# Brane Inflation: Observational Signatures and Non-Gaussianities

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# Collaborators

- Reheating in D-brane inflation:

  D.Chialva, GS, B. Underwood

- Non-Gaussianities in CMB:

  X.Chen, M. Huang, S. Kachru, GS

- DBI Inflation in Warped Throats:

  S.Kecskemeti, J.Maiden, GS, B.Underwood

# Two popular themes in String Phenomenology:

- Construct realistic particle physics models:

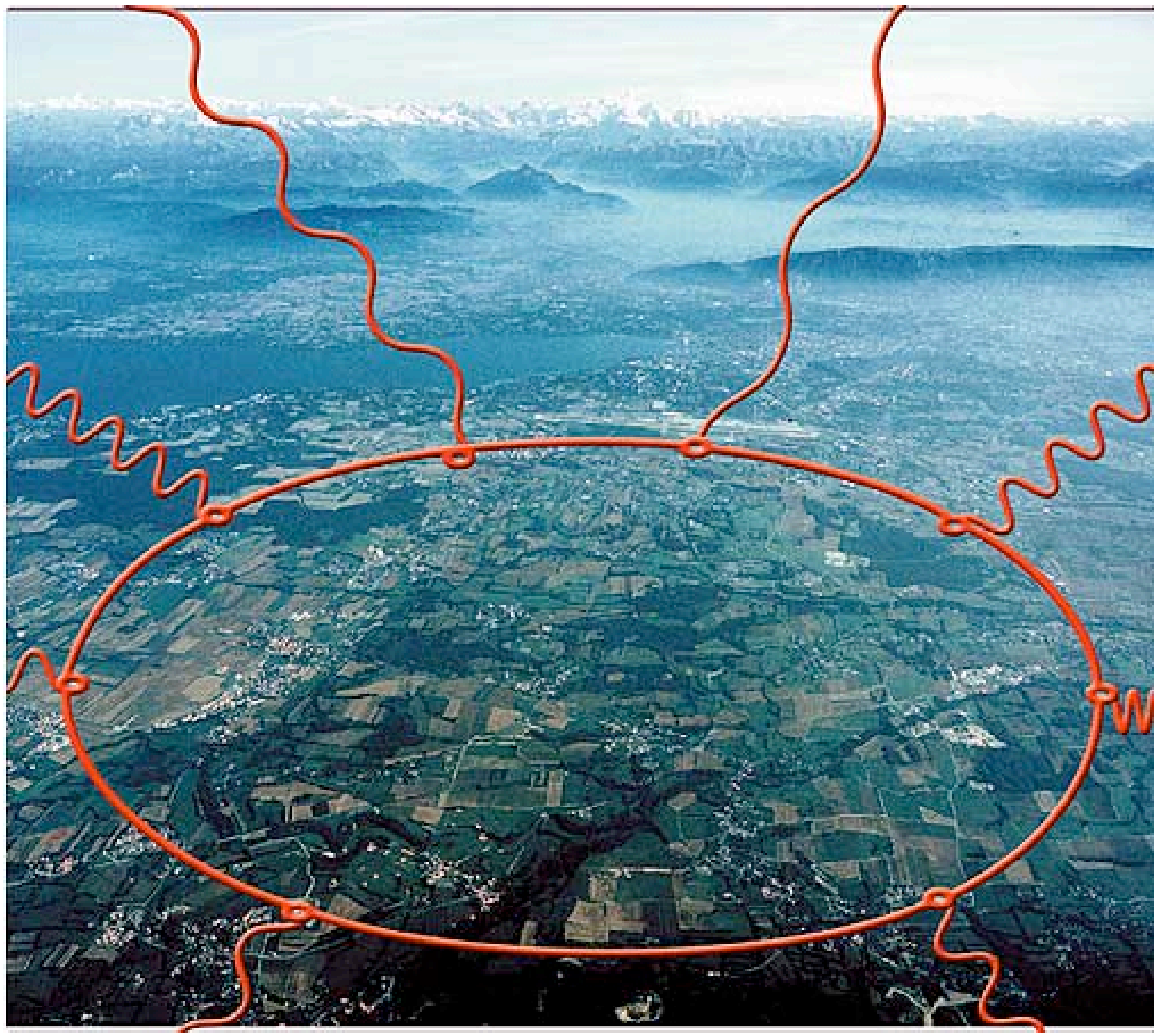
  Not enough (realistic) vacua

- Landscape (statistics, wave function, swampland, ...):

  Too many vacua.

String theory: great scenario generator!

SUSY, brane world, ...

