MINIMAL STANDARD HETEROTIC STRING MODELS

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DATA → STANDARD MODEL

$$SU(3) \times SU(2) \times U(1)_Y \longrightarrow SU(5) \longrightarrow SO(10)$$

$$\left[\begin{pmatrix} \\ e \end{pmatrix} + D^c_L\right] + \left[U^c_L + \begin{pmatrix} u \\ d \end{pmatrix} + E^c_L\right] + N^c_L$$

$$\bar{5} \quad + \quad 10 \quad\quad 1 \quad\quad \overline{16}$$

STANDARD MODEL -> UNIFICATION

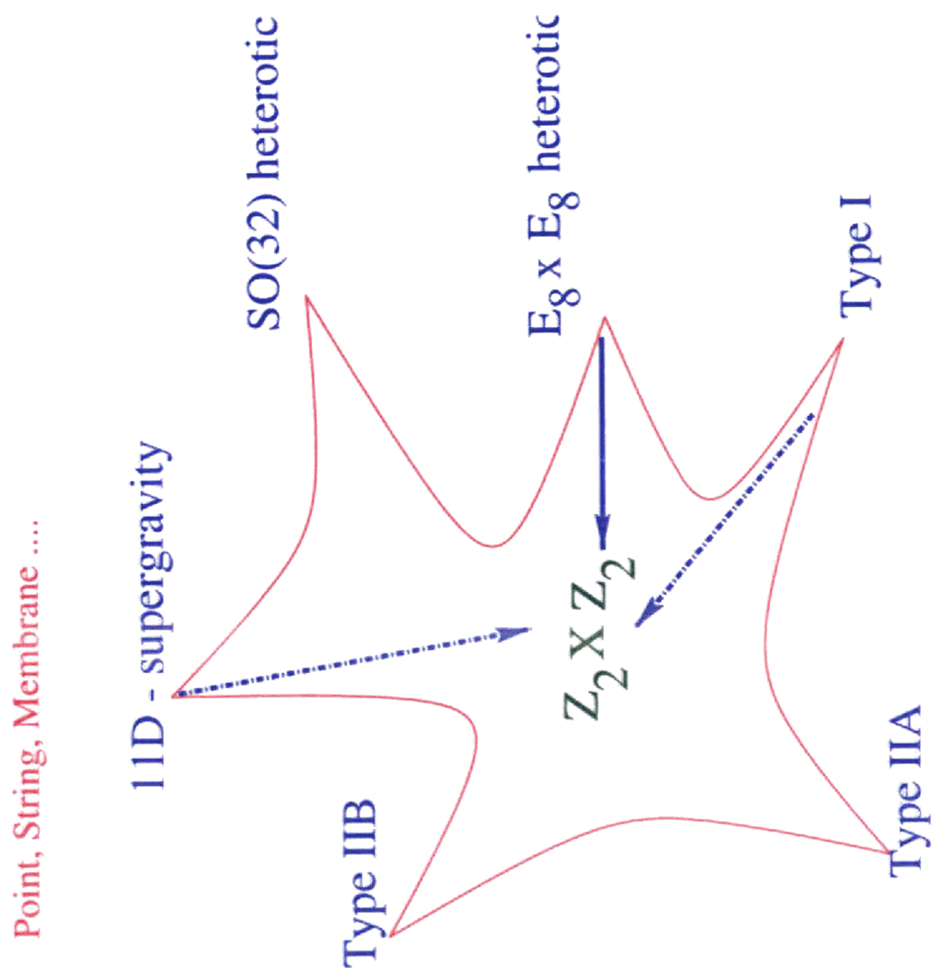
ADDITIONAL EVIDENCE:

Logarithmic running, proton longevity, neutrino masses

PRIMARY GUIDES:

3 generations

SO(10) embedding



Free Fermionic Construction

World-sheet content (light–cone gauge)

Left-Movers: $\psi^{\mu}_{1,2}$, χ_i, y_i, ω_i $(i = 1, \cdots, 6)$

Right-Movers

$$\bar{\phi}_{A=1,\cdots,44} = \begin{cases} \bar{y}_i \ , \ \bar{\omega}_i & i = 1, \cdots, 6 \\ \bar{\eta}_i & i = 1, 2, 3 \\ \bar{\psi}_{1,\cdots,5} & \\ \bar{\phi}_{1,\cdots,8} & \end{cases}$$

$$f \longrightarrow -e^{i\pi\alpha(f)} f$$

$$V \longrightarrow V \qquad Z = \sum_{all\ spin\ structures} c\binom{\vec{\alpha}}{\vec{\beta}} Z\binom{\vec{\alpha}}{\vec{\beta}}$$

Models $\longleftrightarrow$ Basis vectors $+$ one–loop phases

Fermionic $Z_2 \times Z_2$ orbifolds

Models $\longrightarrow$ set of boundary condition basis vectors

The NAHE set $\{1, S, b_1, b_2, b_3\}$

$\longrightarrow$ $Z_2 \times Z_2$ orbifold compactification

Gauge Group : $SO(10) \times SO(6)^{1,2,3} \times E_8$

beyond the NAHE set Add $\{\alpha, \beta, \gamma\}$

number of generations is reduced to three

$$SO(10) \longrightarrow SU(3) \times SU(2) \times U(1)_{T_{3_R}} \times U(1)_{B-L}$$

$$U(1)_Y = \tfrac{1}{2}(B-L) + T_{3_R} \in SO(10) \ !$$

$$SO(6)^{1,2,3} \longrightarrow U(1)^{1,2,3} \times U(1)^{1,2,3}$$

SUPERSTRING DERIVED STANDARD–LIKE MODEL

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar\psi^{1,...,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,...,8}$
1	1	1	1	1	1,...,1	1	1	1	1,...,1
S	1	1	1	1	0,...,0	0	0	0	0,...,0
b_1	1	1	0	0	1,...,1	1	0	0	0,...,0
b_2	1	0	1	0	1,...,1	0	1	0	0,...,0
b_3	1	0	0	1	1,...,1	0	0	1	0,...,0

