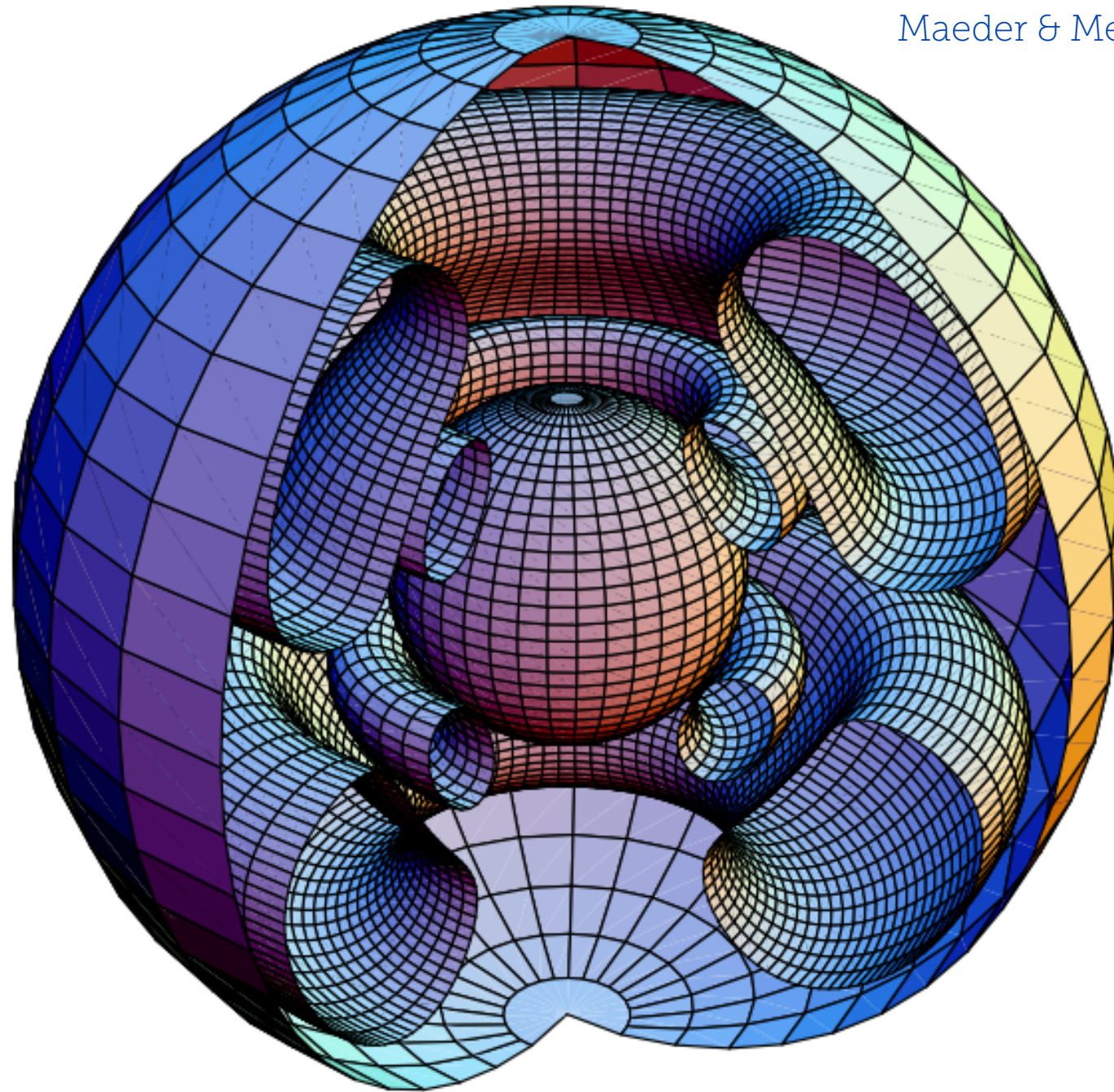


Stellar Rotation

Observational Constraints & Missing Physics

Maeder & Meynet 2002



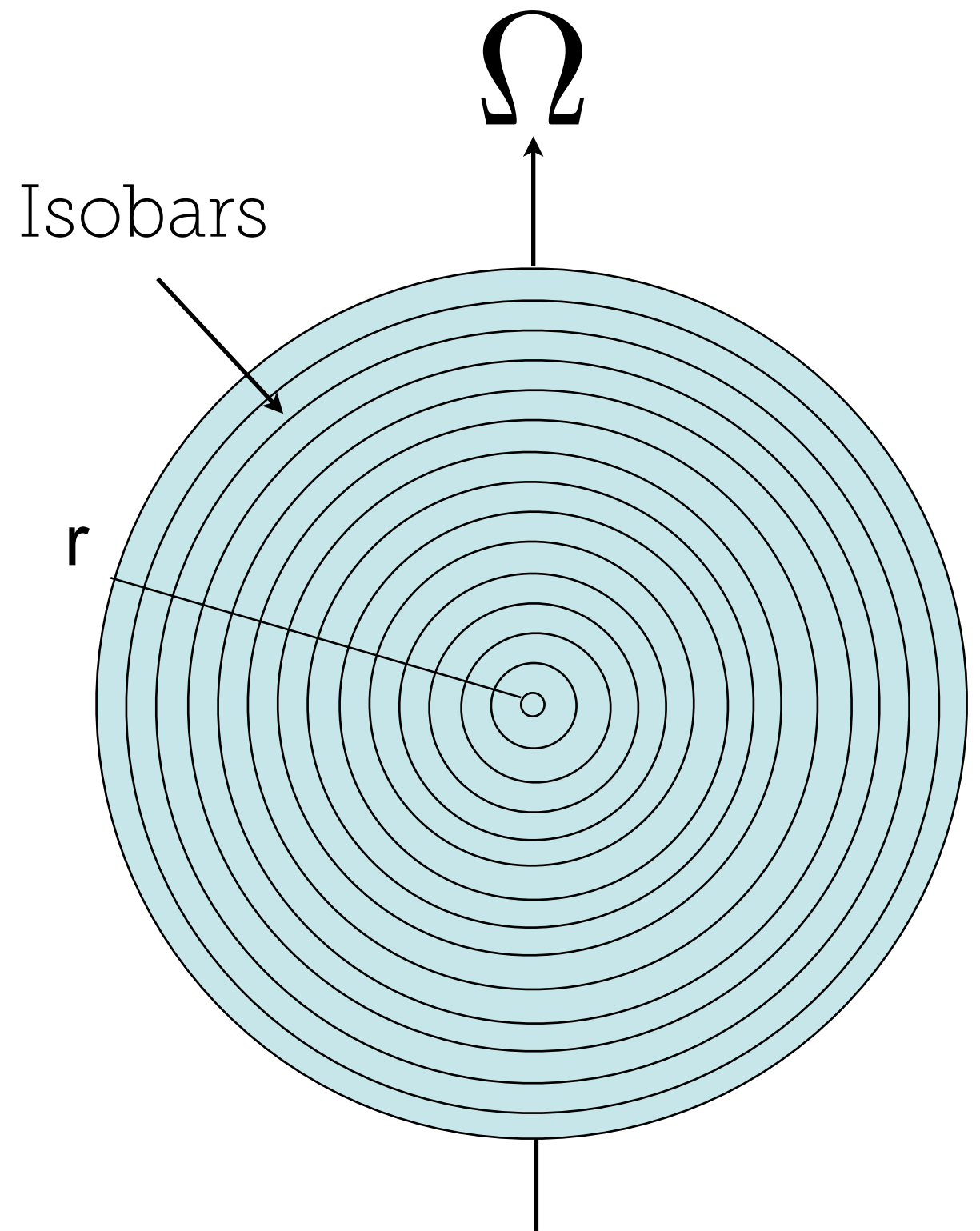
Matteo Cantiello
Center for Computational Astrophysics, Flatiron Institute

All relies on the Shellular Approximation

(allows 1D stellar evolution)

$$\omega = \omega(r)$$

ρ and Composition
only function of the r
coordinate, as each
shell is assumed to be
efficiently mixed by
strong horizontal
turbulence

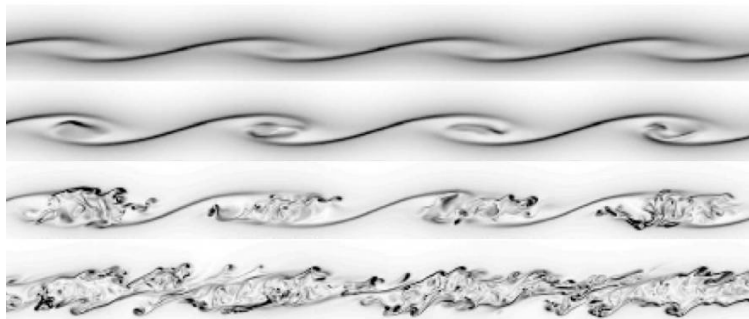


Zahn (1975), Chaboyer & Zahn (1992), Meynet & Maeder 1997

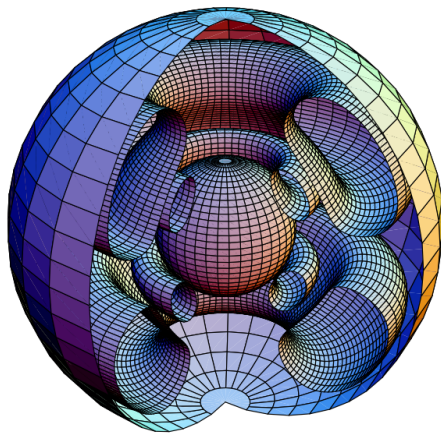
Angular Momentum Transport

Different classes of mechanisms have been proposed:

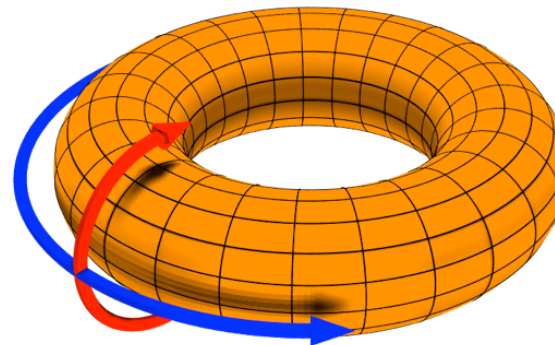
e.g. Heger et al. 2000



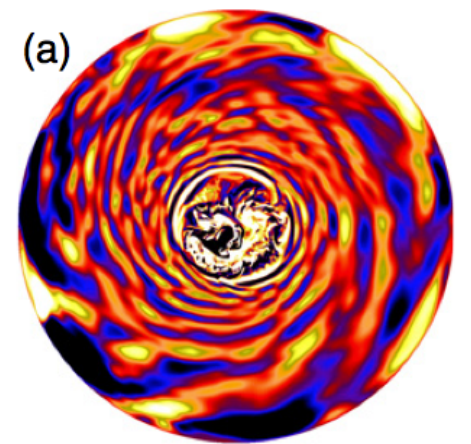
e.g. Maeder & Meynet 2002



e.g. Spruit 2002



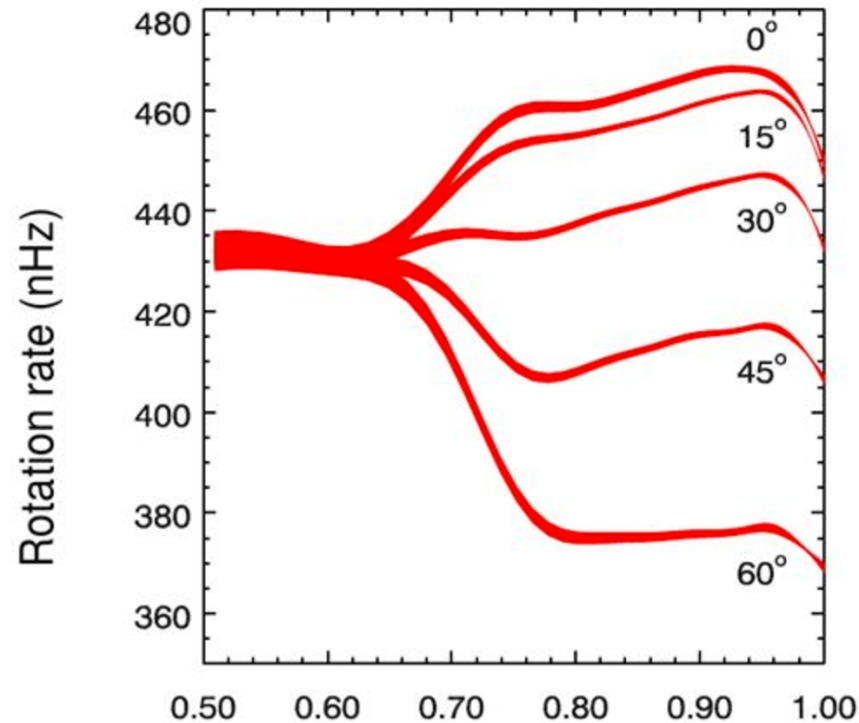
e.g. Rogers et al. 2013



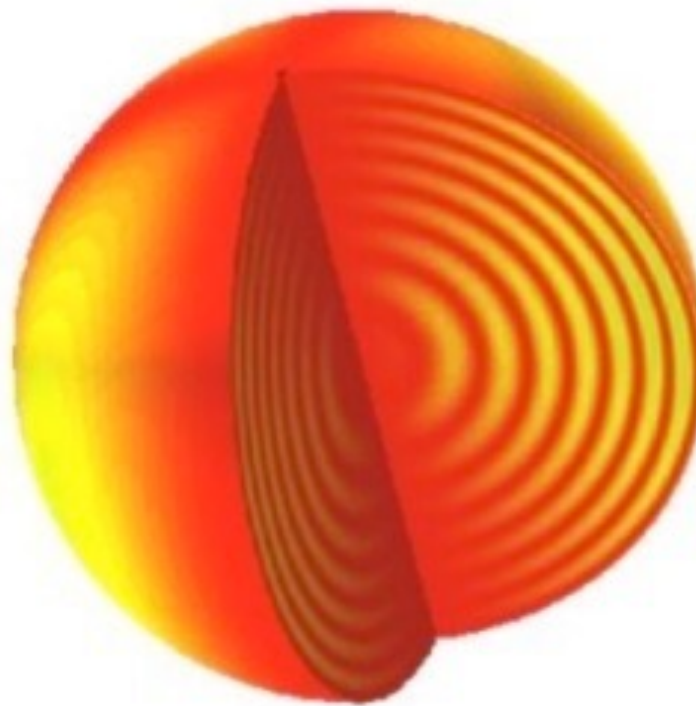
- Hydrodynamics instabilities
- Rotationally induced circulations
- Magnetic torques
- Internal gravity waves

Angular Momentum Transport: Observational Tests

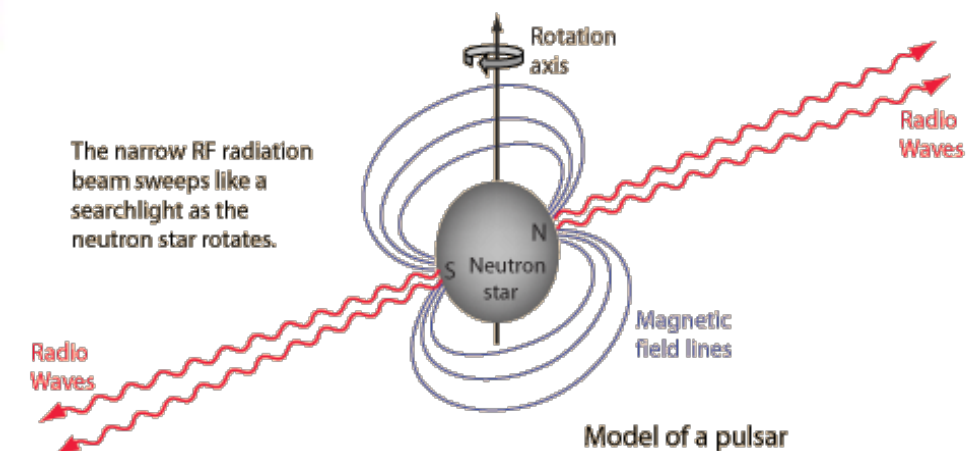
The Sun



Red Giants

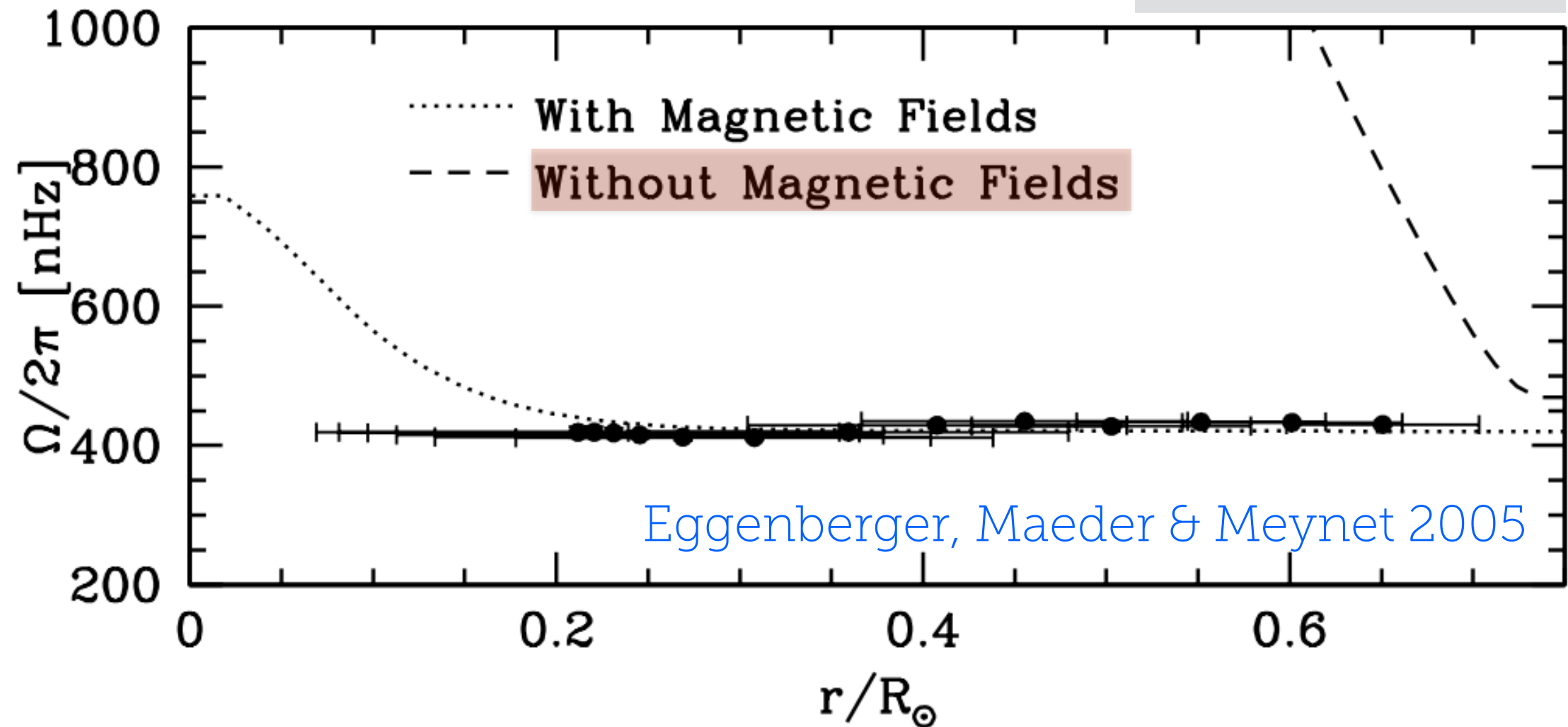


Remnants



Solar rotation profile

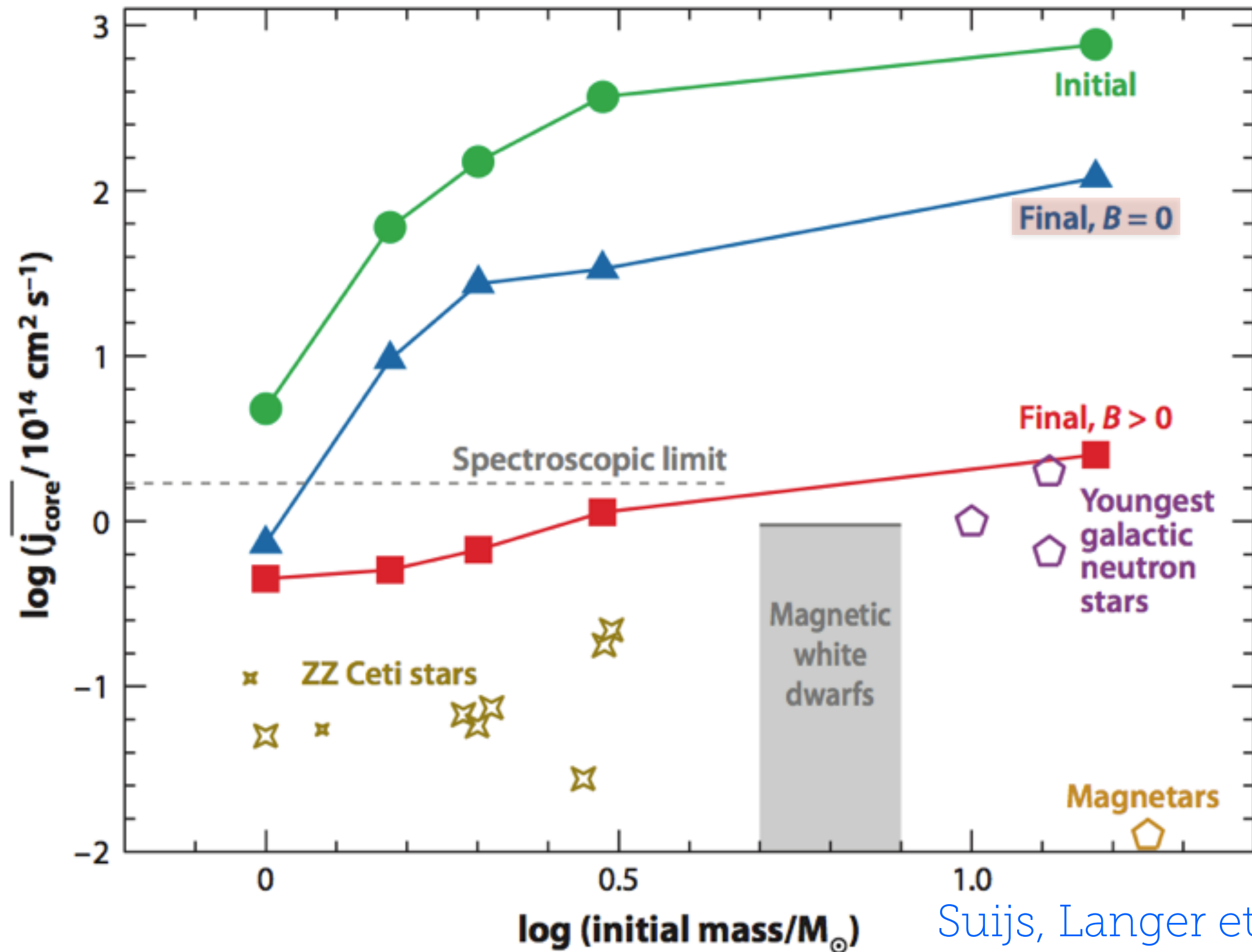
Geneva Code



Model including only rotational effects [...] results in a large differential rotation reaching a factor of about 20 between the angular velocity at the surface and in the stellar core at the age of the Sun, in contradiction with the flat rotation profile of the Sun