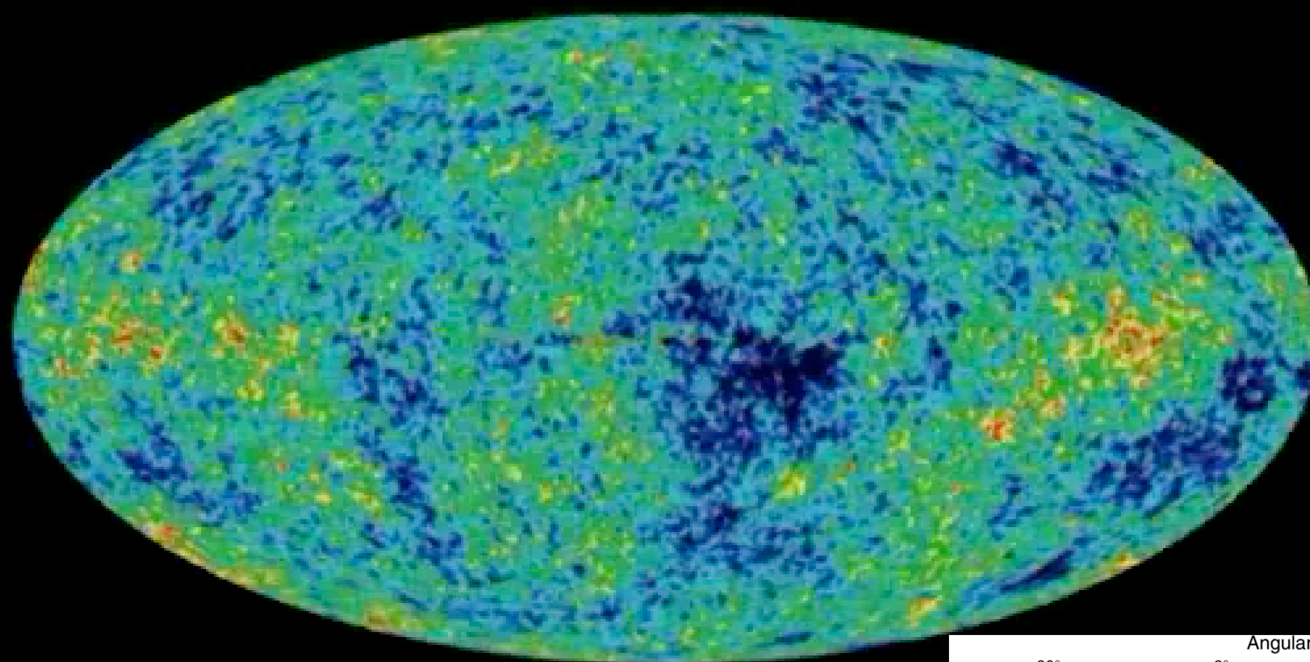


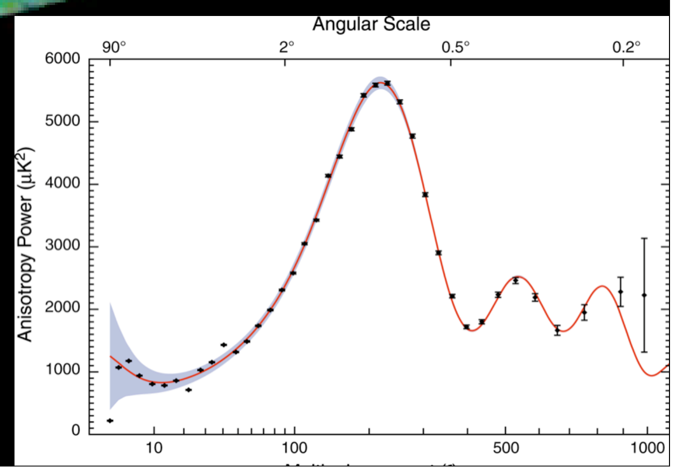
... in the year 1BC

... in the year 1B

... in the year 1BLHC

# WMAP3






Strong and growing evidence
for inflation

# Goals and Motivation

- Construct & study well motivated inflationary scenarios (incorporate SM, reheating, ...)
- Look for distinctive observational signatures
- Building realistic models

Many interesting possibilities
with branes and fluxes

# Brane Inflation

Dvali and Tye

Animation by A. Miller

$D\overline{D}$ Inflation

[Burgess, Majumdar, Nolte, Quevedo, Rajesh, Zhang];[Dvali, Shafi, Solganik], [Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi] and many others.

# Brane Inflation

- Is this scenario viable/robust?

  e.g., number of e-folds, reheating, ...

- Observational signatures/constraints?

  e.g., cosmic strings (Tye's talk), non-Gaussianities, ...

- Model building?

  constraints on compactification geometry?