	$y^{3,...,6}$	$\bar y^{3,...,6}$	$y^{1,2}, \omega^{5,6}$	$\bar y^{1,2}, \bar\omega^{5,6}$	$\omega^{1,...,4}$	$\bar\omega^{1,...,4}$
1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1
S	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0
b_1	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0
b_2	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0
b_3	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar\psi^{1,...,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,...,8}$
α	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
γ	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$ 0 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0

	y^3y^6	$y^4\bar y^4$	$y^5\bar y^5$	$\bar y^3\bar y^6$	$y^1\omega^5$	$y^2\bar y^2$	$\omega^6\bar\omega^6$	$\bar y^1\bar\omega^5$	$\omega^2\omega^4$	$\omega^1\bar\omega^1$	$\omega^3\bar\omega^3$	$\bar\omega^2\bar\omega^4$
α	1	0	0	0	0	0	1	1	0	0	1	1
β	0	0	1	1	1	0	0	0	0	1	0	1
γ	0	1	0	1	0	1	0	1	1	0	0	0

With the choice of generalized GSO coefficients:

$$C\binom{b_i}{b_j} = C\binom{b_i}{S} = C\binom{\mathbf{1}}{\mathbf{1}} = -1 .$$

$$c\binom{b_4}{b_j, \beta} = -c\binom{b_4}{\mathbf{1}} = -c\binom{\beta}{\mathbf{1}} = c\binom{\beta}{b_j} =$$

$$-c\binom{\beta}{\gamma} = c\binom{\gamma}{b_2} = -c\binom{\gamma}{b_1, b_3, \alpha, \gamma} = -1$$

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The massless spectrum

Three twisted generations $b_1, \quad b_2, \quad b_3$

Untwisted Higgs doublets
$h_{1_{1,0,0}} \qquad \bar{h}_{1_{-1,0,0}}$
$h_{2_{0,1,0}} \qquad \bar{h}_{2_{0,-1,0}}$
$h_{3_{0,0,1}} \qquad \bar{h}_{3_{0,0,-1}}$

Sector $b_1 + b_2 + \alpha + \beta$ $\quad h_{\alpha\beta_{-\frac{1}{2},-\frac{1}{2},0,0,0,0}} \qquad \bar{h}_{\alpha\beta_{\frac{1}{2},\frac{1}{2},0,0,0,0}}$

$\oplus$ $SO(10)$ singlets

Sectors $b_j + 2\gamma \quad j = 1, 2, 3 \quad \longrightarrow$ hidden matter multiplets

"standard" $SO(10)$ representations

NAHE + $\{ \alpha , \beta , \gamma \} \rightarrow$ exotic matter

$(Q_Y; Q_{\rm e.m.}) \; ; \; Q_{Z'} \quad = \quad \frac{1}{2} \quad SO(10) \quad$ charges

$Q_Y = +\frac{1}{2} \qquad Q_{e.m} = +\frac{1}{2}.$

F	SEC	$SU(3)_C \times SU(2)_L$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SO(4) \times SU(3)$	Q_H	Q_7	Q_8	Q_9
H_1	$\pm\beta$	(1,1)	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	(2,1)	$\frac{3}{4}$	$\frac{1}{4}$	0	0
H_2		(1,1)	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	(2,1)	$-\frac{3}{4}$	$-\frac{1}{4}$	0	0
H_3	$I \pm \beta$	(1,1)	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	0	(1,1)	$-\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$
H_4		(1,1)	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	(1,1)	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$
H_5		(1,1)	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	(1,1)	$-\frac{3}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$
H_6		(1,1)	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	0	(1,1)	$\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$
H_7	$1 + b_4 \pm \beta$	(1,1)	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$
H_8		(1,1)	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	(1,1)	$\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$
H_9		(1,1)	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$
H_{10}		(1,1)	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	(1,1)	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$
H_{11}	$I + 1 + b_4 \pm \beta$	(1,1)	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	0	$\frac{1}{2}$	(1,1)	$\frac{3}{4}$	$\frac{1}{4}$	0	0
H_{12}		(1,1)	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	0	$\frac{1}{2}$	(1,1)	$\frac{3}{4}$	$\frac{1}{4}$	0	0
H_{13}		(1,1)	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$-\frac{1}{4}$	0	0
H_{14}		(1,1)	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$-\frac{1}{4}$	0	0
H_{15}	$b_1 + b_2 + b_3 + \alpha$	(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	(1,1)	$\frac{3}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	0
H_{16}	$\pm\beta$	(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	0
H_{17}		(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	(1,1)	$\frac{3}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	0
H_{18}		(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	0
H_{19}	$b_2 + b_3 + b_4 + \alpha$	(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	(1,1)	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0
H_{20}	$\pm\beta$	(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0
H_{21}		(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	(1,1)	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0
H_{22}		(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0

Table 2. Massless states and their quantum numbers. H indicates that these states form chiral representations of the Hidden group.

