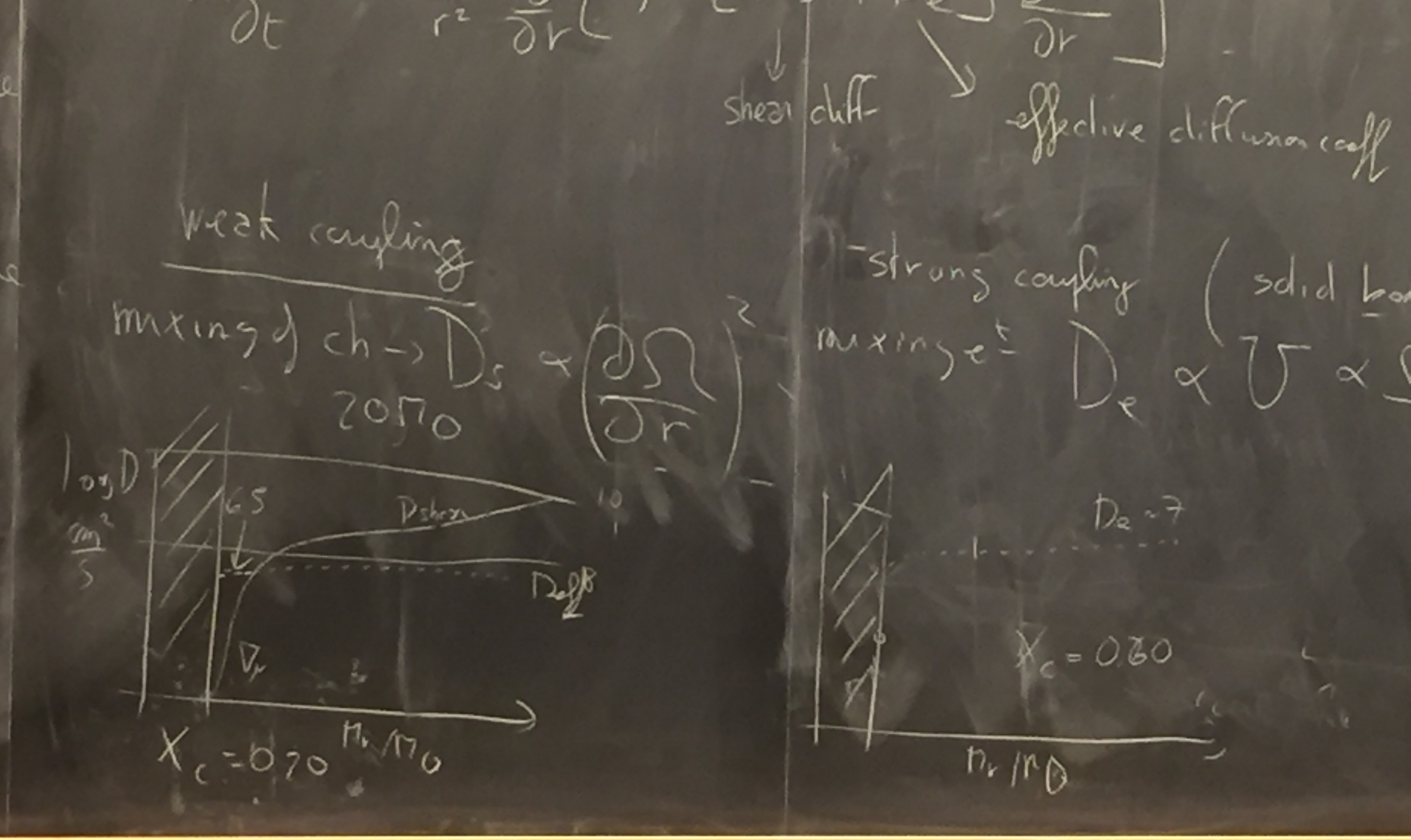


Where we stopped yesterday
(blackboard courtesy Georges Meynet)

Radboud Universiteit Nijmegen

KU LEUVEN

From a Kepler LC to core overshoot, near-core rotation, and envelope mixing

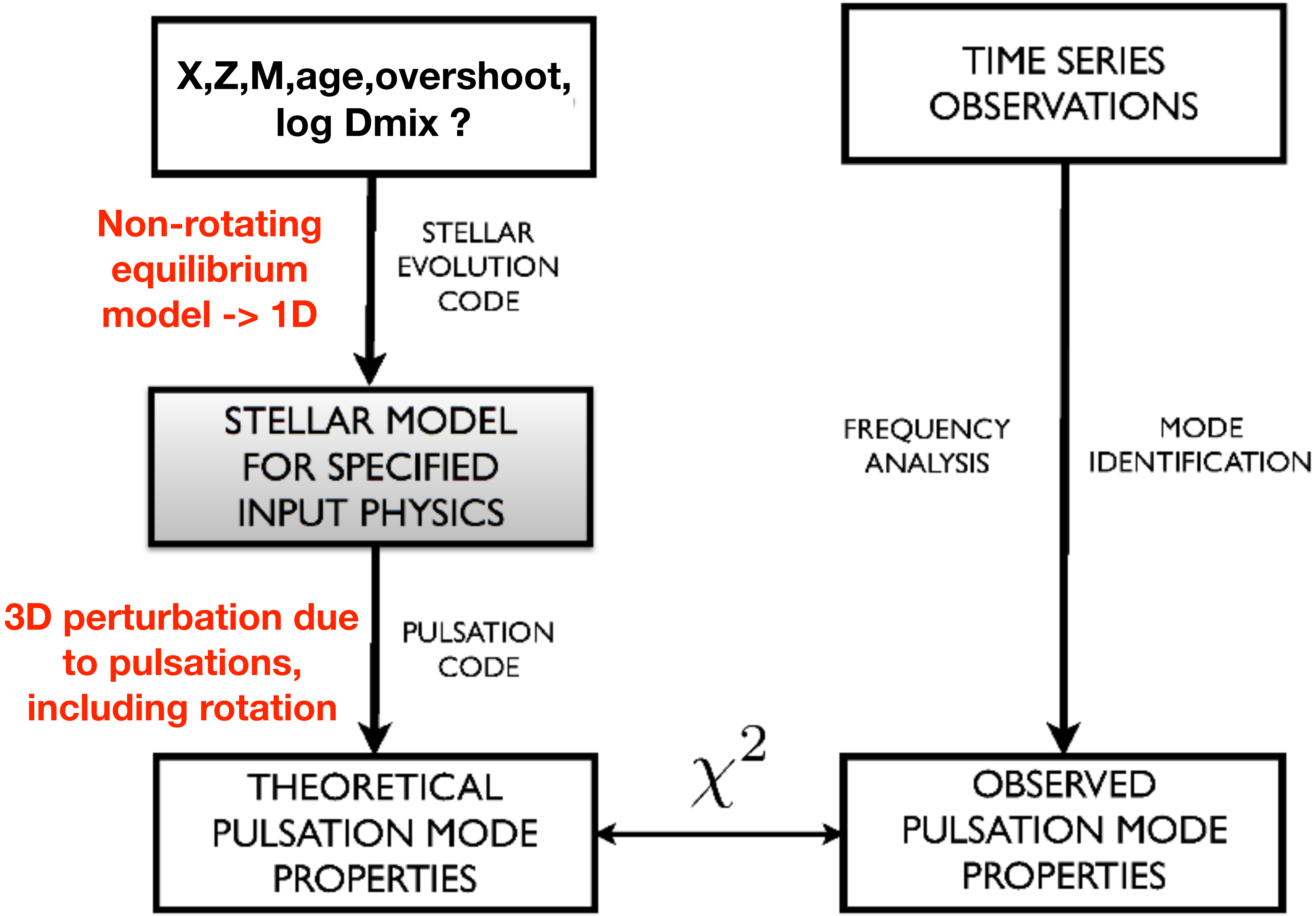
Starring
May Gade Pedersen
Timothy Van Reeth