# Warped Throats

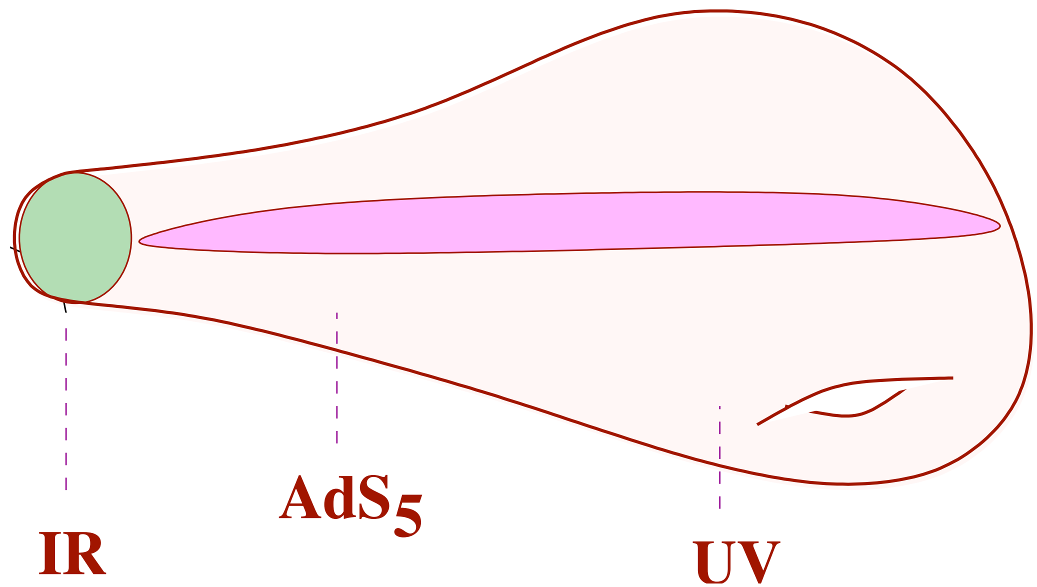
Hierarchies from fluxes

Giddings, Kachru, Polchinski
...

$S^3$ size $e^{-\frac{K}{M g_s}}$

$\updownarrow$

Strong dynamics scale

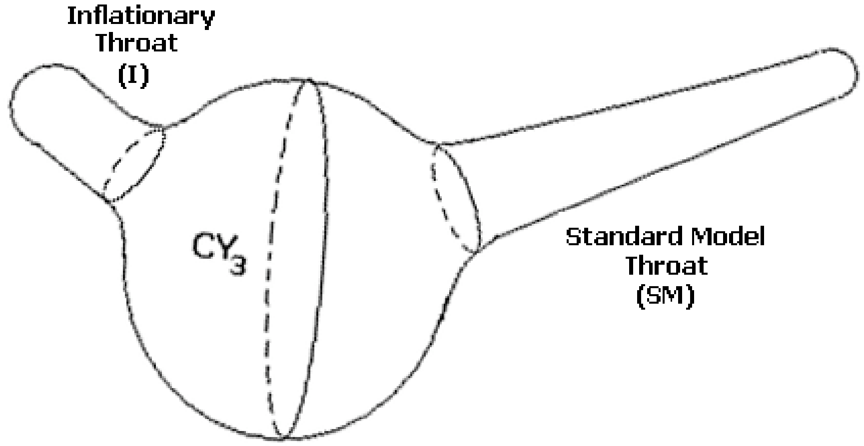



e.g., Klebanov, Strassler

"warped deformed conifold"

