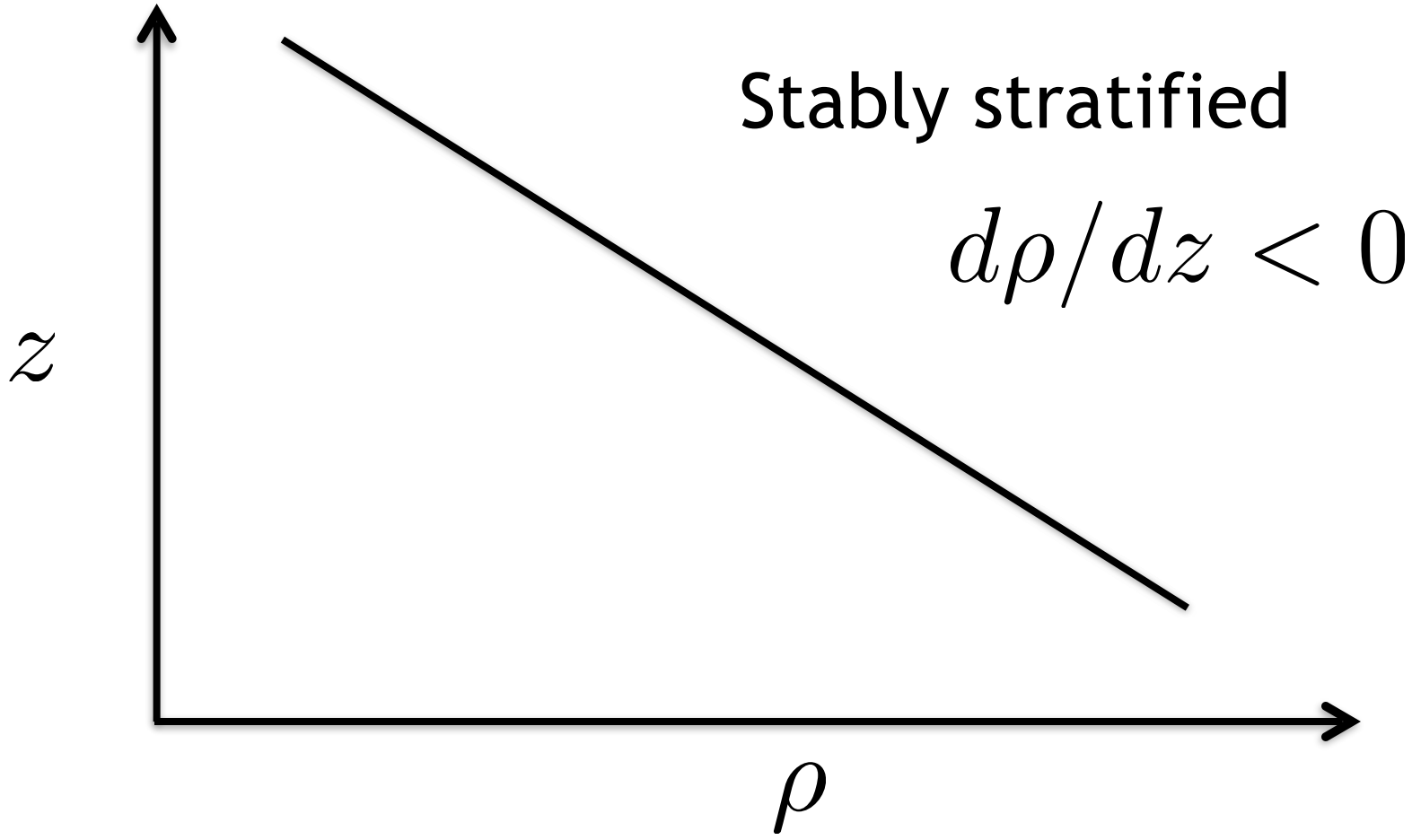
Stochastic Excitation of Gravity Waves

Daniel Lecoanet

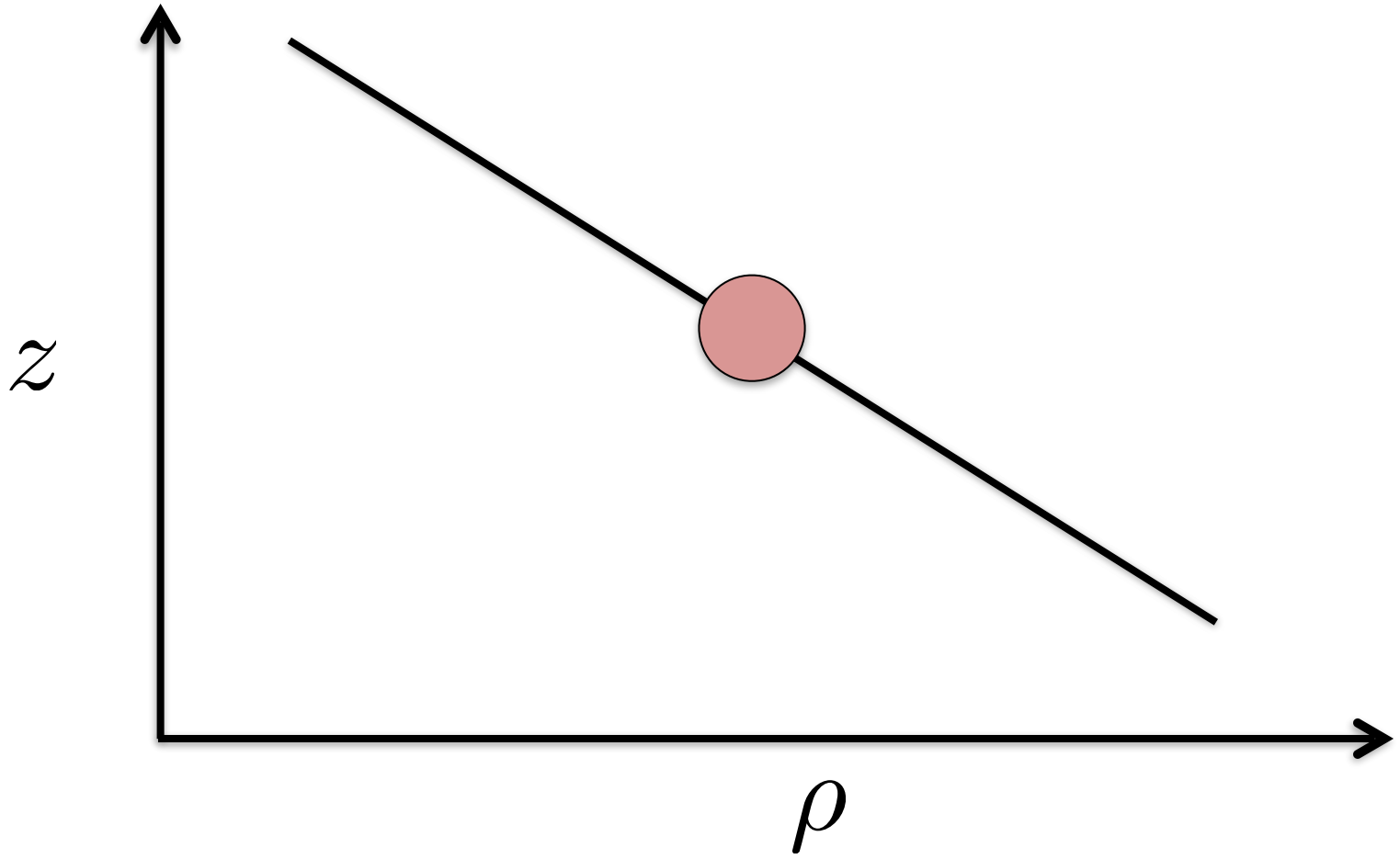
Keaton Burns, Michael Le Bars
Geoff Vasil, Ben Brown, Eliot Quataert,
Jeff Oishi, Jim Fuller

