Unification in Intersecting Brane Models

Kang-Sin Choi
University of Bonn

based on

- KSC, J. E. Kim, hep-th/0508149
- KSC, hep-th/0603186, hep-th/06090nn

contents

1. group unification, adjoint embedding
2. recombination between different SUSY vacua
3. coupling unification

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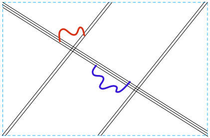
Intersecting brane world

The origin of MSSM can be explained by intersecting brane world?
Open string $\rightarrow$ gauge theory, chiral fermions [Berkooz, Douglas, Leigh]
Typical setup: D6s on T^6 wrapping 3-cycle = 3 + (3+1)D



ex. 2 families of quarks $(\mathbf{3},\mathbf{2})$ under $SU(3)\times SU(2)$

- # families = # intersections
- Yukawa hierarchy

  ⋮

Unification?

- Gauge group and representation
- Gauge coupling $g_{\rm YM}^2 \propto V_{\rm cycle}^{-1}$ $\quad -\frac{1}{4g^2}\int d^{p+1}xF^2 \longrightarrow -\frac{V_{p-3}}{4g^2}\int d^4xF^2$

Must be!

- Spontaneous symmetry breaking from the first principle (unified theory)
- Running gauge coupling from EW scale

Adjoint embedding

T-dual: tilted brane $\leftrightarrow$ magnetic flux $\quad A_1 = 0, A_2 = F_{12}X^1 = X'^2$

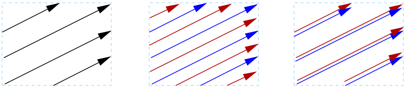
(partial) bound states of various D-branes "toron" ['t Hooft],[Guralnik, Ramgoolam]

- D1-D1 intersec.: D0-D2 bound state,
- D2-D2 intersec.: D0-D4 bound states, marginal
- D3-D3 intersec.: $\overline{\mathrm{D}}$0-$\overline{\mathrm{D}}$0-$\overline{\mathrm{D}}$0-D4 bound states *cf.* no D0-D6 bound state

$$\oplus(n^a, m^a) \leftrightarrow F_{12} = \begin{pmatrix} \frac{m^1}{n^1}\mathbf{1}_{n^1} & & \\ & \ddots & \\ & & \frac{m^k}{n^k}\mathbf{1}_{n^k} \end{pmatrix}$$

$$n = \sum n^a, m = \int \mathrm{Tr} F_{12} = \sum_a c_1^a$$

moduli space, gauge group $U(p), p = \gcd(n, m)$ embedded in $U(n)$



(a) (b) (c)

Specified by torus moduli + Chern #

- No moduli for cycles
- SUSY replace higher order Chern # with lower order ones

Bifundamentals

chiral bifundamental rep [Berkooz, Douglas, Leigh] from adjoint

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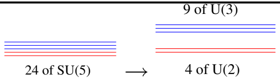
<td>
</td>
<td>parallel separation
adjoint Higgs</td>
<td>$\begin{pmatrix} \mathbf{8} & (\mathbf{3},\mathbf{2}) \\ (\bar{\mathbf{3}},\bar{\mathbf{2}}) & \mathbf{3} \end{pmatrix}$</td>
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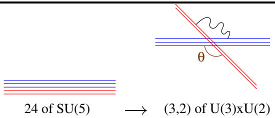
<td>
</td>
<td>“rotation”
bifundamental
chiral</td>
<td>$\begin{pmatrix} \mathbf{8} & (\mathbf{3},\mathbf{2}) \\ (\bar{\mathbf{3}},\bar{\mathbf{2}}) & \mathbf{3} \end{pmatrix}$
gaugino $\tilde{X}, \tilde{Y}$ = quark q</td>
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due to the property of Dirac Op.