# Warped Reheating

## Reheating by DD annihilation

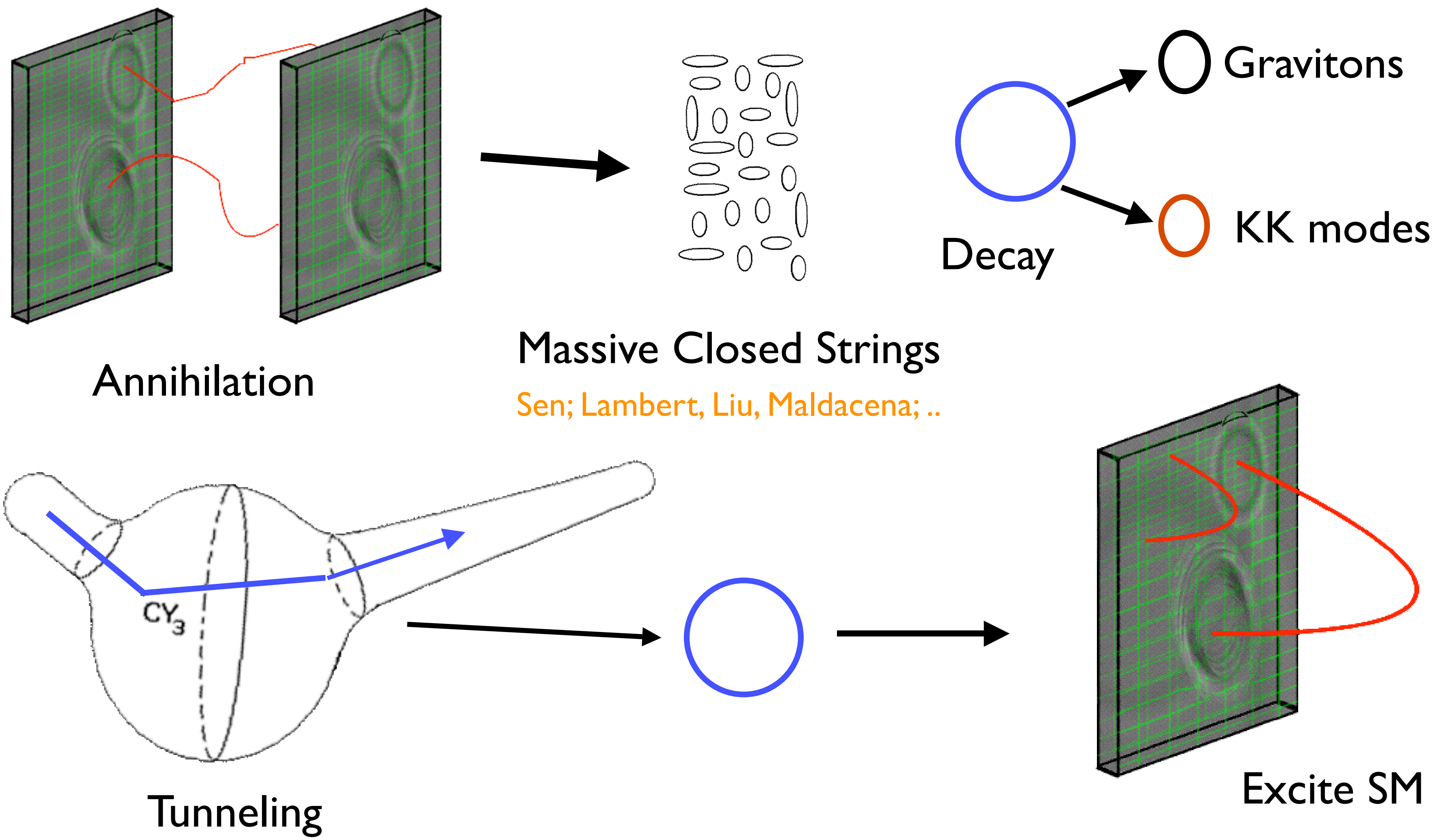



Shiu, Tye, Wasserman

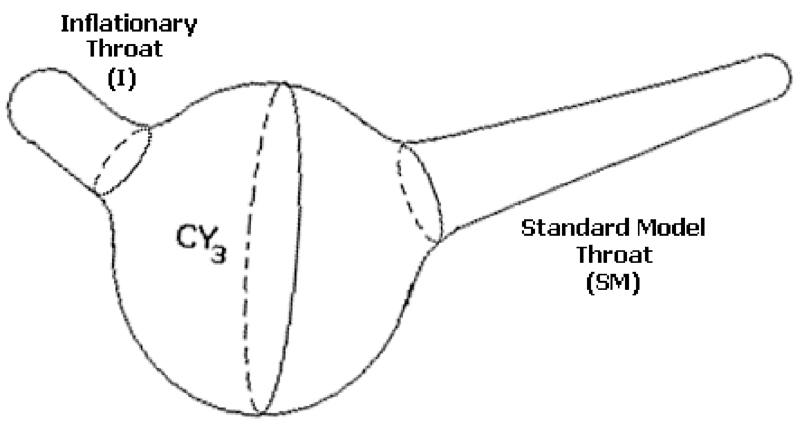
Barnaby, Burgess, Cline
Kofman and Yi
Chialva, Shiu, Underwood
Frey, Mazumdar, Myers
Chen and Tye
Langfelder
...

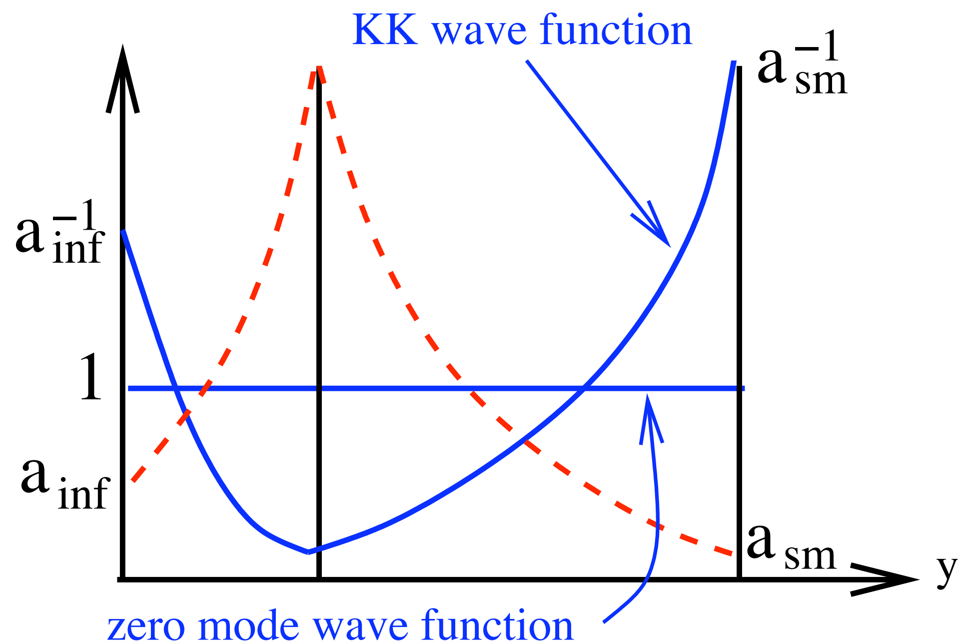
- Accommodate different hierarchies.
- Cosmic strings spatially separated from SM branes: not susceptible to breakage.
- Reheating via tunneling is efficient, can avoid overproduction of gravitational waves.