KITP, 7 April 2017

MAMSIE

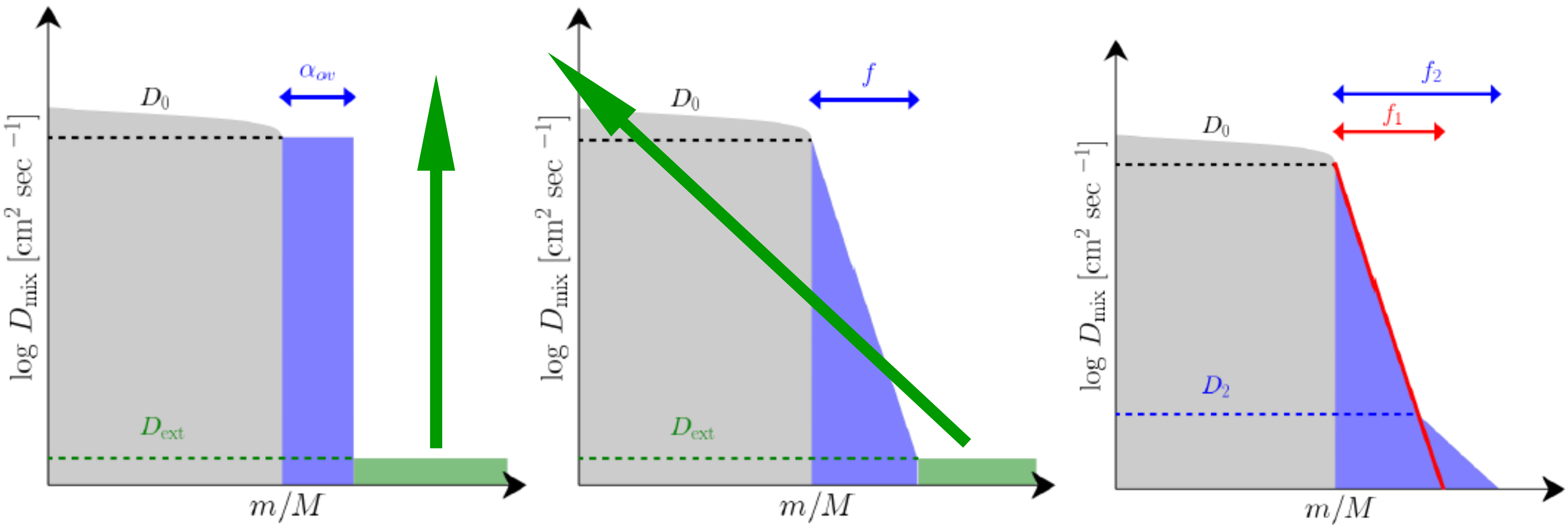
erc 10 years

Forward seismic modelling: simplest case



Core overshoot: value & shape

Core overshoot + envelope mixing

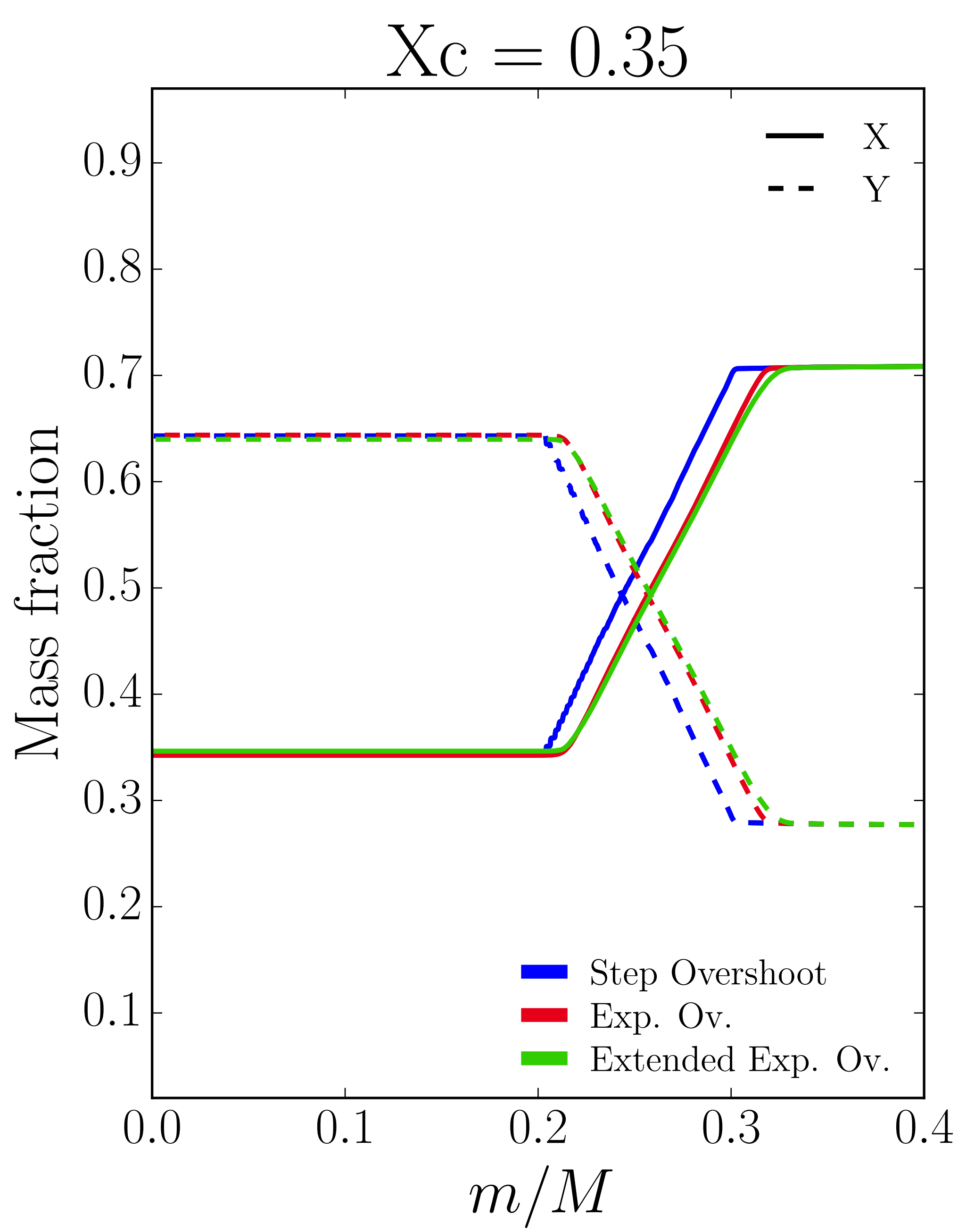


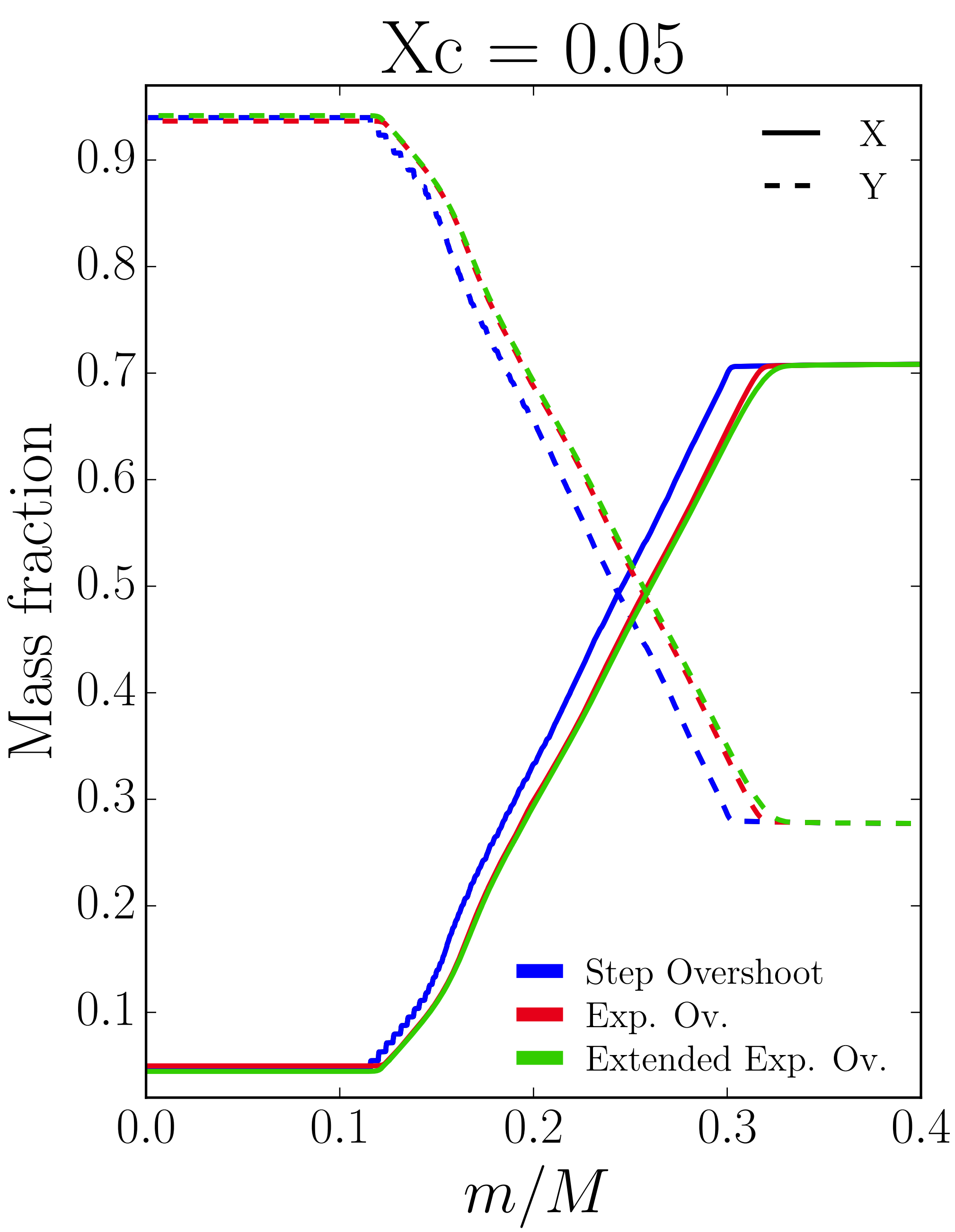
PhD Thesis, May Gade Pedersen:
plug in results from 2D/3D hydrodynamical simulations
(cf. tutorials Tami, Daniel next Thursday)