$Q = \frac{1}{3} U(1)_C + \frac{1}{2} U(1)_L.$

F	SEC	$SU(3)_C \times SU(2)_L$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SO(4) \times SU(3)$	Q_H	Q_7	Q_8	Q_9
H_{23}	$1+b_1+b_2+\alpha$	(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	$\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$
H_{24}	$\pm\beta$	(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	$\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$
H_{25}		(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	$\frac{1}{2}$	(2,1)	$-\frac{3}{4}$	$\frac{1}{4}$	0	$-\frac{1}{2}$
H_{26}	$1+b_3+b_4+\alpha$	(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{2}$	0	(1,1)	$\frac{3}{4}$	$-\frac{1}{4}$	0	$\frac{1}{2}$
H_{27}	$\pm\beta$	(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{2}$	0	(1,1)	$\frac{3}{4}$	$-\frac{1}{4}$	0	$\frac{1}{2}$
H_{28}		(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{2}$	0	(2,1)	$\frac{3}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$
H_{29}	$b_3+\alpha\pm\beta$	(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	$\frac{1}{2}$	(1,1)	$\frac{3}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	0
H_{30}		(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	0	$-\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	0
H_{31}		(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$\frac{1}{4}$	$-\frac{1}{2}$	0
H_{32}		(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{2}$	(1,1)	$-\frac{3}{4}$	$-\frac{3}{4}$	$\frac{1}{2}$	0
H_{33}		(3,1)	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	$-\frac{1}{2}$	(1,1)	$\frac{3}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	0
H_{34}		(1,2)	$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	0	$-\frac{1}{2}$	(1,1)	$\frac{3}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	0
H_{35}		(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0	$\frac{1}{2}$	$(1,\bar{3})$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0
H_{36}	$b_1+b_2+b_4+\alpha$	(1,1)	$-\frac{3}{4}$	$\frac{1}{2}$	$-\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{2}$	0	(1,1)	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0
H_{37}	$\pm\beta$	(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{4}$	0	$-\frac{1}{2}$	0	(1,1)	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0
H_{38}		(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	0	$\frac{1}{2}$	0	(1,1)	$\frac{3}{4}$	$-\frac{1}{4}$	$-\frac{1}{2}$	0
H_{39}		(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$	0	(1,1)	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	0
H_{40}		$(\bar{3},1)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	0	(1,1)	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0
H_{41}		(1,2)	$-\frac{3}{4}$	$-\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{2}$	0	(1,1)	$-\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	0
H_{42}		(1,1)	$\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$	0	(1,3)	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{2}$	0

Table 2. (Continued)

$\frac{1}{2} \in \beta.$

F	SEC	$SU(3)_C \times SU(2)_L$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SO(4) \times SU(3)$	Q_H	Q_7	Q_8	Q_9
V_1	$b_1+2\beta$	(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	(1,1)	$\frac{3}{2}$	$\frac{1}{2}$	0	0
V_2		(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1,1)	$-\frac{3}{2}$	$-\frac{1}{2}$	0	0
V_3		(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$(1,\bar{3})$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0
V_4		(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1,3)	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
V_5	$I+b_1+2\beta$	(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1,1)	0	0	$\frac{1}{2}$	$\frac{1}{2}$
V_6		(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1,1)	0	0	$\frac{1}{2}$	$\frac{1}{2}$
V_7		(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
V_8		(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
V_9		(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(2,1)	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
V_{10}		(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	(2,1)	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
V_{11}	$b_2+2\beta$	(1,1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	(1,1)	$\frac{3}{2}$	$\frac{1}{2}$	0	0
V_{12}		(1,1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	(1,1)	$-\frac{3}{2}$	$-\frac{1}{2}$	0	0
V_{13}		(1,1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$(1,\bar{3})$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0
V_{14}		(1,1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	(1,3)	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
V_{15}	$I+b_2+2\beta$	(1,1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	(1,1)	0	0	$\frac{1}{2}$	$\frac{1}{2}$
V_{16}		(1,1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	(1,1)	0	0	$\frac{1}{2}$	$\frac{1}{2}$
V_{17}		(1,1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
V_{18}		(1,1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
V_{19}		(1,1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	(2,1)	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
V_{20}		(1,1)	0	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	(2,1)	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
V_{21}	$b_3+2\beta$	(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	(1,1)	$\frac{3}{2}$	$\frac{1}{2}$	0	0
V_{22}		(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	(1,1)	$-\frac{3}{2}$	$-\frac{1}{2}$	0	0
V_{23}		(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$(1,\bar{3})$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0
V_{24}		(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	(1,3)	$\frac{1}{2}$	$-\frac{1}{2}$	0	0
V_{25}	$I+b_3+2\beta$	(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	(1,1)	0	0	$\frac{1}{2}$	$\frac{1}{2}$
V_{26}		(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	(1,1)	0	0	$\frac{1}{2}$	$\frac{1}{2}$
V_{27}		(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
V_{28}		(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$
V_{29}		(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	(2,1)	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
V_{30}		(1,1)	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	(2,1)	0	0	$-\frac{1}{2}$	$\frac{1}{2}$

Table 1. Massless states and their quantum numbers. V indicates that these states form vector representations of the Hidden group.