# A Cartoon of Reheating



# Warped Reheating




c.f. Dimopoulos, Kachru, Kaloper, Lawrence, Silverstein

- Production rate, interaction cross sections among KK modes enhanced relative to gravitons.
- For moderate warping of inflationary throat, KK preferably tunnel rather than decay to gravitons.

# Is brane inflation robust?

Helps flatten the potential

Casual speed limit



e.g., KKLMMT, ...

Silverstein, Tong;
Alishahiha, Silverstein, Tong

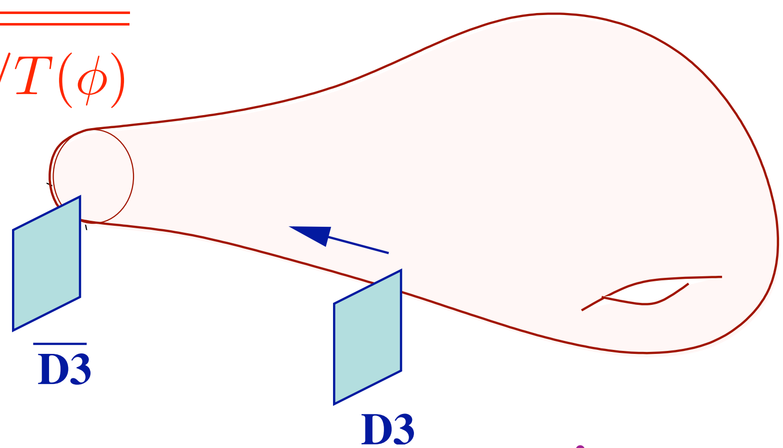
- Derivative terms sum to a DBI action:

$$S = -\int d^4x\, a^3(t) \left[ T(\phi)\sqrt{1-\dot{\phi}^2/T(\phi)} + V(\phi) - T(\phi) \right]$$

$$T(\phi) = T_3 h^4(\phi)$$

- Casual speed limit: $\dot{\phi}^2 \leq T(\phi)$ warp factor

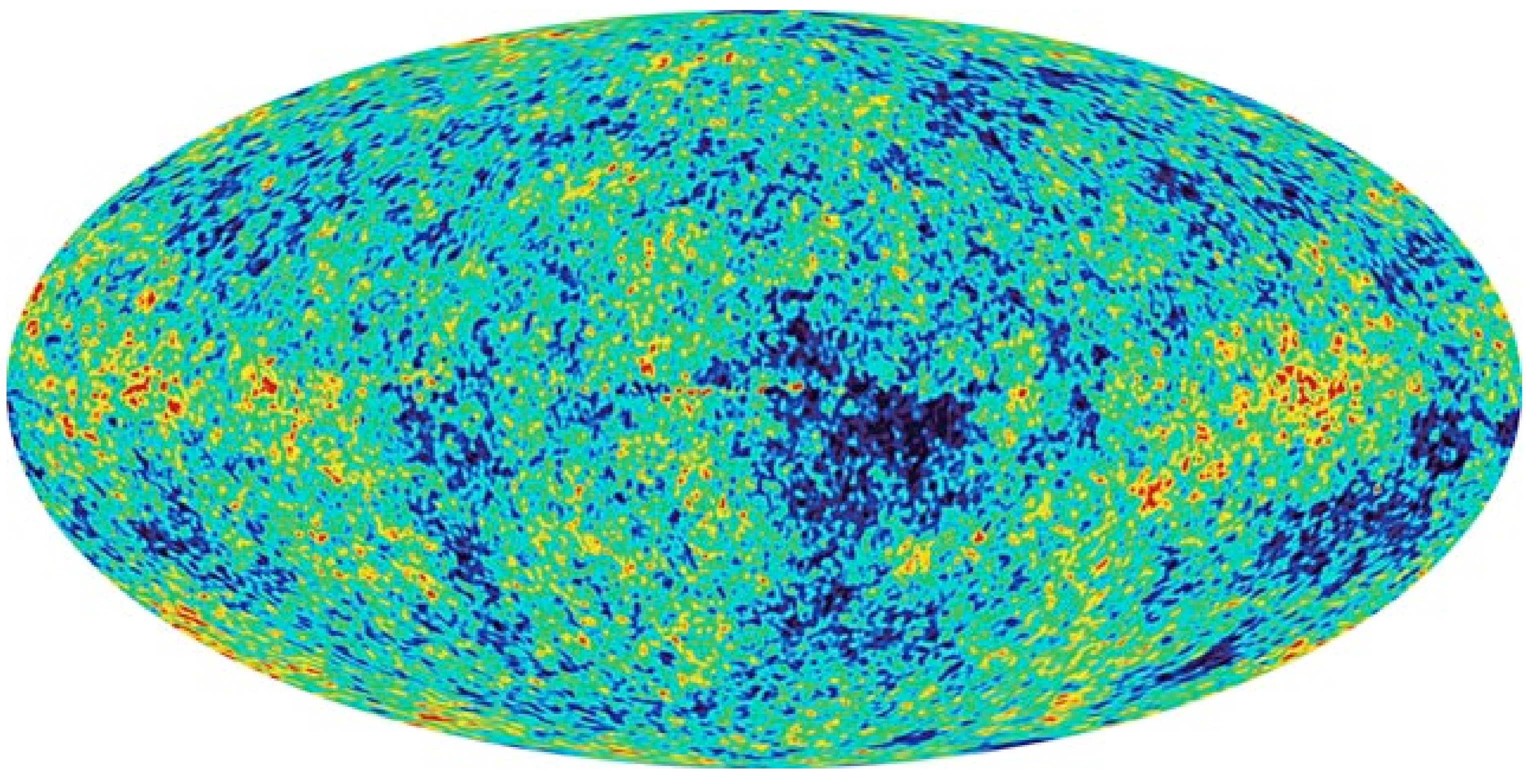
$$\gamma = \frac{1}{\sqrt{1-\dot{\phi}^2/T(\phi)}}$$