Rotation Rate of Compact Remnants

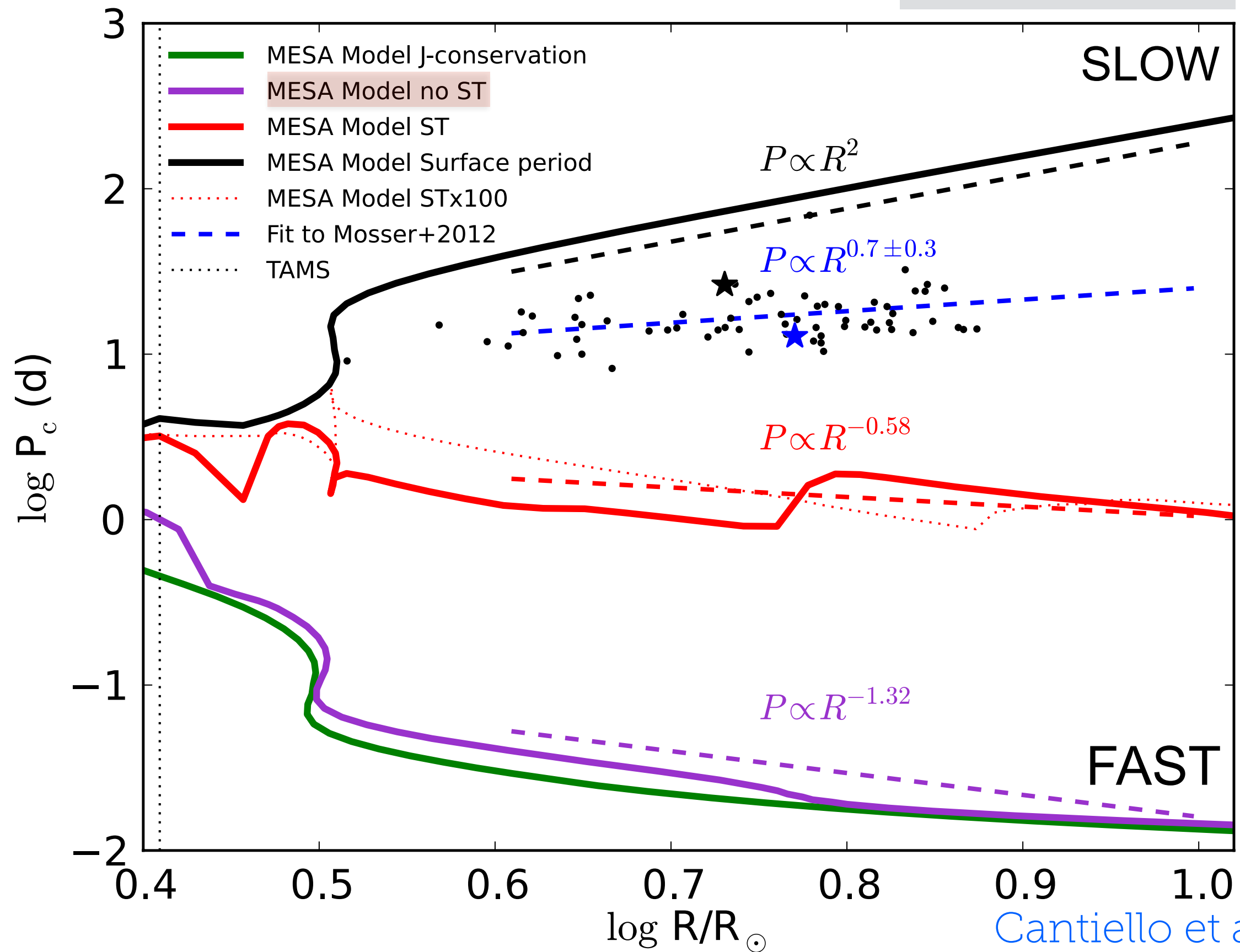
STERN Code



Suijs, Langer et al. 2008

Red Giants Core Rotation

MESA Code



Challenges for the Theory of Stellar Rotation*

Chemical Mixing

- ★ Hunter Diagram still not understood (Norbert)
- ★ Be stars show no surface enrichment (Thomas)

Angular Momentum

- ★ Can not explain solar rotation profile (off by factor ~ 20)
- ★ Can not explain spin rate of RG cores (off by factor of $\sim 100+$)
- ★ Can not explain the spin rate of compact remnants (off by factor of $\sim 100+$)

These results are mostly independent on the details of the implementation of rotational mixing in 1D stellar evolution codes (e.g. diffusion vs advection-diffusion schemes)

* As discussed by Georges (hydrodynamic instabilities + meridional circulation)

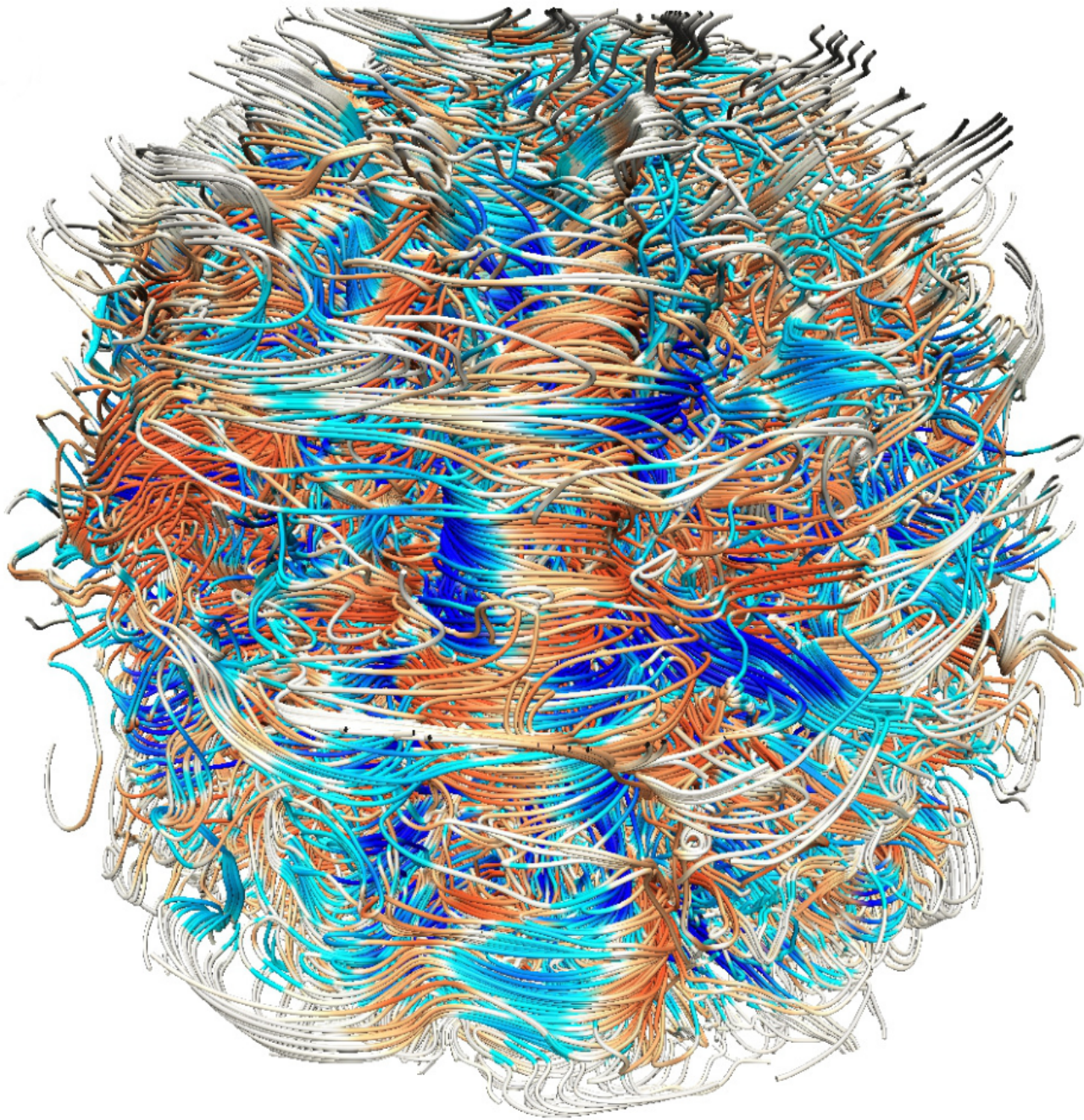
Either some of the
assumptions behind the
theory are wrong,
or there is missing
physics dominating the
problem

Stellar Rotation: Missing Physics?

Magnetic Fields

- ★ Tayler-Spruit Magnetic fields [Spruit, Braithwaite...]
- ★ Stellar MRI [Spada, Gellert...]
- ★ Core Dynamo-Generated magnetic fields [Augustson...]

Magnetic coupling



From Fossil or
Dynamo Fields

Tayler-Spruit (Spruit
2002)

MRI (Spada+ 2016)

Core convection:
Fields could be
ubiquitous (Fuller, MC+
2015, Stello MC+ 2016)

Augustson et al. 2016

See e.g. Mader & Meynet (2014)

Stellar Rotation: Missing Physics?