Core overshoot: value & shape

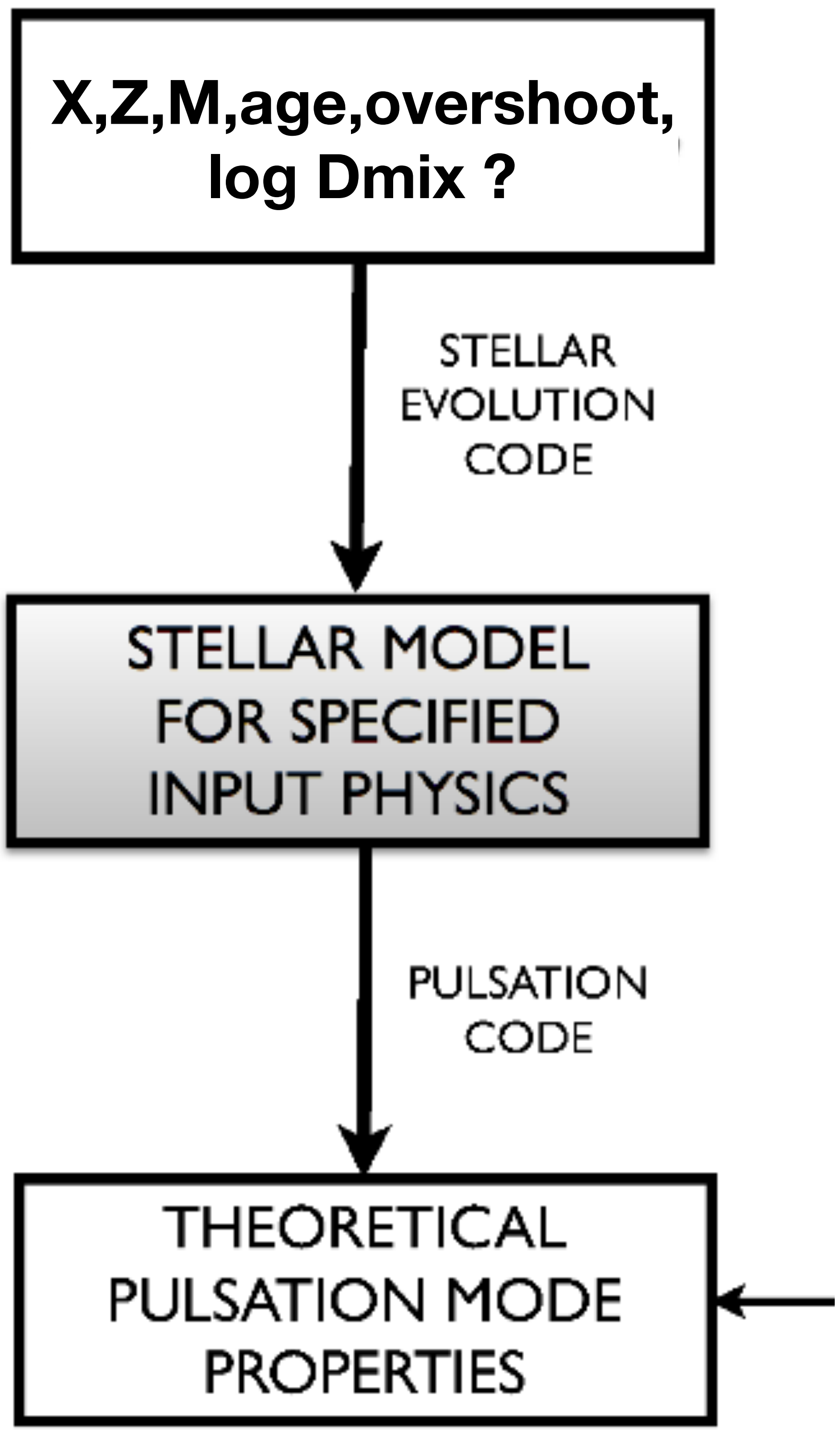
H and He profiles

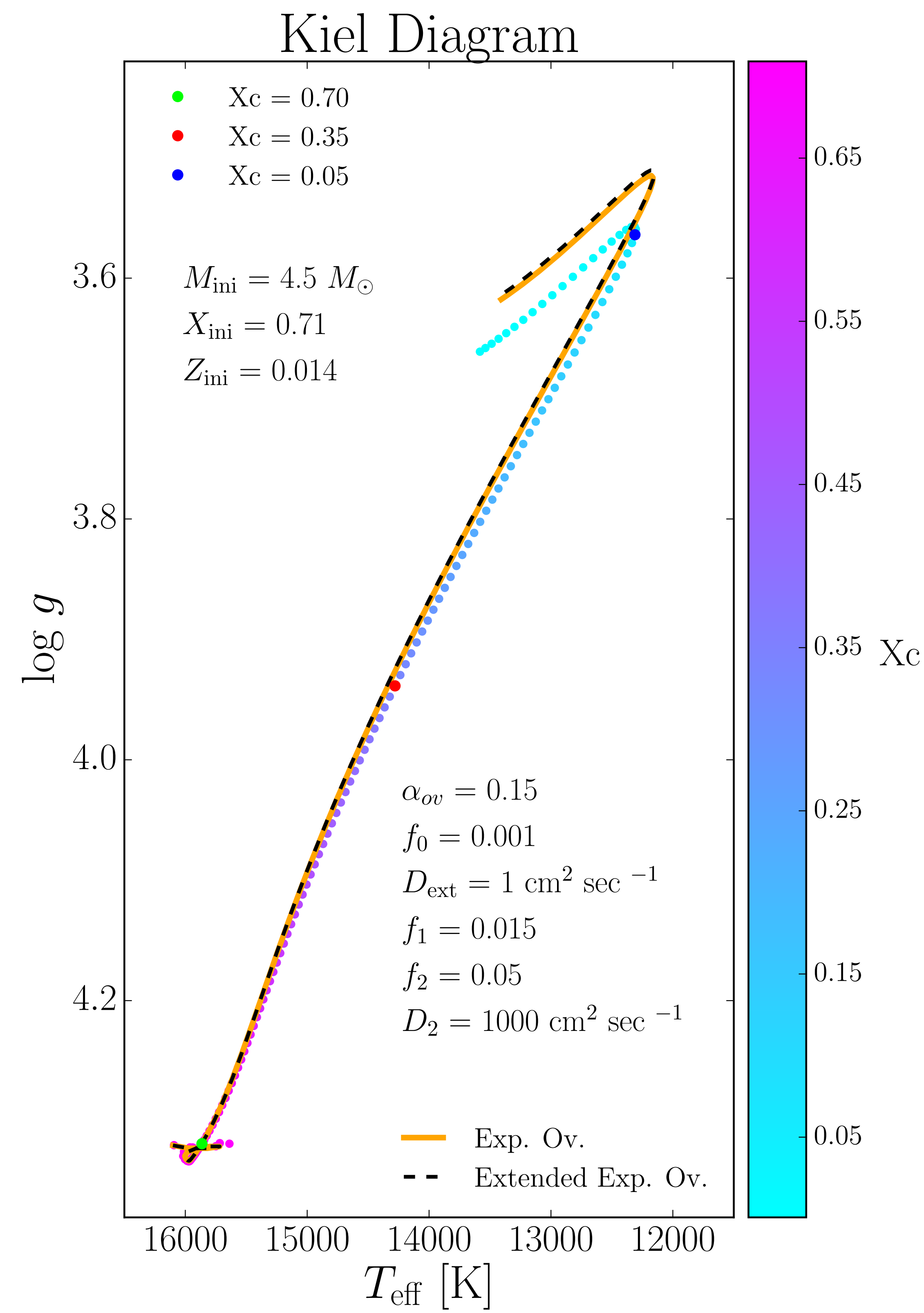
PhD Thesis, May Gade Pedersen





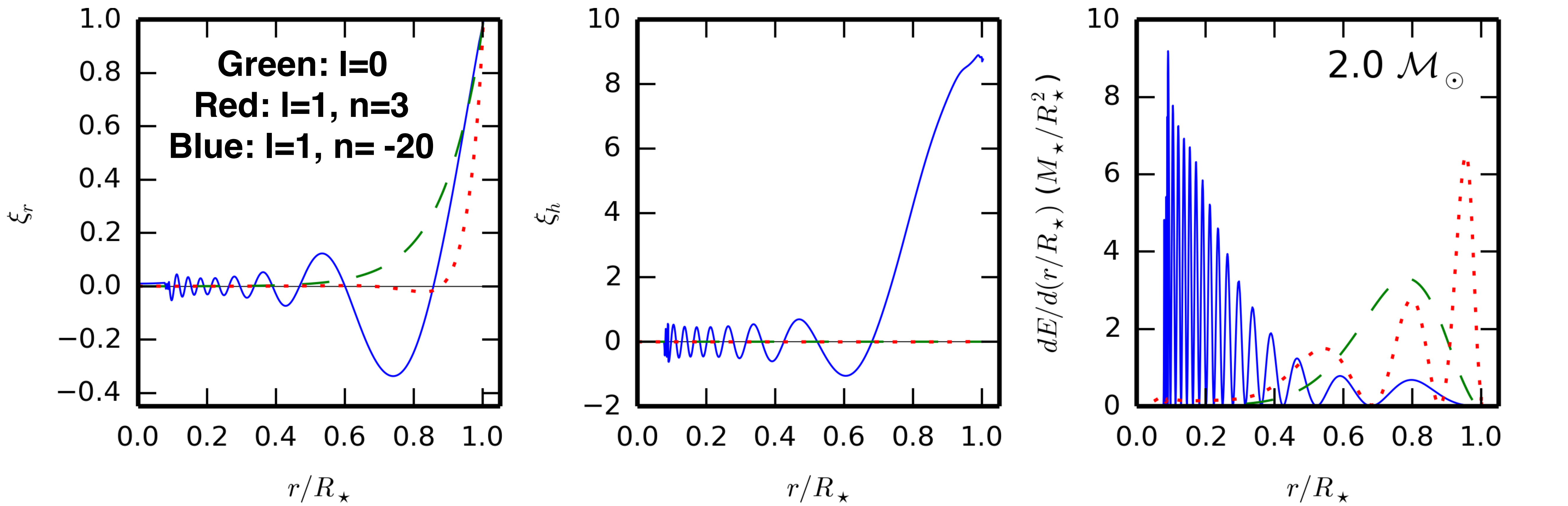
Seismic modelling: interior, not HRD or Kiel





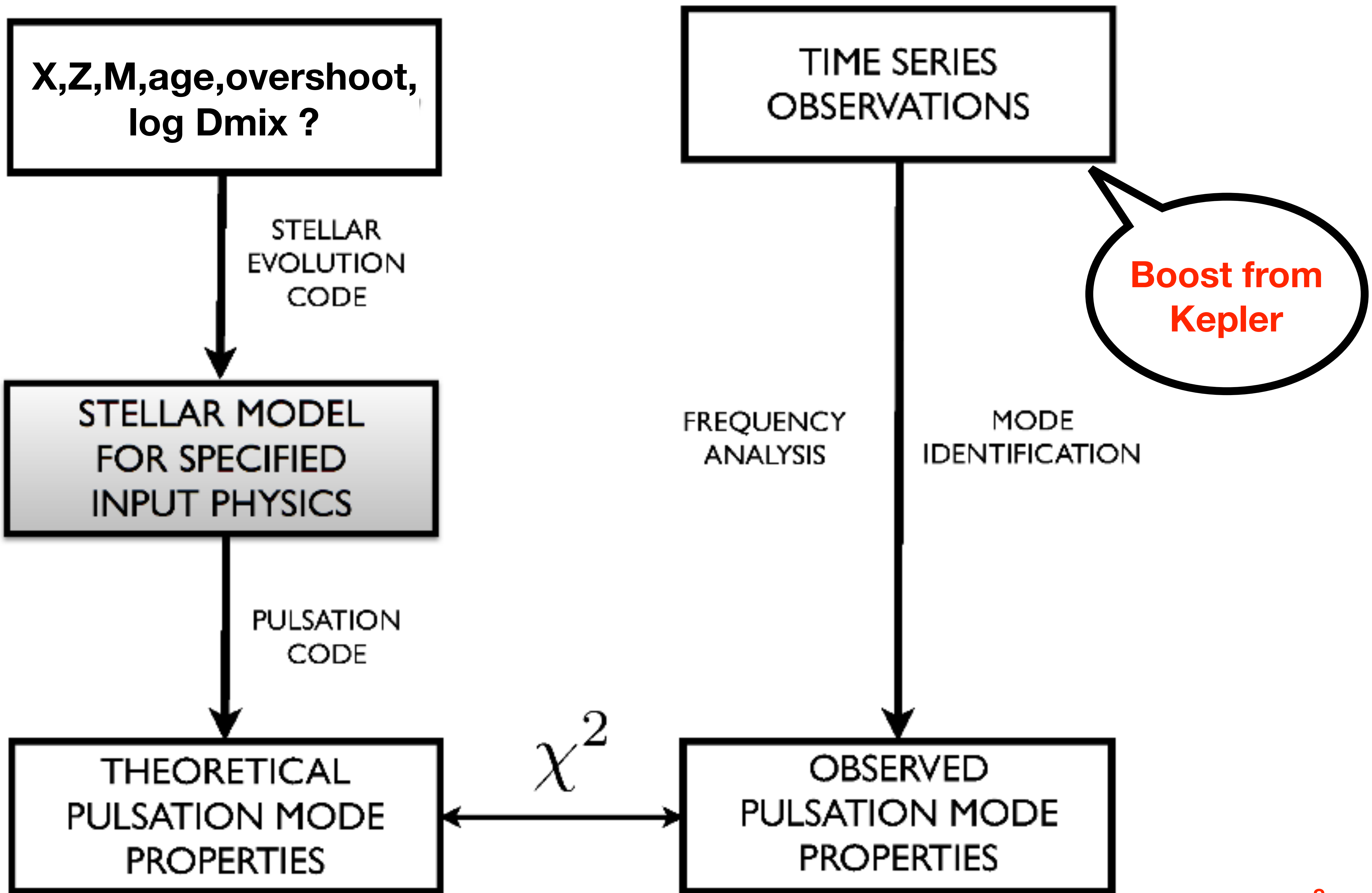
Beauty of gravity modes (only since Kepler)

Allow probing of near-core regions & determine X, Z, M, age, overshoot, log D_{mix} in B & F-type stars

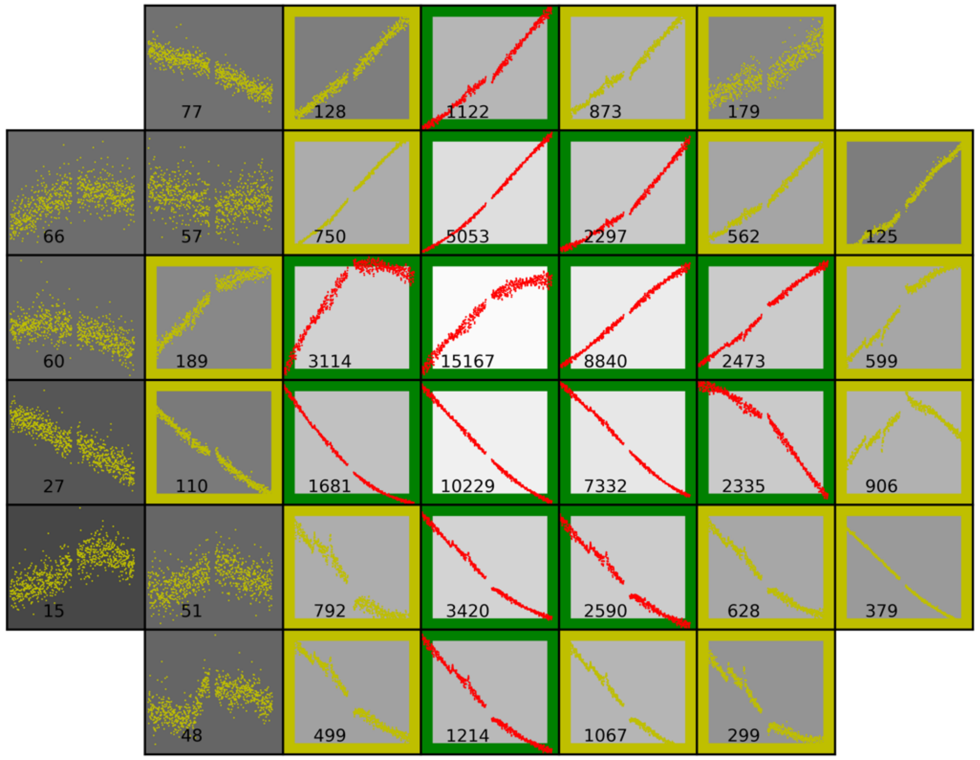


PhD thesis Valentina Schmid (2016)

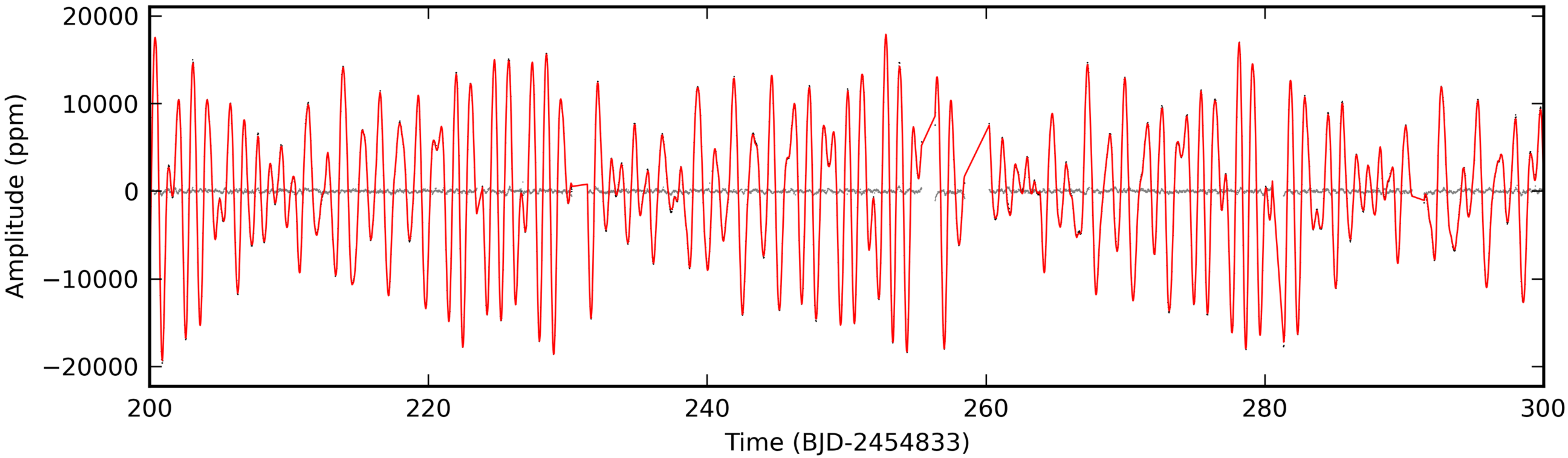
Forward seismic modelling: simplest case