Relativistic even when $\dot{\phi}$ is small.

- Slow-roll + DBI : inflation is robust Shandera & Tye

# Non-Gaussianities



# Non-Gaussianities

- Power spectrum: $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle \sim \delta^3(\mathbf{k}_1 + \mathbf{k}_2) \frac{P_k^\zeta}{k_1^3}$

- Bi-spectrum contain much richer info:

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

size ~ $f_{NL}$ and shape.

- Slow-roll: full functional form derived in Maldacena 02, Acquaviva et al 02

$$f_{NL} \sim \mathcal{O}(\epsilon)$$

- DBI inflation for $\gamma >> 1$ : Alishahiha, Silverstein, Tong; Chen; Chen, Huang, Kachru, GS

$$f_{NL} \sim 0.32\gamma^2$$

# Non-Gaussianities

- For a general single field Lagrangian:

$$\mathcal{L}(\phi, X) \qquad \text{where} \qquad X = \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi$$

- Bi-spectrum depends on **5 parameters**: [Chen, Huang, Kachru, GS]

$$c_s^2 = \frac{\mathcal{L}_{,X}}{\mathcal{L}_{,X} + 2X\mathcal{L}_{,XX}} \equiv \frac{1}{\gamma^2} \text{ for DBI} \quad \lambda/\Sigma = \frac{X^2\mathcal{L}_{,XX} + \frac{2}{3}X^3\mathcal{L}_{,XXX}}{X\mathcal{L}_{,X} + 2X^2\mathcal{L}_{,XX}}$$

and slow variation parameters:

$$\begin{aligned} \epsilon &= -\frac{\dot{H}}{H^2} \\ \eta &= \frac{\dot{\epsilon}}{\epsilon H}, \\ s &= \frac{\dot{c}_s}{c_s H}. \end{aligned}$$

# Shape of Non-Gaussianities

$$F(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = (2\pi)^4 (P_k^\zeta)^2 \frac{1}{\prod_i k_i^3} \times (\mathcal{A}_\lambda + \mathcal{A}_c + \mathcal{A}_\epsilon + \mathcal{A}_\eta + \mathcal{A}_s)$$

where

$$\mathcal{A}_\lambda = \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right) \frac{3k_1^2 k_2^2 k_3^2}{2K^3} ,$$

$$\mathcal{A}_c = \left(\frac{1}{c_s^2} - 1\right) \left( -\frac{1}{K} \sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2} \sum_{i\neq j} k_i^2 k_j^3 + \frac{1}{8} \sum_i k_i^3 \right) ,$$

$$\mathcal{A}_\epsilon = \frac{\epsilon}{c_s^2} \left( -\frac{1}{8} \sum_i k_i^3 + \frac{1}{8} \sum_{i\neq j} k_i k_j^2 + \frac{1}{K} \sum_{i>j} k_i^2 k_j^2 \right) ,$$

$$\mathcal{A}_\eta = \frac{\eta}{c_s^2} \left( \frac{1}{8} \sum_i k_i^3 \right) ,$$

$$\mathcal{A}_s = \frac{s}{c_s^2} \left( -\frac{1}{4} \sum_i k_i^3 - \frac{1}{K} \sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2} \sum_{i\neq j} k_i^2 k_j^3 \right) .$$

and $K = k_1 + k_2 + k_3$, $\Sigma = XP_{,X} + 2X^2 P_{,XX}$, $\lambda = X^2 P_{,XX} + \frac{2}{3} X^3 P_{,XXX}$.

# Correction Terms

- Solution to the quadratic part of the action:

$$u_k(y) \;\rightarrow\; -\frac{\sqrt{\pi}}{2\sqrt{2}}\,\frac{H}{\sqrt{\epsilon c_s}}\,\frac{1}{k^{3/2}}(1+\frac{\epsilon}{2}+\frac{s}{2})\; e^{i\frac{\pi}{2}(\epsilon+\frac{\eta}{2})}\; y^{3/2} H^{(1)}_{\frac{3}{2}+\epsilon+\frac{\eta}{2}+\frac{s}{2}}((1+\epsilon+s)y)$$

where $y \;\equiv\; \dfrac{c_s k}{aH}$

- Slowly-varying parameters $H$, $c_s$, $\lambda$ and $\epsilon$

$$\begin{aligned} f(\tau) &\approx f(\tau_K) \\ &\rightarrow f(\tau_K) - \frac{\partial f}{\partial t}\frac{1}{H_K}\ln\frac{\tau}{\tau_K} + \mathcal{O}(\epsilon^2 f) \end{aligned}$$