$$\text{multiplicity } I_{ab} = \text{index}_{Q,ab} \not\nabla_6 = \frac{1}{3!(2\pi)^3} \int \mathrm{Tr}_{Q,ab} F^3$$

cf. SUSY determined by bosonic sector
Branching under $U(p_a + p_b) \to U(p_a) \times U(p_b)$

$$(\mathbf{p_a} + \mathbf{p_b})^2 = (\mathbf{p_a^2}, \mathbf{1}) + (\mathbf{1}, \mathbf{p_b^2}) + (\mathbf{p_a}, \mathbf{p_b}) \quad (\overline{\mathbf{p}}_\mathbf{a}, \overline{\mathbf{p}}_\mathbf{b}) \text{ is } CPT \text{ conj.}$$

cf. Dual to M theory on G_2 manifold: $A_{p_a+p_b-1} \to A_{p_a-1} + A_{p_b-1}$ singularities
Further projection associated with orbifold action

DBI energy

Given a SUSY intersecting brane model
(nonabelian) DBI energy = BPS relation [Marino, Minasian, Moore, Strominger]

$$\begin{aligned}
&\tau_9 \mathrm{Tr}\sqrt{(\mathbf{1}+f_{45}^2)(\mathbf{1}+f_{67}^2)(\mathbf{1}+f_{89}^2)} \\
&\quad = \tau_9 V_{45} V_{67} V_{89} \sum_a \left(\mathrm{Tr}\mathbf{1}_{N^a} + \tau_9\left(\mathrm{Tr} f_{45,a} f_{67,a} + \mathrm{Tr} f_{67,a} f_{89,a} + \mathrm{Tr} f_{89,a} f_{45,a}\right)\right) \\
&\quad = \tau_9 V_{45} V_{67} V_{89} \sum_a N_a + \tau_5 \left(V_{89} \sum_a c_{2,a}^{89} + V_{45} \sum_a c_{2,a}^{45} + V_{67} \sum_a c_{2,a}^{67}\right).
\end{aligned} \tag{1}$$

$f = 2\pi\alpha' F, \quad \tau_5 = (4\pi^2\alpha')^2 \tau_9, \quad V_{45} V_{67} \mathrm{Tr} F_{45} F_{67} = N \frac{m_1 m_2}{n_1 n_2} = c_2$

tadpole condition: RR charge sum(s) to be 32 (incl. O-images), depending on the # orientifolds

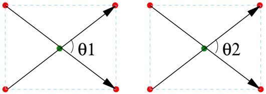
- $\mathbf{Z}_M$ with M odd 1, with M even 2 [Gimon, Polchinski]
- $\mathbf{Z}_M \times \mathbf{Z}_N$ with M, N even 4 [Berkooz, Leigh]
- maximal rank 16 for each

Every SUSY setup has the same total energy with

type I compactification

vacua connected?

Recombination of BPS cycles

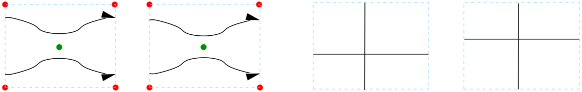


$T^2 \times T^2$ ws scalar
$m_1^2 = \theta_1 - \theta_2 = -m_2^2$

- $\theta_1 \neq \theta_2$ unstable tachyon
- $\theta_1 = \theta_2 \leftrightarrow F_{12} = F_{34}$
  $\leftrightarrow$ local Cauchy–Riemann condition — Any complex curves in $\mathbf{C}^2$ is sLag

1/4 BPS = stable SUSY = minimal vol.

Both cases recombine toward minimal vol.



- marginal deformation, same volume [CKS, Kim] [Erdminger et al] [Douglas, Zhou]
- T-dual to D4-D0 bound state
- U-dual to (F,Dp) bound state = string junction [CKS, Kim]

charged scalar VEV also induces recombination = Higgs mech [Cremedes, Ibanez, Marchesano]
1/8-cycle recombination is also marginal due to SUSY.

Type I compactification on orientifolds

[Cvetic, Shiu, Uranga 01] (intersecting) = [Berkooz, Leigh 96] (parallel)

- IIA on $T^6/(\mathbf{Z}_2 \times \mathbf{Z}_2)$ — 4 O6 planes
- All cycles are 1/4-BPS, with O6-image

above 1/4 recombination!