Funded by: NSF, Hertz, ARC, others

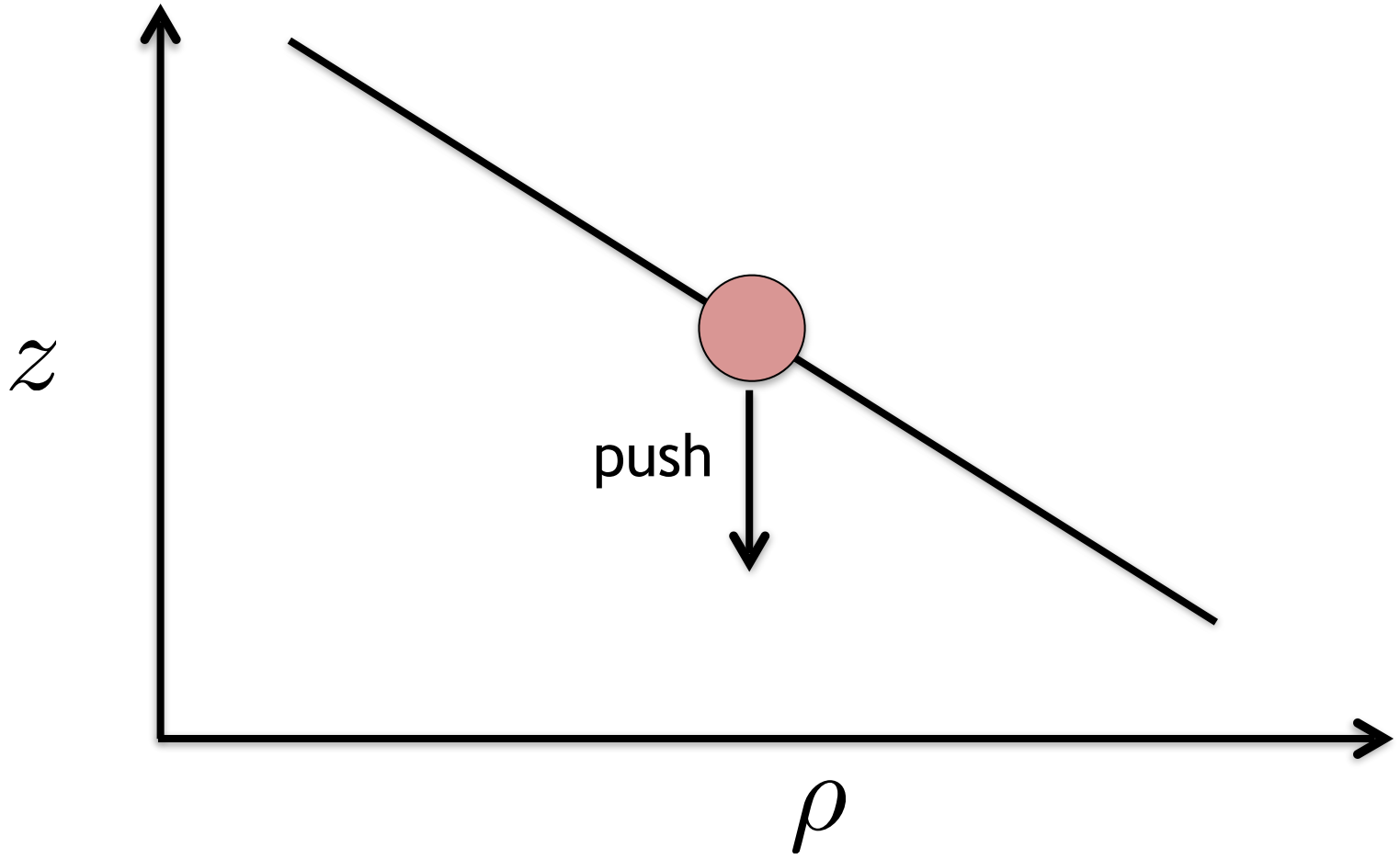
Buoyancy Wave



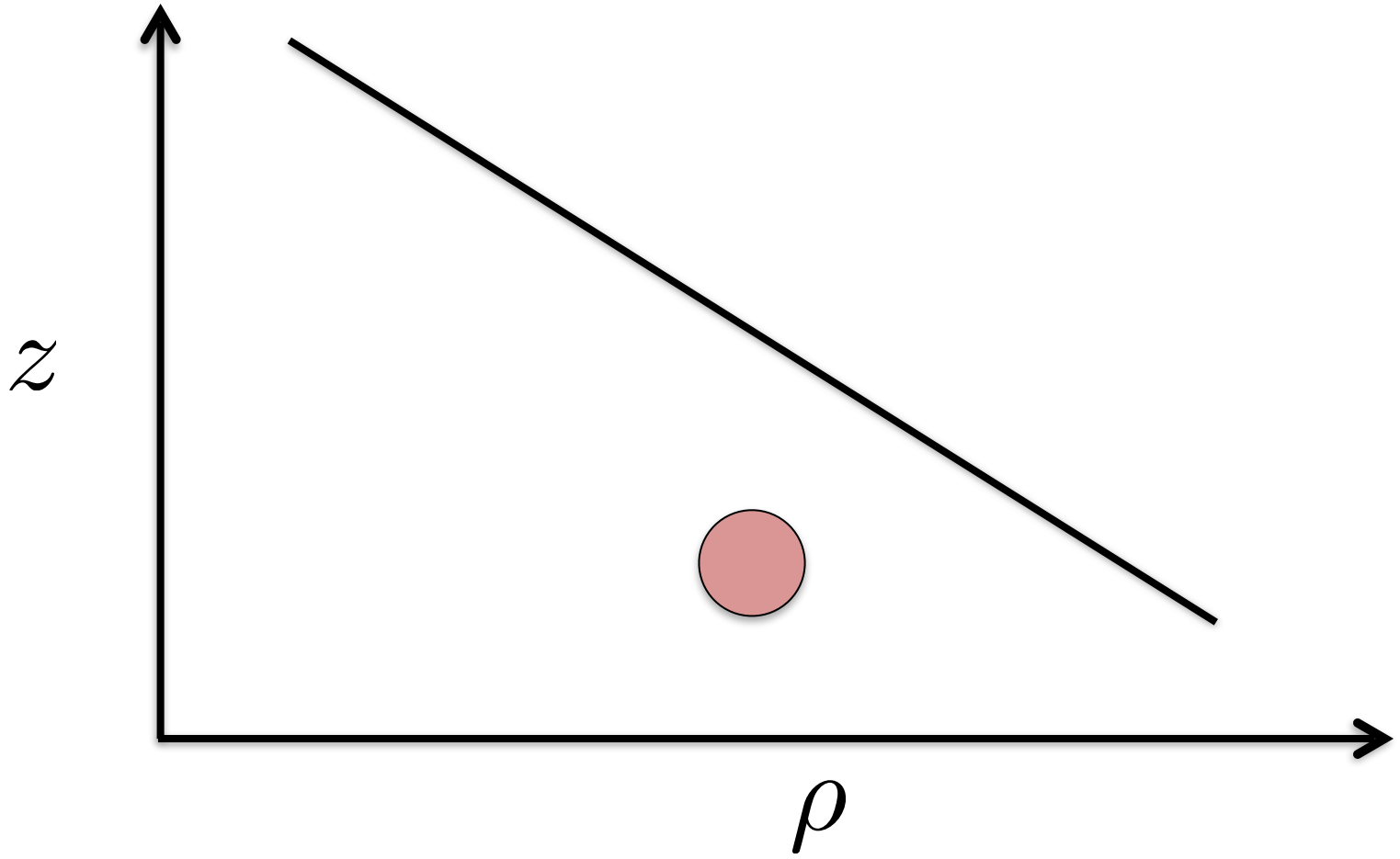
Buoyancy Wave



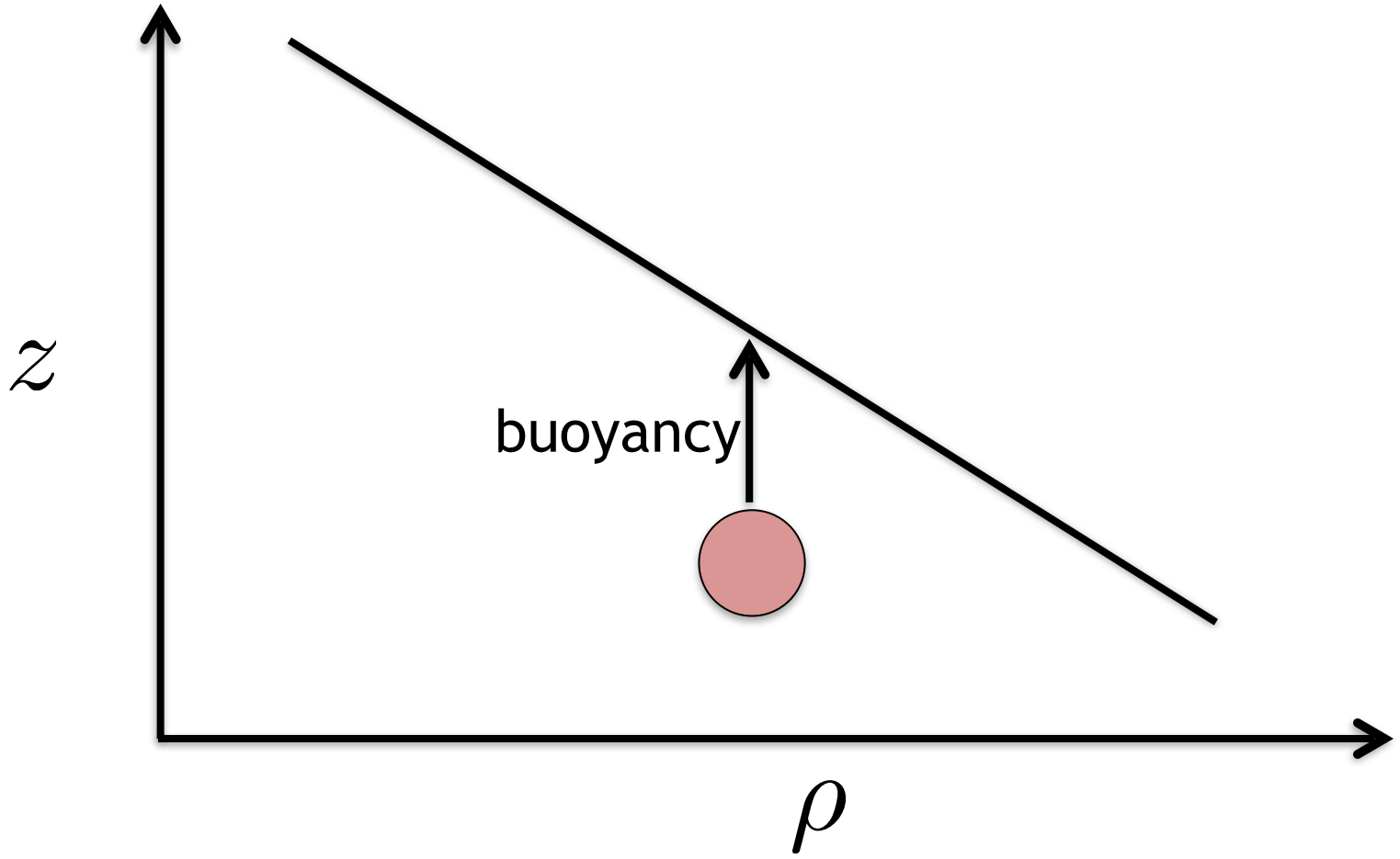
Buoyancy Wave



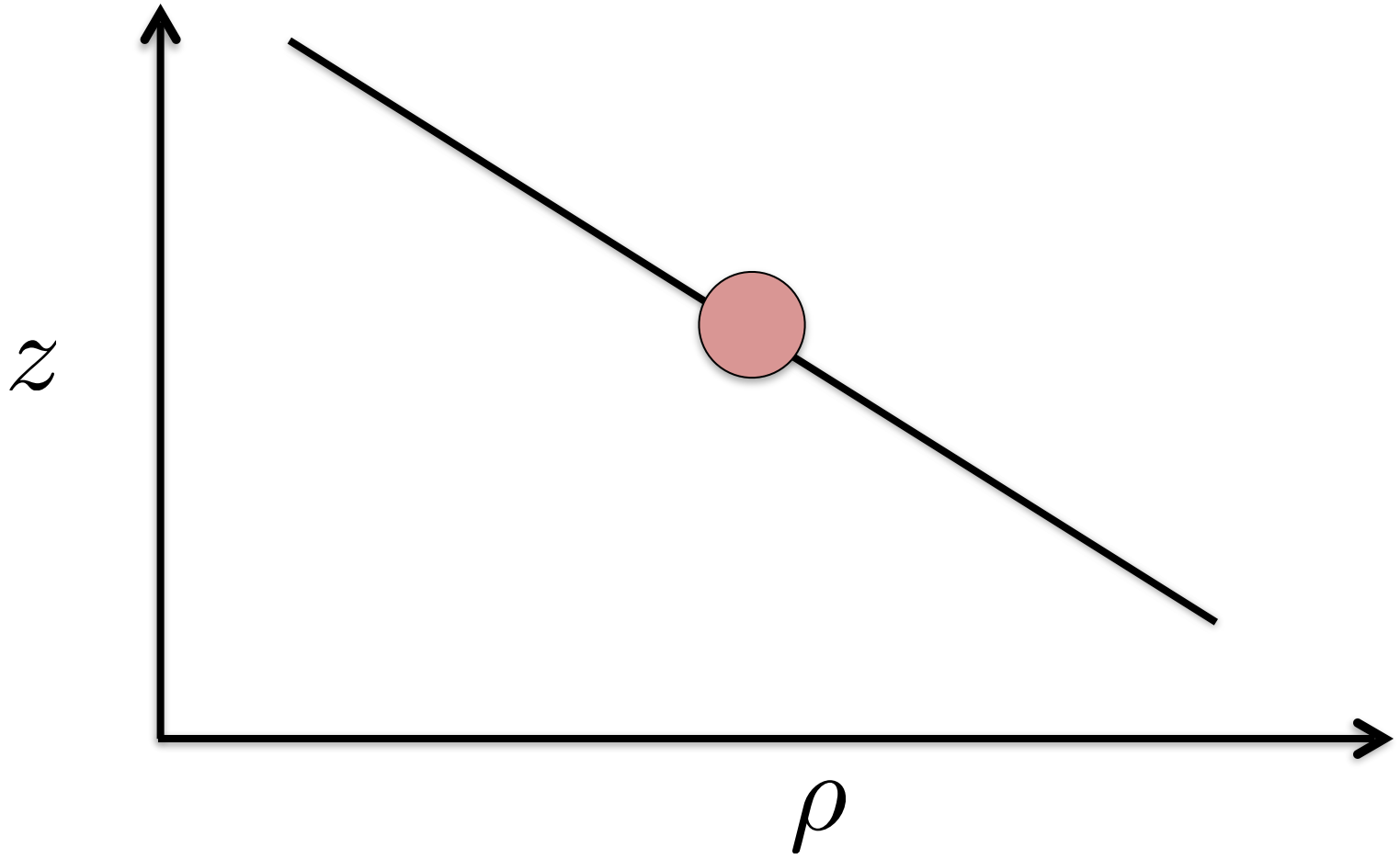
Buoyancy Wave



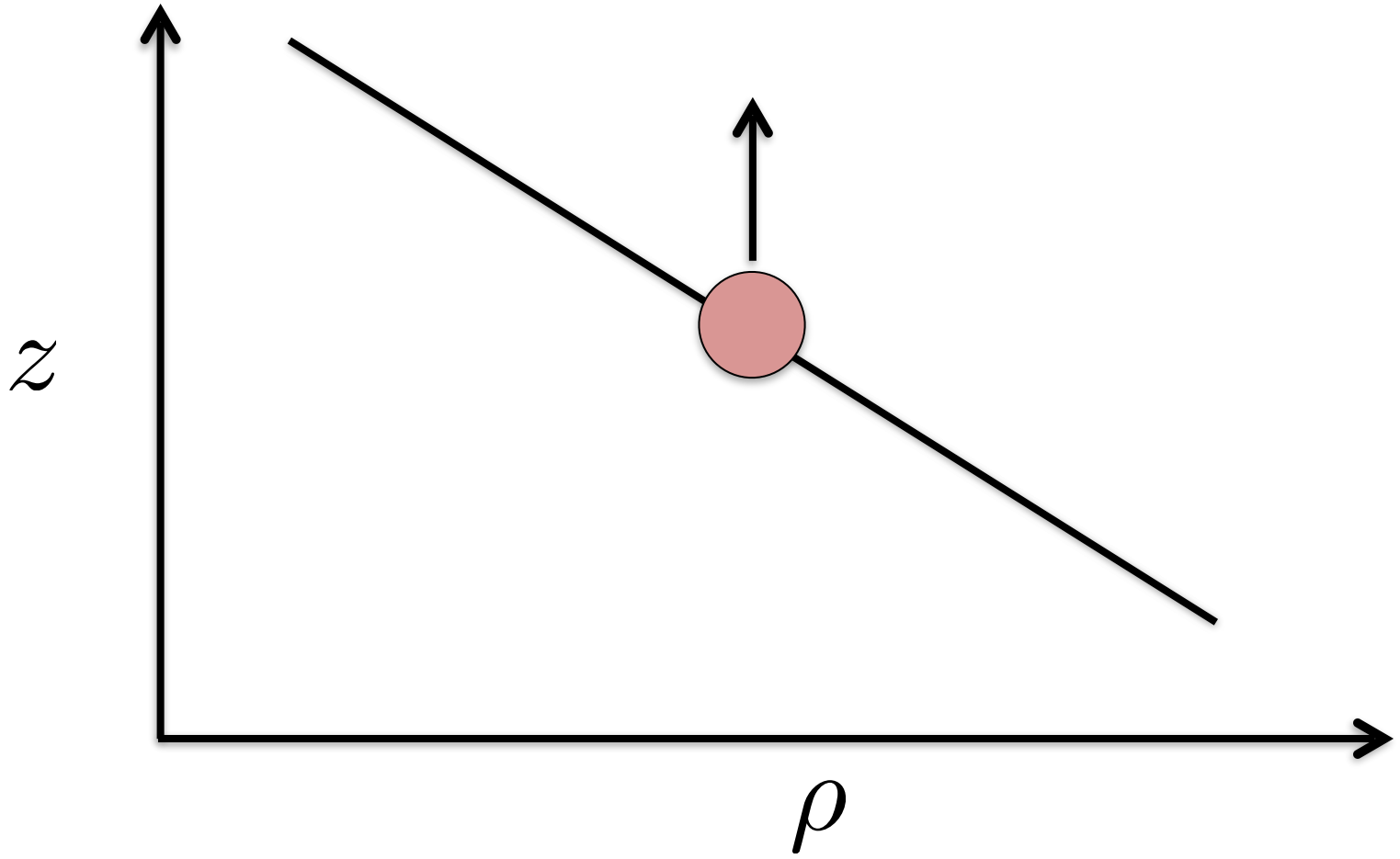
Buoyancy Wave



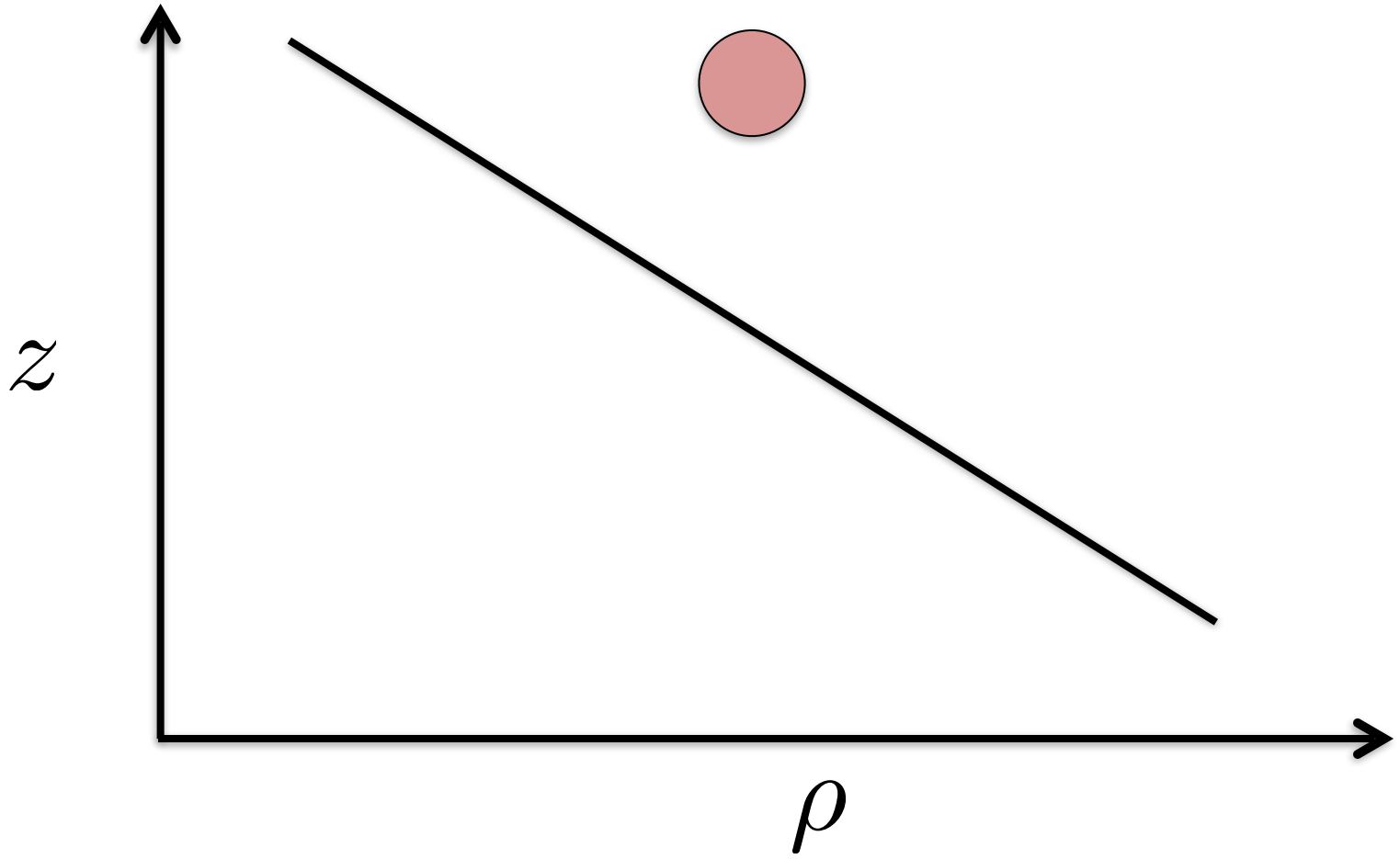
Buoyancy Wave



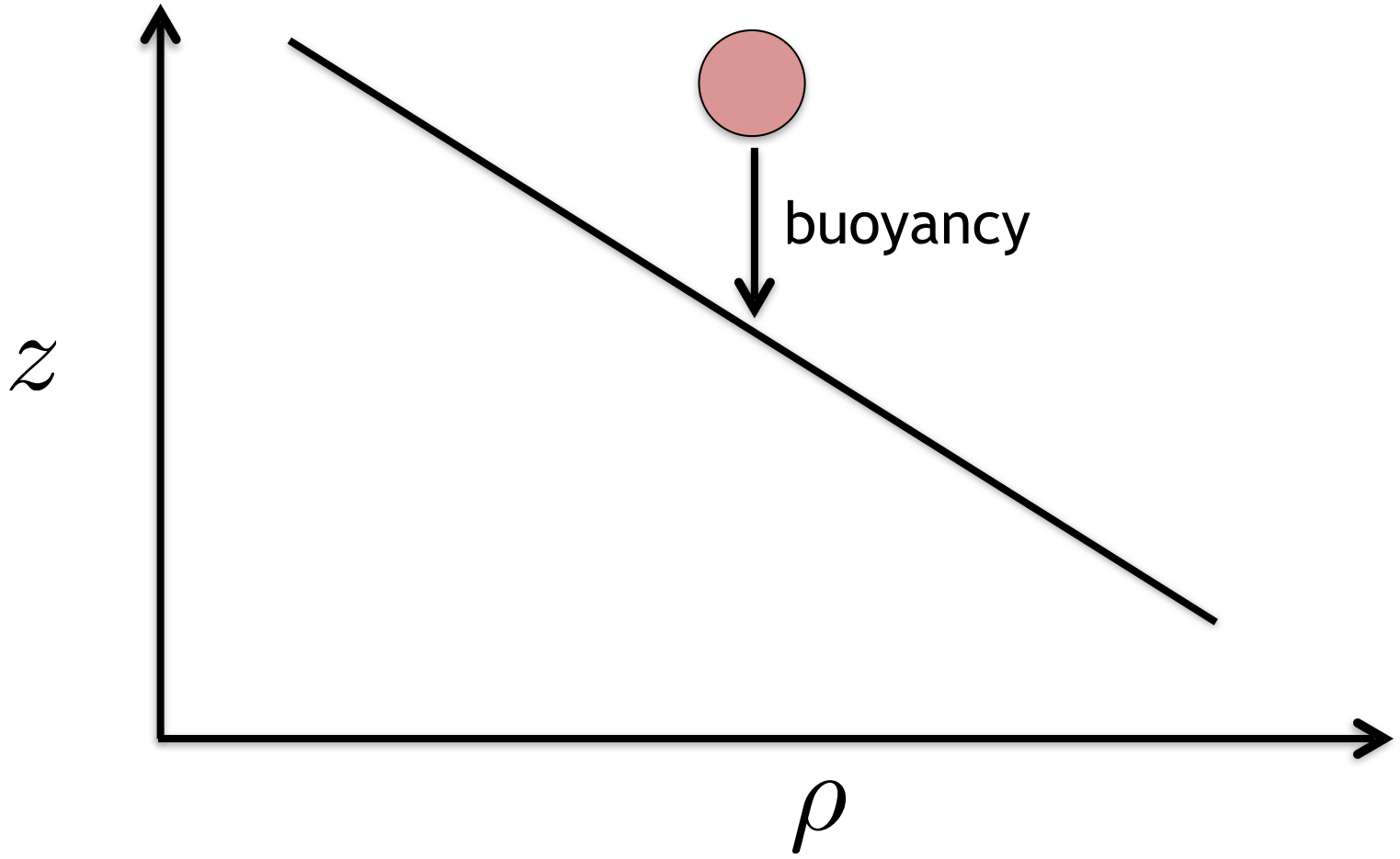
Buoyancy Wave



Buoyancy Wave



Buoyancy Wave



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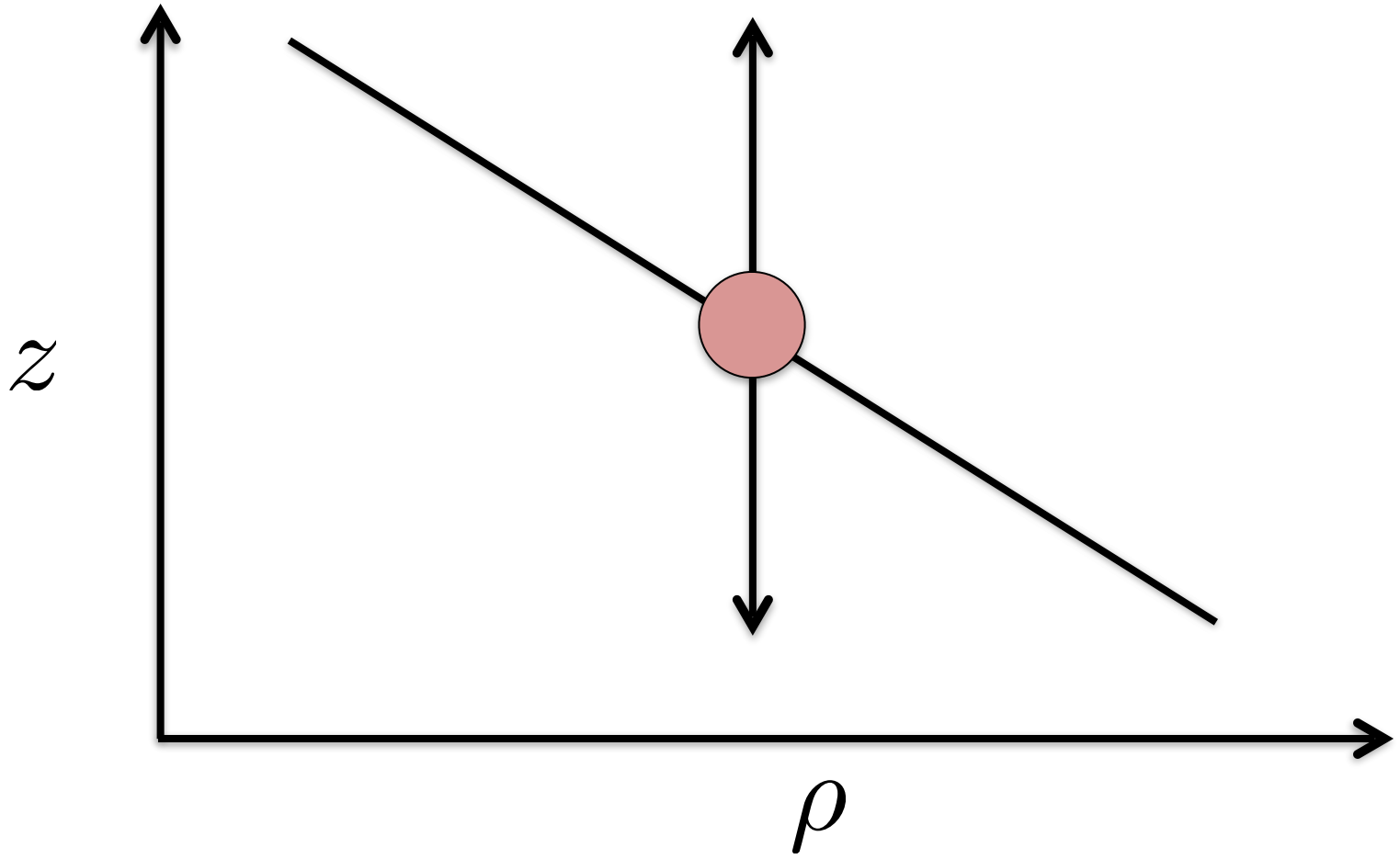