- The scale factor

$$\begin{aligned} a &\approx -\frac{1}{H_K\tau} \\ &\rightarrow -\frac{1}{H_K\tau} - \frac{\epsilon}{H_K\tau} + \frac{\epsilon}{H_K\tau}\ln(\tau/\tau_K) + \mathcal{O}(\epsilon^2) \end{aligned}$$

# Final Results

$$F(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) \;=\; (2\pi)^4(\tilde{P}^{\zeta}_K)^2\frac{1}{\prod_i k_i^3}\times(\mathcal{A}_\lambda+\mathcal{A}_c+\mathcal{A}_o+\mathcal{A}_\epsilon+\mathcal{A}_\eta+\mathcal{A}_s)$$

$$\begin{aligned}
\mathcal{A}_\lambda &= \left(\frac{1}{c_s^2}-1-\frac{\lambda}{\Sigma}[2-(3-2\mathbf{c}_1)l]\right)_K\frac{3k_1^2k_2^2k_3^2}{2K^3}\ ,\\
\mathcal{A}_c &= \left(\frac{1}{c_s^2}-1\right)_K\left(-\frac{1}{K}\sum_{i>j}k_i^2k_j^2+\frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3+\frac{1}{8}\sum_i k_i^3\right)\ ,\\
\mathcal{A}_o &= \left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)_K(\epsilon F_{\lambda\epsilon}+\eta F_{\lambda\eta}+sF_{\lambda s})\\
&+\left(\frac{1}{c_s^2}-1\right)_K(\epsilon F_{c\epsilon}+\eta F_{c\eta}+sF_{cs})\ ,\\
\mathcal{A}_\epsilon &= \epsilon\left(-\frac{1}{8}\sum_i k_i^3+\frac{1}{8}\sum_{i\neq j}k_ik_j^2+\frac{1}{K}\sum_{i>j}k_i^2k_j^2\right)\ ,\\
\mathcal{A}_\eta &= \eta\left(\frac{1}{8}\sum_i k_i^3\right)\ ,\\
\mathcal{A}_s &= sF_s\ .
\end{aligned}$$

# Experimental Bound

- WMAP ansatz for the primordial non-Gaussianities

$$\zeta(x) = \zeta_g(x) - \frac{3}{5} f_{NL}(\zeta_g(x)^2 - \langle \zeta_g^2(x) \rangle$$

  here $\zeta_g(x)$ is purely Gaussian with vanishing three point functions.

- The size of non-Gaussianities is measured by the parameter $f_{NL}$ in the above ansatz. Current experimental bound (from WMAP3) is

$$-54 < f_{NL} < 114 \quad \text{at } 95\% \text{ C.L.}$$

  Future experiments can eventually reach the sensitivity of $f_{NL} \lesssim 20$ (WMAP) and $f_{NL} \lesssim 5$ (PLANCK).

- However, the experimental bound depends on the **shape** of $F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$.

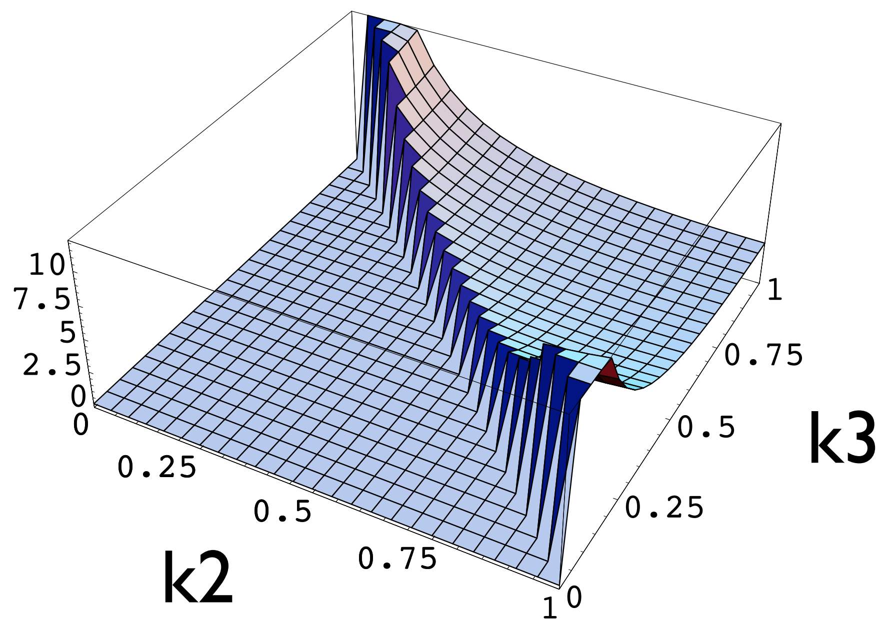
  Creminelli, Nicolis, Senatore, Tegmark, and Zaldarriaga