$Q_Y = \frac{1}{3} Q_C + \frac{1}{2} Q_L.$

F	SEC	$SU(3)_C \times SU(2)_L$	Q_C	Q_L	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	$SO(4) \times SU(3)$	Q_H	Q_7	Q_8	Q_9
V_{31}	$b_4 + 2\beta$	(1,1)	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	(1,1)	$\frac{3}{2}$	$-\frac{1}{2}$	0	0
V_{32}		(1,1)	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1,1)	$-\frac{3}{2}$	$\frac{1}{2}$	0	0
V_{33}		(1,1)	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	$(1,\bar{3})$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0
V_{34}		(1,1)	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1,3)	$\frac{1}{2}$	$\frac{1}{2}$	0	0
V_{35}	$I + b_4 + 2\beta$	(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1,1)	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
V_{36}		(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1,1)	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
V_{37}		(1,1)	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1,1)	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
V_{38}		(1,1)	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0	(1,1)	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
V_{39}		(1,1)	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	(2,1)	0	0	$\frac{1}{2}$	$\frac{1}{2}$
V_{40}		(1,1)	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	(2,1)	0	0	$\frac{1}{2}$	$\frac{1}{2}$
V_{41}	$1 + b_1 + \alpha + 2\beta$	(1,1)	0	1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	(1,1)	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
V_{42}		(1,1)	0	−1	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	(1,1)	0	$-\frac{1}{2}$	0	$\frac{1}{2}$
V_{43}		(1,1)	0	1	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	(1,1)	0	$-\frac{1}{2}$	0	$\frac{1}{2}$
V_{44}		(1,1)	0	−1	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	(1,1)	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
V_{45}		(1,2)	0	0	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	(1,1)	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
V_{46}		(1,2)	0	0	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	(1,1)	0	$-\frac{1}{2}$	0	$\frac{1}{2}$
V_{47}	$I + \alpha + 2\beta$	(1,1)	0	1	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	(1,1)	0	$-\frac{1}{2}$	0	$\frac{1}{2}$
V_{48}		(1,1)	0	−1	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	(1,1)	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
V_{49}		(1,1)	0	1	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	(1,1)	0	$\frac{1}{2}$	0	$-\frac{1}{2}$
V_{50}		(1,1)	0	−1	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	(1,1)	0	$-\frac{1}{2}$	0	$\frac{1}{2}$
V_{51}		(1,2)	0	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	0	(1,1)	0	$-\frac{1}{2}$	0	$\frac{1}{2}$
V_{52}		(1,2)	0	0	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	(1,1)	0	$\frac{1}{2}$	0	$-\frac{1}{2}$

Table 1. (Continued)

$$Q_Y = \frac{1}{3} Q_C + \frac{1}{2} Q_L$$

$$Q_{e.m.} = T_{3_L} + Q_Y.$$

Calculation of Mass Terms

nonvanishing correlators

$$\langle V_1^f V_2^f V_3^b \cdot \cdots V_N^b \rangle$$

gauge & string invariant

"anomalous" $U(1)_A$

$$\mathrm{Tr} Q_A \neq 0 \Rightarrow D_A = 0 = A + \sum Q_k^A |\langle \phi_k \rangle|^2$$
$$D_j \;\; = 0 = \qquad \sum Q_k^j |\langle \phi_k \rangle|^2$$

$$\langle W \rangle = \langle \frac{\partial W_N}{\partial \eta_i} \rangle = 0 \quad N = 3 \cdots$$

Supersymmetric vacuum $\langle F \rangle = \langle D \rangle = 0.$

nonrenormalizable terms → effective renormalizable operators

$$V_1^f V_2^f V_3^b \cdot \cdots V_N^b \;\; \rightarrow \;\; V_1^f V_2^f V_3^b \frac{\langle V_4^b \cdots V_N^b \rangle}{M^{N-3}}$$

Examine the superpotential in FNY model (NPB 335 (1990) 347

$$W_2 = \frac{1}{\sqrt{2}}\{H_1H_2\phi_4 + H_3H_4\bar\phi_4 + H_5H_6\bar\phi_4 + (H_7H_8 + H_9H_{10})\phi_4' + (H_{11}+H_{12})(H_{13}+H_{14})\bar\phi_4' + V_{41}V_{42}\bar\phi_4 + V_{43}V_{44}\bar\phi_4 + V_{45}V_{46}\phi_4 + (V_{47}V_{48}+V_{49}V_{50})\bar\phi_4' + V_{51}V_{52}\phi_4'\}$$

VEVs of $\bar\phi_4, \bar\phi_4', \phi_4, \phi_4'$ give mass to all exotic states at N=3 (PRD 46 (1993) 3204)

Cleaver, Faraggi, Nanopoulos $\rightarrow$ Classification of flat directions PLB 455 (1999) 135; ...

Example: $\{\phi_{12}, \phi_{23}, \bar\phi_{56}, \phi_4, \phi_4', \bar\phi_4, \bar\phi_4', H_{15}, H_{30}, H_{31}, H_{38}\}$

All Standard Model charged states beyond MSSM $\rightarrow \approx M_{\rm string}$

MINIMAL STANDARD HETEROTIC STRING MODEL

Top quark mass prediction

cubic level $\longrightarrow$ vector γ selects trilevel Yukawa coupling

$$\Delta_j = |\gamma(U(1)_{L_{j+3}}) - \gamma(U(1)_{R_{j+3}})| = 0, 1 \qquad j = 1, 2, 3$$

$\Delta_j = 1 \Rightarrow u_jQ_j\bar h_j$ $\qquad$ $\Delta_j = 0 \Rightarrow d_jQ_jh_j \ ; \ e_jL_jh_j$

		y_3y_6	$y_4\bar y_4$	$y_5\bar y_5$	$\bar y_3\bar y_6$
b_1	:	y_3y_6	$y_4\bar y_4$	$y_5\bar y_5$	$\bar y_3\bar y_6$
γ	:	1	0	0	0

$\Delta_1 = 1$

y_3y_6	$y_4\bar y_4$	$y_5\bar y_5$	$\bar y_3\bar y_6$
1	0	0	1

$\Delta_1 = 0$

A superstring selection mechanism

Models with $\qquad \Delta_{1,2,3} = 1$

$\Rightarrow$ Only $U_L^CQ\bar h$ type Yukawa at

N=3

Top quark mass

At the cubic level

$$M = \begin{array}{c} \\ \bar{h}_1 \\ \bar{h}_2 \\ \bar{h}_3 \\ \bar{h}_{\alpha\beta} \end{array} \begin{array}{c} \begin{array}{cccc} h_1 & h_2 & h_3 & h_{\alpha\beta} \end{array} \\ \begin{pmatrix} 0 & \Phi_{12} & \bar{\Phi}_{13} & 0 \\ \bar{\Phi}_{12} & 0 & \bar{\Phi}_{23} & 0 \\ \Phi_{13} & \Phi_{23} & 0 & \Phi_{\alpha\beta} \\ 0 & 0 & \bar{\Phi}_{\alpha\beta} & 0 \end{pmatrix} \end{array}$$

$$\langle F, D\rangle = 0 \;\Rightarrow\; \langle \Phi_{12}, \bar{\Phi}_{12}\rangle = 0 \;;\; \langle \Phi_{\alpha\beta}, \Phi_{13}, \bar{\Phi}_{13}, \bar{\Phi}_{23}\rangle \neq 0$$

two pairs of massless Higgs eigenstates.