Magnetic Fields

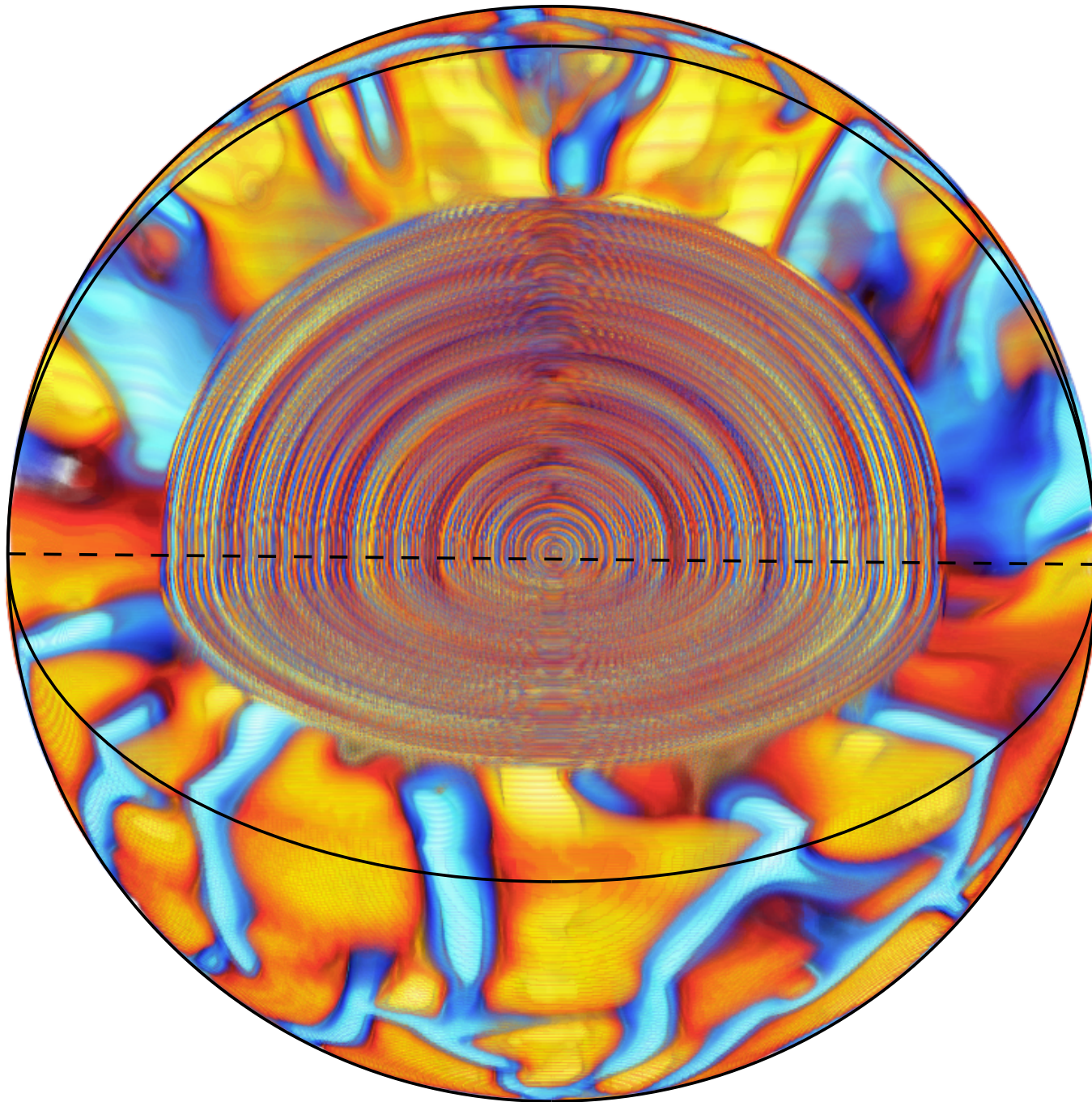
- ★ Tayler-Spruit Magnetic fields [Spruit, Braithwaite...]
- ★ Stellar MRI [Spada, Gellert...]
- ★ Core Dynamo-Generated magnetic fields [Augustson...]

Waves

- ★ Internal Gravity Waves [Fuller, Rogers, Alvan...]
- ★ Modes in stellar pulsators [Townsend, Belkacem]

Internal Gravity Waves

Alvan et al. 2014



IGW: Excited by turbulent convection

They carry angular momentum

Spectrum: Not well understood. But likely Kolmogorov-like with a steep exponent

Dissipation: Radiative dissipation usually dominates in stellar interiors

See e.g.: Charbonnel & Talon 2005, Goldreich & Kumar 1990, Lecoanet & Quatert 2013, Mathis et al. 2014, Rogers et al. 2013
Fuller, Lecoanet, MC et al. 2014 Fuller, MC et al. 2015

Stellar Rotation: Missing Physics?

Magnetic Fields

- ★ Tayler-Spruit Magnetic fields [Spruit, Braithwaite...]
- ★ Stellar MRI [Spada, Gellert...]
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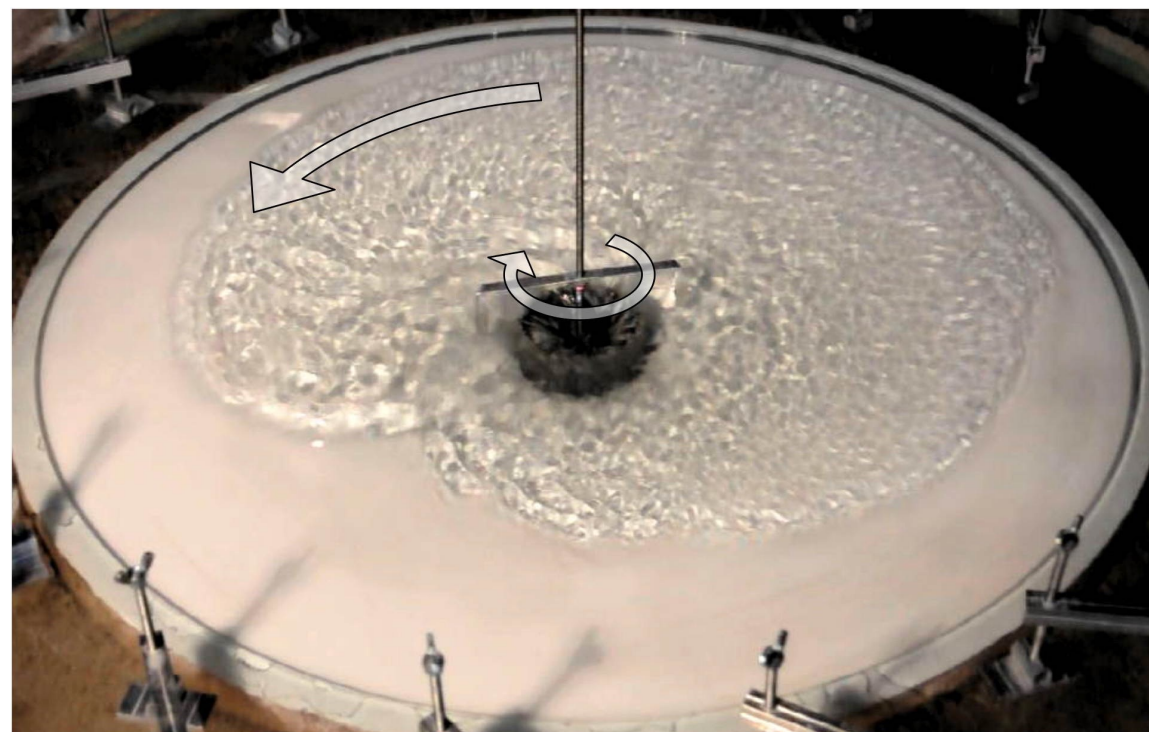
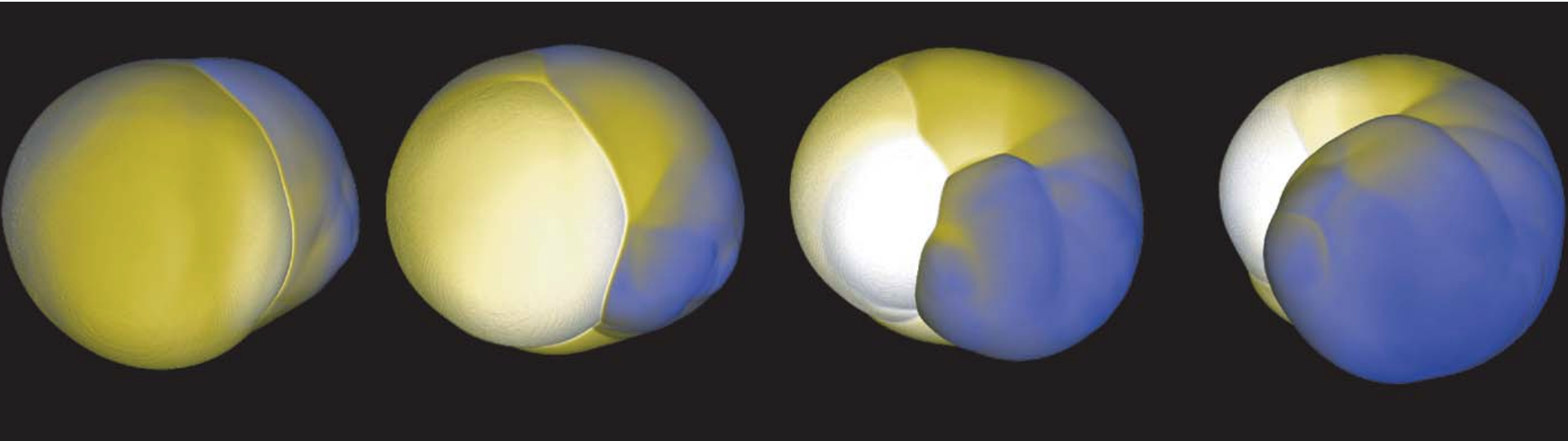
Waves

- ★ Internal Gravity Waves [Fuller, Rogers, Alvan...]
- ★ Modes in stellar pulsators [Townsend, Belkacem]

Other

- ★ SASI (in PNS) [Foglizzo]

SASI



e.g. Foglizzo et al. 2007

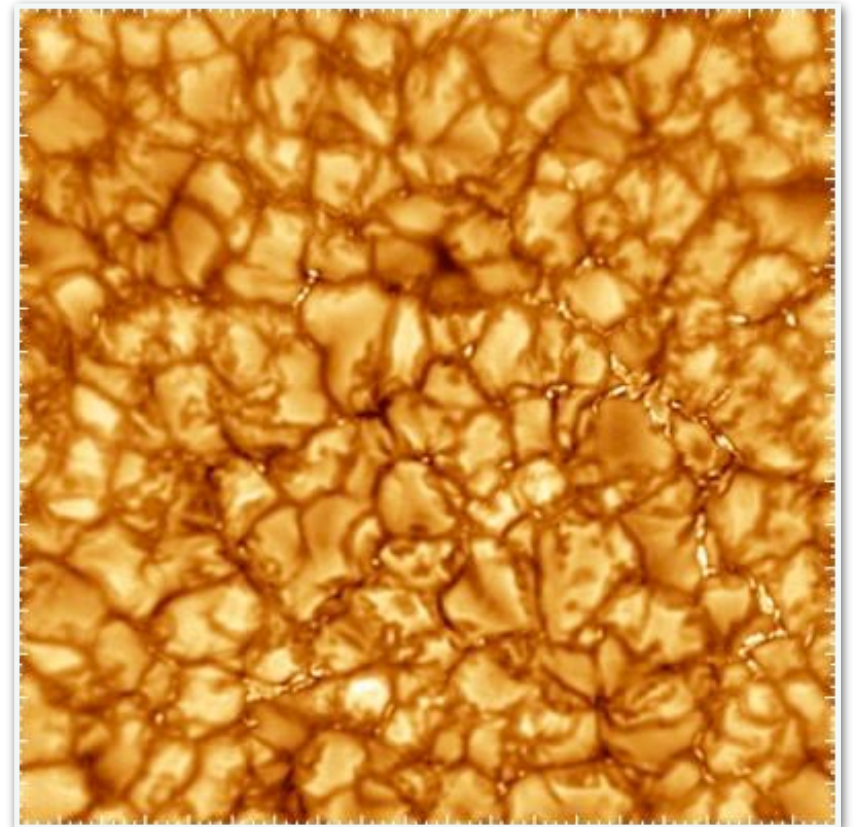
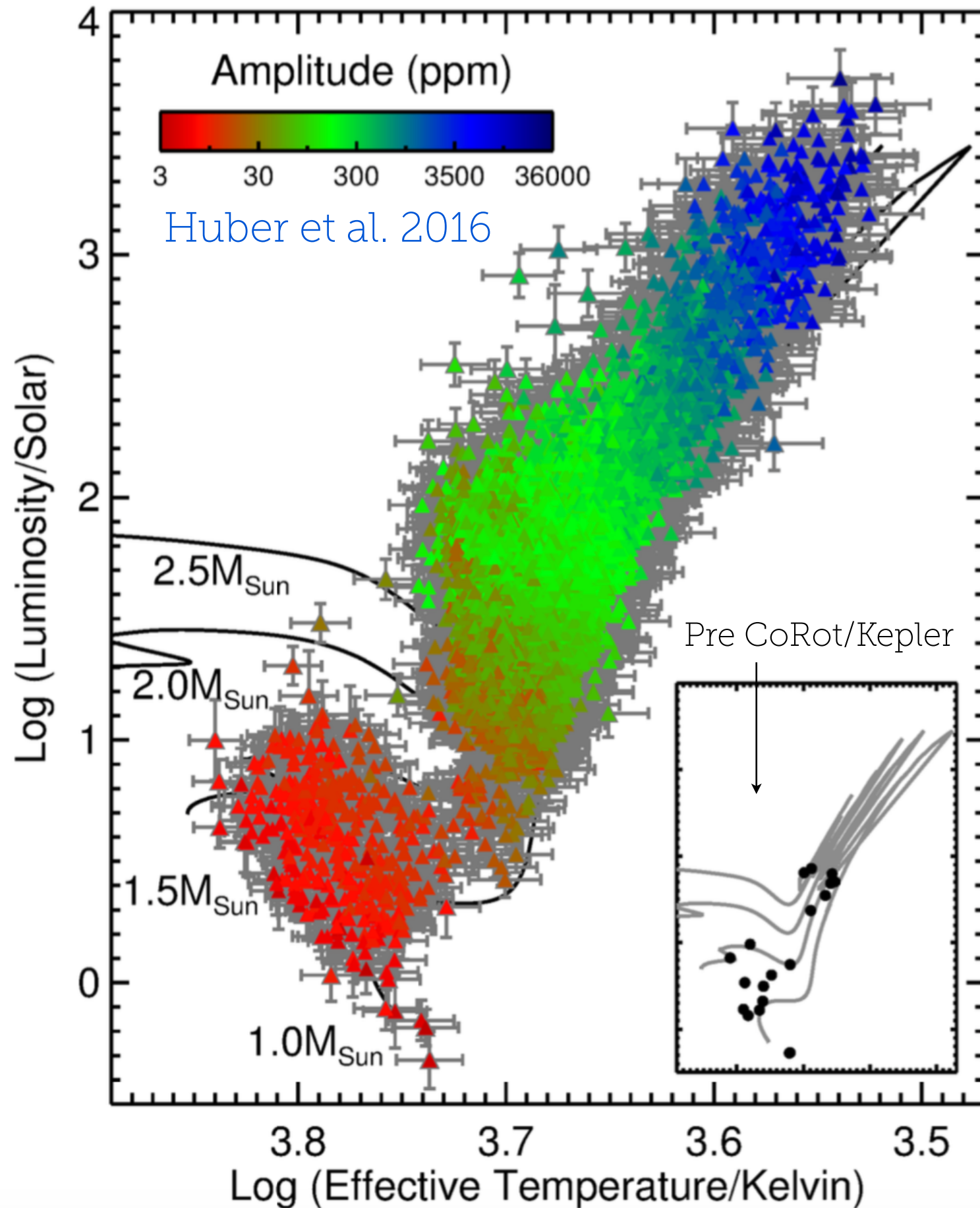
1D Stellar Rotation