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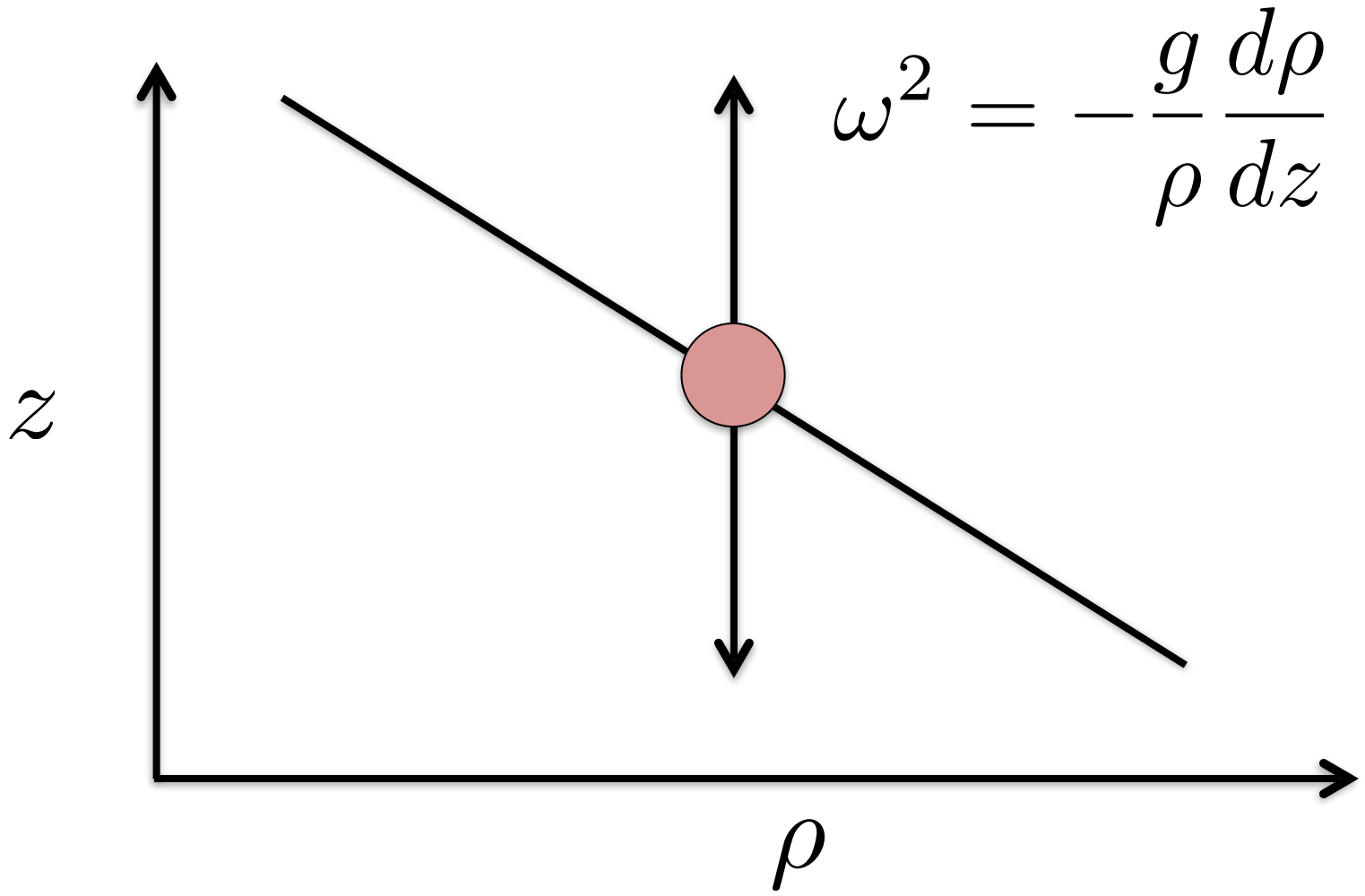
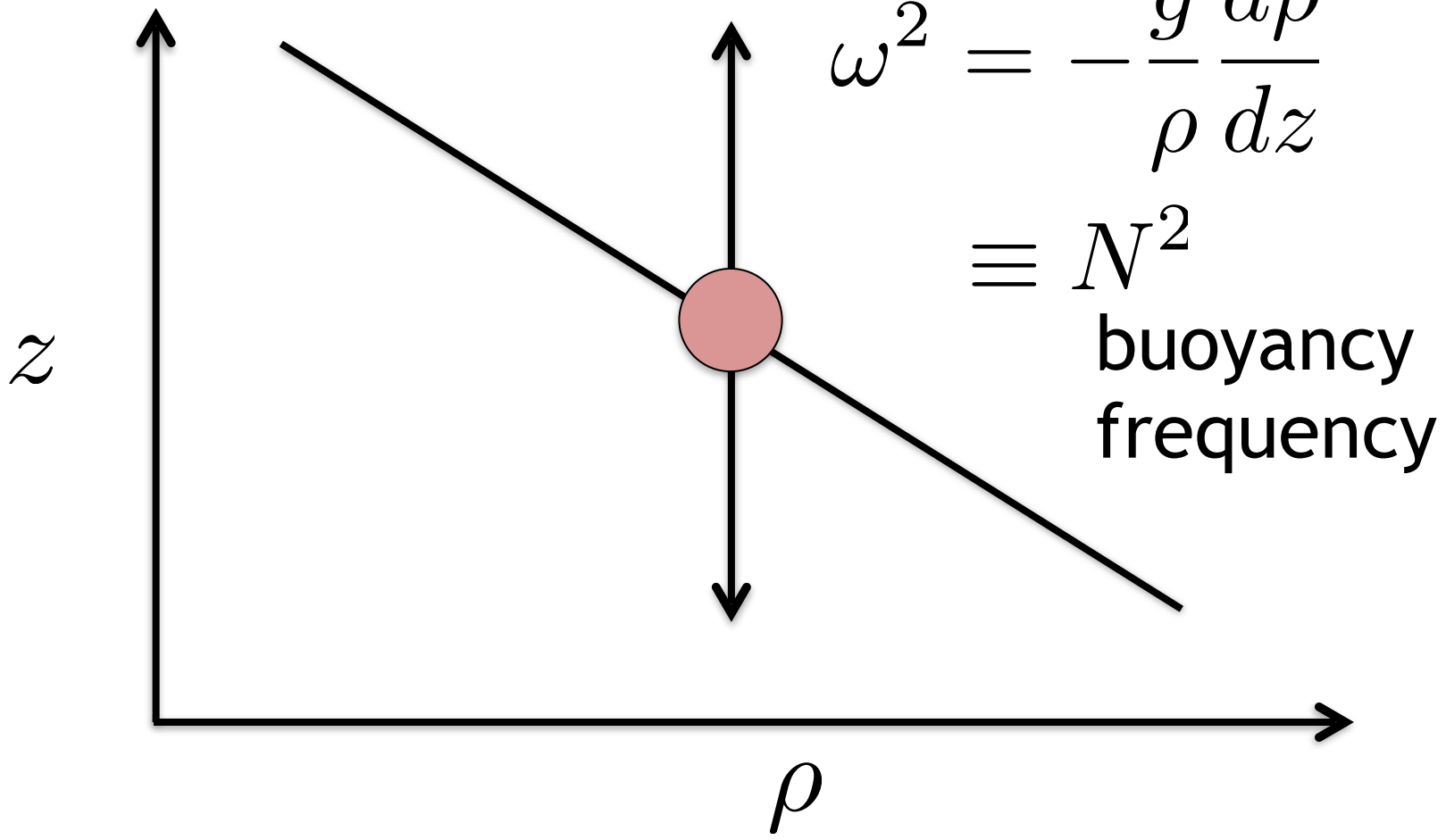
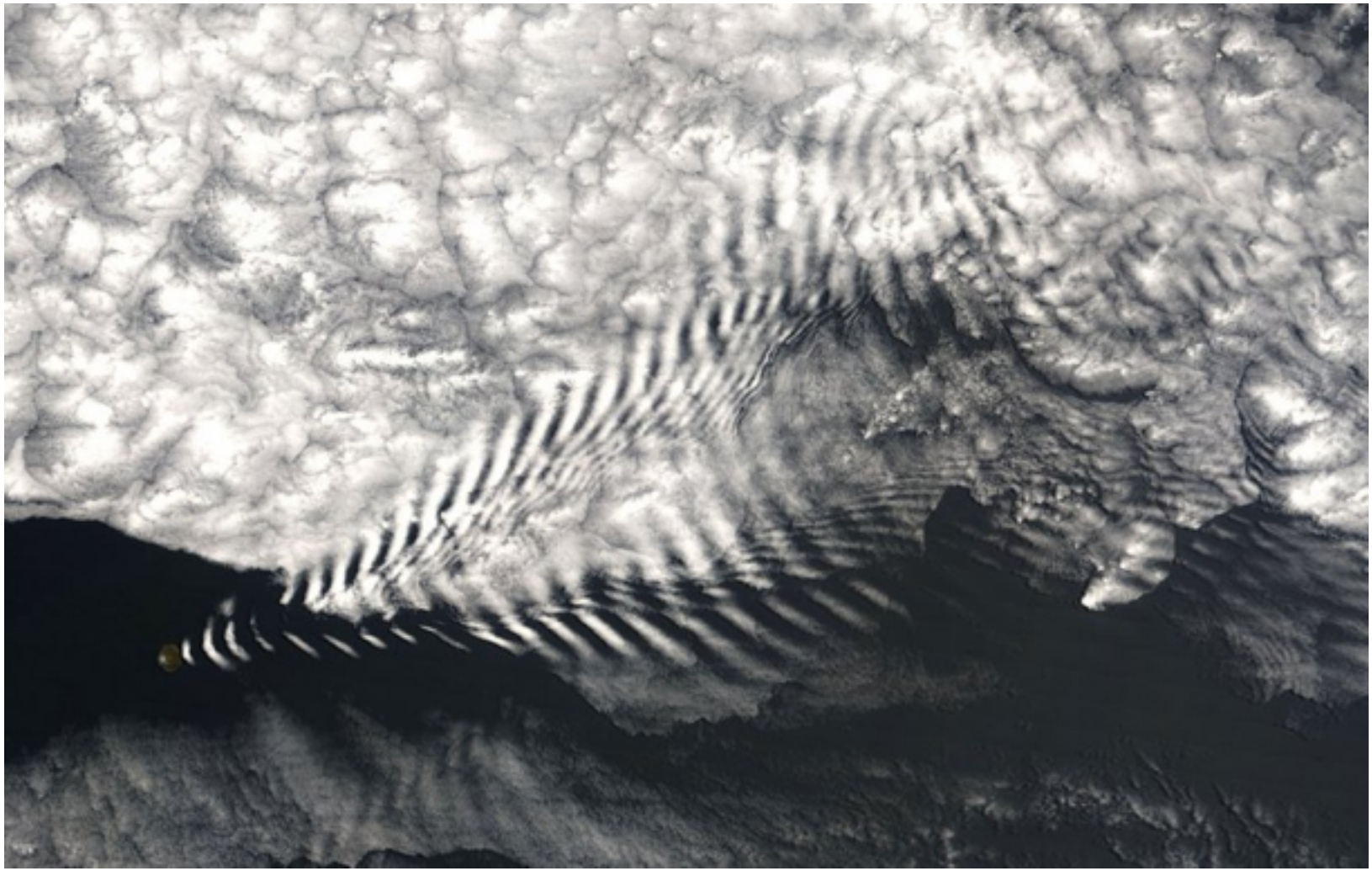


Buoyancy Wave



http://en.wikipedia.org/wiki/File:Wave_cloud.jpg

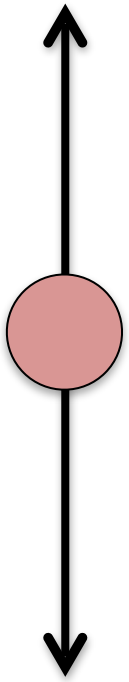


Buoyancy Wave

oscillations only associated
with vertical motions

Buoyancy Wave

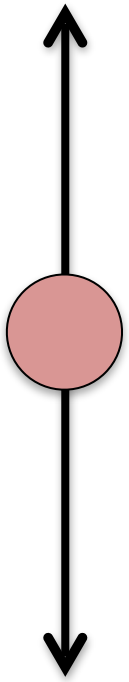
oscillations only associated with vertical motions



$$\omega^2 = N^2$$

Buoyancy Wave

oscillations only associated with vertical motions

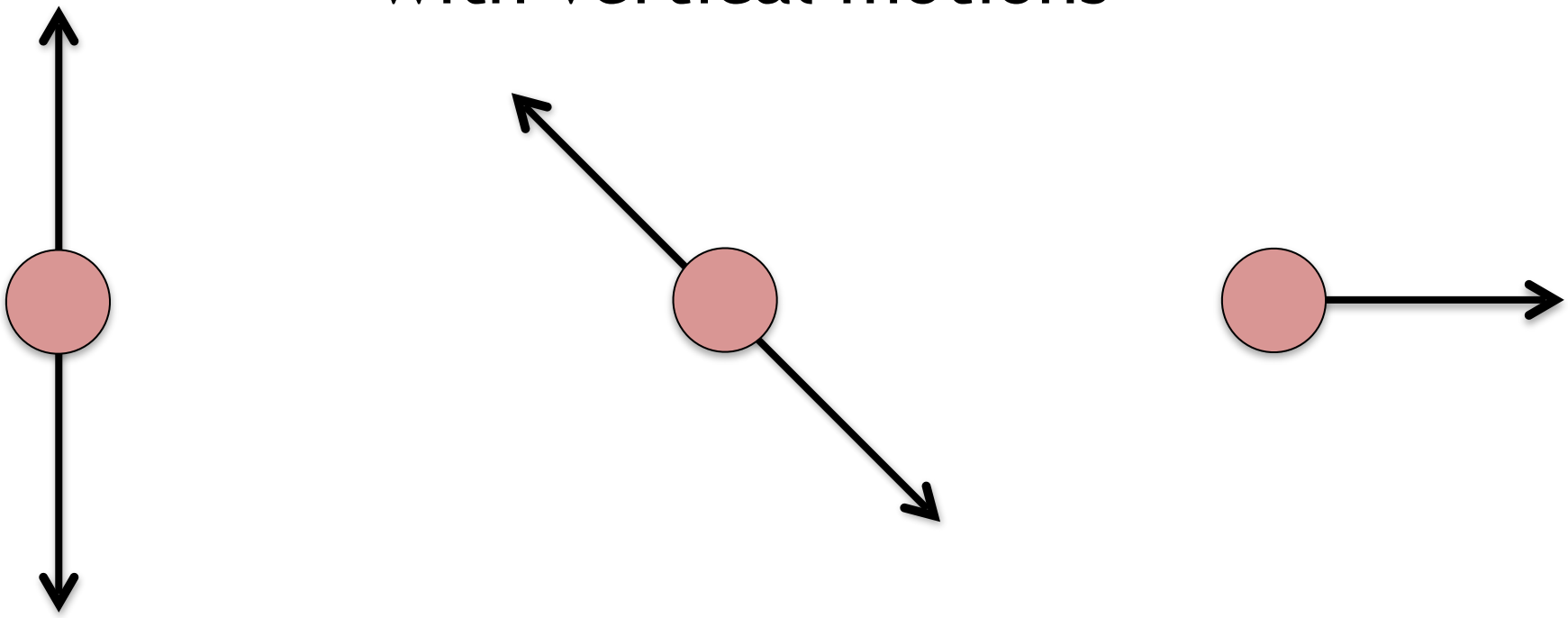


$\omega^2 = N^2$

$\omega^2 = 0$

Buoyancy Wave

oscillations only associated with vertical motions



$\omega^2 = N^2$

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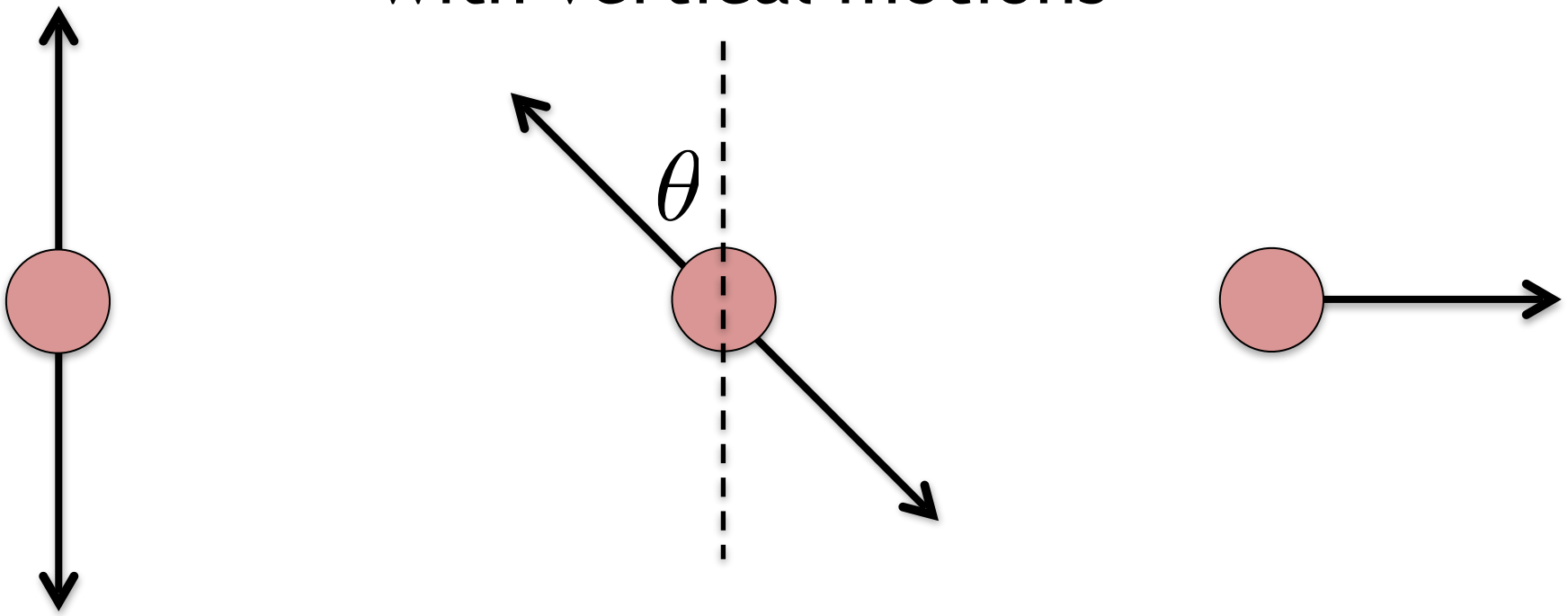
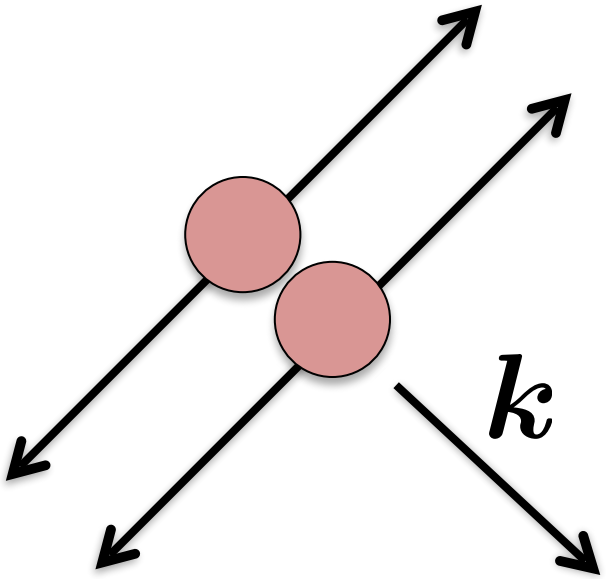
Buoyancy Wave

oscillations only associated with vertical motions



$\omega^2 = N^2$ $\omega^2 = N^2 \cos(\theta)^2$ $\omega^2 = 0$

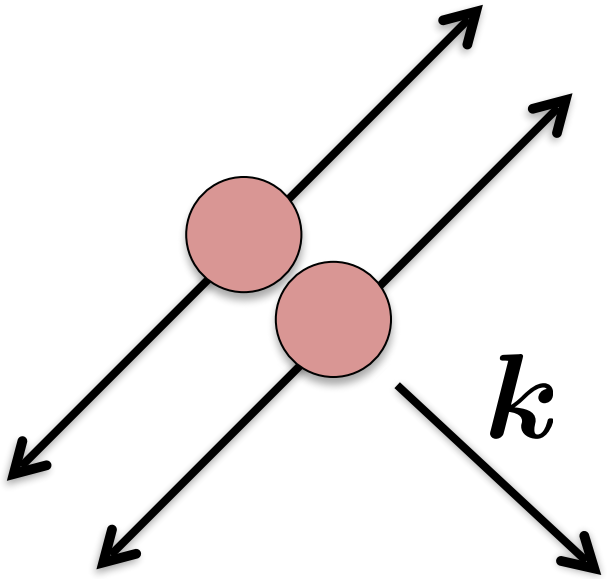
Buoyancy Wave



Buoyancy Wave

Dispersion relation:

$$\omega^2 = N^2 \frac{k_x^2 + k_y^2}{k_x^2 + k_y^2 + k_z^2} = N^2 \frac{k_\perp^2}{k^2}$$



Buoyancy Wave

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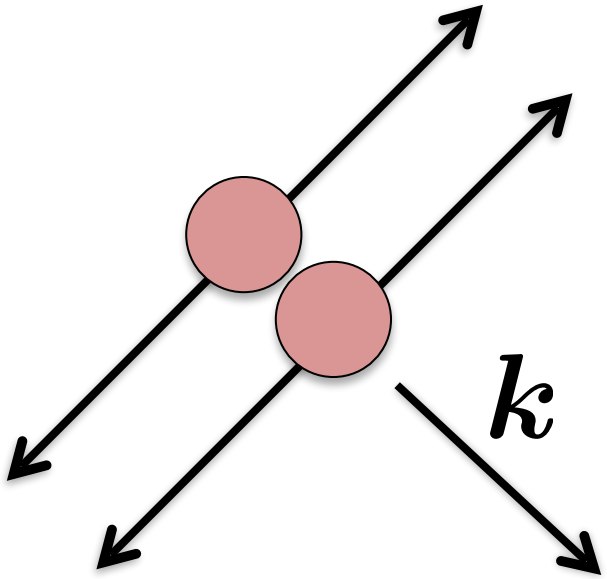
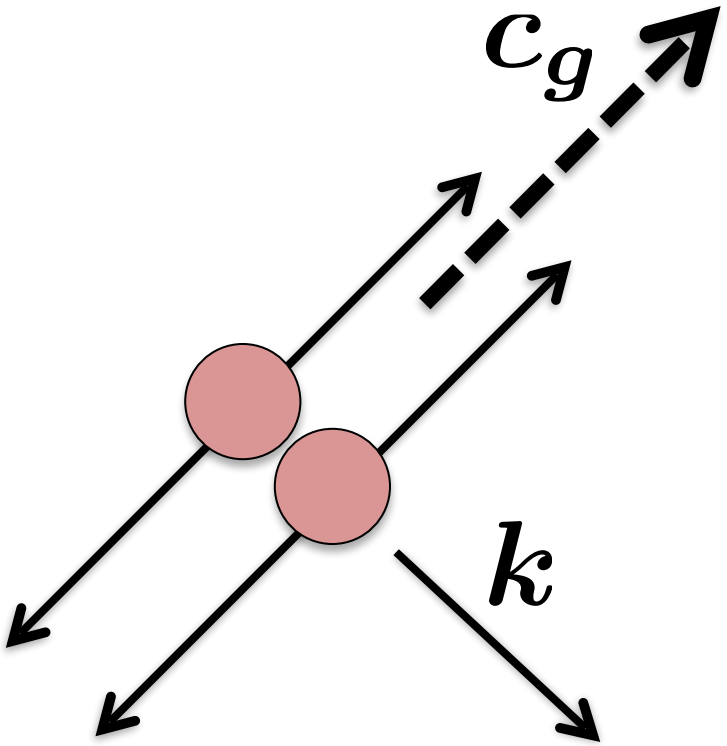


Low frequency => $k_z \gg k_\perp$

Buoyancy Wave



Why should I care about gravity waves?

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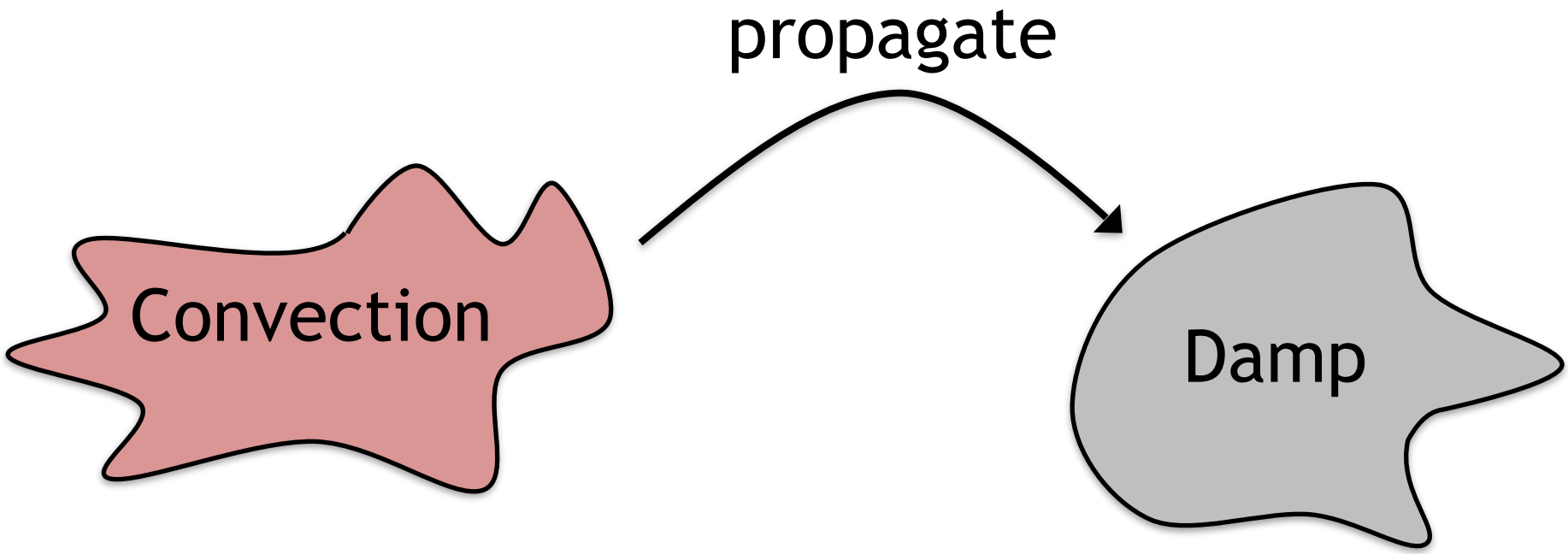


Why should I care about gravity waves?