At N=5 $\quad \bar{h}_2 h_{\alpha\beta}\bar{\Phi}_{\alpha\beta}\left(\frac{\Lambda_{Z'}}{M}\right)^2 \quad h_2 \bar{h}_{\alpha\beta}\Phi_{\alpha\beta}\left(\frac{\Lambda_{Z'}}{M}\right)^2$

$\Longrightarrow$ At low energies, one pair, $\bar{h}_1$ or $\bar{h}_2$ and $h_{\alpha\beta}$

$\Longrightarrow$ only $\lambda_t t^c_L Q \bar{h}$ at the cubic level

lighter quarks and leptons $\rightarrow$ nonrenormalizable terms

$\Longrightarrow$ $m_t(M_Z) \approx 175 - 180\text{GeV}$ PLB 274 (1992) 47

But ... construct string model with directly only one Higgs pair?

PLB 274 (1992) 47

Top quark mass prediction

only $\quad \lambda_t = \langle t^c Q_t \bar{h}_1\rangle = \sqrt{2}g \neq 0 \quad$ at $N=3$

mass of lighter quarks and leptons $\rightarrow$ nonrenormalizable terms

$$W_4 \longrightarrow b^c Q_b h_{\alpha\beta}\Phi_1 + \tau^c L_\tau h_{\alpha\beta}\Phi_1$$

$$\Longrightarrow \quad \lambda_b = \left(c_b \frac{\langle\phi\rangle}{M}\right) \qquad \lambda_\tau = \left(c_\tau \frac{\langle\phi\rangle}{M}\right)$$

$$\longrightarrow \quad \lambda_b = \lambda_\tau = 0.35 g^3 \sim \frac{1}{8}\lambda_t$$

Evolve λ_t , λ_b to low energies

$$m_t = \lambda_t v_1 = \lambda_t \frac{v_0}{\sqrt{2}}\sin\beta \qquad m_b = \lambda_b v_2 = \lambda_b \frac{v_0}{\sqrt{2}}\cos\beta$$

where $\quad v_0 = \frac{2m_W}{g_2(M_Z)} = 246\text{GeV} \quad$ and $\quad v_1^2 + v_2^2 = \frac{v_0^2}{2}$

$$m_t = \lambda_t(m_t)\,\frac{v_0}{\sqrt{2}}\,\frac{\tan\beta}{(1+\tan^2\beta)^{\frac{1}{2}}} \Longrightarrow$$

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PHYSICS LETTERS B

Hierarchical top–bottom mass relation in a superstring derived standard-like model

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I propose a mechanism in a class of superstring standard-like models which explains the mass hierarchy between the top and bottom quarks. At the trilinear level of the superpotential only the top quark gets a nonvanishing mass term while the bottom quarks and tau lepton mass terms are obtained from nonrenormalizable terms. I construct a model which realized this mechanism. In this model the bottom quark and tau lepton Yukawa couplings are obtained from quartic order terms. I show that $\lambda_b=\lambda_\tau\sim\frac{1}{8}\lambda_t$ at the unification scale. A naive estimate yields $m_t\sim175$–180 GeV.

One of the unresolved puzzles of the standard model is the mass splitting between the top quark and the lighter juarks and leptons. Especially difficult to understand within the context of the standard model is the big splitting in the heaviest generation. Experimental limits [1] indicate the top mass to be above 80 GeV, while the oottom and tau lepton masses are found at 5 GeV and 1.78 GeV respectively. Possible extensions to the standard nodel are grand unified theories. Although the main prediction of GUTs, proton decay, has not yet been observed, calculations of $\sin^2\theta_w$ and of the mass ratio m_b/m_τ support their validity. Recent calculations seem to support supersymmetric GUTs versus nonsupersymmetric ones [2]. In spite of the success of SUSY GUTs in confronting LEP data [2], an understanding of the mass splitting between the top quark and the lighter quarks and leptons is still lacking. The next level in which such an understanding may be developed is in the context of superstring theory [3].

In this paper I show how the mass splitting between the top quark and the rest of the quarks and leptons is explained in a class of superstring standard-like models [4]. The splitting is achieved by the following mechanism. At the trilinear level of the superpotential only $+\frac{2}{3}$ charged quarks obtain a nonvanishing Yukawa coupling, while the remaining quarks and leptons obtain their mass terms from nonrenormalizable terms. To illustrate this mechanism I present a toy model. In this model only $+\frac{2}{3}$ charged quarks obtain trilinear mass terms while nonvanishing mass terms for $-\frac{1}{3}$ charged quarks and for charged leptons appear at the quartic level. The model contains an anomalous U(1) symmetry. Application of the Dine–Seiberg–Witten mechanism [5] to cancel the anomaly leaves a trilevel mass term only for the top quark. The bottom and tau lepton mass terms are obtained from the quartic order terms. An SO(10) singlet field in these terms obtains a nonvanishing VEV by the application of the Dine–Seiberg–Witten mechanism. Thus, the quartic terms become effective trilinear terms with a suppression factor of $\langle\Phi\rangle/M_{Pl}$. I explicitly demonstrate how this mechanism is realized in the model. In the standard-like models, a close connection may exist between this mechanism and the requirement of F and