Either some of the assumptions behind the theory are wrong, or there is missing physics dominating the problem

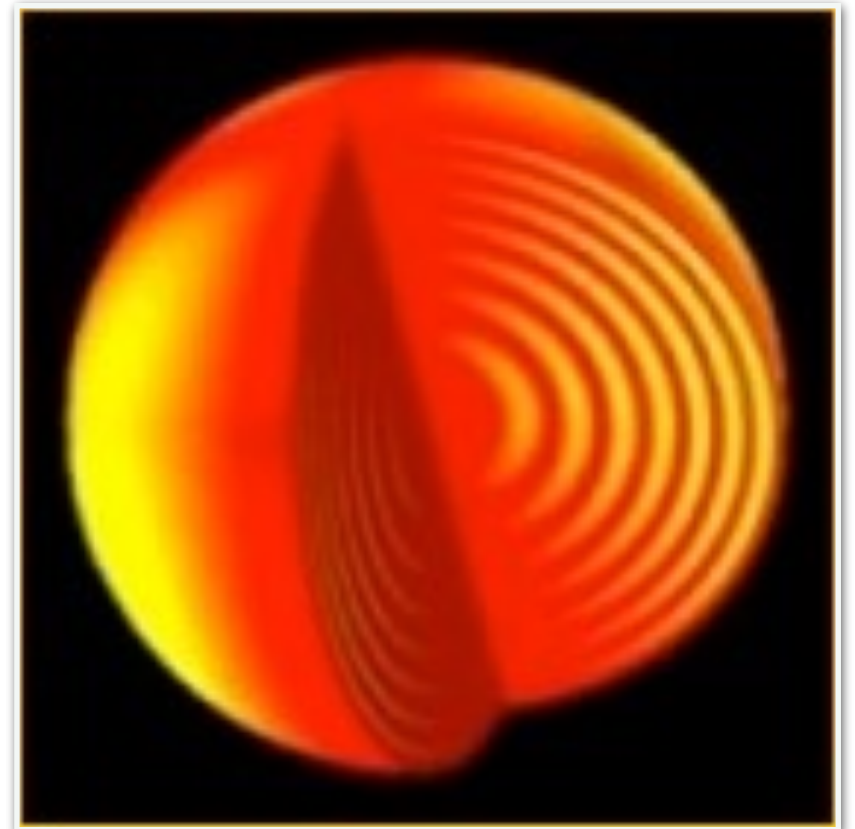
Backup Slides

Mixed Modes

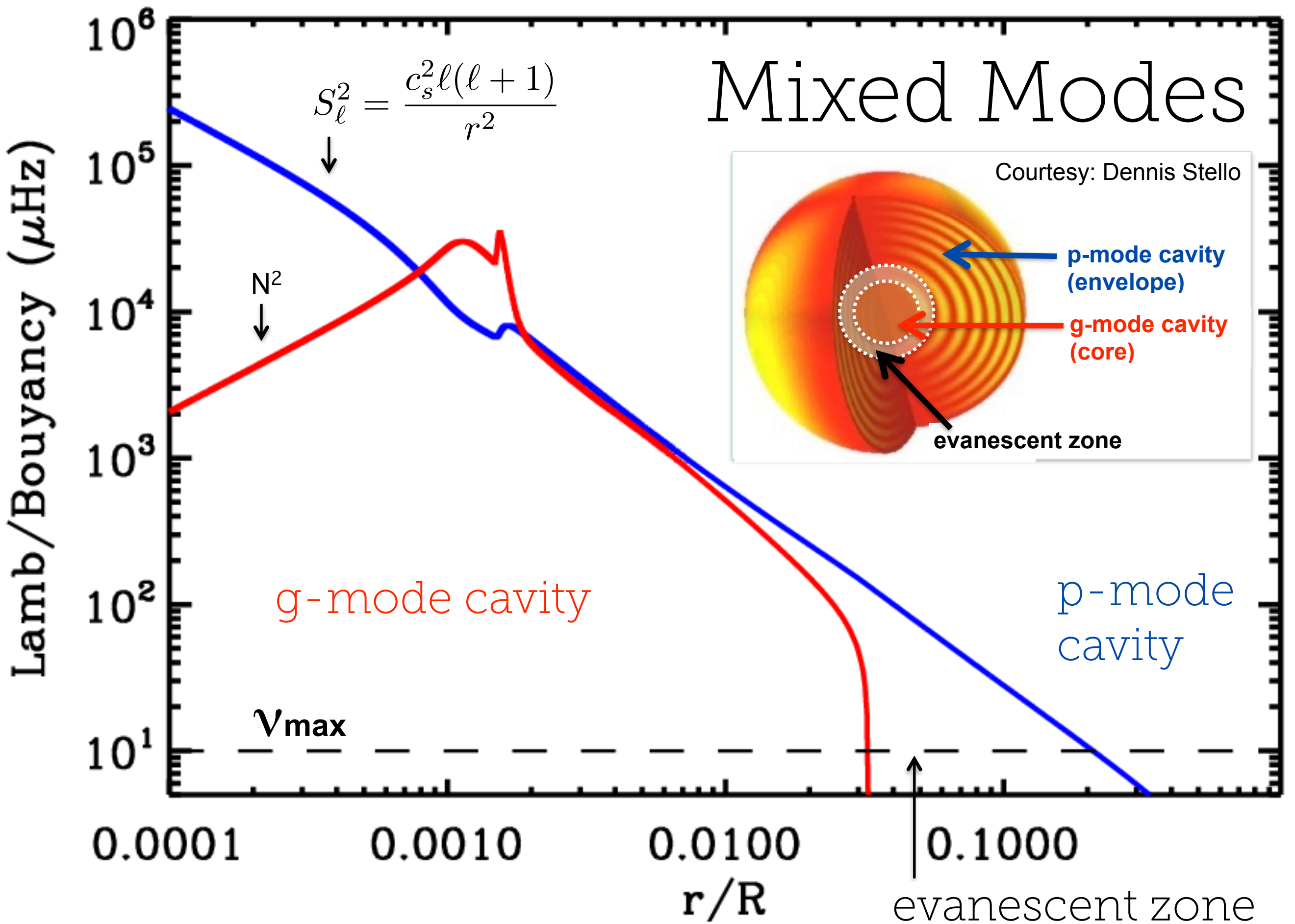
Solar-Like Oscillations



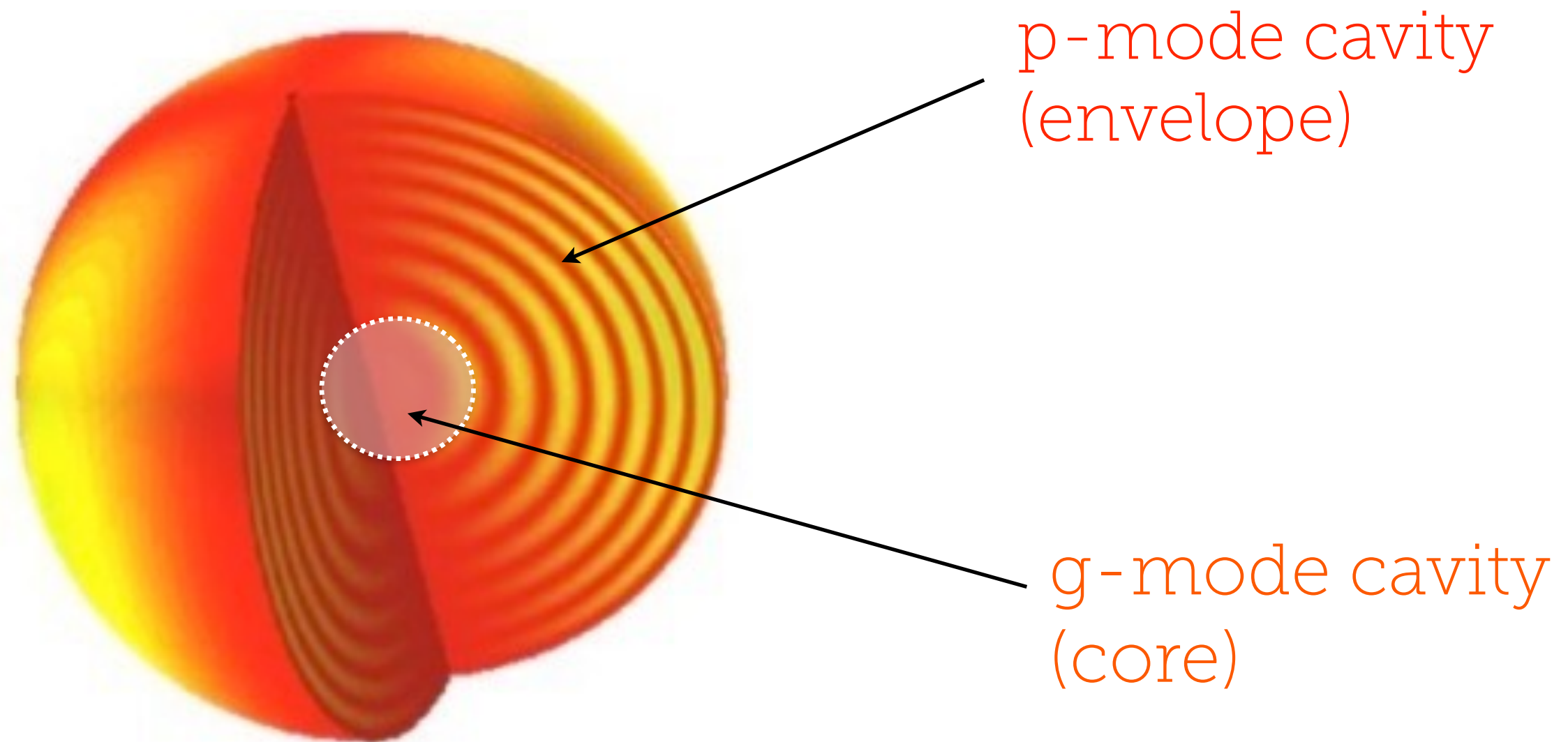
Credit: BBSO/NJIT



Dupret et al. 2009



Mixed Modes

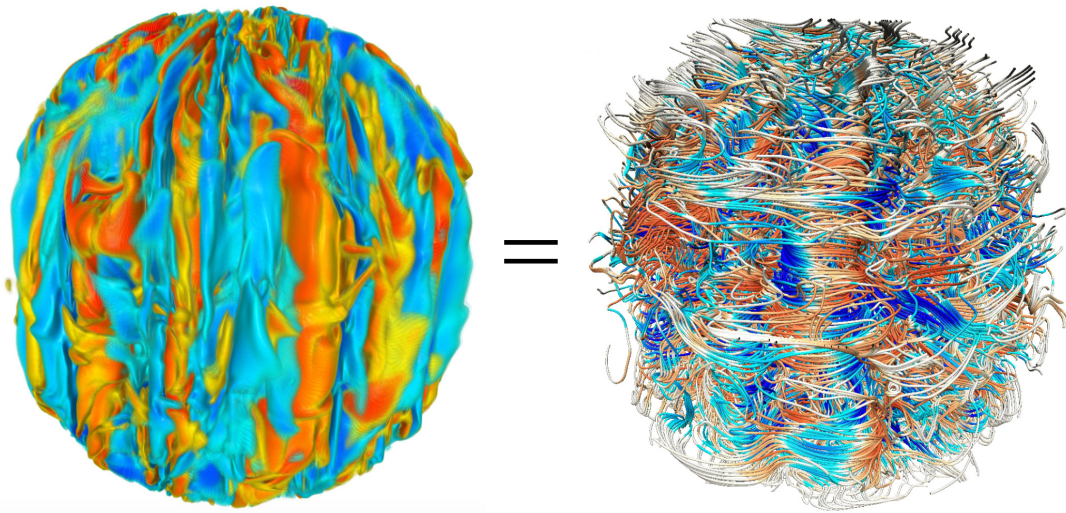


Since a mixed mode lives both as a p-mode (in the envelope) and as a g-mode (in the core), if observed at the surface can give informations about conditions (e.g. rotation rate) in different regions of the star!

B-fields

B-Fields 101

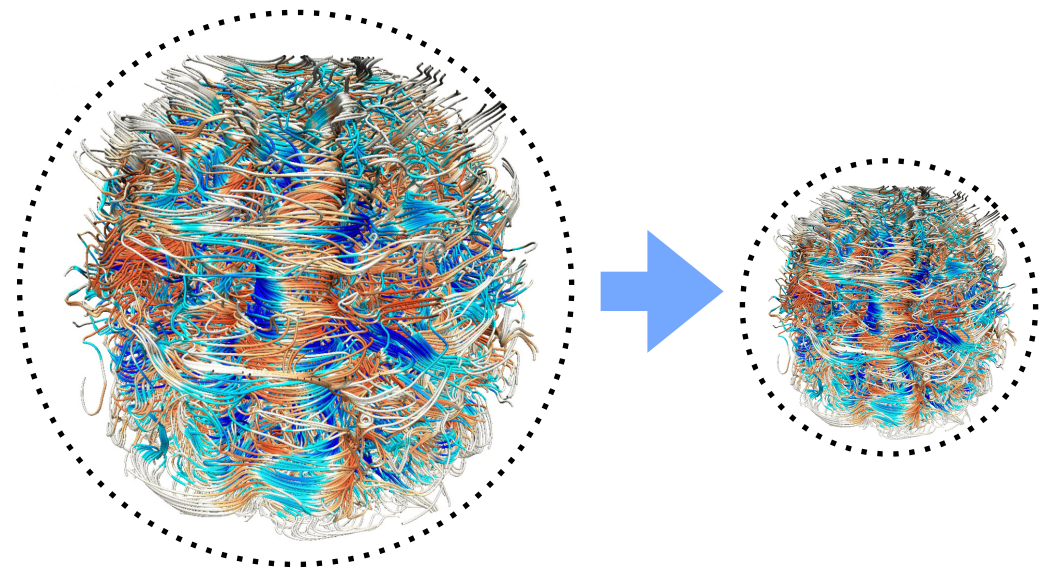
Energy Equipartition



$$\frac{1}{2}\rho v_{\text{con}}^2 \simeq \frac{B_{\text{eq}}^2}{8\pi}$$



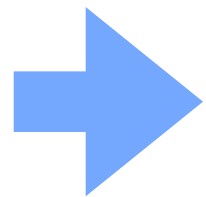
Magnetic Flux Freezing & Conservation



$$B_{\text{RG}} = B_{\text{MS}} \left(\frac{r_{\text{MS}}}{r_{\text{RG}}} \right)^2$$

In the presence of strong B-fields, magnetic tension forces can become comparable to buoyancy

Lorentz Force \sim Buoyancy Force



Critical Field Strength

$$B_c = \sqrt{\frac{\pi\rho}{2}} \frac{\omega^2 r}{N}$$



Fuller + Cantiello et al. (Science 2015)

Lecoanet et al. (2016)