Kepler LCs from raw pixel data (MAST)



Pápics et al. (2014):
LC: 29.42 min (sum of 270 exposures)
Nyquist frequency: 24.47/d



Fourier analysis

Fourier transform of $x(t)$:

$$F(f) \equiv \int_{-\infty}^{+\infty} x(t) \exp(2\pi \mathrm{i}\, f t) dt$$

Fourier transform $F(f)$ of sum of harmonic functions with frequencies $f_1, \ldots, f_n$ and amplitudes $A_1, \ldots, A_n$:

$$x(t) = \sum_{k=1}^{n} A_k \exp(2\pi \mathrm{i}\, f_k t) : \quad F(f) = \sum_{k=1}^{n} A_k \delta(f - f_k)$$

For $x(t) =$ sine with frequency f_1, $F(f) \neq 0$ for $f = \pm f_1$
For $x(t) =$ sum of n harmonic functions with frequencies $f_1, \ldots, f_n$, $F(f) =$ sum of δ-functions $\neq 0$ for $\pm f_1, \ldots, \pm f_n$

Real data set: $x(t)$ known for a discrete number of times t_j, $j=1, \ldots, N$

Discrete Fourier transform

$$F_N(f) \equiv \sum_{j=1}^{N} x(t_j) \exp(2\pi \mathrm{i}\, f t_j)$$

$F_N \neq F$! but connected through window function: $w_N(t) \equiv \frac{1}{N} \sum_{j=1}^{N} \delta(t - t_j)$

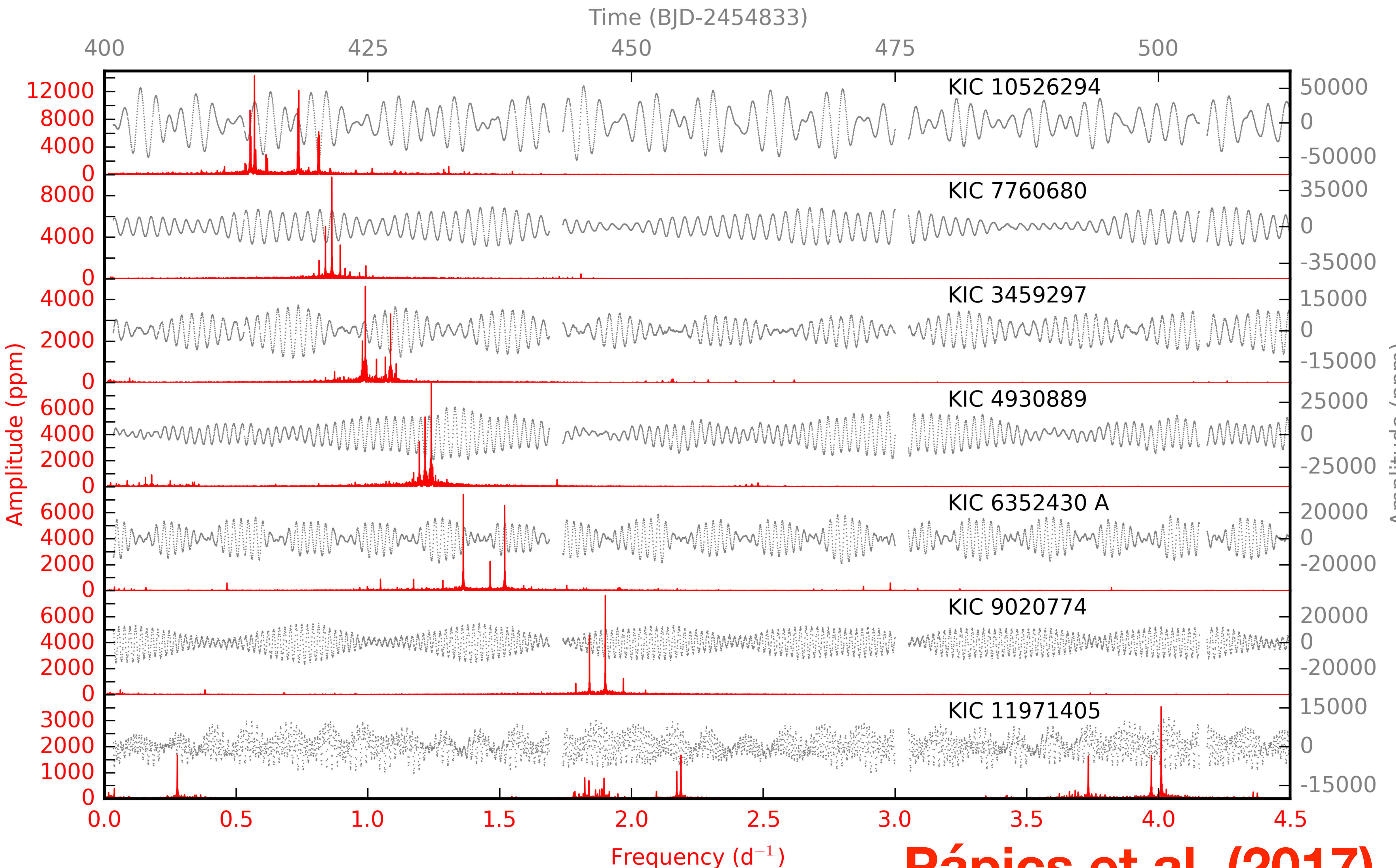
Hence: $$\frac{F_N}{N} = \int_{-\infty}^{+\infty} x(t) w_N(t) \exp(2\pi \mathrm{i}\, f t) dt$$

Discrete Fourier transform of window function = spectral window $W_N(f)$:

$$W_N(f) = \frac{1}{N} \sum_{j=1}^{N} \exp(2\pi \mathrm{i}\, f t_j)$$

Discrete Fourier transform = convolution of spectral window and Fourier transform: $F_N(f)/N = F(f) * W_N(f)$

Detecting gravity-mode oscillations



Pápics et al. (2017)

Prewhitening & residuals

Least-squares fitting with *f* fixed: $x_i(t_i) = A\sin\left[2\pi(ft_i + \psi)\right] + C$

Variance reduction in ∈ *[0,1]*:

$$1 - \frac{\sum_{i=1}^{N}\left\{x_i - \left[A\sin\left(2\pi(ft_i+\psi)\right) + C\right]\right\}^2}{\sum_{i=1}^{N}\left(x_i - \overline{x}\right)^2}$$

Search for new frequencies in **residuals** $R_i(f) \equiv x_i - x_i^c(f)$ with

$$x_i^c(f) \equiv A\sin\left[2\pi(ft_i + \psi)\right] + C$$

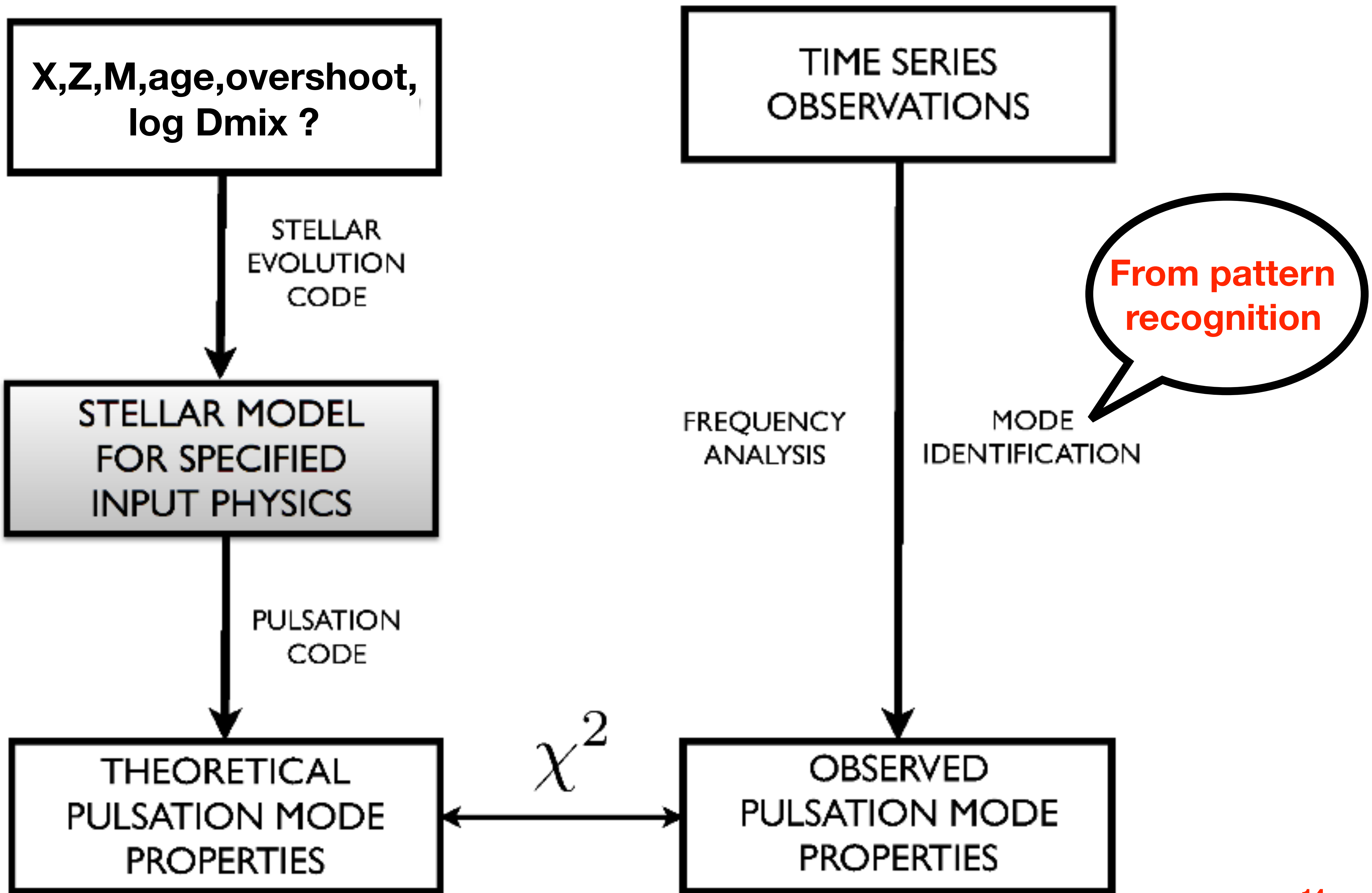
and so on. BUT: frequency is only known up to certain precision:

optimising f within uncertainty interval is necessary: $\sigma_f = \dfrac{\sqrt{6}\sigma_R}{\pi\sqrt{N}AT}$
do NLLS fitting + prewhitening