Realistic free fermionic models

Phenomenology of the Standard Model and string unification

1. Top quark mass $\sim 175-180$ GeV — PLB 274 (1992) 47
2. Generation mass hierarchy — NPB 407 (1993) 57
3. CKM mixing — NPB 416 (1994) 63 with Edi Halyo
4. Stringy see–saw mechanism — PLB 307 (1993) 311 with Edi Halyo
5. Gauge coupling unification — NPB 457 (1995) 409 with Keith Dienes
6. Proton stability — NPB 428 (1994) 111
7. Squark degeneracy — NPB 526 (1998) 21 with jogesh Pati
8. Minimal Superstring Standard Model — PLB 455 (1999) 135 with Cleaver & Nanopoulos

STRINGY DOUBLET–TRIPLET SPLITTING

NAHE $\rightarrow$ $(5+\bar{5})_j = 10_j$ of $SO(10)$

$\alpha \rightarrow SO(10) \rightarrow SO(6)\times SO(4)$

$y_3\bar{y}_3$	$y_4\bar{y}_4$	$y_5\bar{y}_5$	$y_6\bar{y}_6$	y_3y_6	$y_4\bar{y}_4$	$y_5\bar{y}_5$	$\bar{y}_3\bar{y}_6$
1	0	0	1	1	0	0	0

$$\Delta_j = |\alpha_L(T_2^j) - \alpha_R(T_2^j)|$$

$\Delta_j = 0 \rightarrow D_j, \bar{D}_j$ $\Delta_j = 1 \rightarrow h_j, \bar{h}_j$

$$\Delta_{1,2,3} = 1 \Rightarrow h_j \quad \bar{h}_j \qquad j = 1, 2, 3$$

A superstring solution to the GUT hierarchy problem

AEF, Elisa Manno and Cristina Timirgaziu

	y^3y^6	$y^4\bar{y}^4$	$y^5\bar{y}^5$	$\bar{y}^3\bar{y}^6$	$y^1\omega^6$	$y^2\bar{y}^2$	$\omega^5\bar{\omega}^5$	$\bar{y}^1\bar{\omega}^6$	$\omega^1\omega^3$	$\omega^2\bar{\omega}^2$	$\omega^4\bar{\omega}^4$	$\bar{\omega}^1\bar{\omega}^3$
α	1,	0,	0,	0	0,	0,	1,	1	0,	0,	1,	0
β	0,	0,	1,	1	1,	0,	0,	1	0,	1,	0,	1
γ	0,	1,	0,	1	0,	1,	0,	0	1,	0,	0,	1

supplemented with adequate gauge symmetry breaking

$\rightarrow$ 3 gen $\oplus$ $SU(3)\times SU(2)\times U(1)^2$ model

Only one pair of Higgs doublets h_1 , $\bar{h}_1$

$\Longrightarrow$ only $\lambda_t t_1 Q_1 \bar{h}_1$ at the cubic level

Classification of fermionic $Z_2 \times Z_2$ orbifolds (FKNR, FKR)

Basis vectors: consistent modular blocks 4,8 periodic fermions

$$\begin{aligned}
1 &= \{\psi^\mu,\ \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} \mid \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\} \\
S &= \{\psi^\mu, \chi^{1,\dots,6}\}, \\
z_1 &= \{\bar{\phi}^{1,\dots,4}\}, \\
z_2 &= \{\bar{\phi}^{5,\dots,8}\}, \\
e_i &= \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\},\ i = 1,\dots,6,
\end{aligned}$$

$N=4$ Vacua

$$\begin{aligned}
b_1 &= \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, & N=4 \to N=2 \\
b_2 &= \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}, & N=2 \to N=1
\end{aligned}$$

Vector bosons: NS, $z_{1,2}$, $z_1 + z_2$, $x = 1 + s + \sum e_i + z_1 + z_2$

impose: only NS vector bosons survive GSO projections

$\Longrightarrow$ Gauge group $SO(10) \times U(1)^3 \times SO(8) \times SO(8)$

Independent phases $c\left[\begin{smallmatrix} v_i \\ v_j \end{smallmatrix}\right] = \exp[i\pi(v_i|v_j)]$: upper block

$$\begin{array}{c} \\ 1 \\ S \\ e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \\ z_1 \\ z_2 \\ b_1 \\ b_2 \end{array}
\begin{array}{c}
\begin{array}{cccccccccccc} 1 & S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & z_1 & z_2 & b_1 & b_2 \end{array} \\
\left(\begin{array}{cccccccccccc}
-1 & -1 & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\
 & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & \pm & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & \pm & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & \pm & \pm & \pm & \pm & \pm \\
 & & & & & & & & \pm & \pm & \pm & \pm \\
 & & & & & & & & & \pm & \pm & \pm \\
 & & & & & & & & & & \pm & \pm \\
 & & & & & & & & & & & \pm \\
 & & & & & & & & & & &
\end{array}\right)
\end{array}$$