- Angular momentum transport
- Chemical transport
- Energy transport

(Tami was supposed to talk about this)

Why should I care about gravity waves?

- **Angular momentum transport**
- Chemical transport
- Energy transport

(Tami was supposed to talk about this)

Angular momentum transport

Wave with frequency ω , azimuthal wavenumber m, and energy E, carries

$$J = \frac{m}{\omega} E$$

Fuller et al (2014)

Angular momentum transport

Wave with frequency ω, azimuthal wavenumber m, and energy E, carries

$$J = \frac{m}{\omega} E$$

Assume typical wave model:

$$t_{\rm waves} \sim \frac{I_{\rm rad}\Omega}{\dot{J}} \sim 10^5 \ {\rm yr}$$

Fuller et al (2014)

Angular momentum transport

Wave transport actually much weaker, because waves damp quickly:

$$L_d = \frac{2r^3\omega^4}{[\ell(\ell+1)]^{3/2}N^3K}$$

Fuller et al (2014)

Angular momentum transport

Wave transport actually much weaker, because waves damp quickly:

$$L_d = \frac{2r^3\omega^4}{[\ell(\ell+1)]^{3/2}N^3K}$$

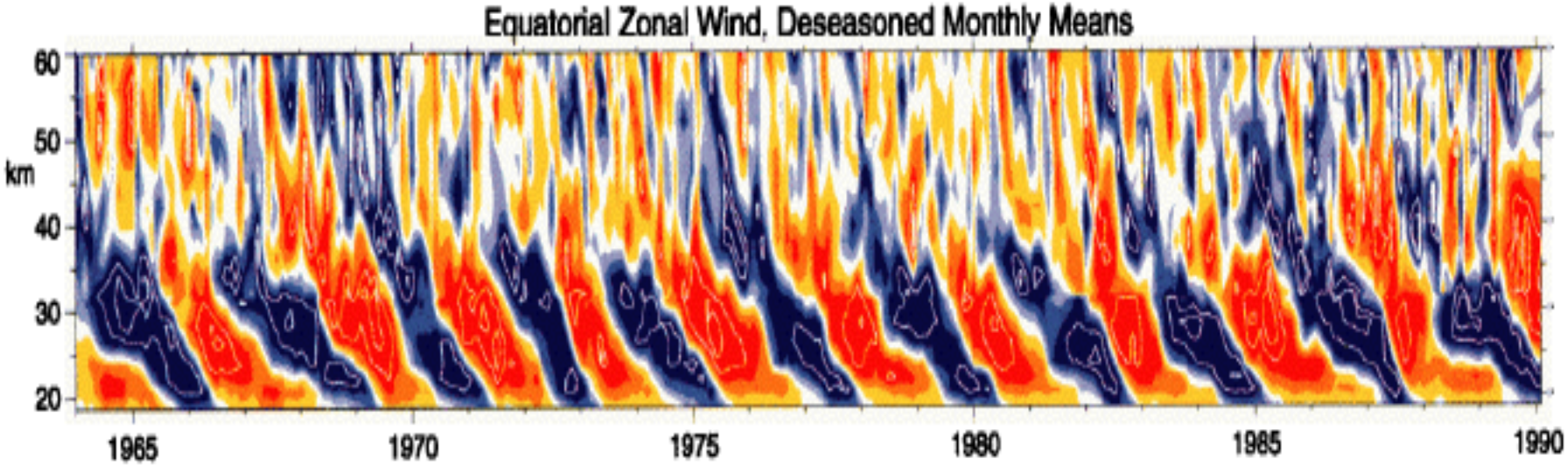
In sun, biggest waves (low freq.) have

$$L_d \sim 0.01 R_\odot$$

Fuller et al (2014)

Angular momentum transport

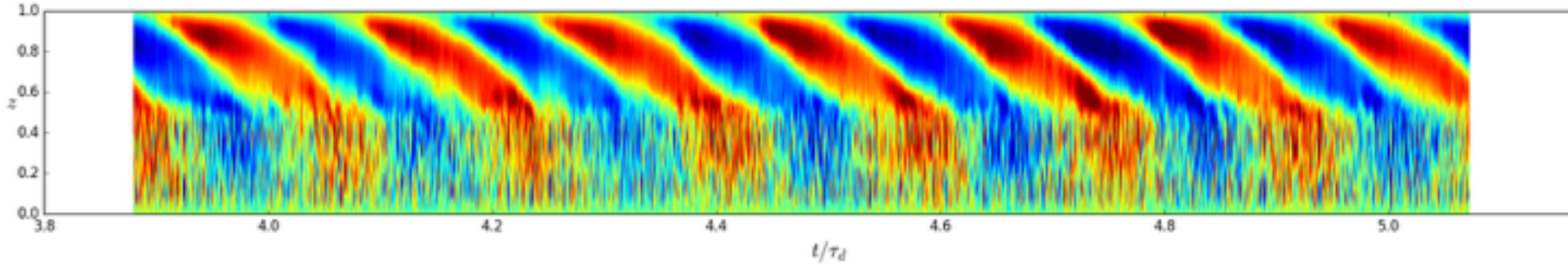
Quasi-biennial oscillation (QBO)

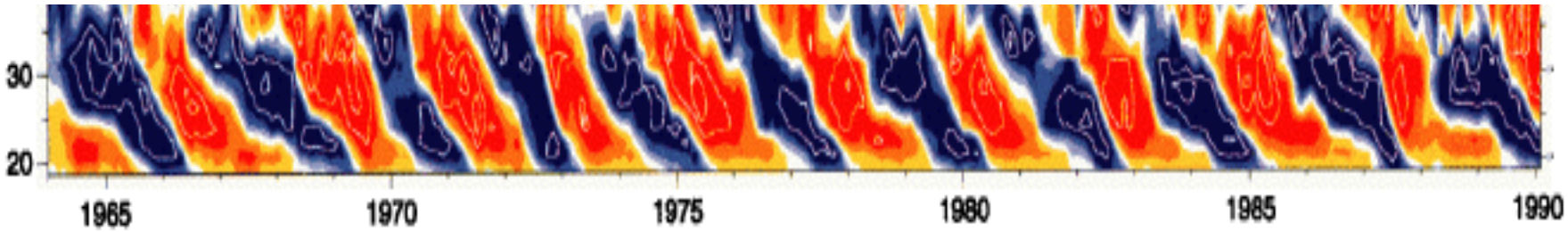


Baldwin+ 2009

Angular momentum transport

Louis Couston, IRPHE, Marseille FR





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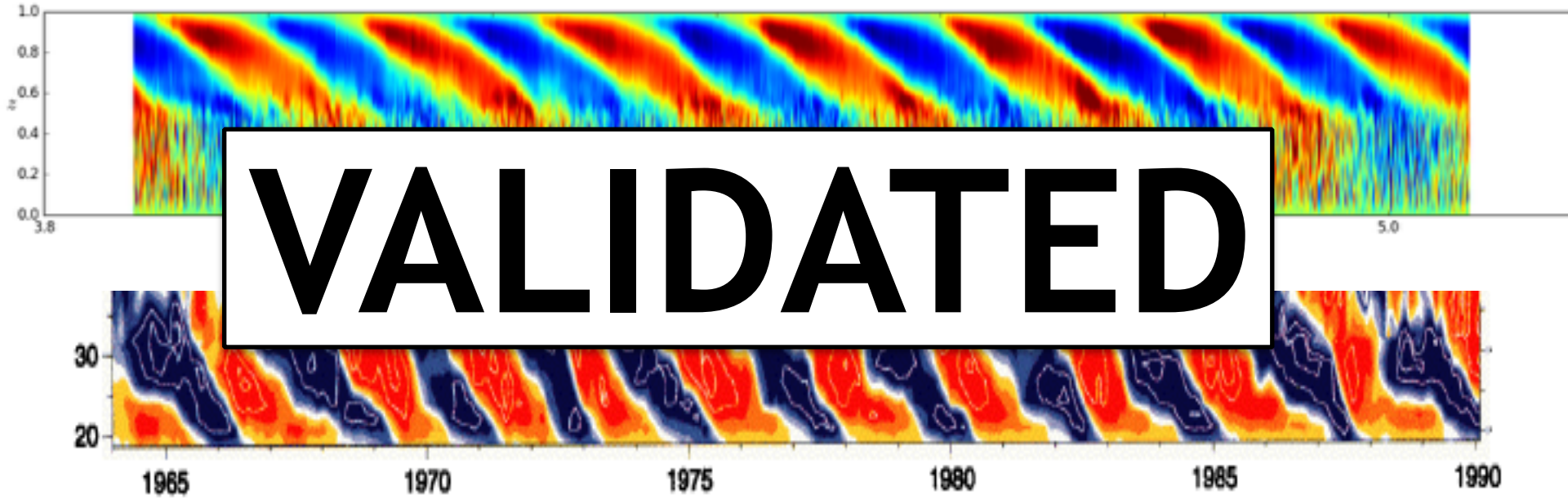
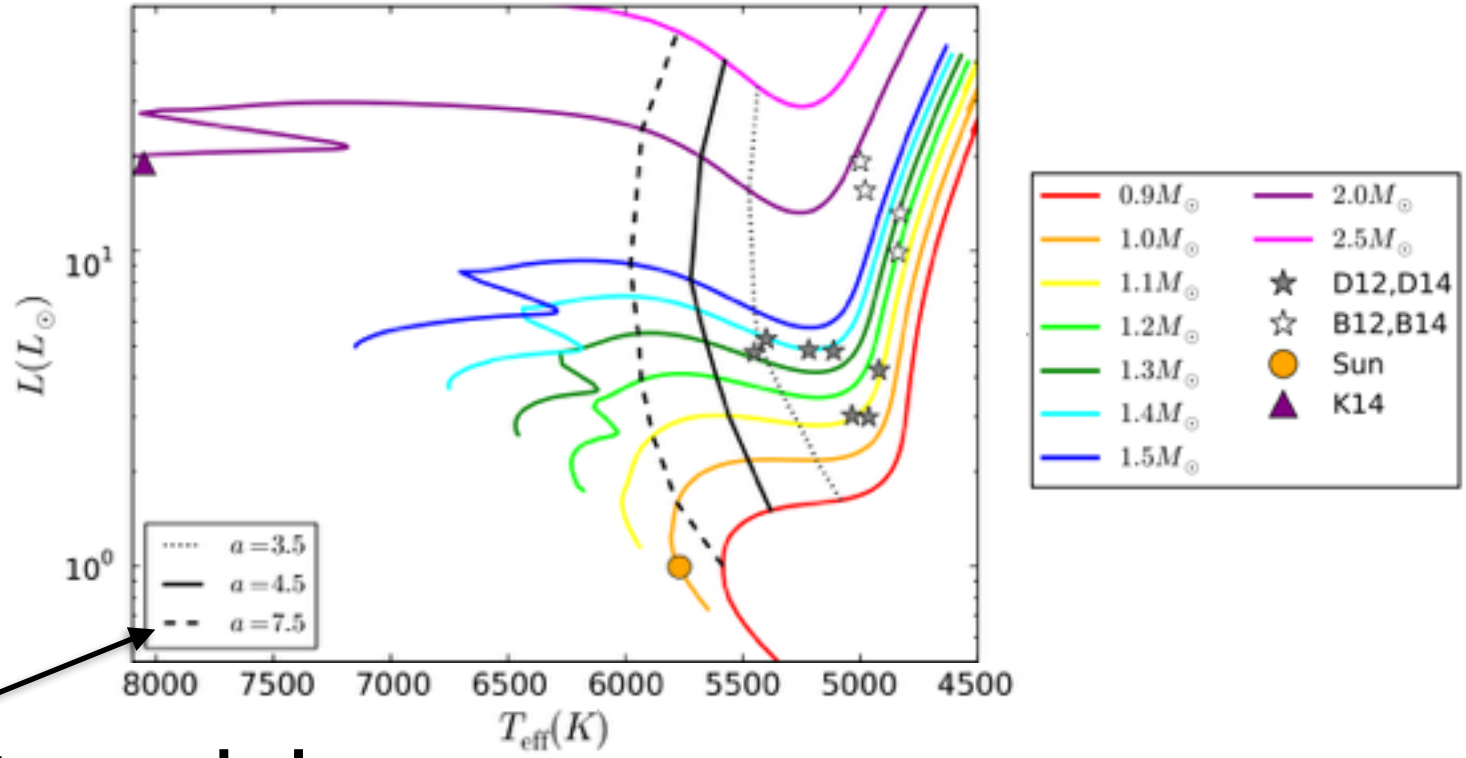


VALIDATED

Baldwin+ 2009

Angular momentum transport



different models

Fuller et al (2014)

Why should I care about gravity waves?

- Angular momentum transport
- Chemical transport
- Energy transport

All depend on model for wave generation

How should we study this problem?

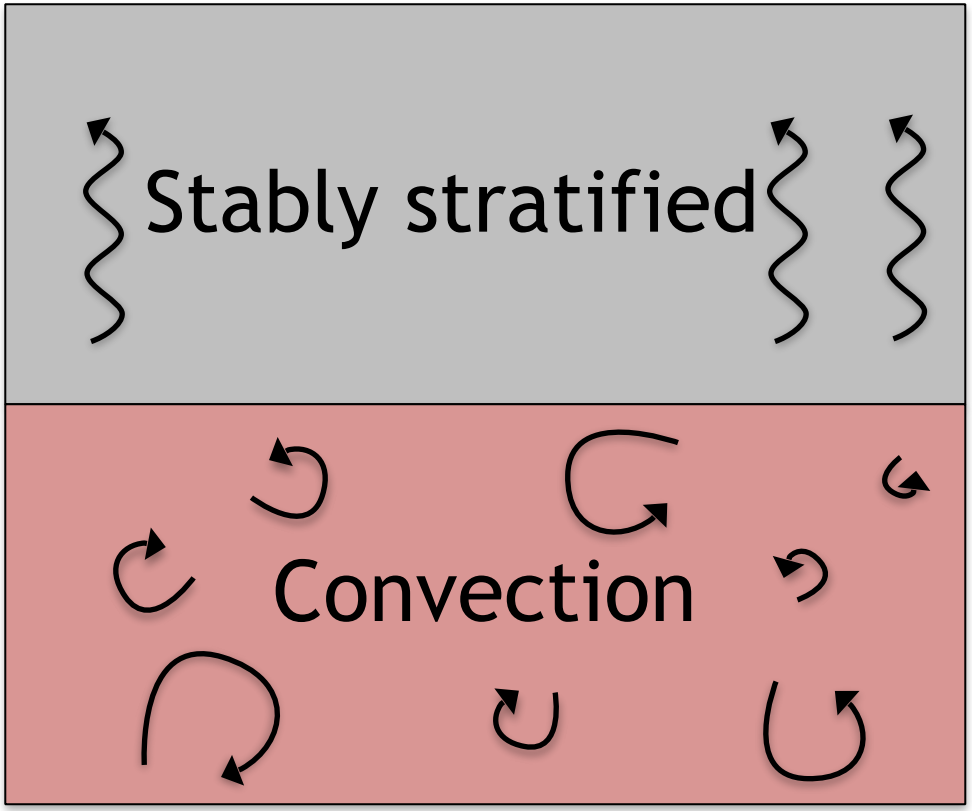
“We should do well-posed experiments to teach us physics.

Use that to improve comparison between observation and theories.

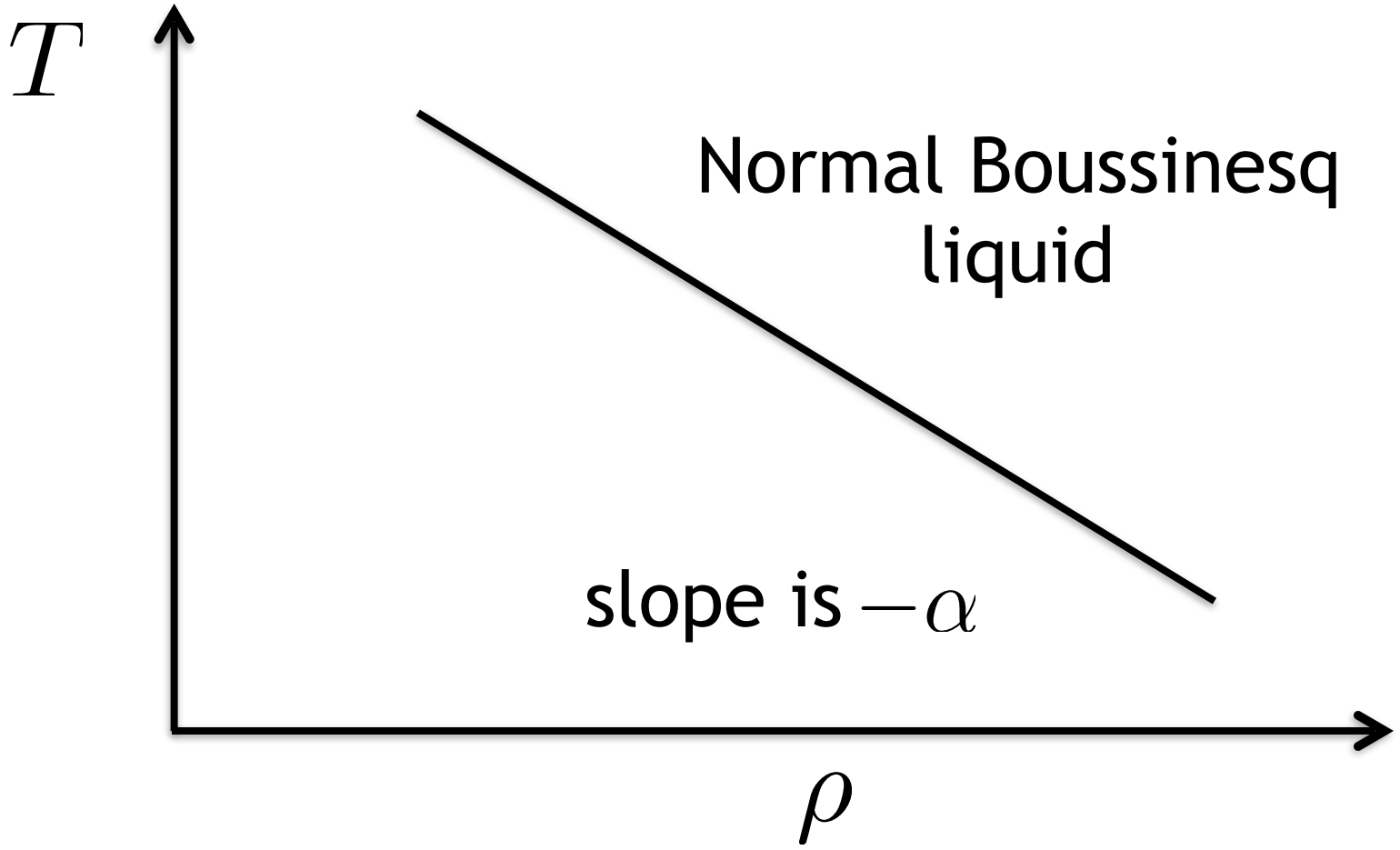
We should give up the fool's goal of simulating reality.”