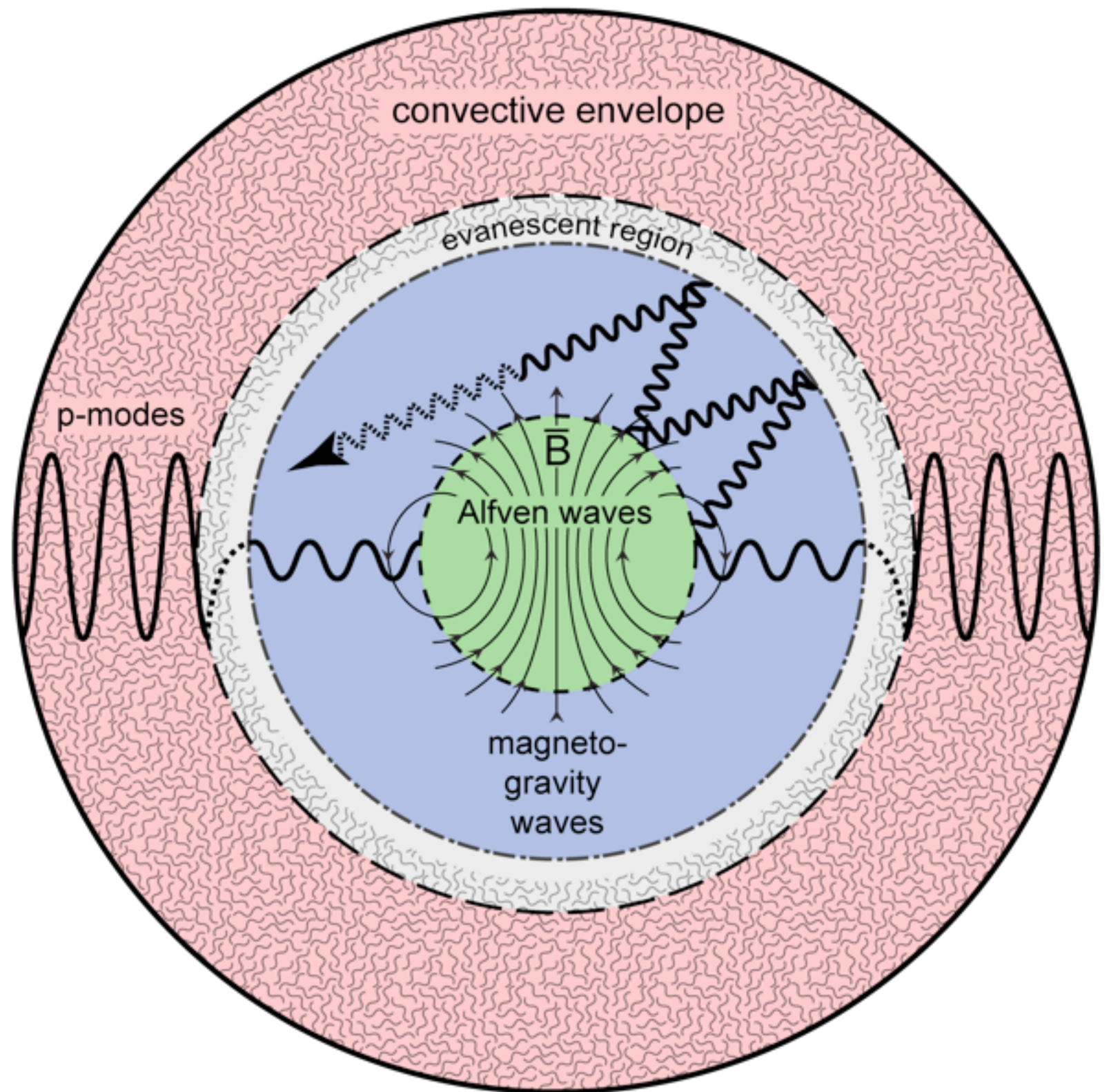
Magnetic Greenhouse Effect

Magnetic fields break spherical symmetry in the core

Dipolar waves "scattered" to high harmonic degrees l

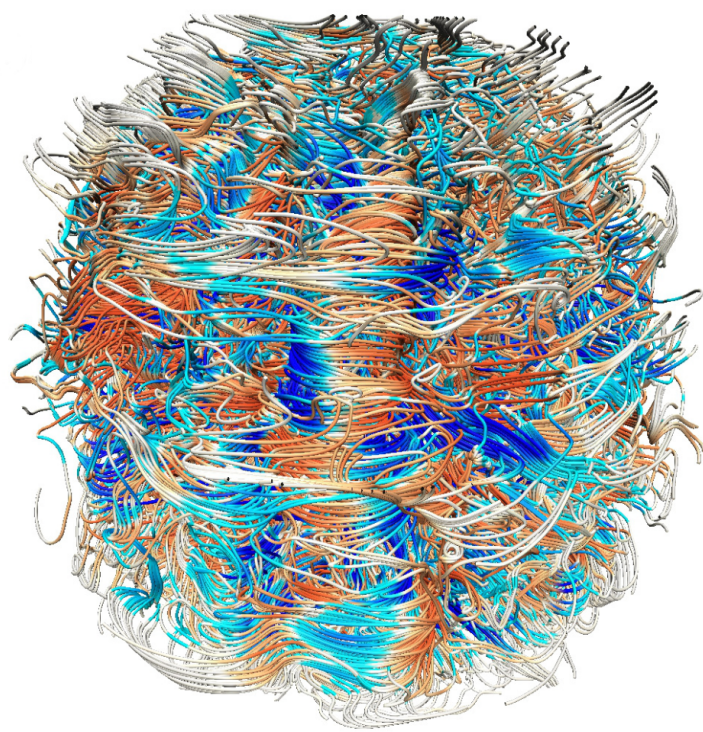
Waves trapped and dissipate quickly

Typical Critical B-field $\sim 10^5$ G

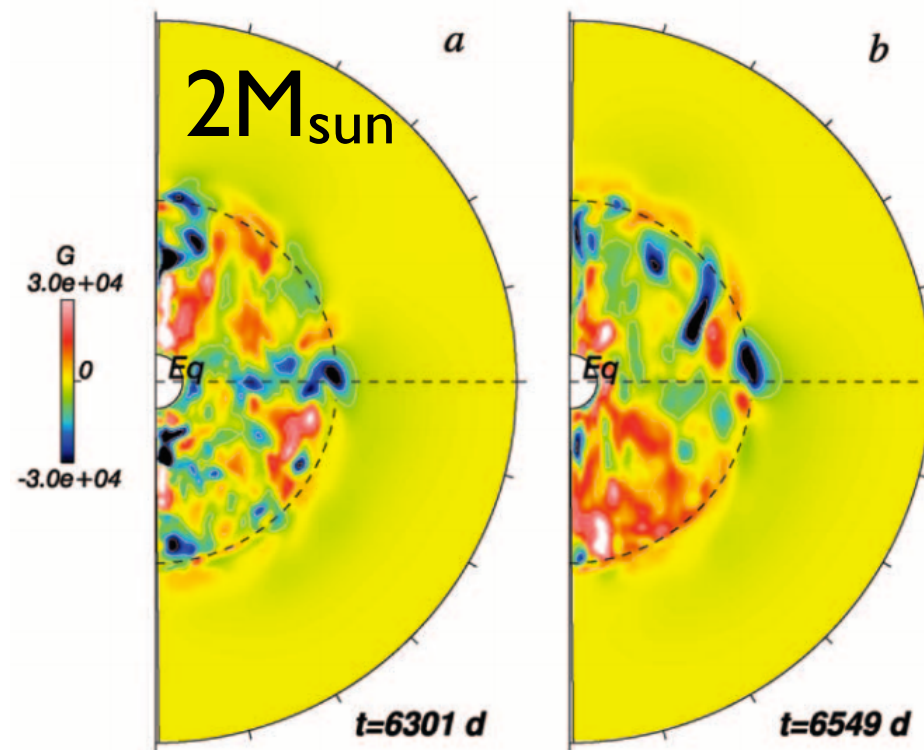


Reese et al. 2004, Rincon & Rieutord 2003,
Lee 2007,2010, Mathis & De Brye 2010,2012

Fuller + Cantiello et al. (Science 2015)
Lecoanet et al. 2016, Cantiello + Fuller et al. 2016



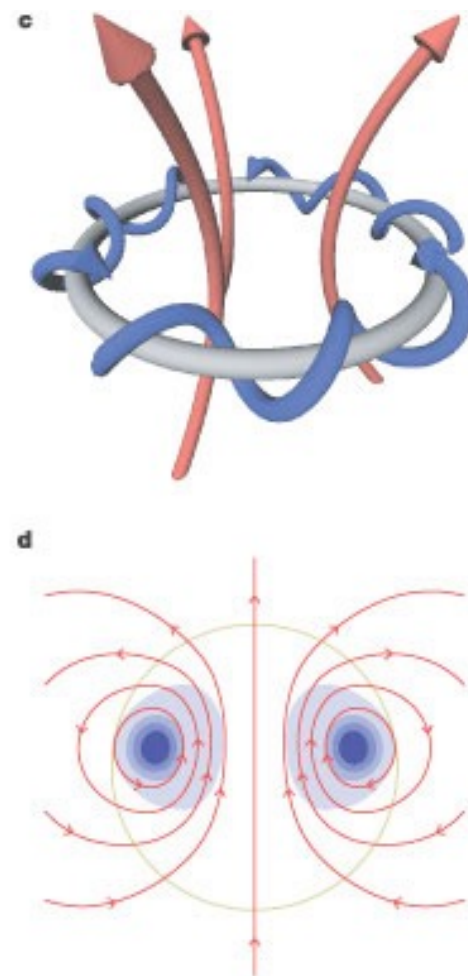
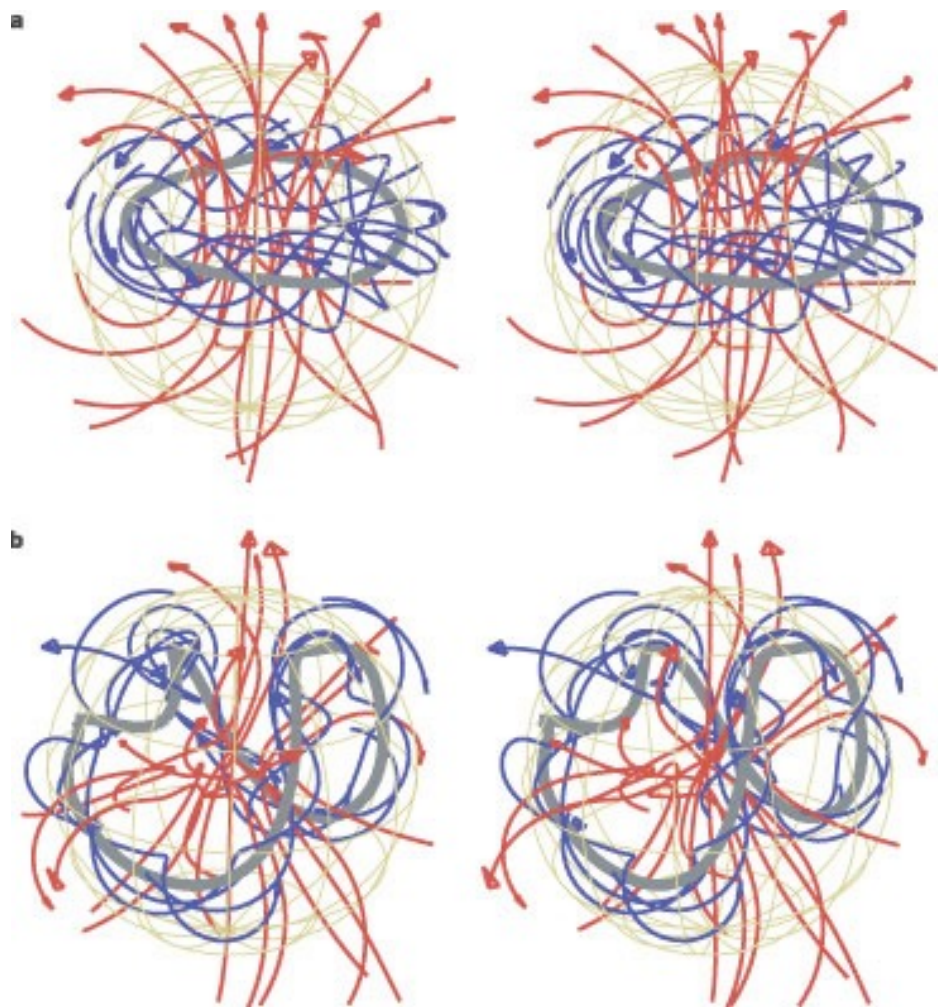
Augustson et al. 2016

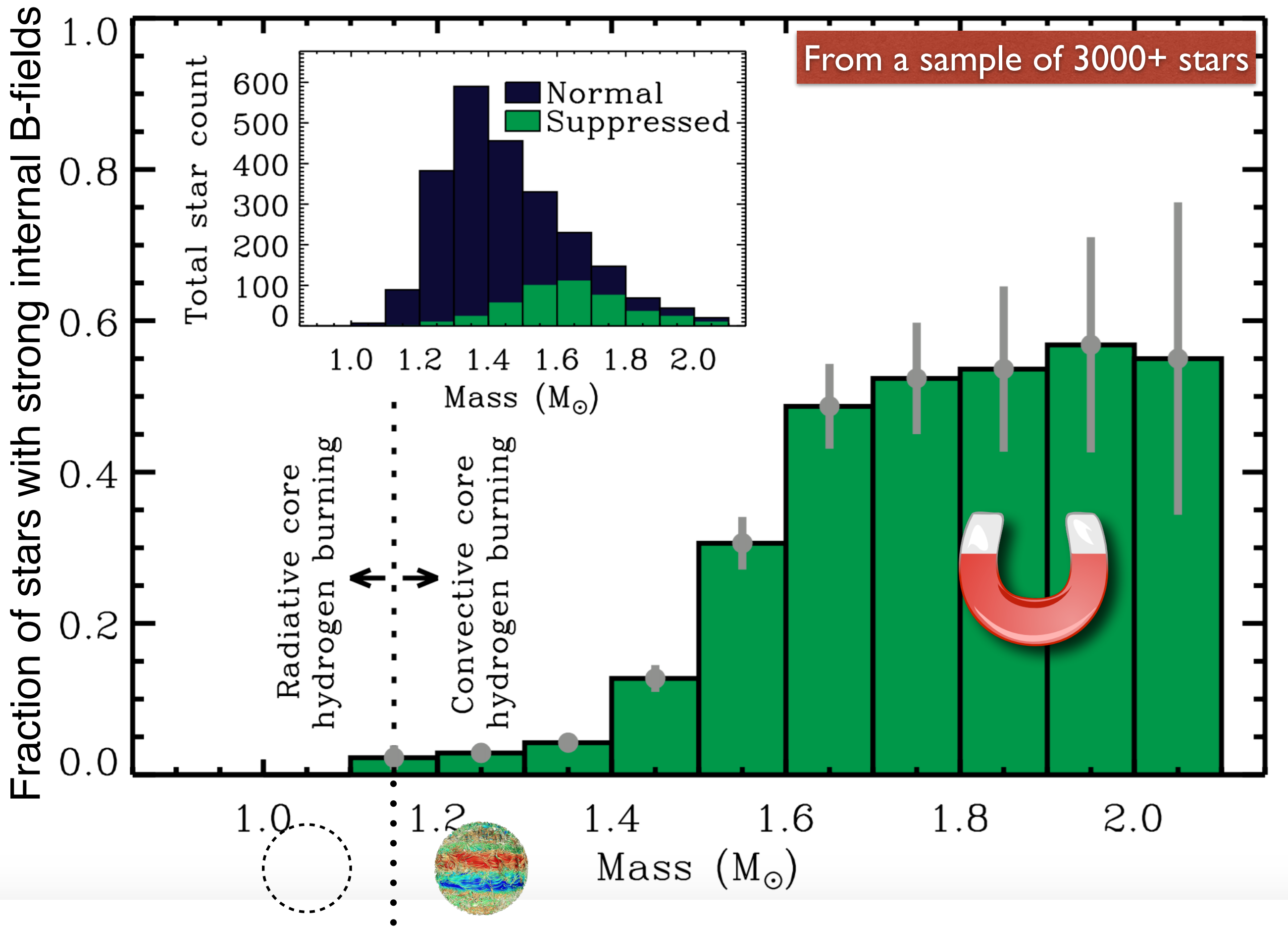


Brun et al. 2005

- Convective core dynamos on the MS: $B_{eq} \sim 10^5$ G
- Magnetic field topology is complex
- Flux conservation can easily lead to $B \sim 10^6 - 10^7$ G on the RG
- Stable magnetic configurations of interlocked poloidal+toroidal fields exist in radiative regions

Kyle Augustson's talk & Poster





But See also Mosser et al. 2016

Stello, Cantiello, Fuller et al. (Nature 2016)

1D Rotation

The Shellular Approximation

Rotation and especially differential rotation generates turbulent motions. On the Earth, we have the example of west winds and jet streams. In a radiative zone, the turbulence is stronger ([Zahn, 1992](#)) in the horizontal than in the vertical direction, because in the vertical direction the stable thermal gradient opposes a strong force to the fluid motions.