Deformable

- By brane recombinations and parallel translation

all can be on top of O6s

- Final group $Sp(8)^4 \in SO(32)^4$, branching $\mathbf{496} \to \mathbf{136} + 3 \cdot \mathbf{120}$
- T_{579}-dual: 4 O6s $\to$ 1 O9 and 3 O5s $\Omega, \theta\Omega, \omega\Omega, \omega\theta\Omega$, orbifold group P invariant or become asymmetric $\hat{P}$

All are type I compactification on P

- requiring SUSY: Dp–D$(p-4)$ bound states — T-dual to D9-D5
- all become D6/O6 by some T-dualities, *cf.* M-theory on G_2 manifold
- on a symmetric or asymmetric orbifold. [Blumenhagen, Gorlich, Kors, Lust]
- branching + projection assoc. w/ orbifold
- Many SUSY vacua are connected

Gauge coupling

fluctuation around D-brane background [A. Hashimoto, Taylor], [Denef, Sevrin, Troost]

$$A_m = \langle A_m \rangle + \delta A_m$$

$$\begin{aligned}&\mathrm{STr}\sqrt{\det(\mathbf{1}+f)}\sqrt{-\det(\mathbf{1}+(\mathbf{1}-f^2)^{-1}\delta F - f(\mathbf{1}-f^2)^{-1}\delta F)}\\ =&\mathrm{STr}\sqrt{\det(\mathbf{1}+f)}((\text{tension}) + (\text{YM with “metric” } (\mathbf{1}-f^2)^{-1}) + (\text{topological}) + \mathcal{O}(\alpha' F)^4)\end{aligned}$$

- expansion nonlocal $(\mathbf{1}-f^2)^{-1}$
- no moduli for cycles: specified by Kähler moduli + quantized flux

$$\mathbf{1}+f^2 = \begin{pmatrix} (1+(\frac{m^1}{n^1})^2)\mathbf{1}_{N^1} & & & \\ & (1+(\frac{m^2}{n^2})^2)\mathbf{1}_{N^2} & & \\ & & \ddots & \\ & & & (1+(\frac{m^k}{n^k})^2)\mathbf{1}_{N^k} \end{pmatrix}.$$

Although there is a single unified YM coupling above M_U low energy coupling may be different

- Recombination occurs at $\mathcal{O}(\alpha' f)$: change the wrapping volume, thus the coupling
- From low energy: a large threshold correction from f

Weak mixing angle

U(1) parts $A_{\mu,U(1)} = \frac{1}{N}\mathrm{Tr}A_{\mu,U(N)}, \quad g^2_{U(1)} = \frac{2}{N}g^2_{U(N)}$.
e.g. Madrid model

$$Q_Y = \frac{1}{3}Q_C - Q_L - Q_R,$$

From normalization of these gauge kinetic terms $\frac{1}{g_Y^2} = \sum_i \frac{N_i C_i^2}{2}\frac{1}{g_i^2}$

$$\frac{1}{g_Y^2} = \frac{1}{6}\frac{1}{g_C^2} + \frac{1}{g_L^2} + \frac{1}{2}\frac{1}{g_R^2} = \frac{5}{3}\frac{1}{g^2},$$

In the unified coupling limit, $g = g_L = g_R = g_C$, we have weak mixing angle at M_U

$$\sin^2\theta_W = \frac{1}{g_Y^2/g^2 + 1} = \frac{3}{8}.$$

This is because $U(1)_{B-L} \times U(1)_R \subset SO(10)$

$$Q_Y = Q_B - Q_L - Q_R = \frac{1}{3}Q_C - Q_L - Q_R,$$

For $USp(2)$ rather than $SU(2)$, $\sin^2\theta_W = \frac{6}{13}$.
cf. no spinorial **16**. A structure $SO(10) \subset SO(32)$.

Conclusions

For intersecting brane models wrapping on compact orbifolds.

- All the representations, including bifundamental, are embedded into an adjoint.
- Tadpole cancellation constrains these as $SO(32)$ adjoint(s).
- Symmetry breaking is done via brane separations and recombinations.
- SUSY vacua are connected to type I compactification
- A unified gauge coupling can become different for subgroups below $M_U \sim \alpha'^{-1/2}$.
- Weak mixing angle $\sin\theta_W^2 = 3/8$.