Apriori 55 independent coefficients $\to$ 2^{55} distinct vacua

Impose: Gauge group $SO(10) \times U(1)^3 \times SO(8)^2$

$\to$ 40 independent coefficients

The twisted matter spectrum:

$$B^1_{\ell^1_3\ell^1_4\ell^1_5\ell^1_6} = S + b_1 + \ell^1_3 e_3 + \ell^1_4 e_4 + \ell^1_5 e_5 + \ell^1_6 e_6$$
$$B^2_{\ell^2_1\ell^2_2\ell^2_5\ell^2_6} = S + b_2 + \ell^2_1 e_1 + \ell^2_2 e_2 + \ell^2_5 e_5 + \ell^2_6 e_6$$
$$B^3_{\ell^3_1\ell^3_2\ell^3_3\ell^3_4} = S + b_3 + \ell^3_1 e_1 + \ell^3_2 e_2 + \ell^3_3 e_3 + \ell^3_4 e_4$$

$l^j_i = 0,1$ $\qquad b_3 = 1 + S + b_1 + b_2 + \sum e_k + \sum z_k$

sectors $B^i_{pqrs} \rightarrow 16$ or $\overline{16}$ of $SO(10)$ with multiplicity $(1,0,-1)$

$B^i_{pqrs} + x \rightarrow 10$ of $SO(10)$ with multiplicity $(1,0)$

Counting: for each B^i_{pqrs}:

Projector:

$$P^{(1)}_{p^1q^1r^1s^1} = \frac{1}{4}\left(1 - c\left[{}^{e_1}_{B^{(1)}_{p^1q^1r^1s^1}}\right]\right)\left(1 - c\left[{}^{e_2}_{B^{(1)}_{p^1q^1r^1s^1}}\right]\right)$$
$$\frac{1}{4}\left(1 - c\left[{}^{z_1}_{B^{(1)}_{p^1q^1r^1s^1}}\right]\right)\left(1 - c\left[{}^{z_2}_{B^{(1)}_{p^1q^1r^1s^1}}\right]\right) \; ; \; P^{(2)} \; ; \; P^{(3)}$$

Chirality:

$$X^{(1)}_{p^1q^1r^1s^1} = -c\left[{}^{b_2+(1-r^1)e_5+(1-s^1)e_6}_{B^{(1)}_{p^1q^1r^1s^1}}\right] \; ; \; X^{(2)} \; ; \; X^{(3)}$$

$$S^{(i)}_{\pm} = \sum_{pqrs} \frac{1 \pm X^{(i)}_{p^iq^ir^is^i}}{2} P^{(i)}_{p^iq^ir^is^i} \; , \; i = 1,2,3$$

similarly for vectorials

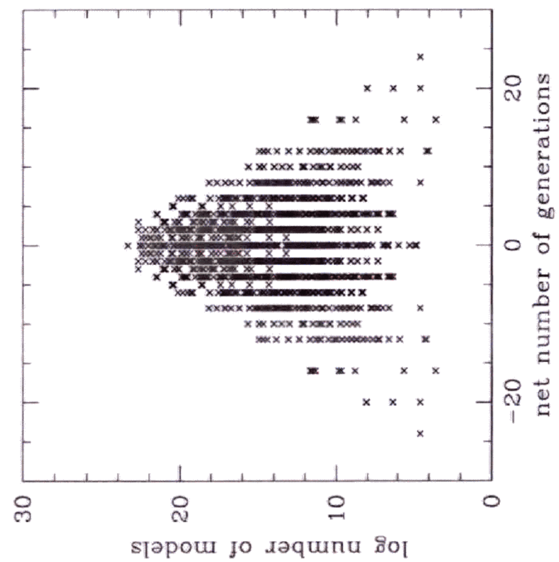
$S = \sum_{i=1}^{3} S^{(i)}_{+} - S^{(i)}_{-}$ and $V = \sum_{i=1}^{3} V^{(i)}$