—Stan, 10-4-2017

IGW Generation by Convection



Equation of state



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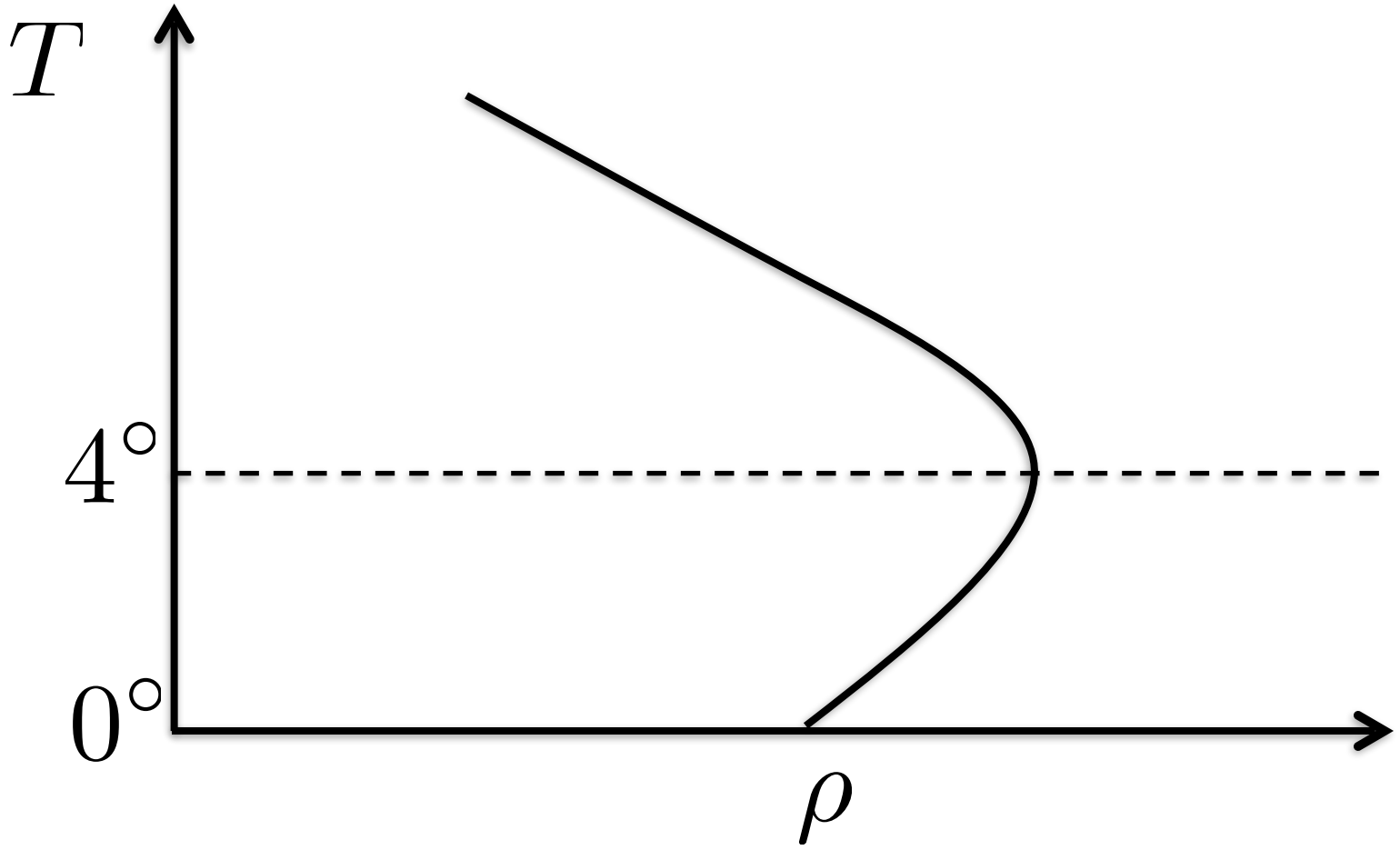
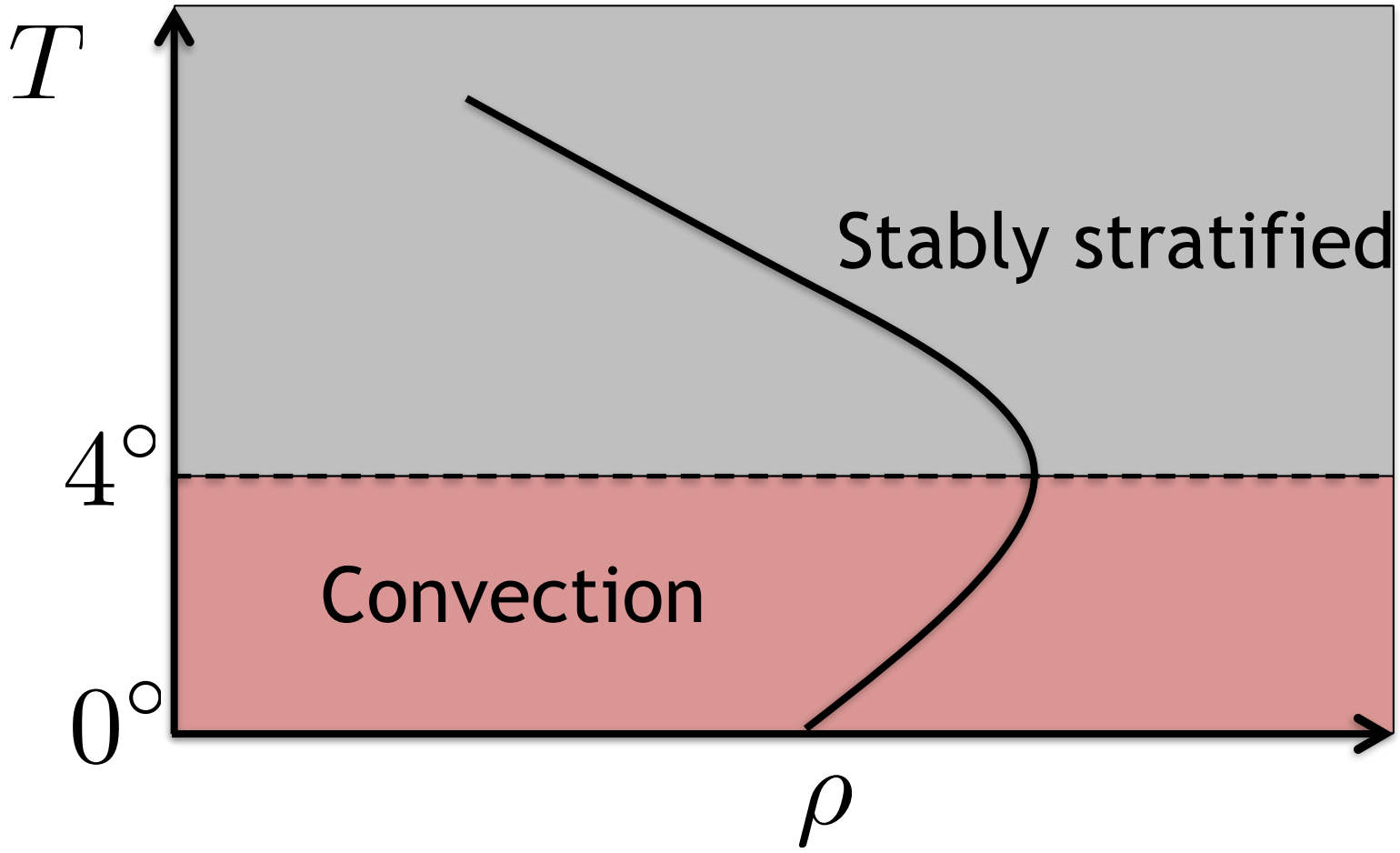
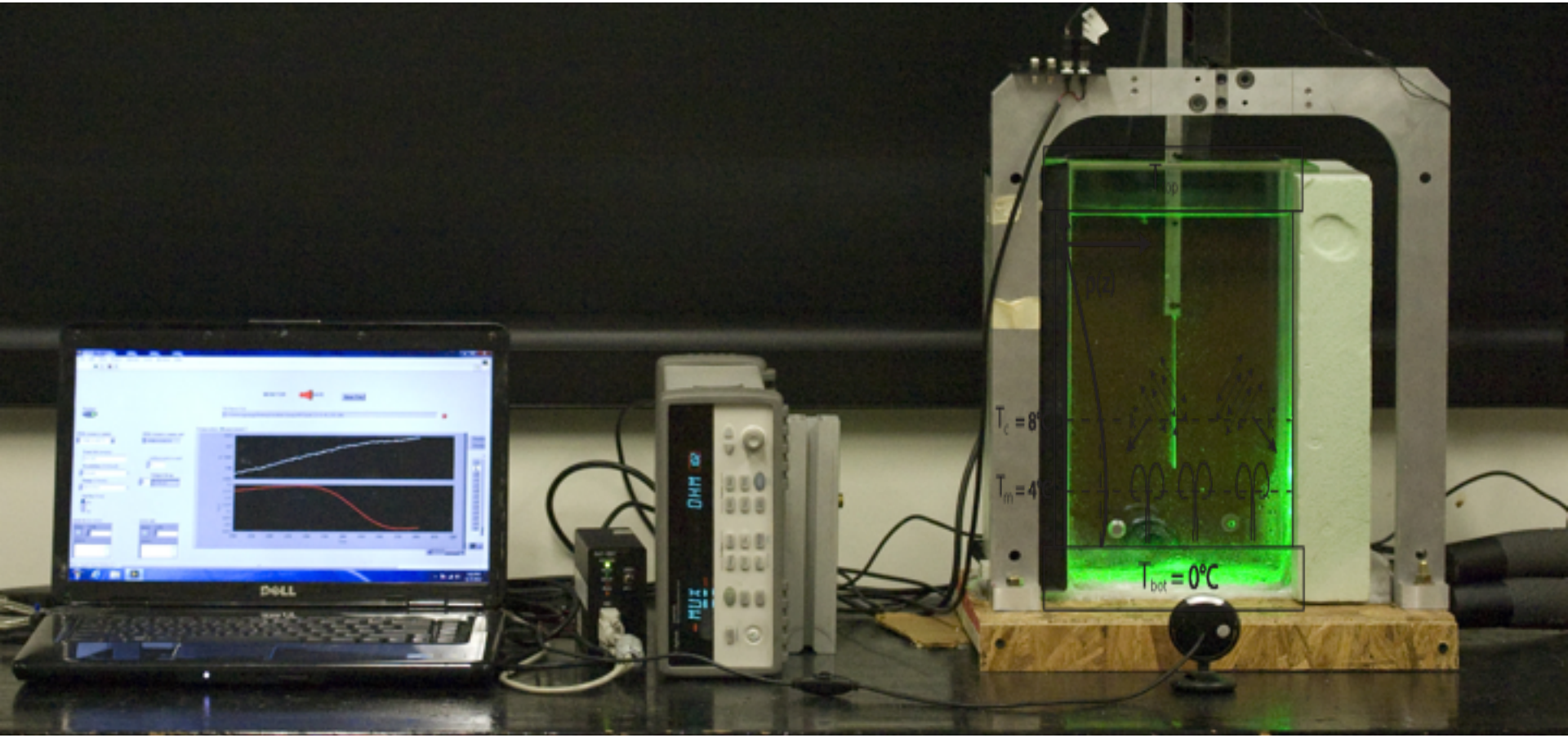


Equation of state



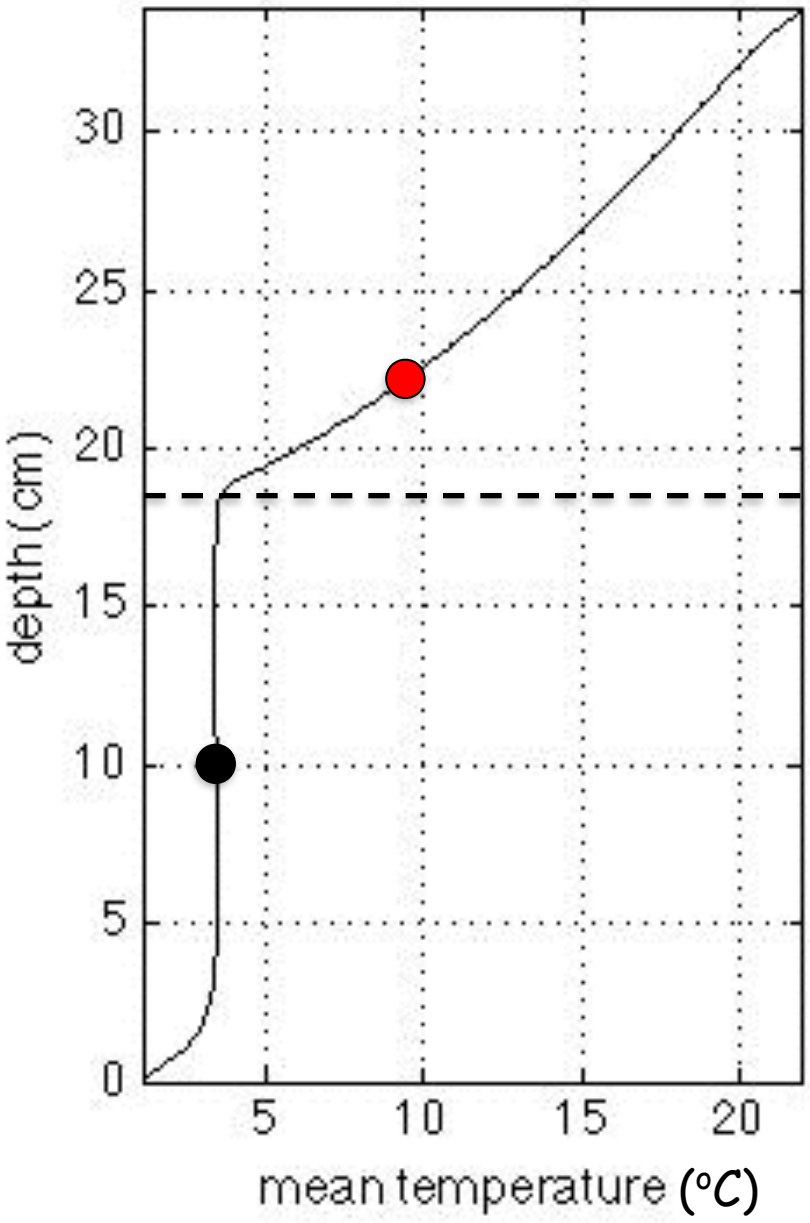
Convection in water around 4°C…



Dimensions: 20 x 4 x 35 cm^3

From Michael Le Bar

Convection in water around 4°C…



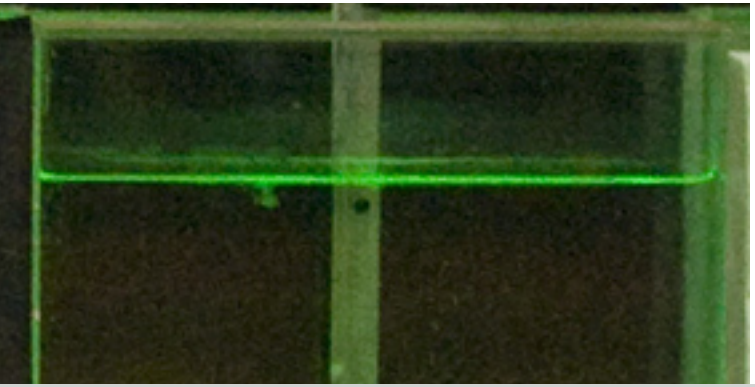
Stratified zone
= heat transfer by conduction

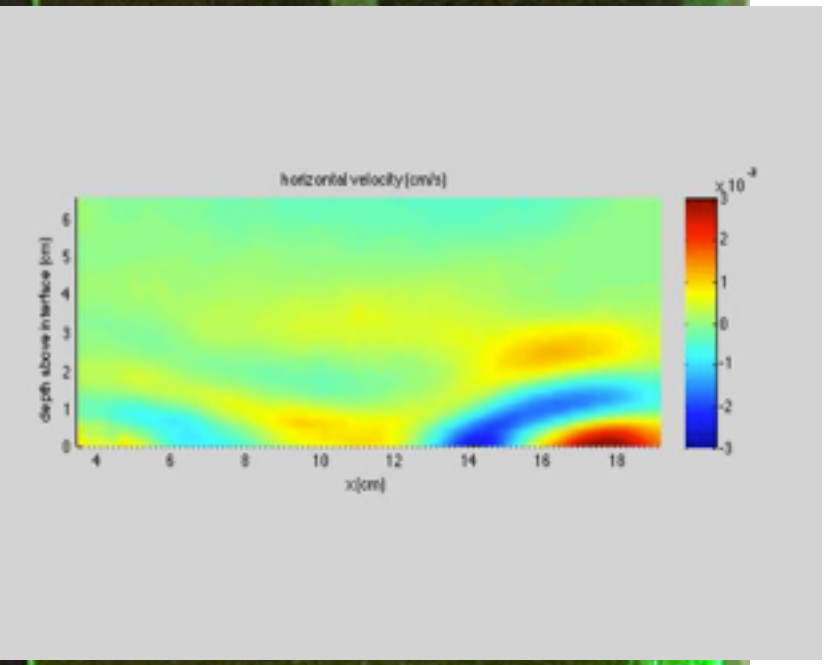
Interface (location function of imposed T° and heat losses)

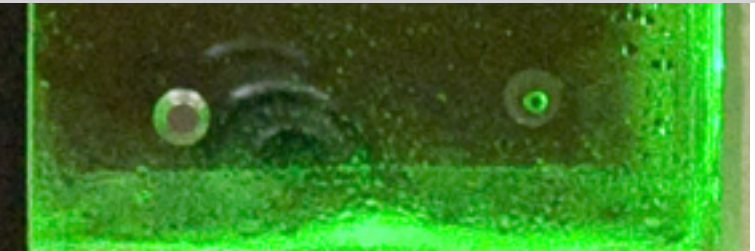
Convective zone
= heat transfer by convection
Mean T° ~3.4°C

From Michael Le Bar

Wave field



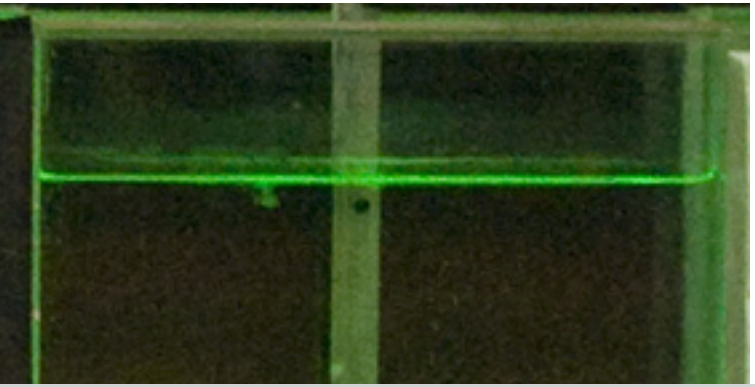


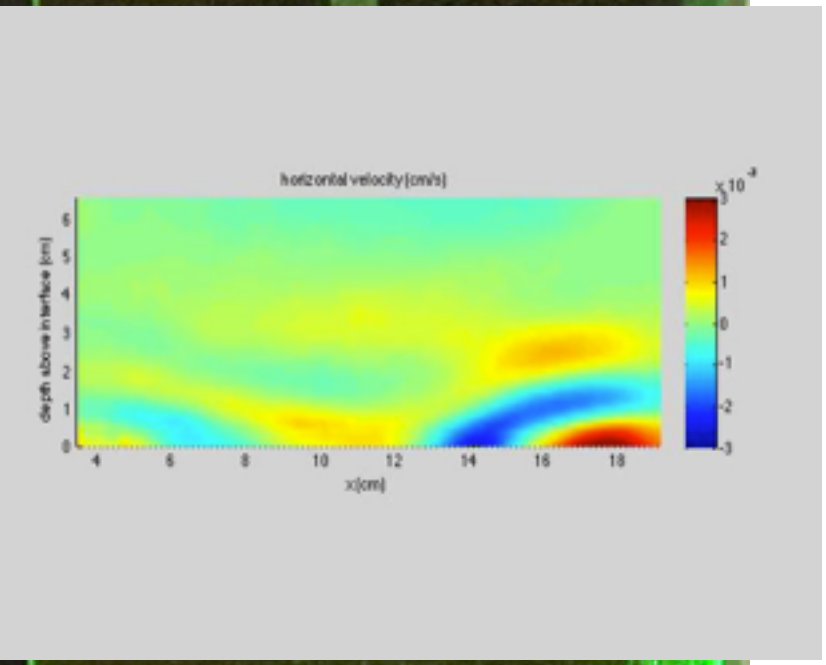


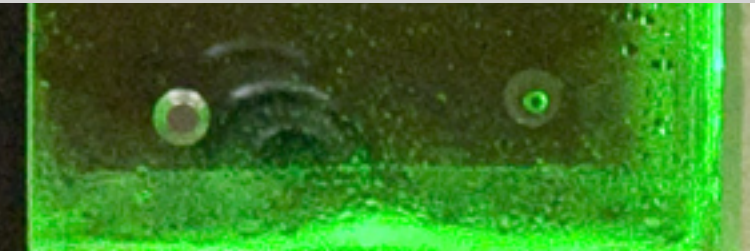
Accelerated x20
(real duration = 13 min)