Radboud Universiteit Nijmegen

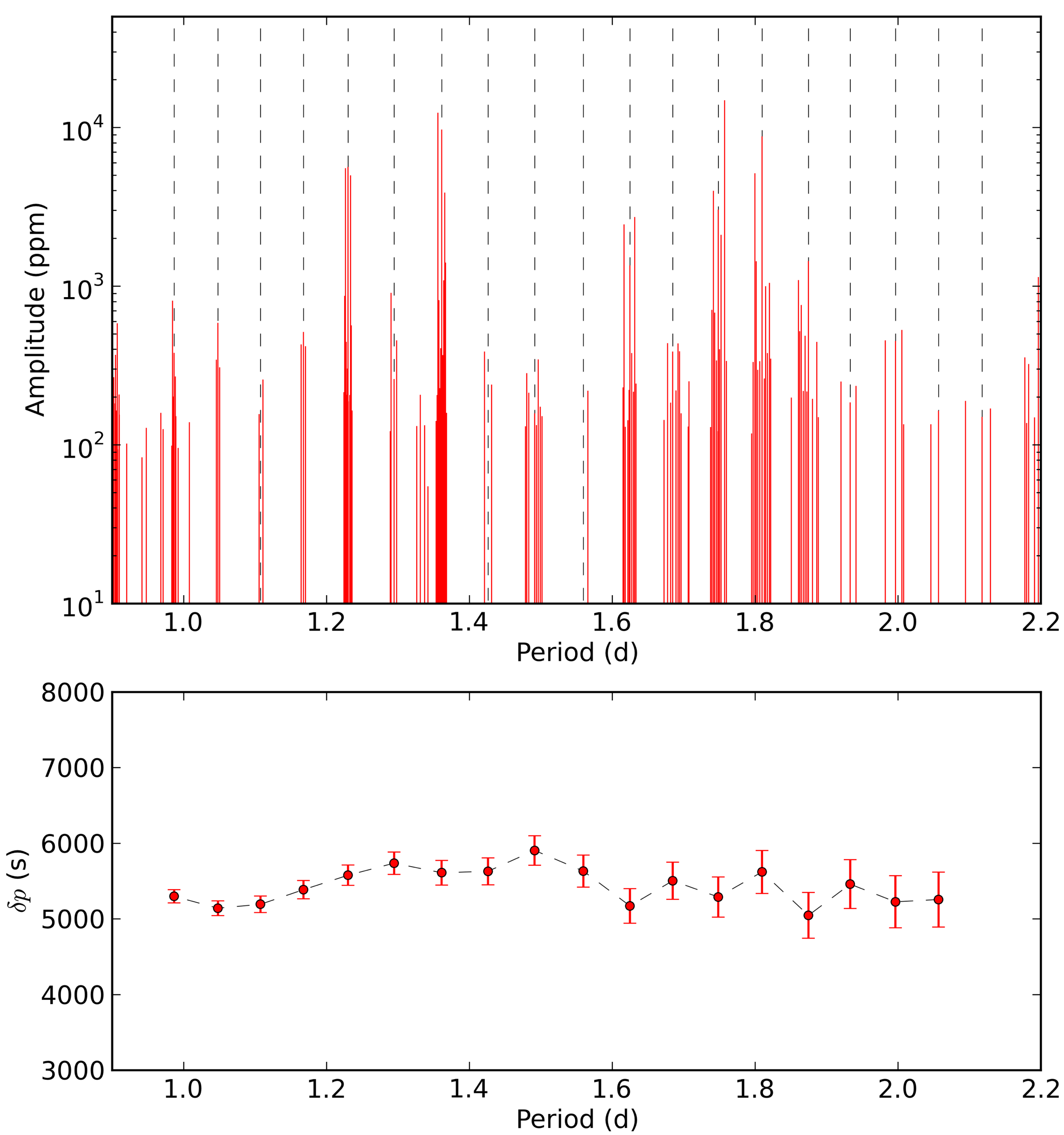
KU LEUVEN

Forward seismic modelling: simplest case

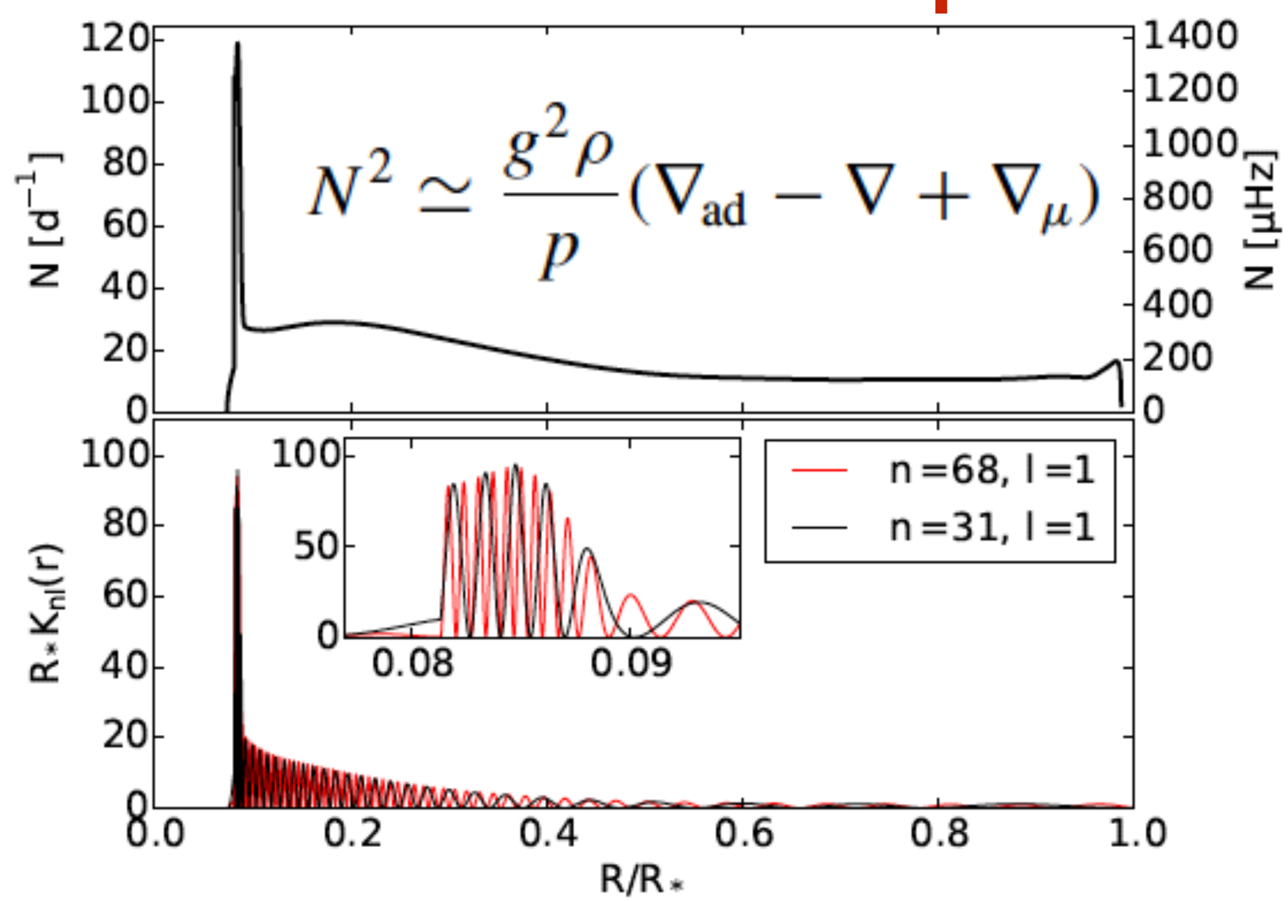


MESA summerschool 2016

$$P_{nl} = \frac{\Pi_0}{\sqrt{l(l+1)}}(n+\alpha_g), \qquad \Delta\Pi_l = \frac{\Pi_0}{\sqrt{l(l+1)}}, \qquad \Pi_0 = 2\pi^2 \left(\int_{r_1}^{r_2} N \frac{\mathrm{d}r}{r} \right)^{-1}$$