obtain algebraic formulas for total numbers of S and V

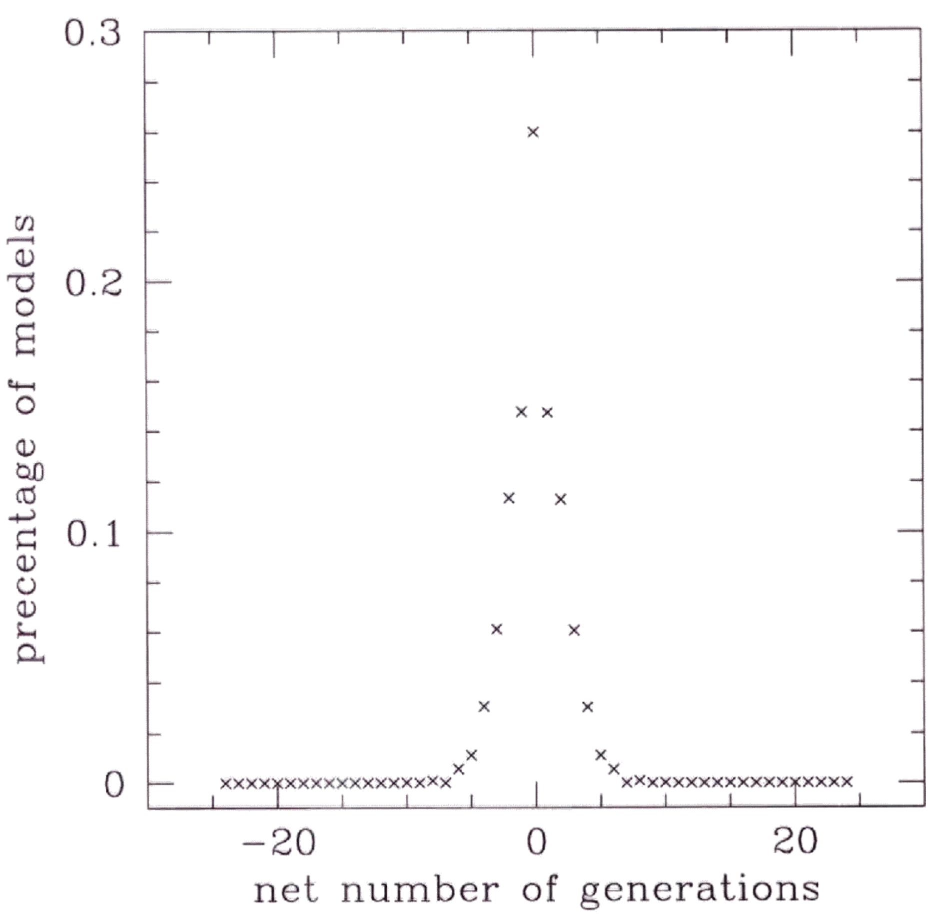
Results

FKR I: Random sampling of phases. $\quad SO(10) \times U(1)^3 \times$ hidden

FKR II: Complete classification. $\quad SO(10) \times U(1)^3 \times SO(8)^2$



Results are similar



∼ 15% with 3 gen.
$7 \cdot 10^9$ models.
} FKR I

Spinor–vector duality:

Duality under exchange of spinors and vectors.

first plane	second plane	third plane	# of models
s $\bar{s}$ v	s $\bar{s}$ v	s $\bar{s}$ v	
2 0 0	0 0 0	0 0 0	1325963712
0 2 0	0 0 0	0 0 0	1340075584
1 1 0	0 0 0	0 0 0	3718991872
0 0 2	0 0 0	0 0 0	6385031168

# of models with $\#(16+\overline{16})$ = # of models with $\#(10)$

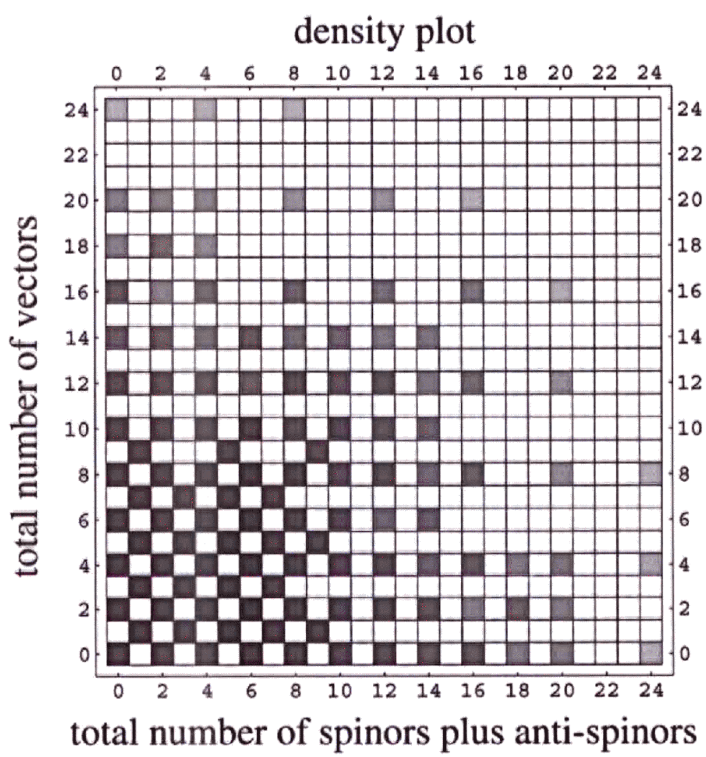
For every model with $\#(16+\overline{16})$ & $\#(10)$

There exist another model in which they are interchanged

Reflects discrete exchange of phases

Spinor - vector duality (FKR II)

Invariance under exchange of #(16 + $\overline{16}$) <-> #(10)



Symmetric under exchange of rows and columns

Self-dual. #(16 + $\overline{16}$) = #(10) without E6 symmetry

1

	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	14424168320	0	19155093504	0	17251226688	0	5722036224	0	1663598208	0	135948288	0	50867584	0
1	0	35042893824	0	54063267840	0	24984354816	0	3050569728	0	113541120	0	0	0	0
2	19155093504	0	138128891904	0	80400635904	0	22541905920	0	2593253376	0	406695936	0	42448896	0
3	0	54063267840	0	128713392128	0	43913576448	0	3064725504	0	0	0	0	0	0
4	17251226688	0	80400635904	0	78871289088	0	11554105344	0	2246205312	0	107403264	0	65853312	0
5	0	24984354816	0	43913576448	0	21663891456	0	856424448	0	67829760	0	0	0	0
6	5722036224	0	22541905920	0	11554105344	0	8043915264	0	937728000	0	104902656	0	387072	0
7	0	3050569728	0	3064725504	0	856424448	0	866942976	0	0	0	0	0	0
8	1663598208	0	2593253376	0	2246205312	0	937728000	0	703000320	0	174673392	0	185902208	0
9	0	113541120	0	0	0	67829760	0	0	0	9904128	0	0	0	0
10	135948288	0	406695936	0	107403264	0	104902656	0	174673392	0	42663936	0	4749312	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	50867584	0	42448896	0	65853312	0	387072	0	185902208	0	4749312	0	6261376	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	1210368	0	2420736	0	387072	0	774144	0	202752	0	405504	0	67584	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	1854336	0	36864	0	1514688	0	0	0	714816	0	0	0	208512	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	36864	0	313344	0	36864	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	33984	0	36864	0	62784	0	0	0	7680	0	0	0	6528	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	576	0	0	0	1152	0	0	0	576	0	0	0	0	0

Conclusions

String theory: exploration of gauge & gravity unification

Phenomenology: Connect data and theory

Construct detailed quasi–realistic models

Fermionic $Z_2 \times Z_2$ orbifolds ->

Minimal Standard Heterotic String Models

Explore properties of "Ensembles" of vacua

Fermionic $Z_2 \times Z_2$ orbifolds ->

Spinor–vector duality