From Michael Le Bar

Wave field







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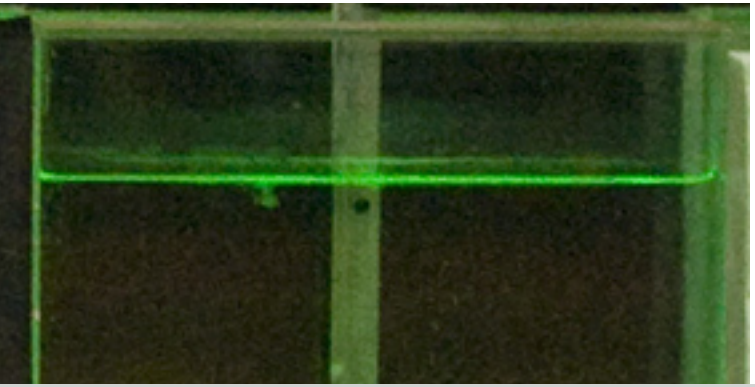
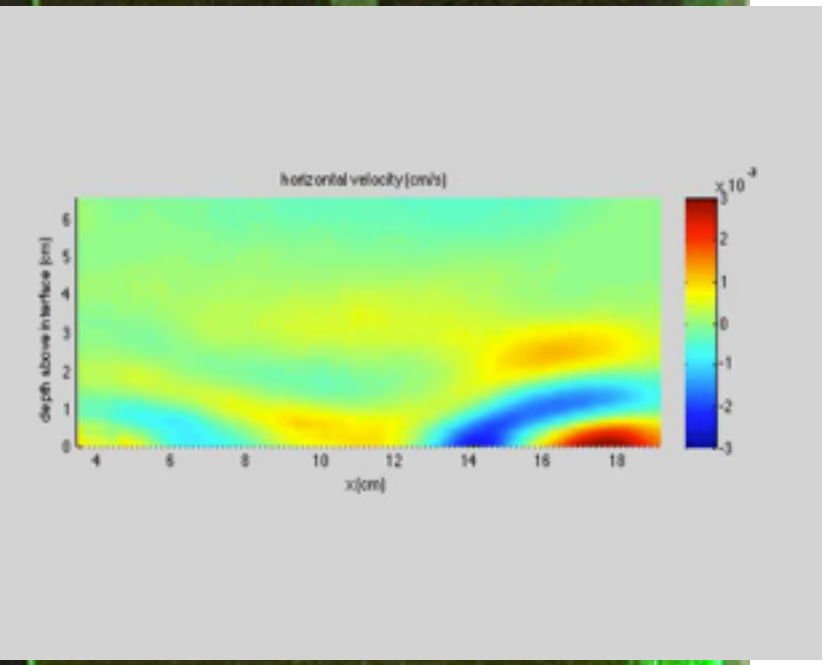
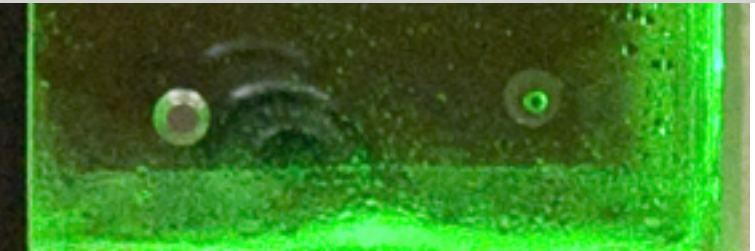






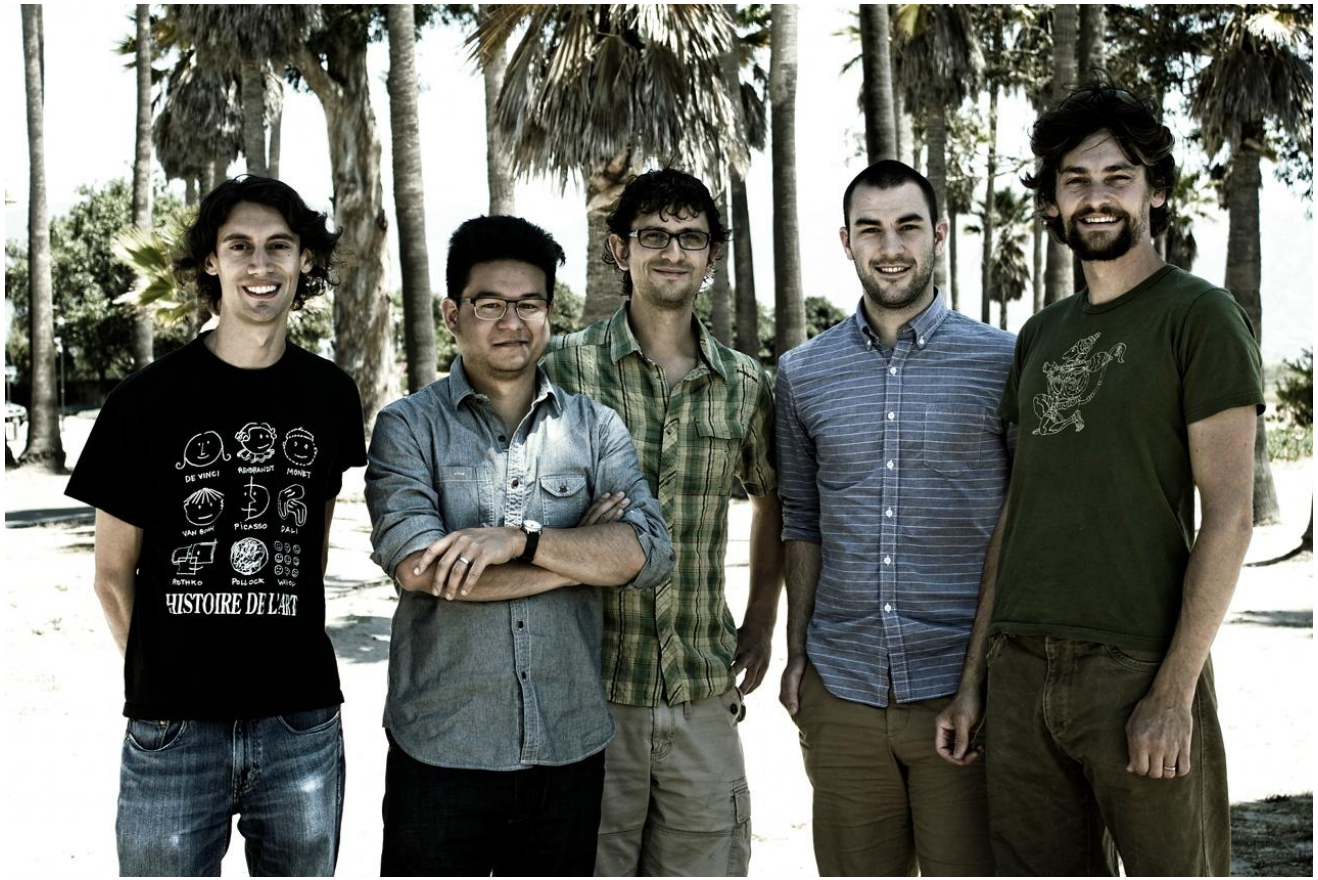
Accelerated x20
(real duration = 13 min)

From Michael Le Bar

Dedalus

Pseudo-spectral
Open-source
Python
****Very flexible equations**

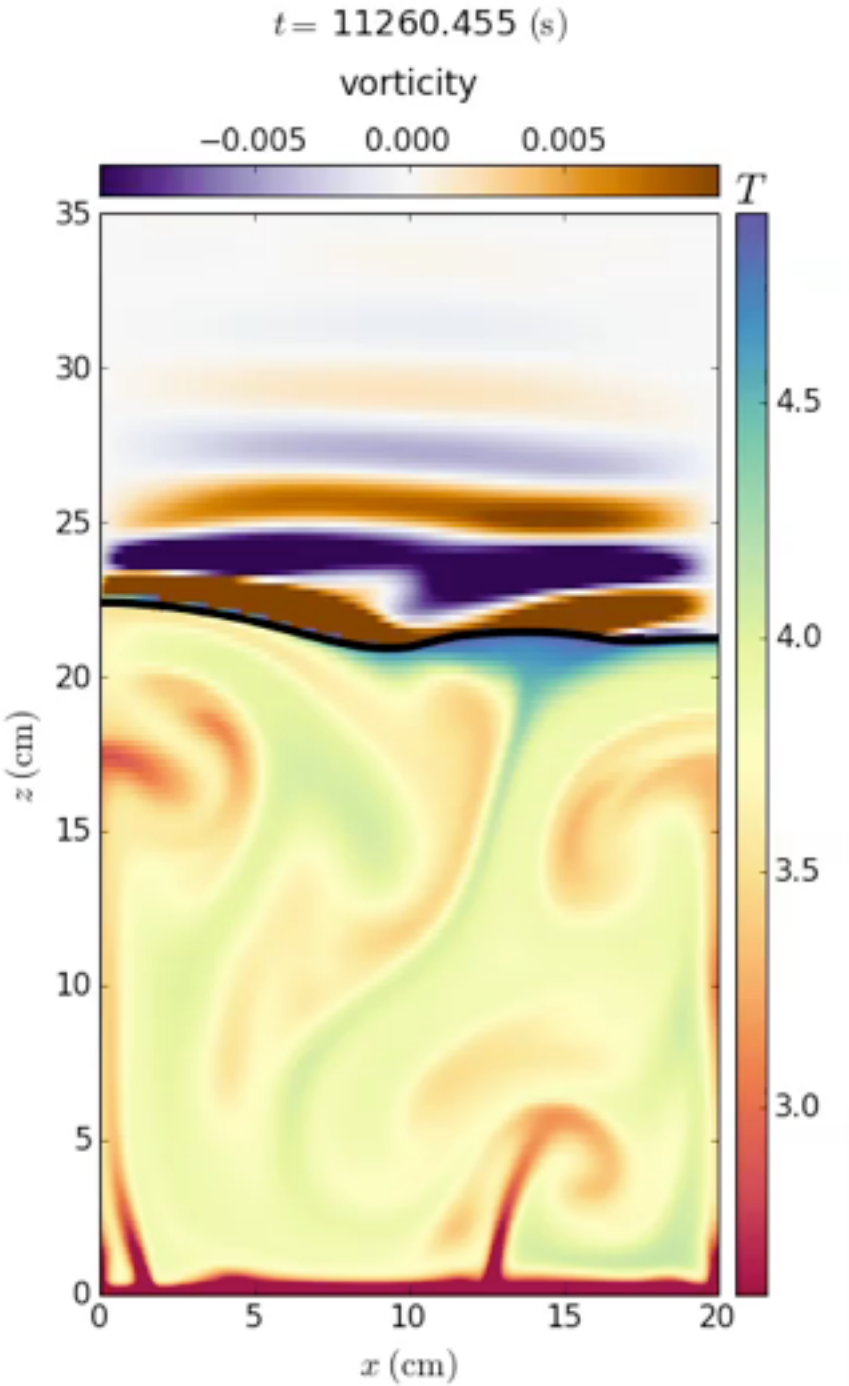
The team so far



Daniel Lecoanet (Princeton) Keaton Burns (MIT)

Jeff Oishi (Bates) Ben Brown (Colorado)

Geoff Vasil (Sydney)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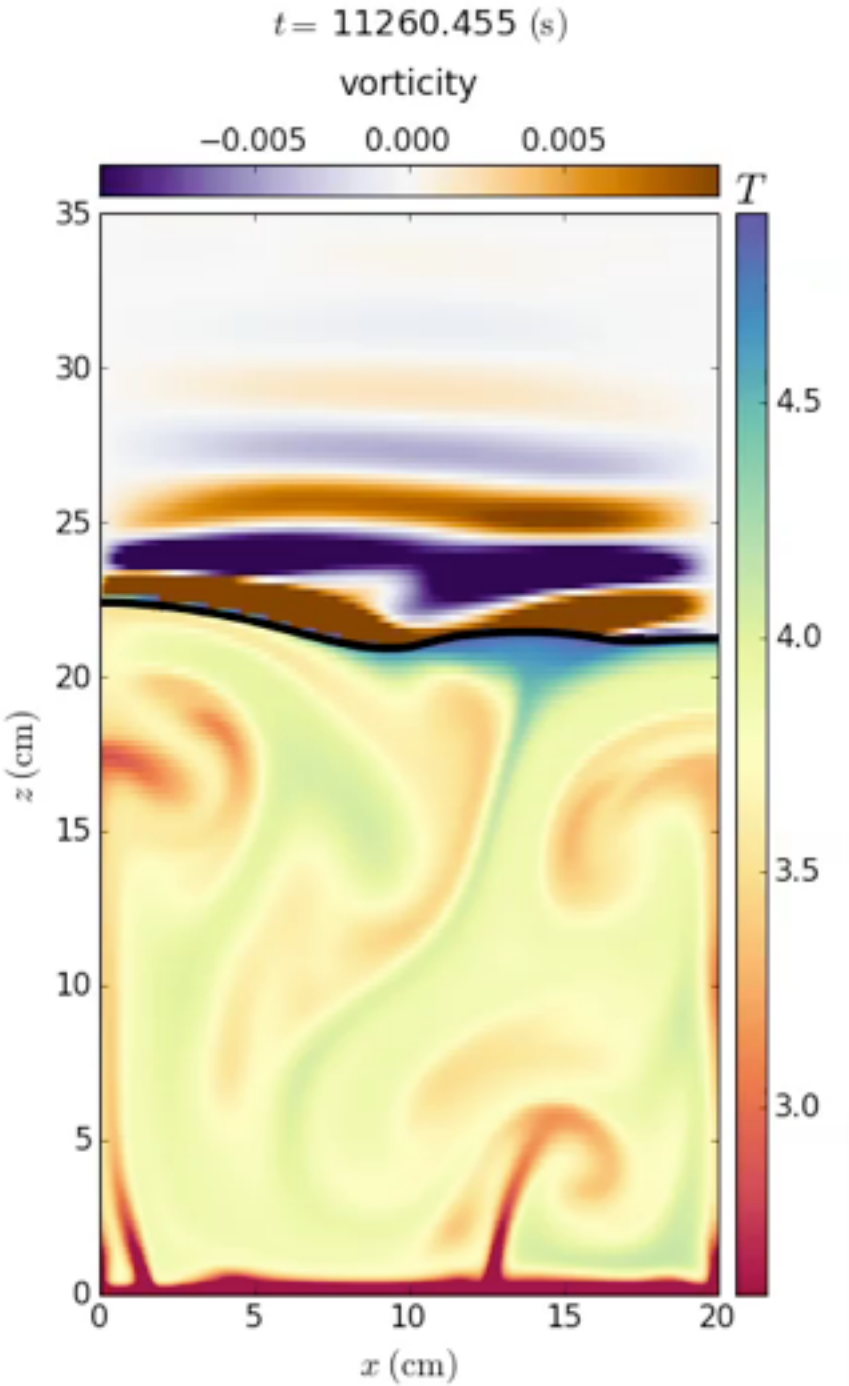
vorticity
−0.005
0.000
0.005
T
35
30
25
20
15
10
5
0
z (cm)
0
5
10
15
20
x (cm)
4.5
4.0
3.5
3.0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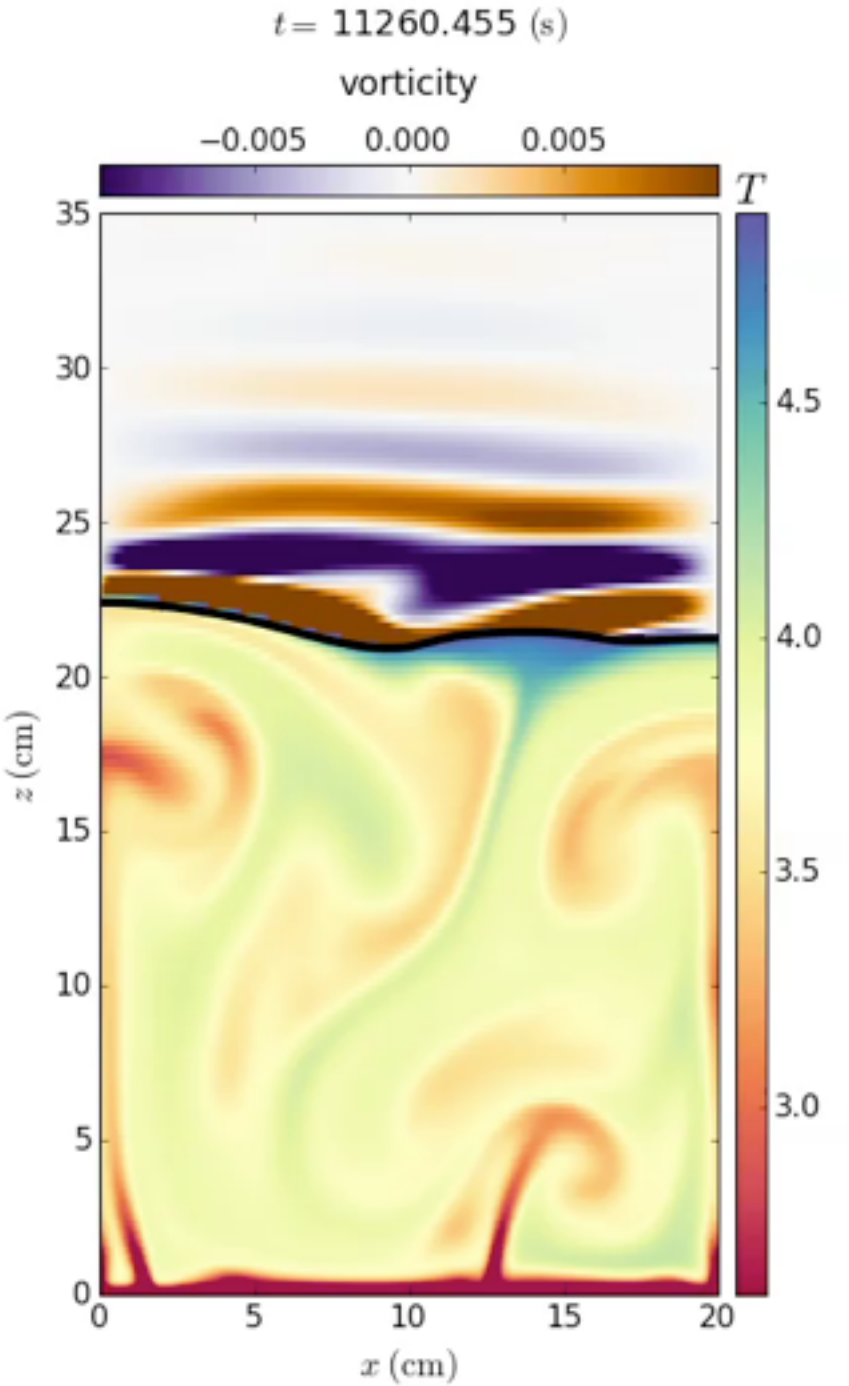
3.0