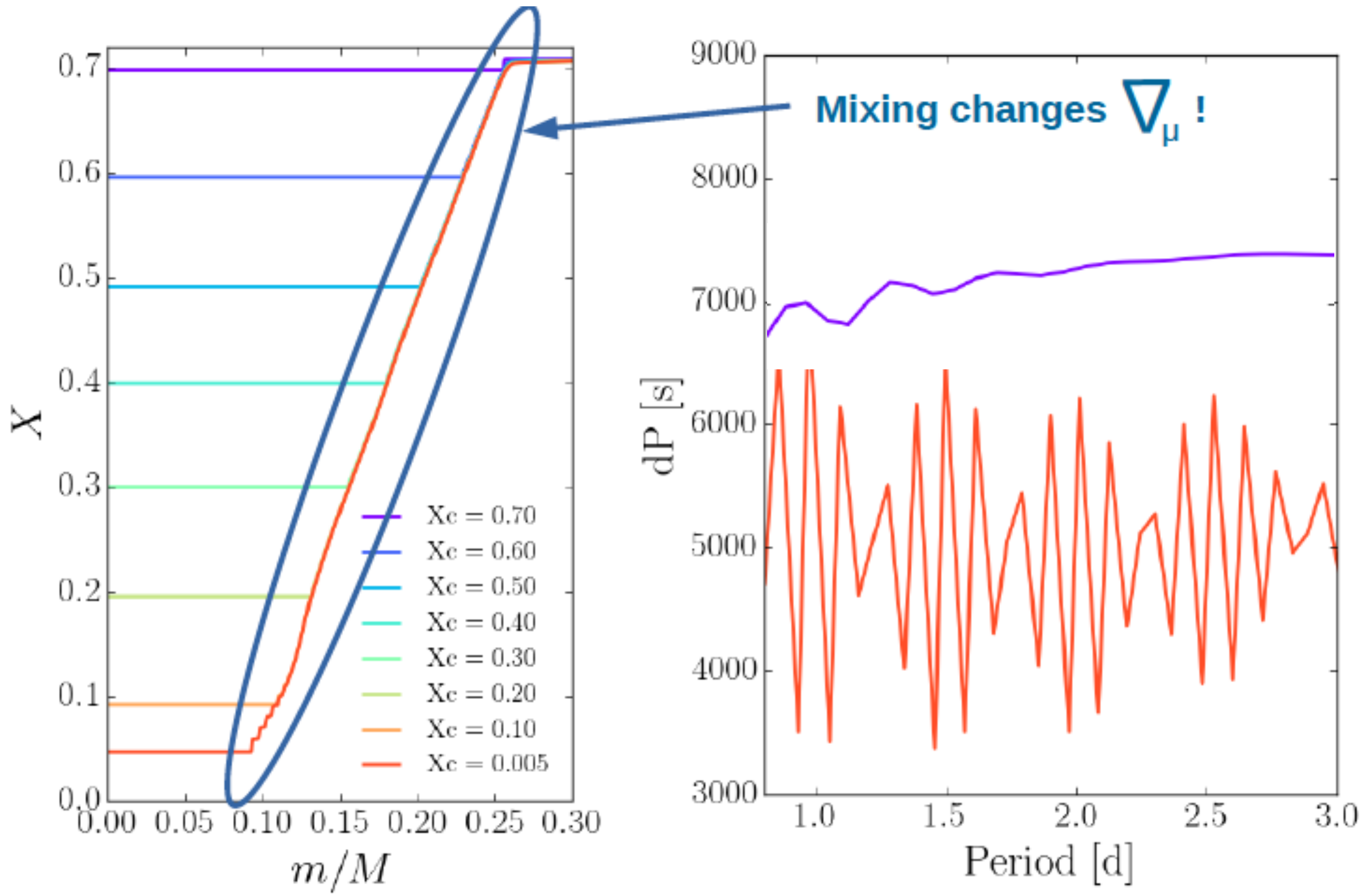


No rotation, no magnetic field, convective core, radiative envelope



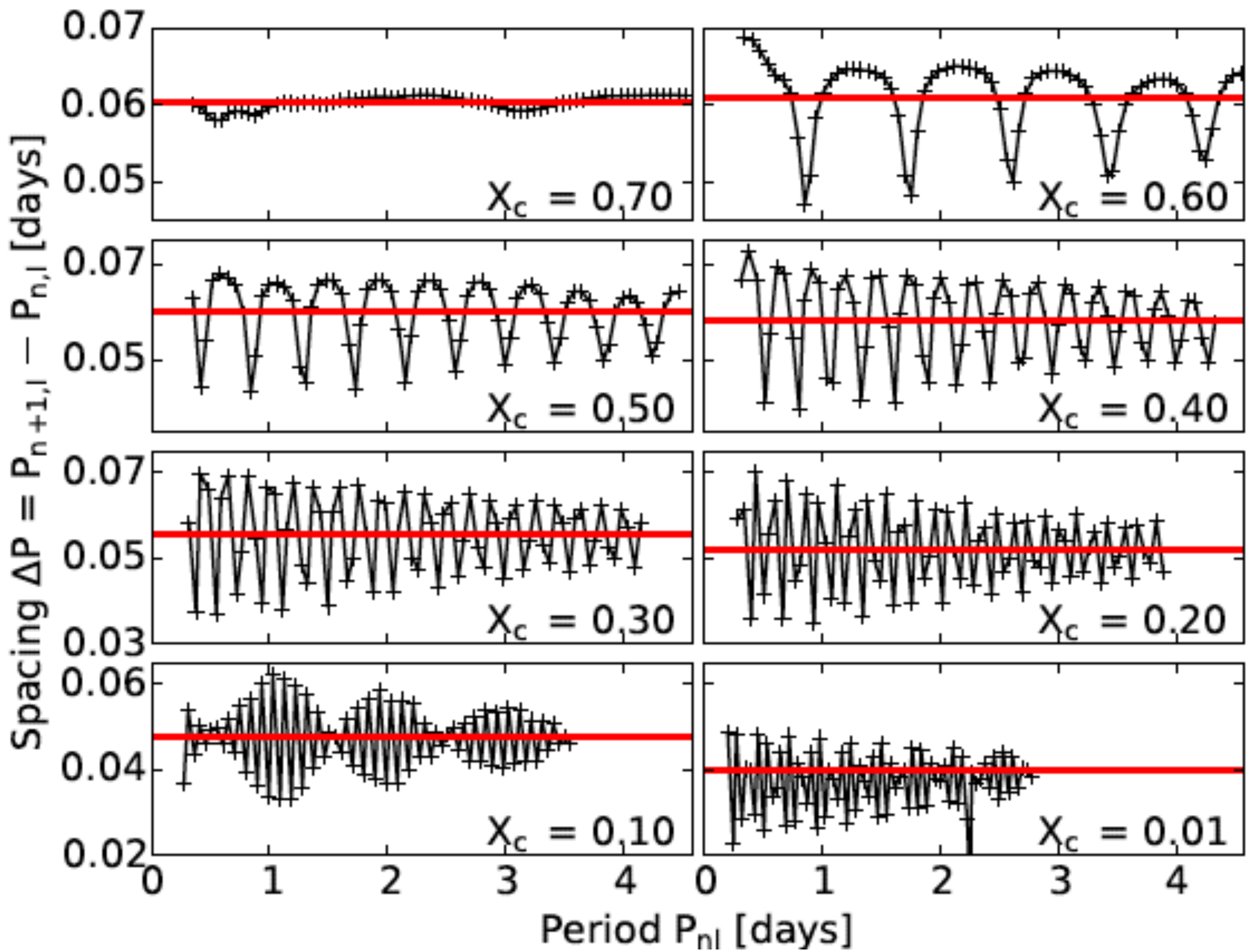
Receding core & μ-gradient zone

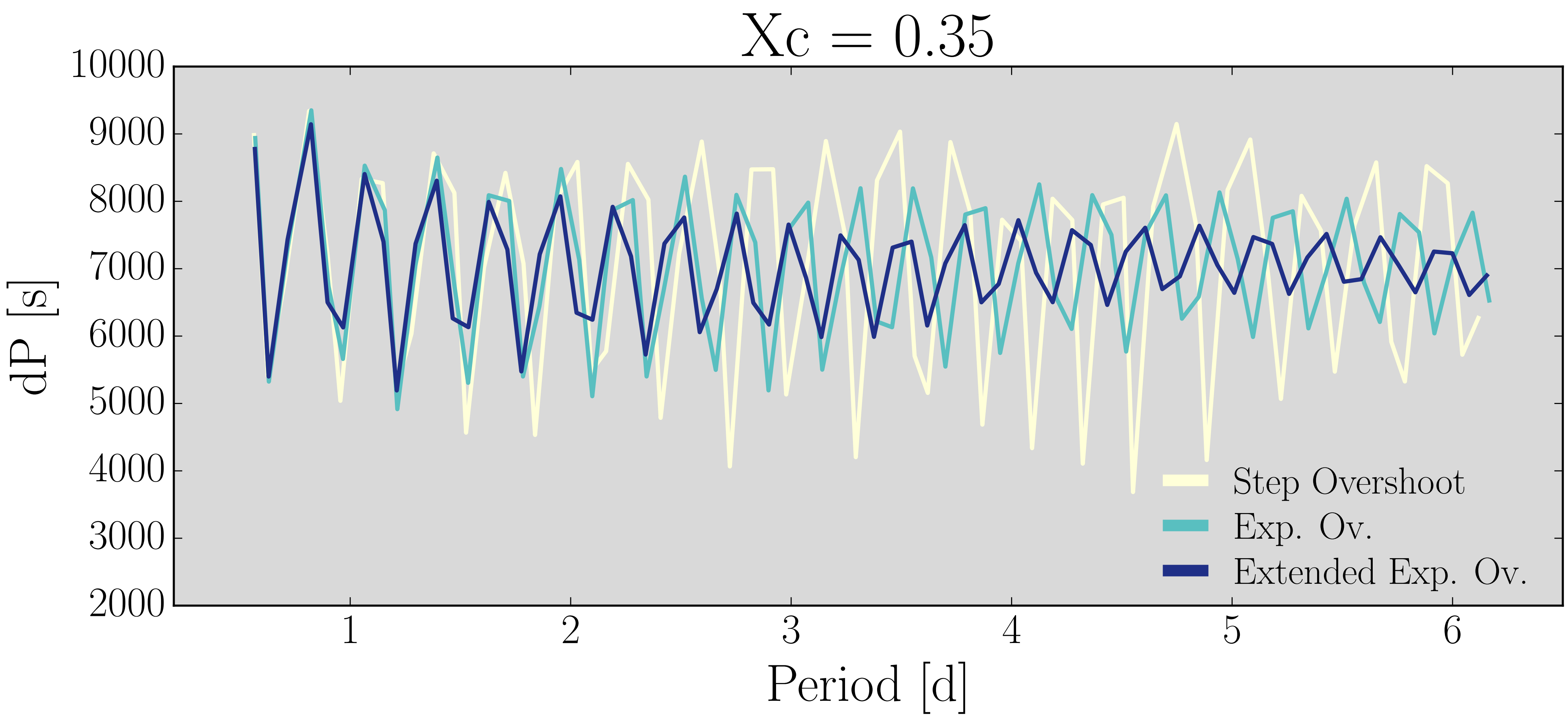
PhD Thesis, May Gade Pedersen



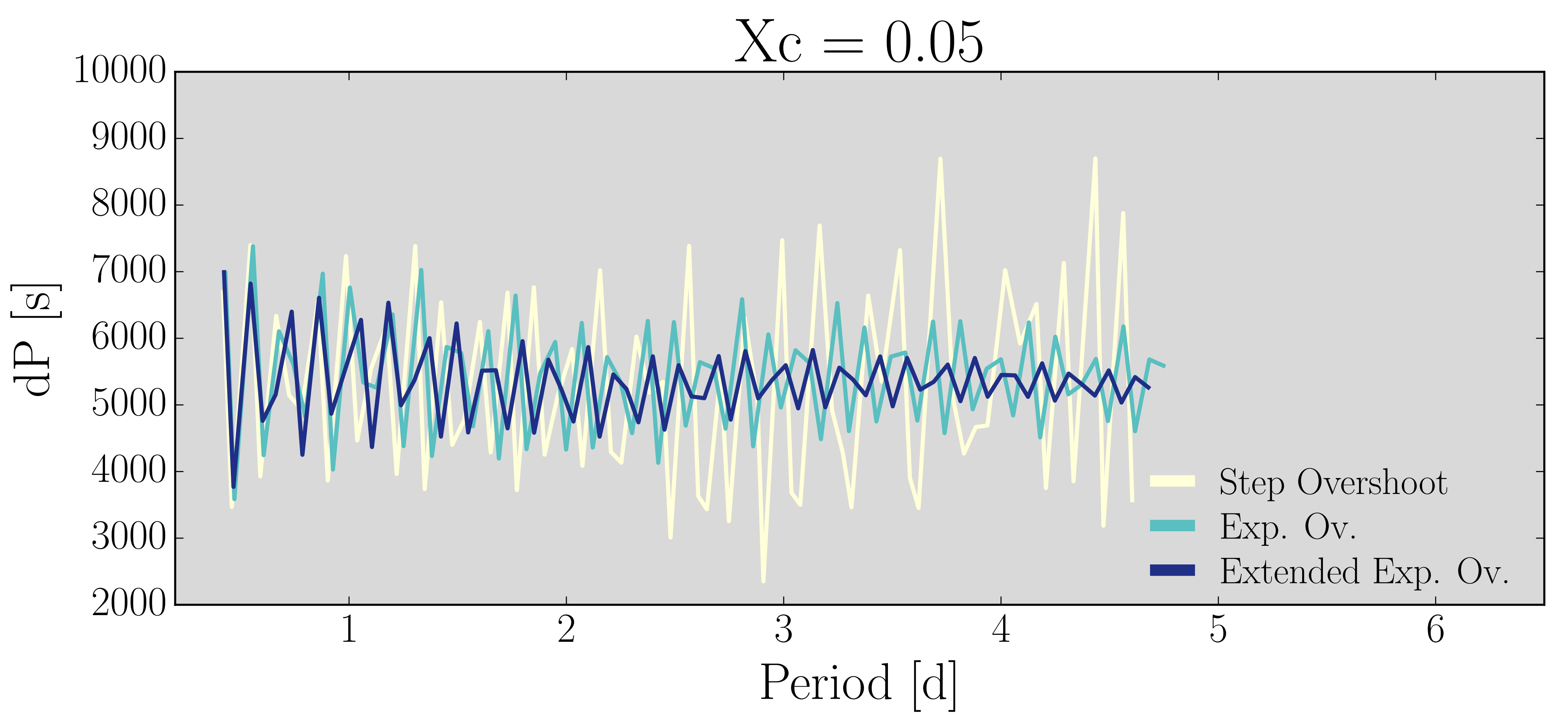
Minilab 2: stellar pulsations

Results for the $3M_\odot$ model



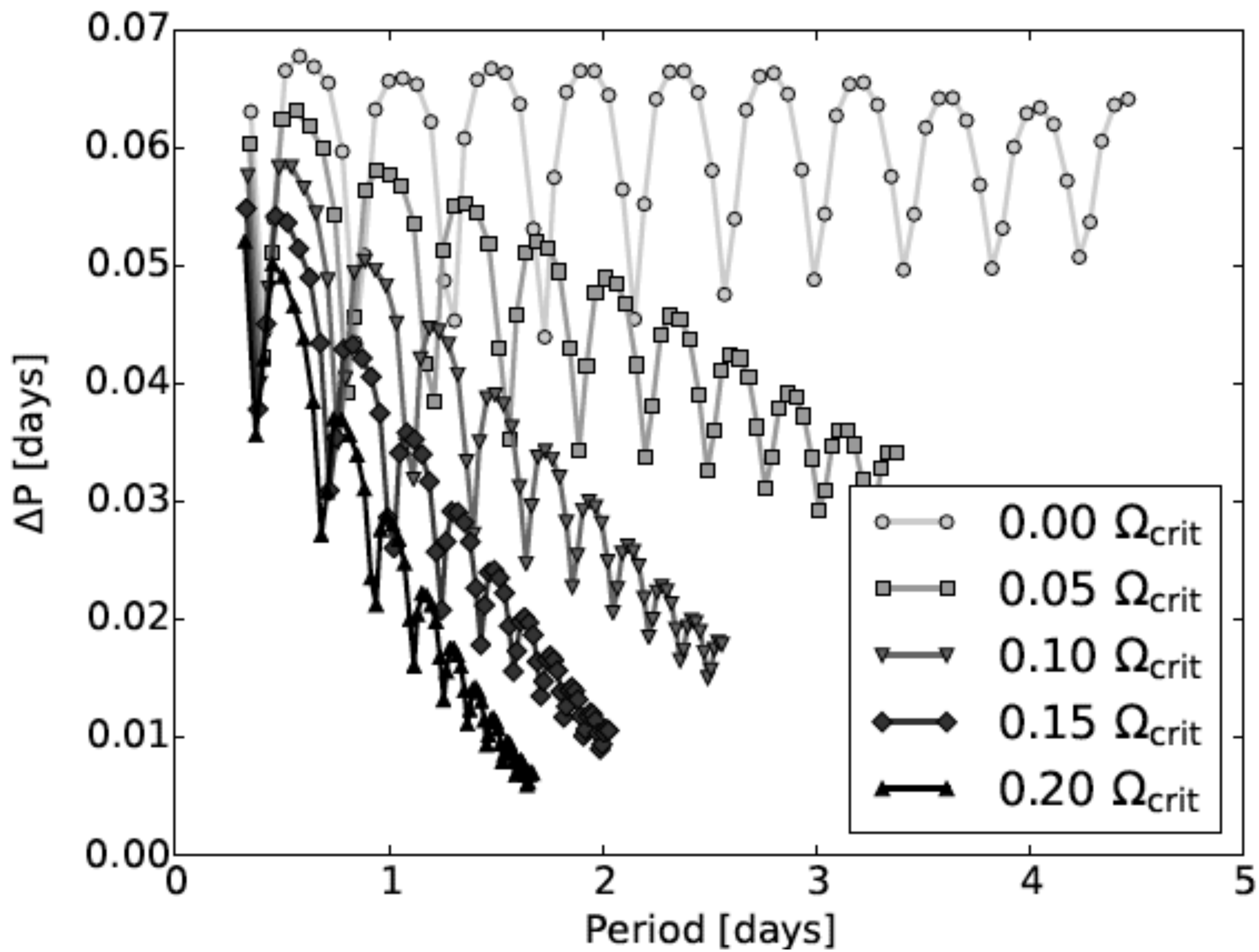


PhD Thesis, May Gade Pedersen



Minilab 3: rigid stellar rotation

Results for the $3M_{\odot}$ model at $X_c = 0.50$