According to [Zahn \(1975\)](#), [Chaboyer & Zahn \(1992\)](#), and [Zahn \(1992\)](#), anisotropic turbulence acts much stronger on isobars than in the perpendicular direction. This enforces a shellular rotation law ([Meynet & Maeder 1997](#)), and it sweeps out compositional differences on isobars. Therefore it can be assumed that matter on isobars is approximately chemically homogeneous. Together with the shellular rotation, this allows us to retain a one-dimensional approximation. The specific angular momentum, j , of a mass shell is treated as a local variable, and the angular velocity, ω , is computed from the specific moment of inertia, i . ([Heger et al. 2000](#))

In this approach, mass shells correspond to isobars instead of spherical shells.

Barotropic Star

$$\frac{1}{\varrho} \vec{\nabla} P = -\vec{\nabla} \Phi + \frac{1}{2} \Omega^2 \vec{\nabla} (r \sin \vartheta)^2$$

$$\Psi = \Phi + V$$

$$\frac{1}{\varrho} \vec{\nabla} P = -\vec{\nabla} \Psi = \vec{g}_{\text{eff}}$$

If Omega is constant (Solid body rotation) or has cylindric symmetry, the centrifugal acceleration can be derived from a potential (V). The eq. of Hydrostatic Equilibrium then implies that the star is Barotropic

Barotropic Star

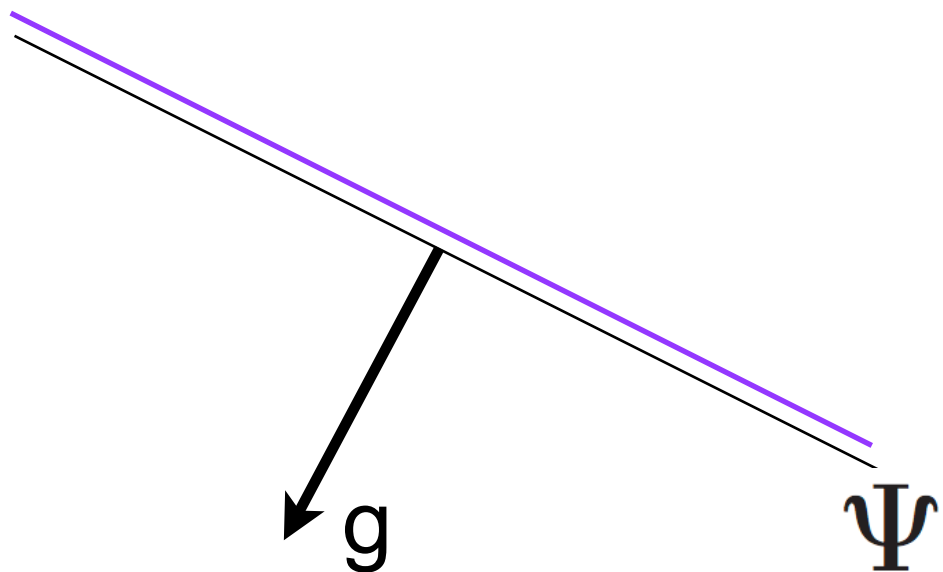
$$\frac{1}{\varrho} \vec{\nabla} P = -\vec{\nabla} \Phi + \frac{1}{2} \Omega^2 \vec{\nabla} (r \sin \vartheta)^2$$

$$\Psi = \Phi + V$$

$$\frac{1}{\varrho} \vec{\nabla} P = -\vec{\nabla} \Psi = \vec{g}_{\text{eff}} \quad \longrightarrow$$

$$P = P(\Psi)$$

$$T = T(\Psi)$$



ϱ, P, T

constant on
equipotentials

Baroclinic Star

$$\frac{1}{\rho} \vec{\nabla} P = -\vec{\nabla} \Phi + \frac{1}{2} \Omega^2 \vec{\nabla} (r \sin \vartheta)^2$$

$$\Psi = \Phi + V$$

$$\frac{1}{\rho} \vec{\nabla} P = -\vec{\nabla} \Psi = \vec{g}_{\text{eff}}$$

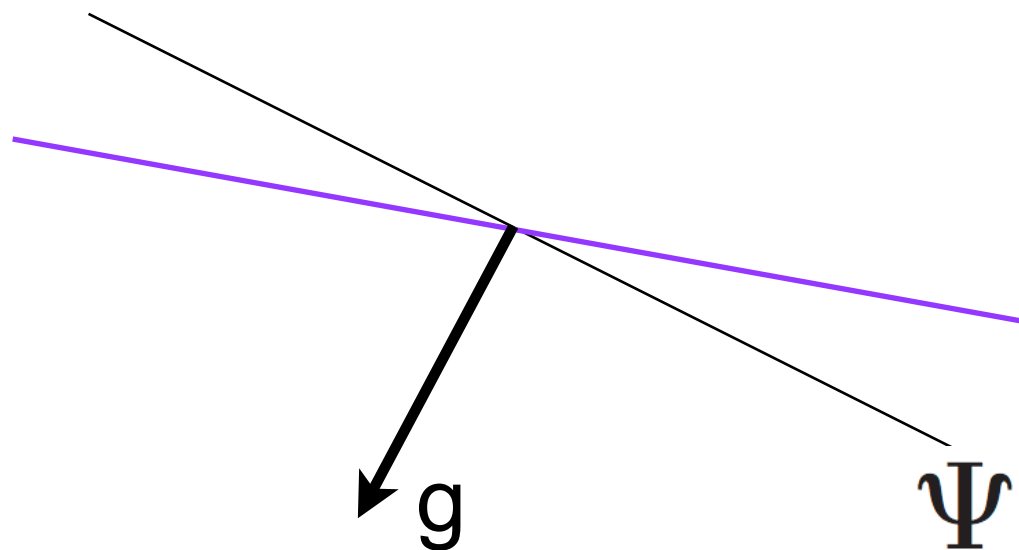
For different rotation laws (e.g. Shellular), the centrifugal acceleration can not be derived from a potential (V).
In this case Isobars and Equipotentials DO NOT coincide.
The star is Baroclinic

Baroclinic Star

$$\frac{1}{\varrho} \vec{\nabla} P = -\vec{\nabla} \Phi + \frac{1}{2} \Omega^2 \vec{\nabla} (r \sin \vartheta)^2$$

$$\Psi \neq \Phi + V$$

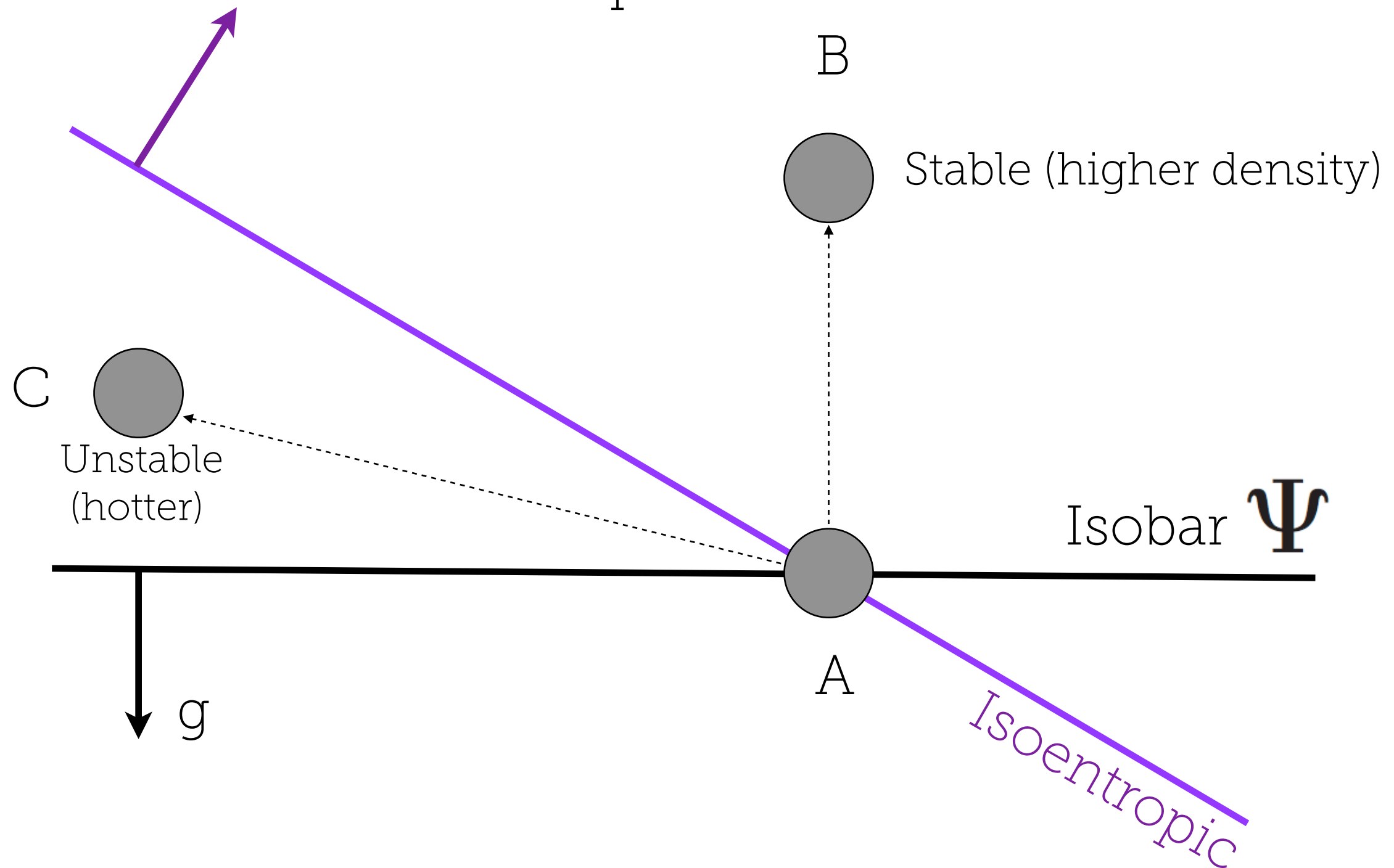
$$\frac{1}{\varrho} \vec{\nabla} P \neq -\vec{\nabla} \Psi = \vec{g}_{\text{eff}} \quad \rightarrow \quad \begin{array}{l} P \neq P(\Psi) \\ T \neq T(\Psi) \end{array}$$



ϱ, P, T
 not constant on
 equipotentials

Baroclinicity leads to instabilities

Assumption: adiabatic displacement



The structure equations of rotating stars

For a star in shellular rotation it is possible to modify the eqs of stellar structure to include the effect of the centrifugal force while keeping the form of the equations very close to that of the non-rotating case. Basically all quantities are redefined on isobars.

Mass conservation

$$\frac{dm_P}{dr_P} = 4\pi r_P^2 \rho$$

Hydrostatic Eq.

$$\frac{dP}{dm_P} = -\frac{Gm_P}{4\pi r_P^4} f_P$$

Energy transport

$$\frac{d \ln T}{d \ln P} = \frac{3\kappa P L_P}{16\pi ac G m_P T^4} \frac{f_T}{f_P}$$

...

$$V_P = 4\pi r_P^3 / 3$$

$$\langle q \rangle \equiv \frac{1}{S_P} \oint_{S_P} q d\sigma$$

$$f_P = \frac{4\pi r_P^4}{G m_P S_P} \langle g_{\text{eff}}^{-1} \rangle^{-1}$$

$$f_T \equiv \left(\frac{4\pi r_P^2}{S_P} \right) (\langle g_{\text{eff}} \rangle \langle g_{\text{eff}}^{-1} \rangle)^{-1}$$

Endal & Sofia 1978