$t =$ 11260.455 (s)
vorticity
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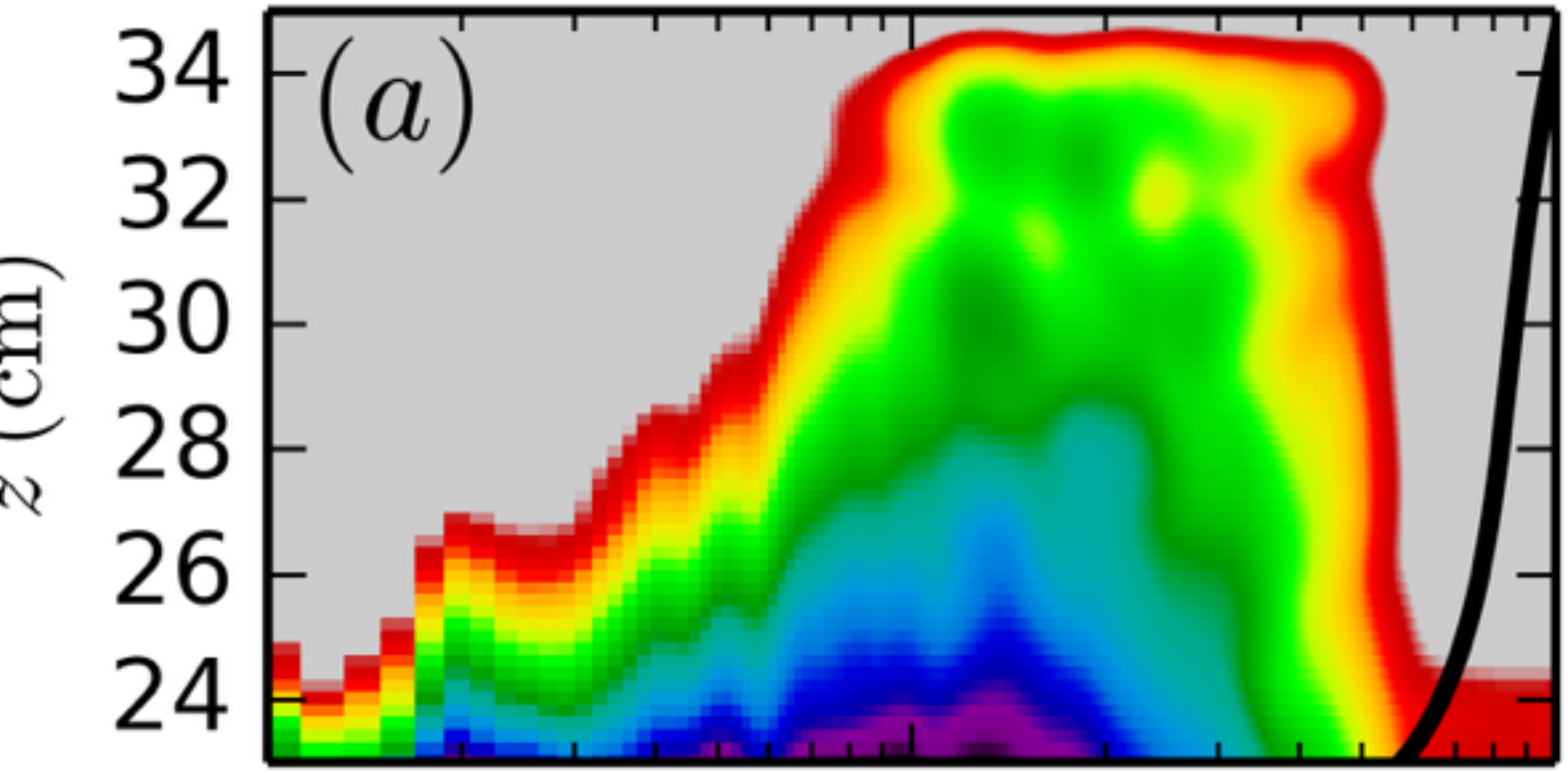
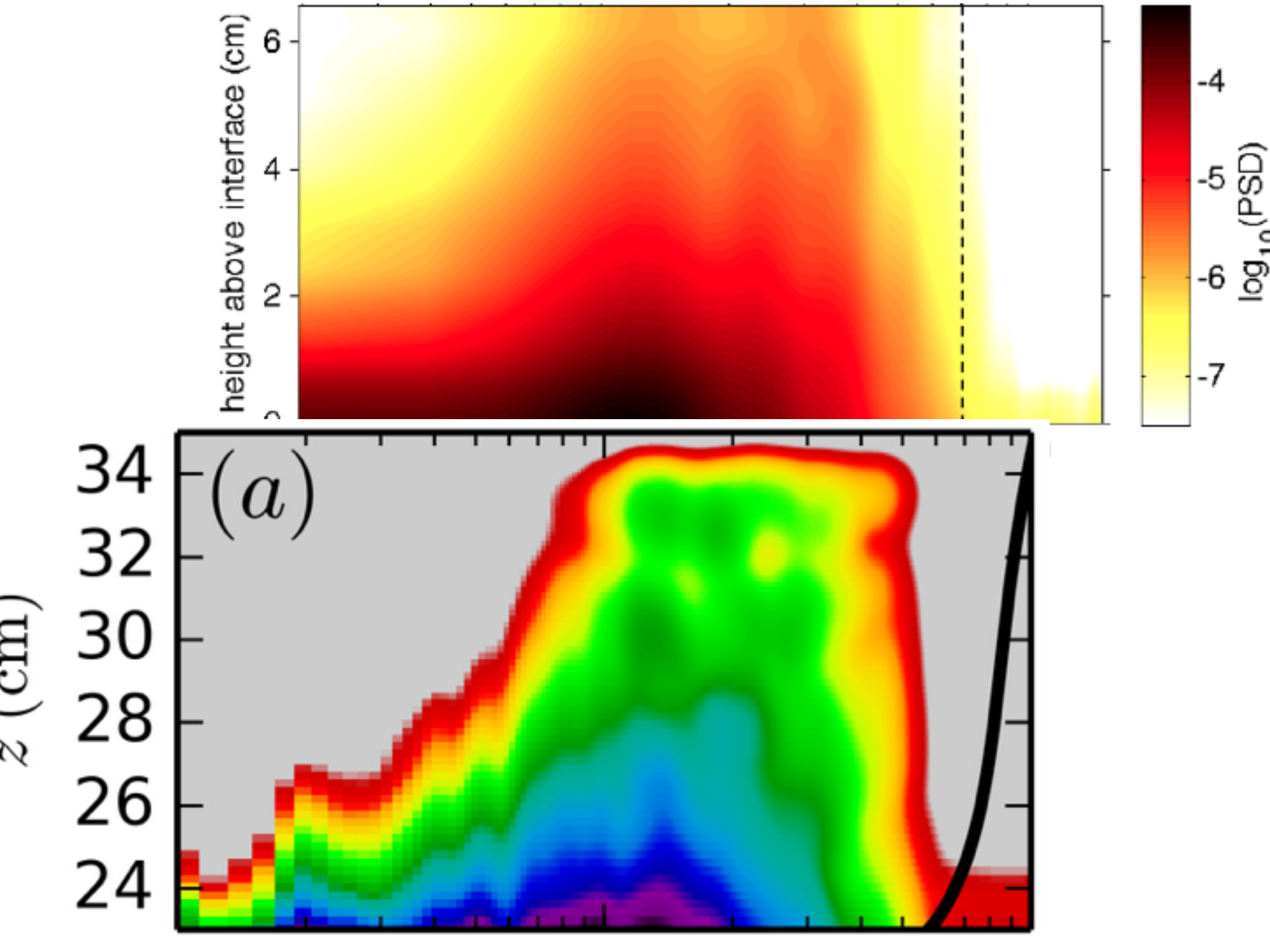
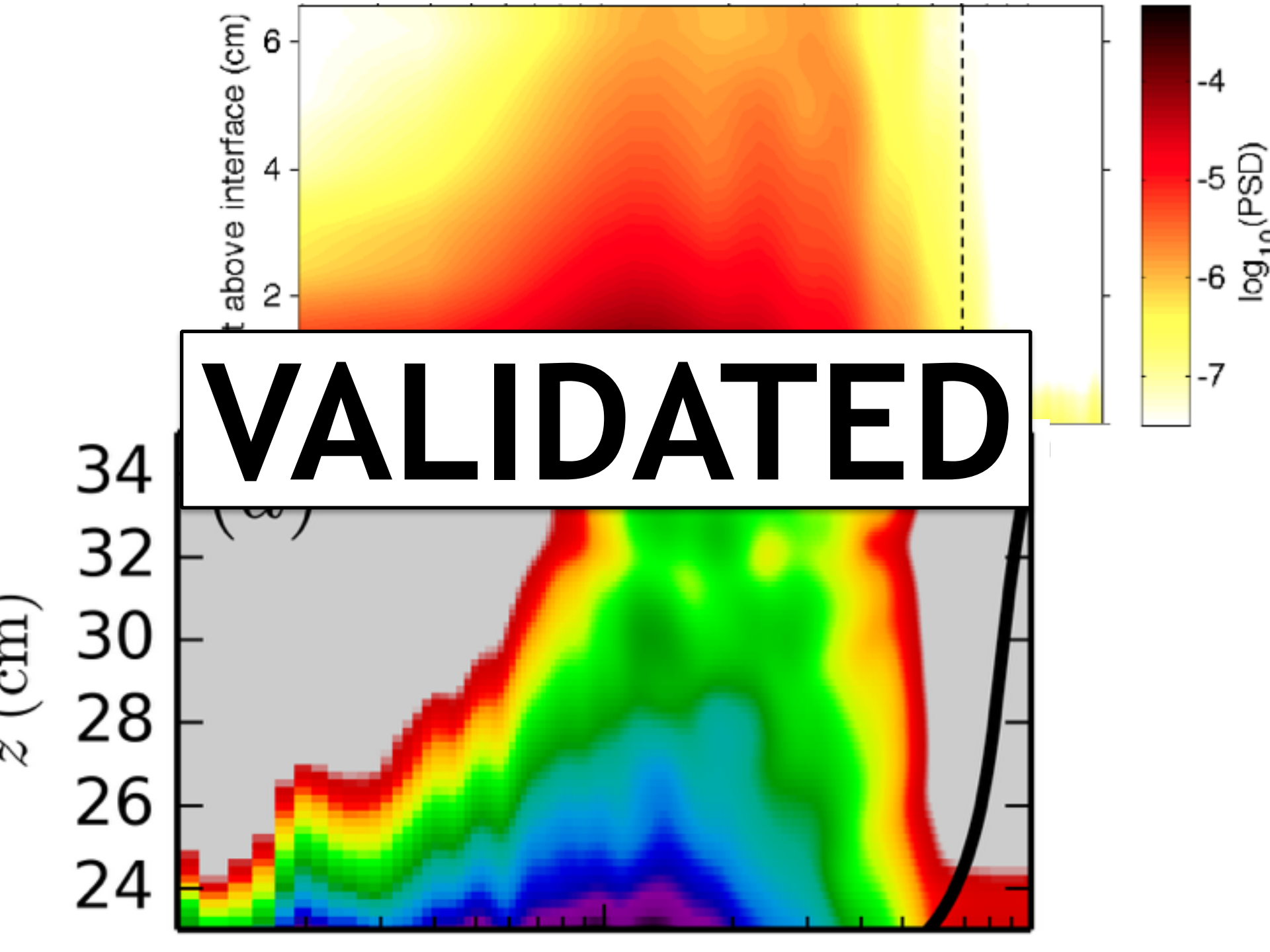

(a)
34
32
30
28
26
24
z (cm)

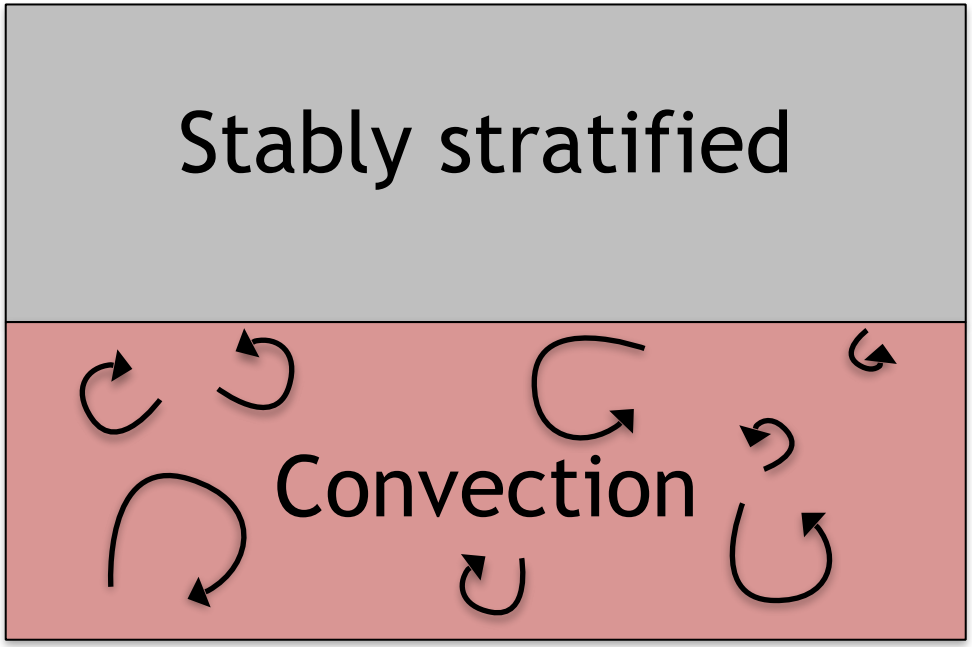

height above interface (cm)
6
4
2
-4
-5
-6
-7
log10(PSD)
(a)
34
32
30
28
26
24
z (cm)


6
4
2
above interface (cm)
-4
-5
-6
-7
log10(PSD)
VALIDATED
34
32
30
28
26
24
z (cm)

Mechanisms for Wave Generation

(e.g., Fritts & Alexander 2003)

Bulk forcing



Lecoanet & Quataert, MNRAS 2013

Bulk Forcing

$$\xi_z \sim \int G \; S$$

IGW amplitude

Green's function

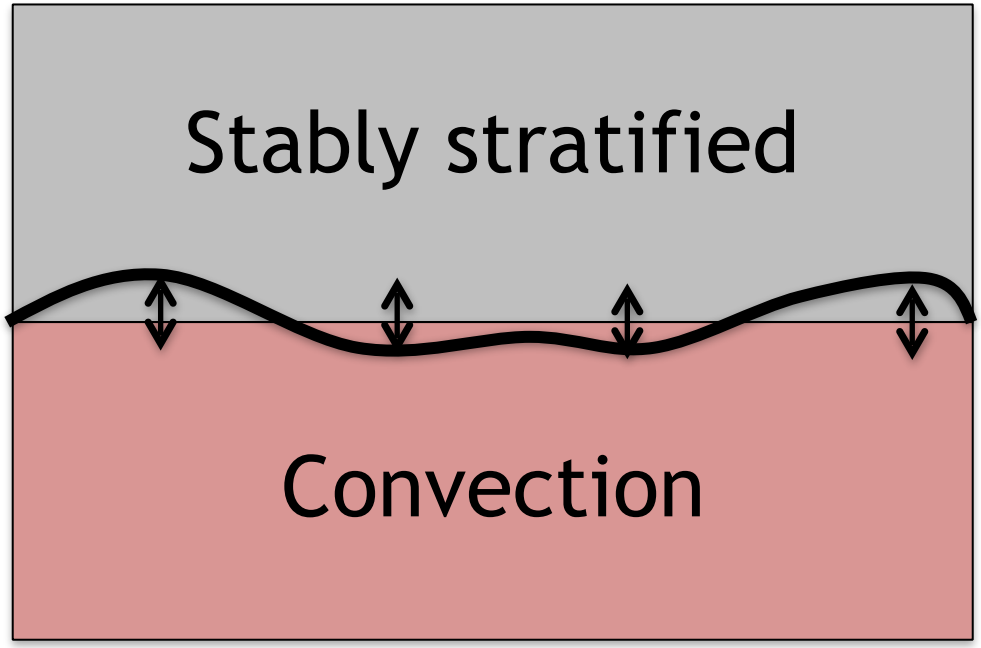
Source term:
Reynolds stress
from convection

Mechanisms for Wave Generation

(e.g., Fritts & Alexander 2003)

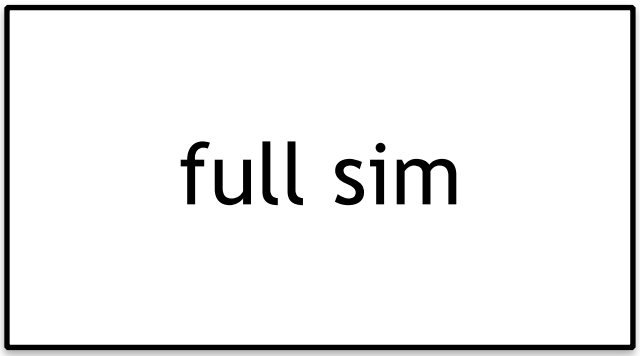
Bulk forcing

Interface oscillator



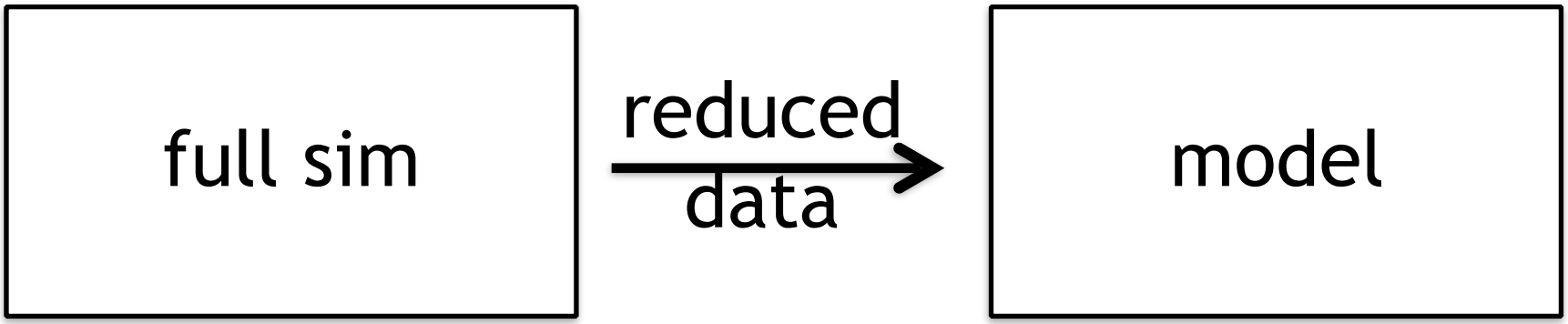
Simulation of the Simulation

Simulation of the Simulation



Simulation of the Simulation

Simulation of the Simulation



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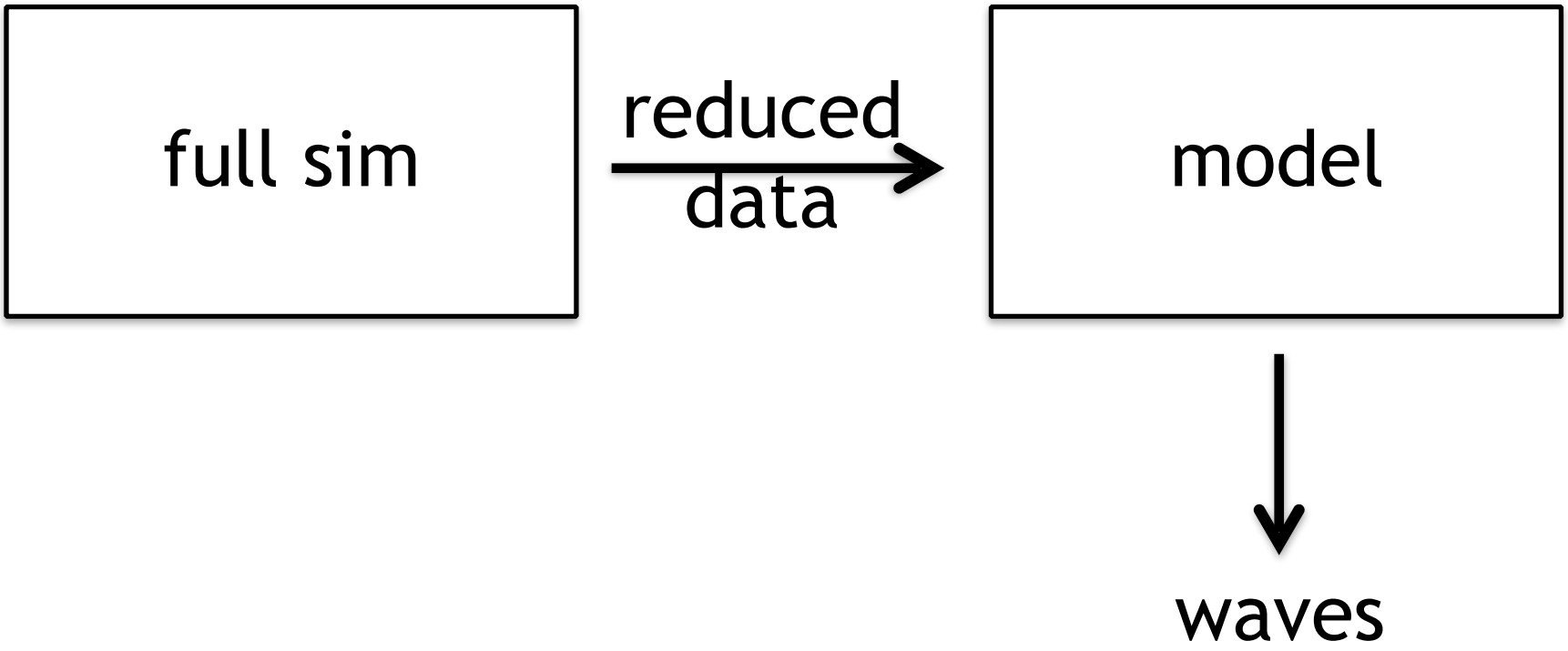
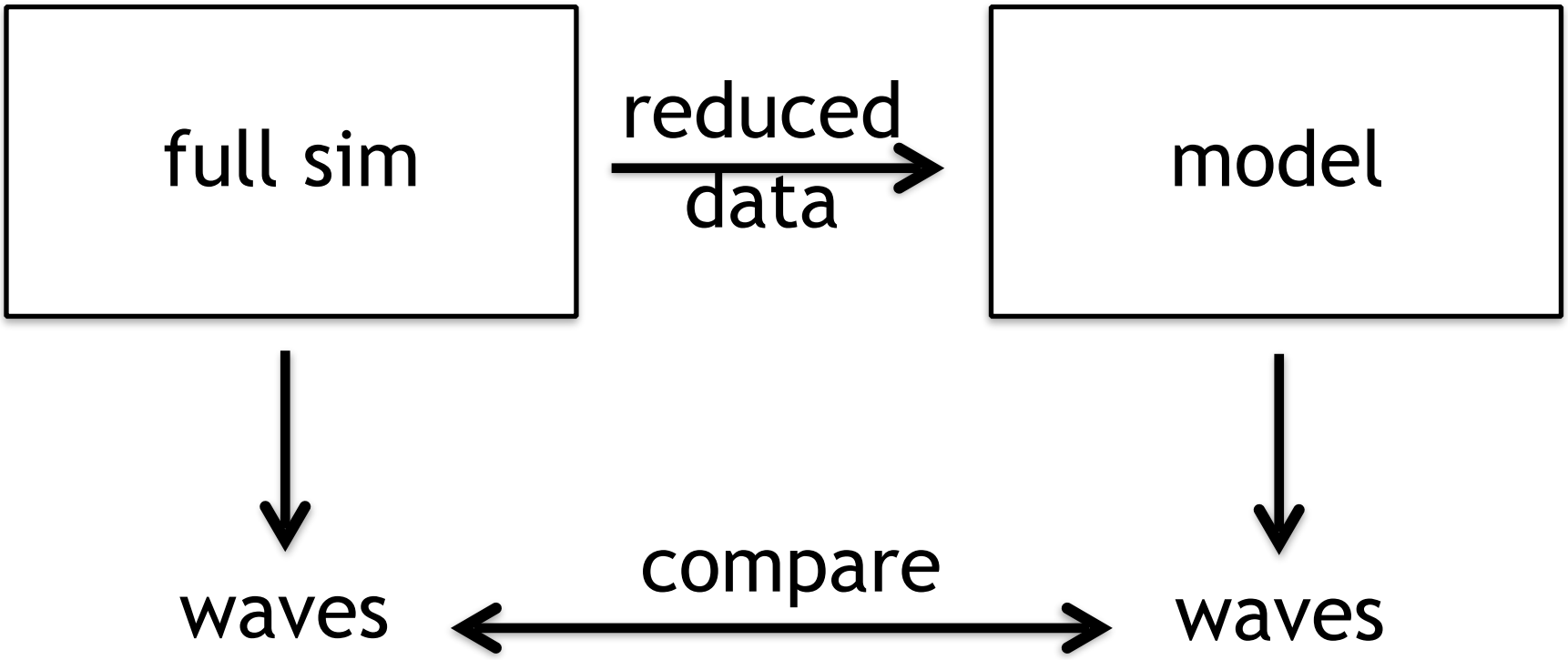