Dips & chemical mixing

Various types of mixing in MESA

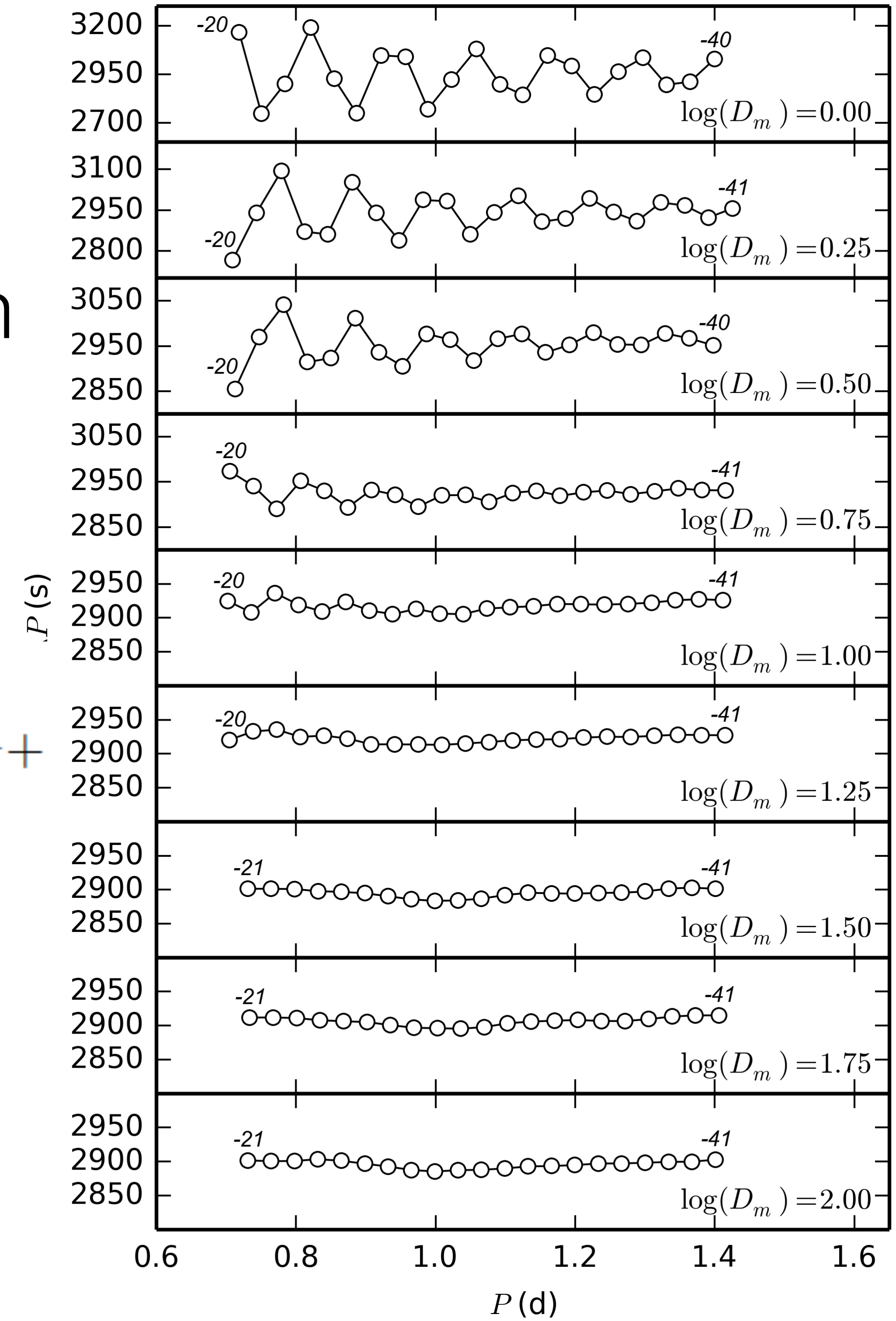
- Mixing disconnected with rotation
- Rotationally induced mixing

$$D = D_{Non-rot} + f_c (D_{DSI,factor} \cdot D_{DSI} + D_{SH,factor} \cdot D_{SH} + D_{SSI,factor} \cdot D_{SSI} + D_{ES,factor} \cdot D_{ES} + D_{GSF,factor} \cdot D_{GSF} + D_{ST,factor} \cdot D_{ST})$$

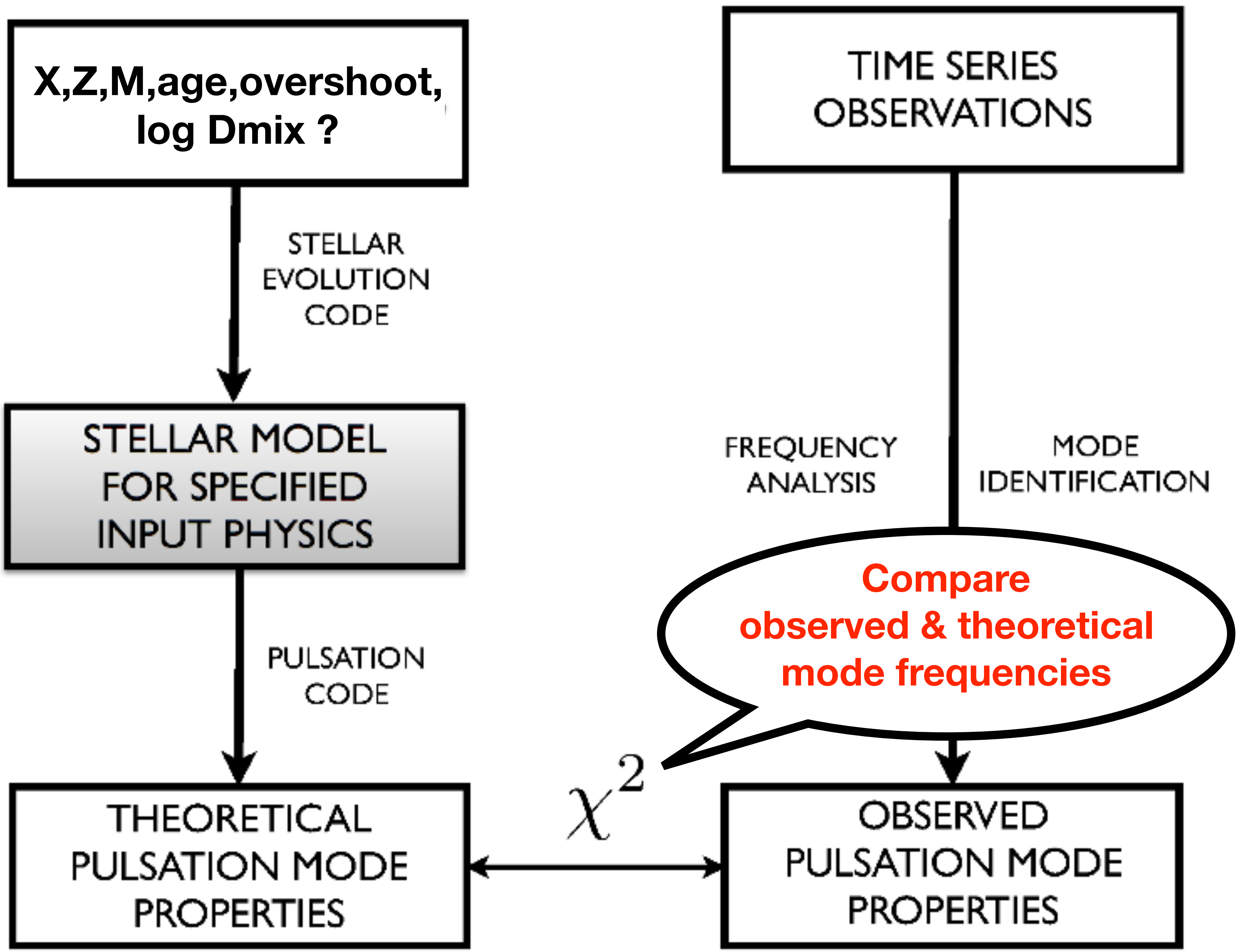
→ **log Dmix**

$M = 1.76\ M_\odot, Z = 0.0125, f_{ov} = 0.005, X_c = 0.20$

Schmid & Aerts (2016)