Simulation of the Simulation



Simulation of the Simulation
Bulk forcing

Solve wave equation in Dedalus:

$$\nabla^2(\partial_t - \nu\nabla^2)\partial_t\xi_z + N^2(z)\partial_x^2\xi_z = S$$

Calculate S from full simulation

$$S = -\nabla^2(\boldsymbol{u}\cdot\boldsymbol{\nabla}u_z) + \partial_z\left[(\partial_{x_i}u_j)(\partial_{x_j}u_i)\right]$$

Use same BC's as full sim

Simulation of the Simulation
Interface forcing

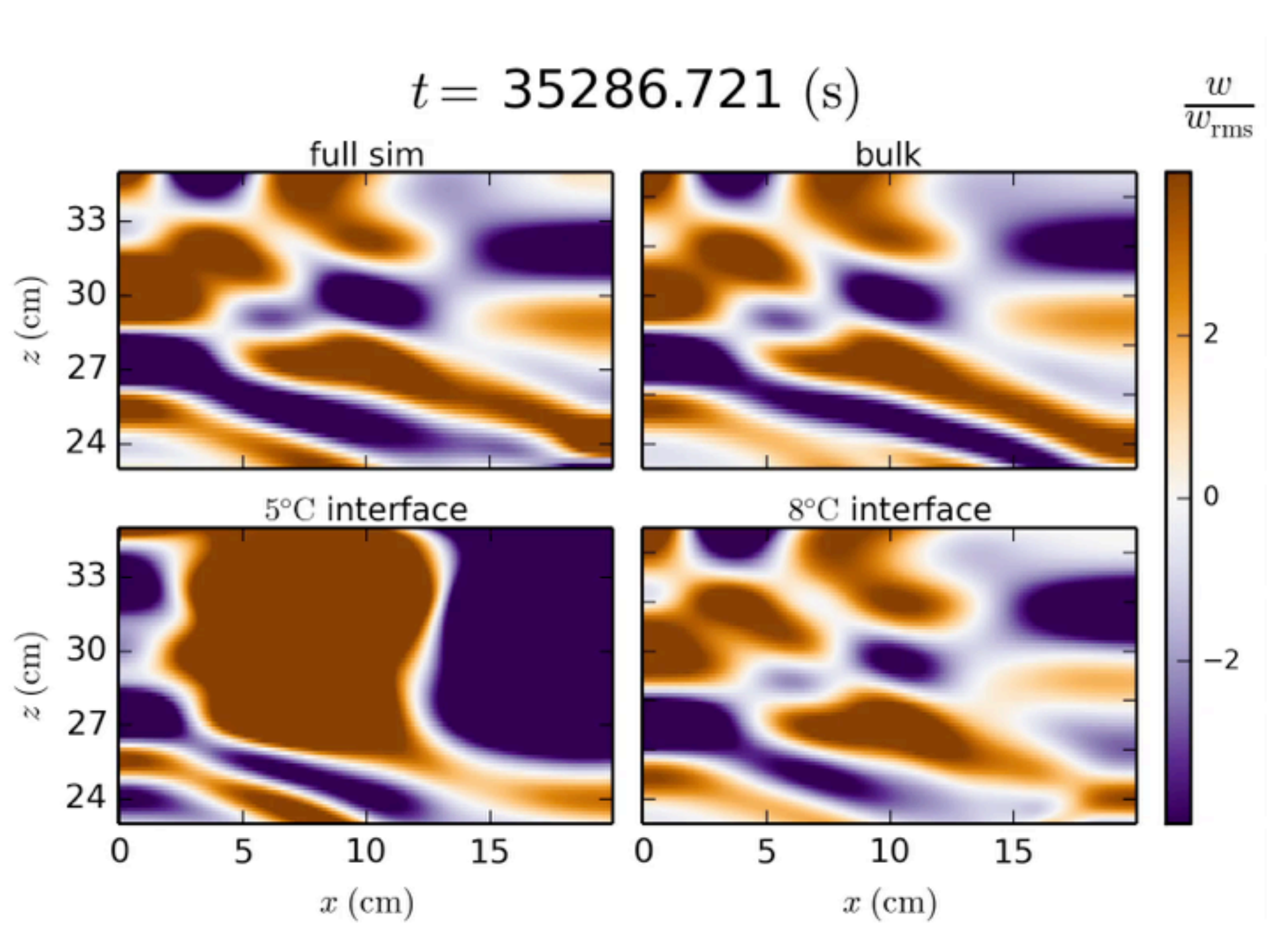
No source term, but force boundaries

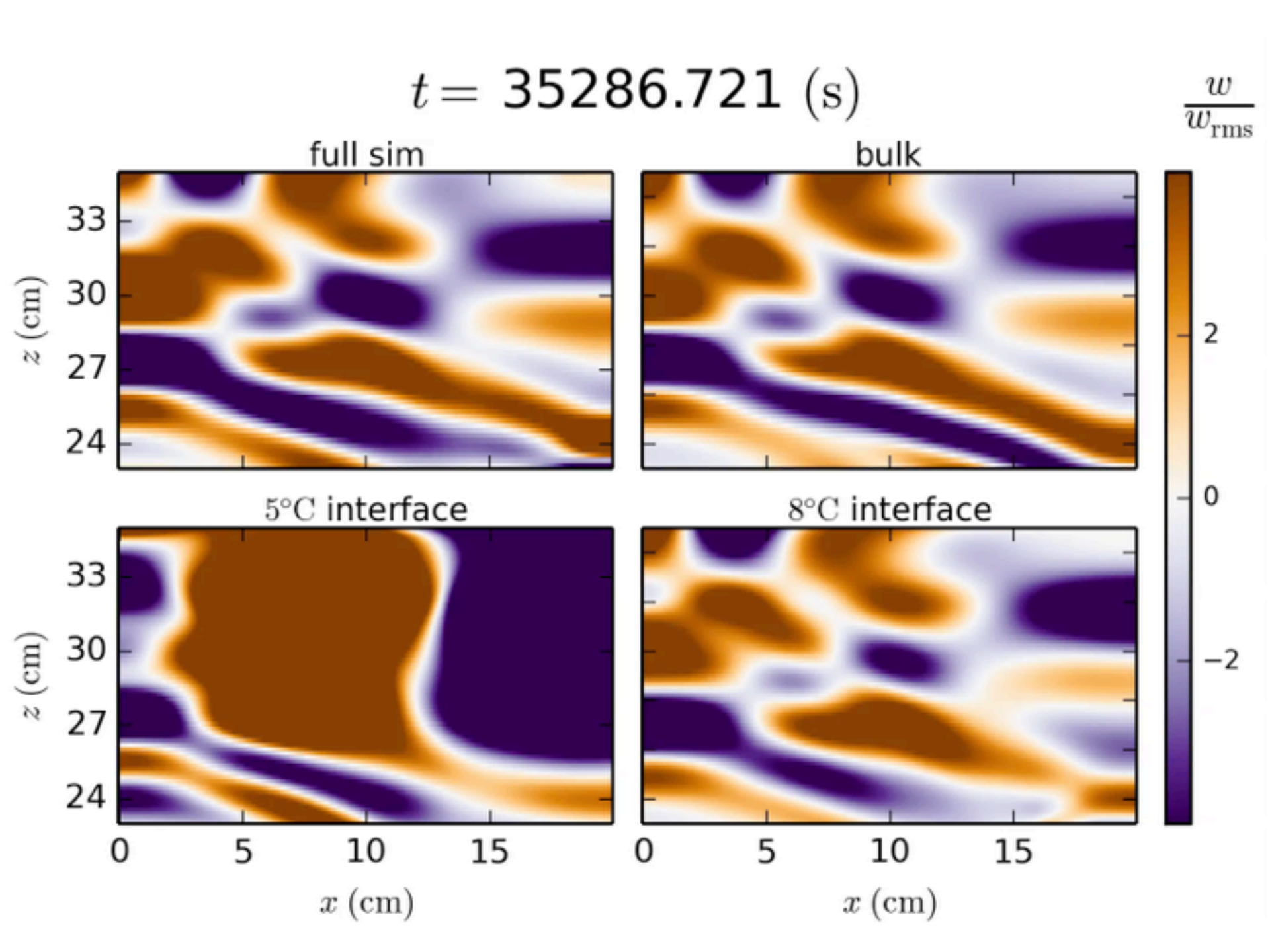
$$\nabla^2(\partial_t - \nu\nabla^2)\partial_t\xi_z + N^2(z)\partial_x^2\xi_z = 0$$

Boundary condition (calculated from full sim):

$$\xi_z(x, z_{\mathrm{int}}) = z_{\mathrm{int}}(x) - \bar{z}_{\mathrm{int}}$$

$$\partial_z^2\xi_z \propto \partial_z u = 0$$


$t = 35286.721$ (s)
$\frac{w}{w_{\mathrm{rms}}}$
full sim
bulk
5°C interface
8°C interface
33
30
27
24
z (cm)
0
5
10
15
x (cm)
2
0
−2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5°C interface
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24
z (cm)
0
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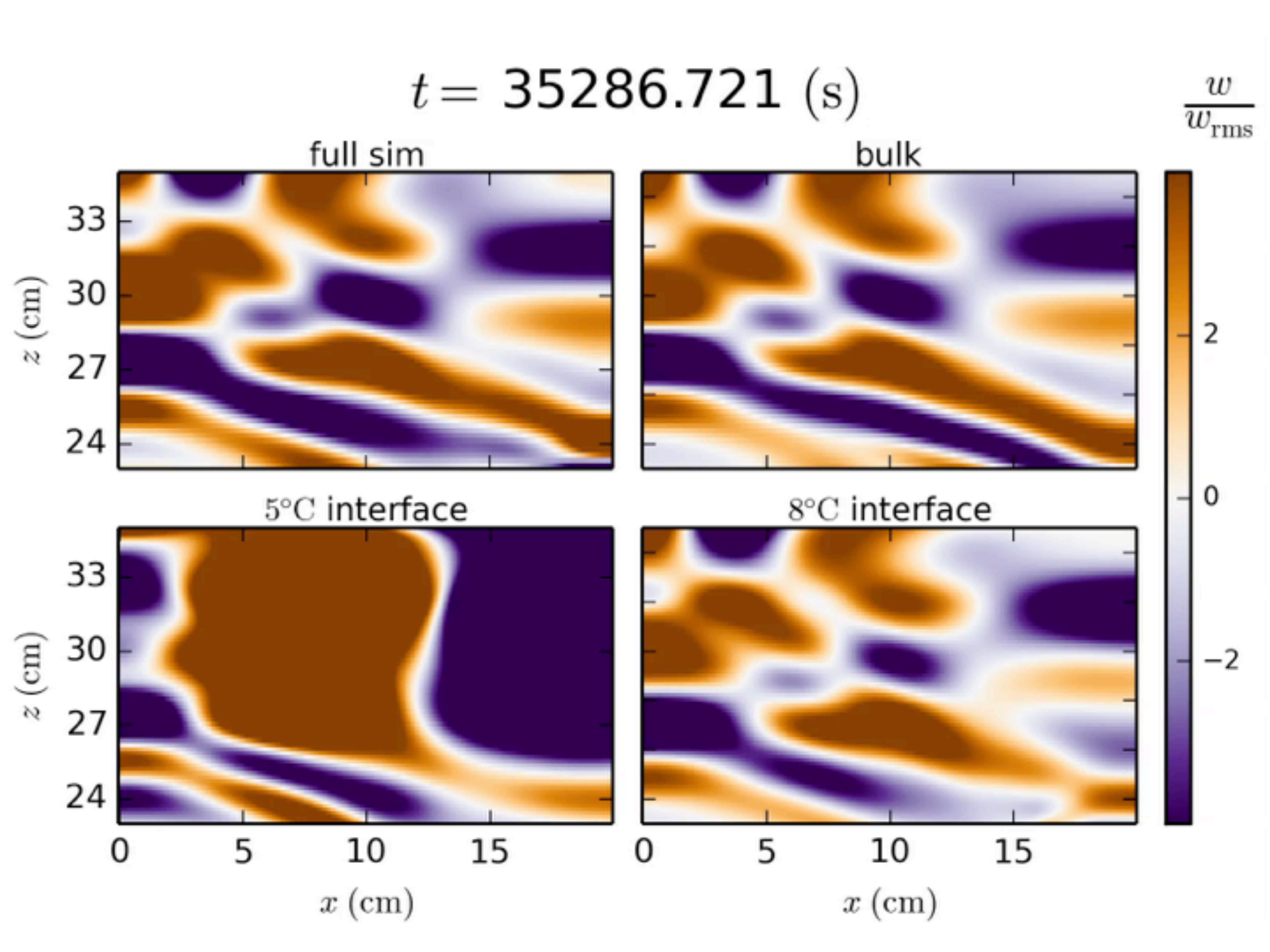
x (cm)
2
0
−2


$t = 35286.721$ (s)
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full sim
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z (cm)
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0
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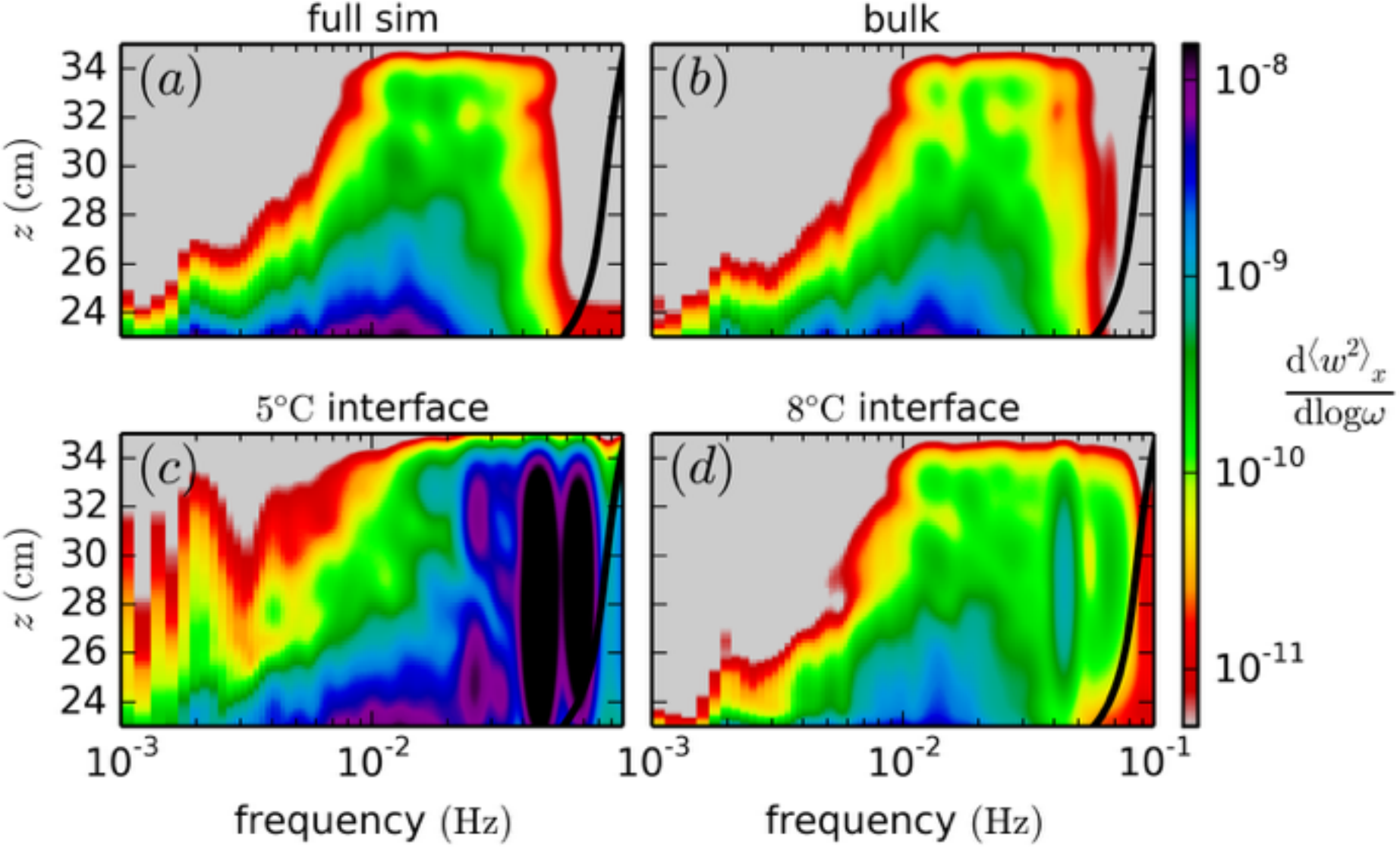
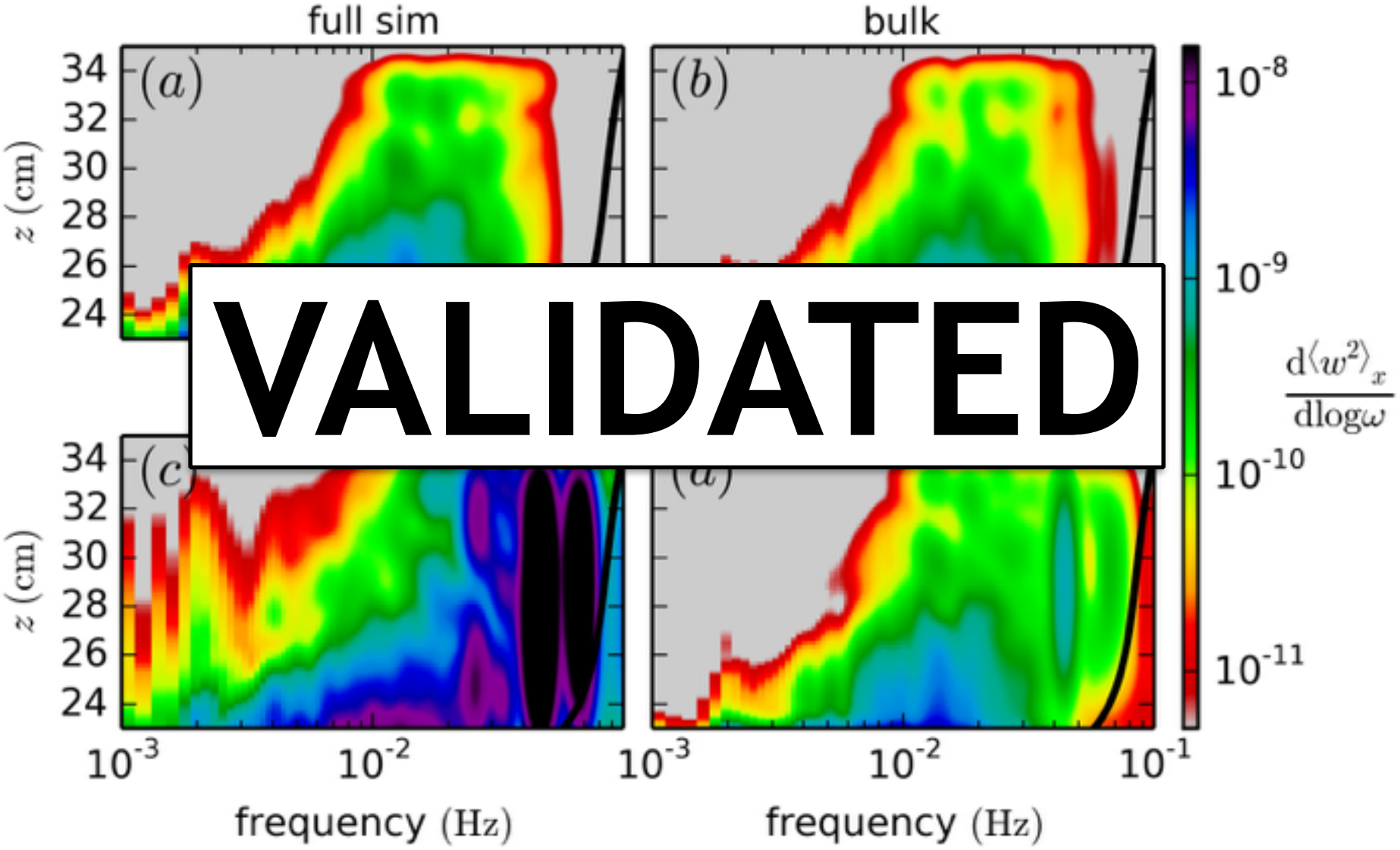

full sim
bulk
5°C interface
8°C interface
(a)
(b)
(c)
(d)
z (cm)
34
32
30
28
26
24
10^-3
10^-2
10^-1
frequency (Hz)
10^-8
10^-9
10^-10
10^-11
d⟨w²⟩_x / dlogω


full sim
bulk
(a)
(b)
(c)
(d)
34
32
30
28
26
24
z (cm)
10^-3
10^-2
10^-1
frequency (Hz)
10^-8
10^-9
10^-10
10^-11
d⟨w²⟩x / dlogω
VALIDATED

Bulk forcing works

But what’s the source term?