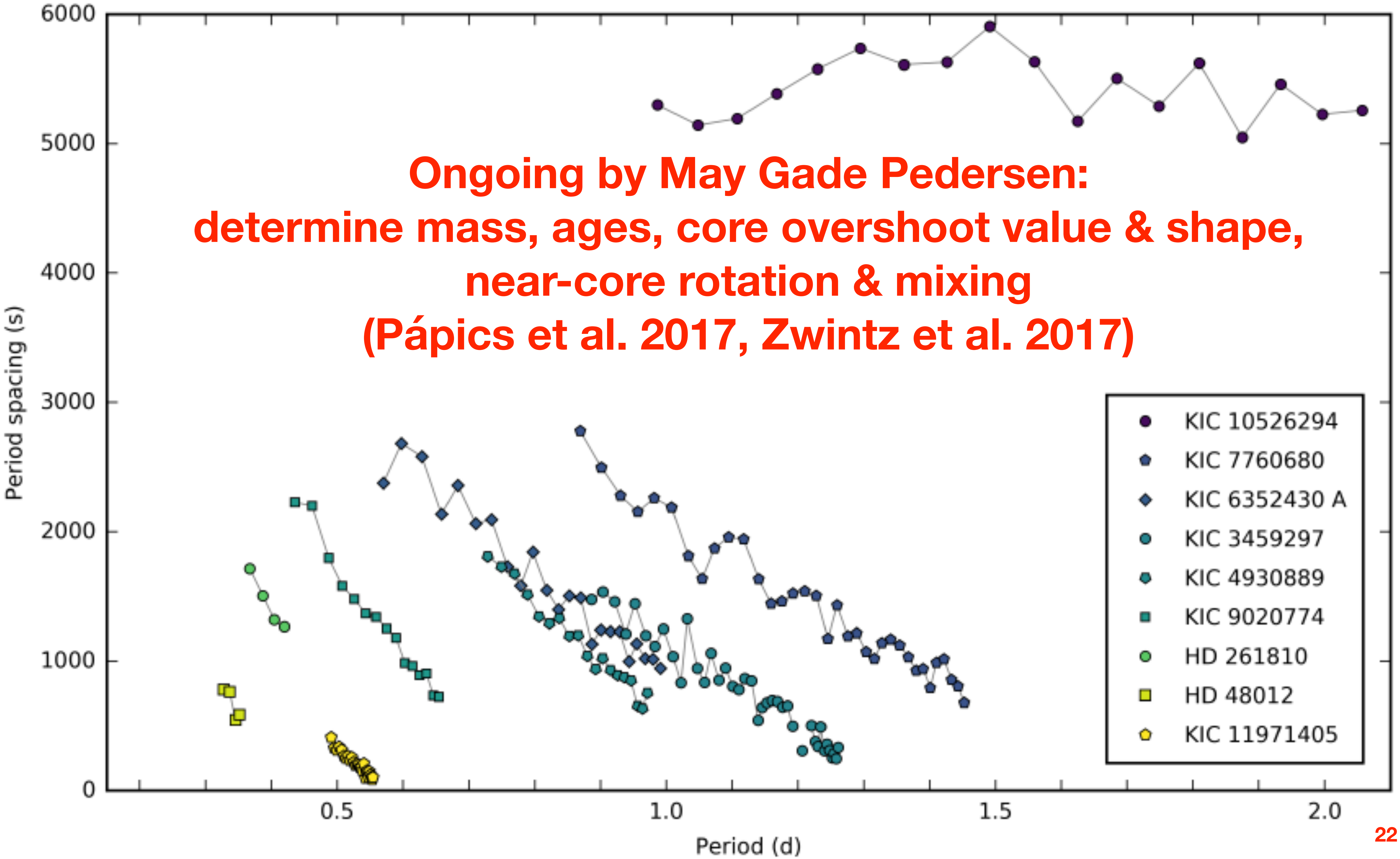
Forward seismic modelling: simplest case



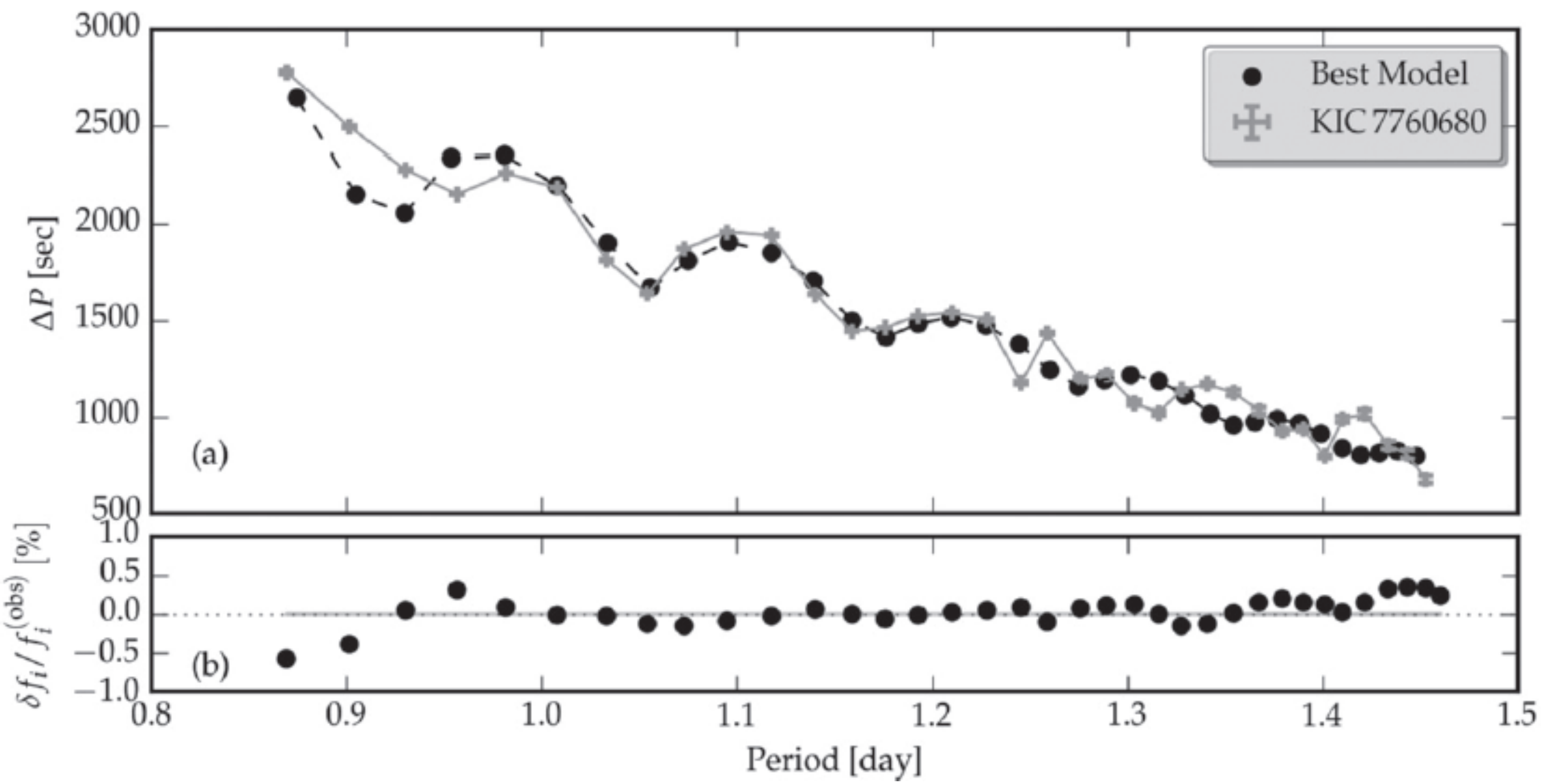
Radboud Universiteit Nijmegen

KU LEUVEN

Gravity-mode period spacings massive stars

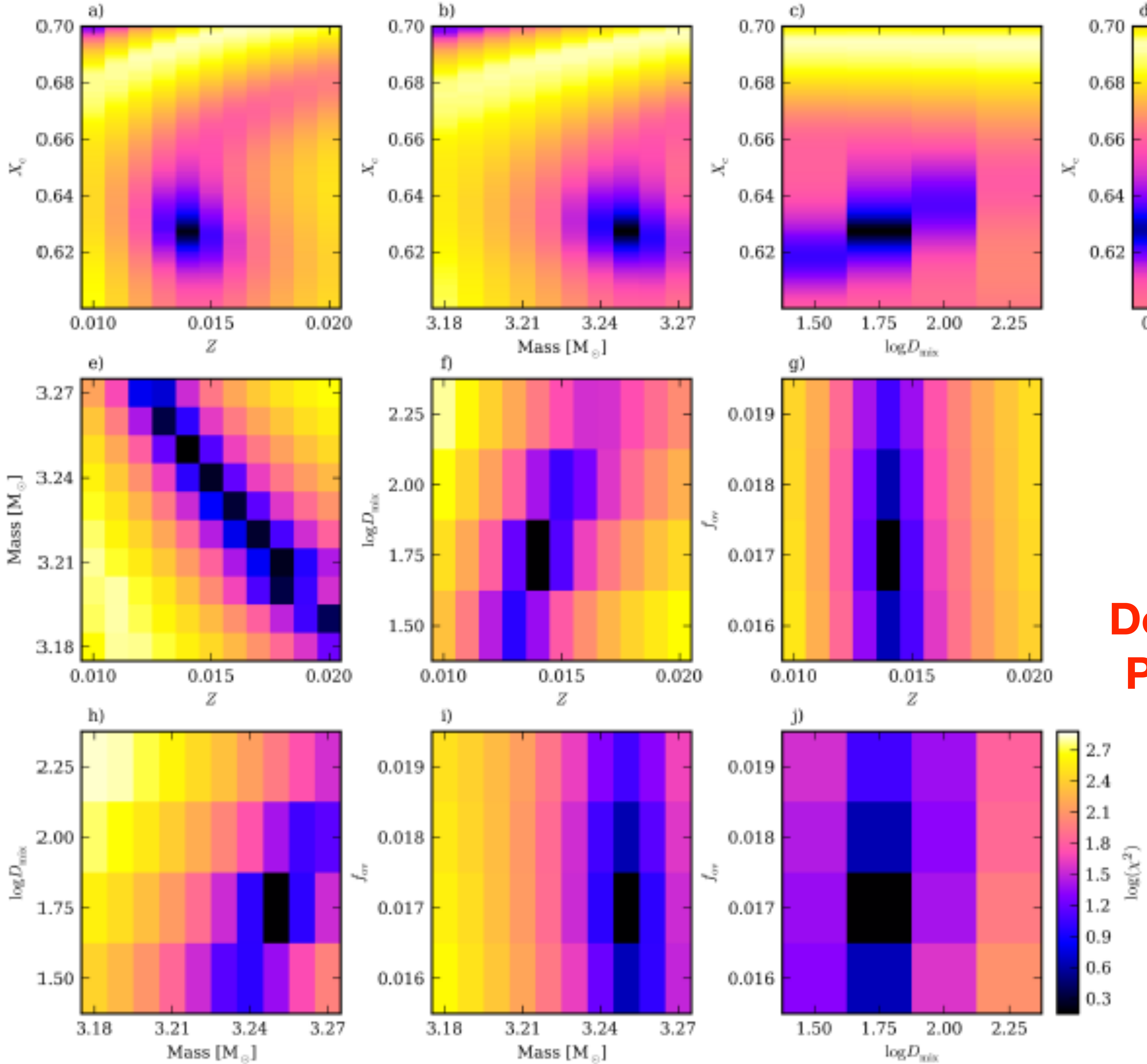


A real star:KIC 7760680

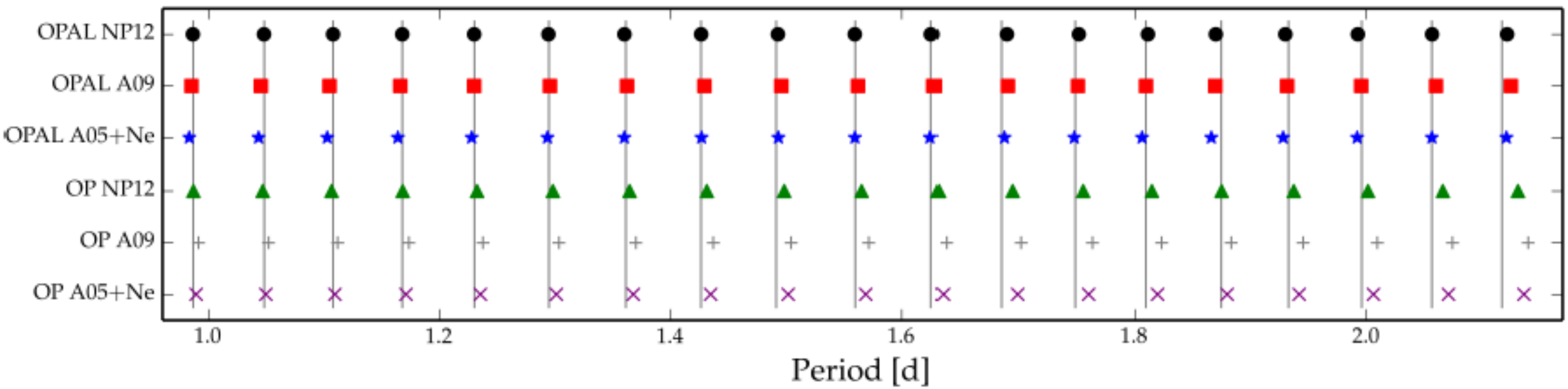


KIC7760680: M=3.25M⊙, X_c=0.50, f_{rot}=0.48/d, f_{ov}=0.024 H_p

log D_{mix}=0.75±0.25 **(Moravveji et al. 2016)**



Degeneracies, Pápics, KITP talk 2015



Dependence on opacities and chemical mixture
(Moravveji et al. 2015)