- Due to the symmetry and scaling property of $F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$, all info about the shape can be viewed by plotting [Babich, Creminelli, Zaldarriaga]

$$F(1, k_2, k_3)k_2^2 k_3^2$$

- For the WMAP ansatz:

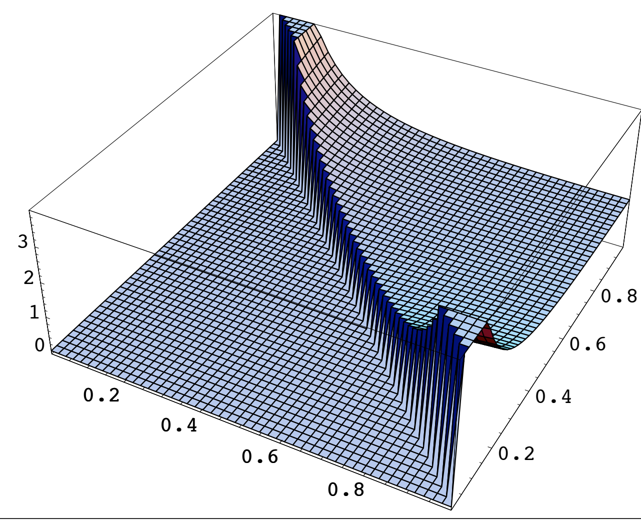
$$F(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \sim f_{NL} \left(P_k^\zeta\right)^2 \frac{k_1^3 + k_2^3 + k_3^3}{k_1^3 k_2^3 k_3^3}$$



# Slow Roll Shapes

The relevant shapes are $F(k_1,k_2,k_3) \sim \frac{1}{\prod_i k_i^3}\mathcal{A}(k_1,k_2,k_3)$ where

$$\begin{aligned}
\mathcal{A}_\epsilon &= \frac{\epsilon}{c_s^2}\left(-\frac{1}{8}\sum_i k_i^3 + \frac{1}{8}\sum_{i\neq j} k_i k_j^2 + \frac{1}{K}\sum_{i>j} k_i^2 k_j^2\right) \ , \\
\mathcal{A}_\eta &= \frac{\eta}{c_s^2}\left(\frac{1}{8}\sum_i k_i^3\right) \\
\mathcal{A}_s &= \frac{s}{c_s^2}\left(-\frac{1}{4}\sum_i k_i^3 - \frac{1}{K}\sum_{i>j} k_i^2 k_j^2 + \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3\right) \ .
\end{aligned}$$



# Consistency Condition

Maldacena

- In the "squeeze triangle limit": one momentum mode is much smaller than the other two:

$$k_3 \ll k_1, k_2 \quad \mathbf{k}_1 \sim -\mathbf{k}_2$$

- During inflation, the comoving Hubble scale decreases with time. The long wavelength mode $k_3$ crosses the horizon much earlier than the other two modes $k_1, k_2$.

- After horizon crossing, the long wavelength mode $k_3$ acts as background whose effect is to introduce a time variation at which $k_{1,2}$ cross the horizon.

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \sim \langle \zeta_{\mathbf{k}_3} \zeta_{\text{-}\mathbf{k}_3} \rangle \frac{d}{d \ln k_1} \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle \sim (n_s - 1) \frac{1}{k_1^3} \frac{1}{k_3^3}$$

# DBI Shape

- Non-Gaussianities are generically quite large

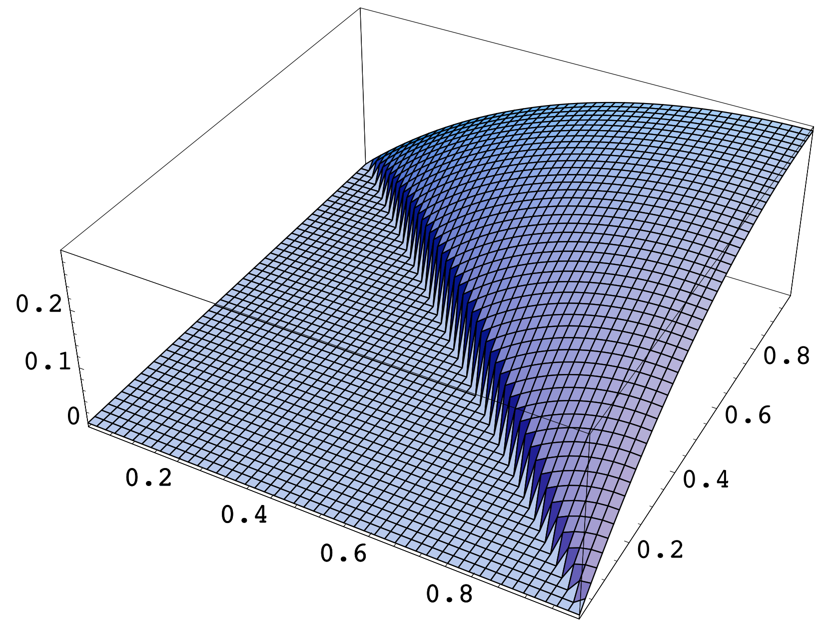
$$f_{NL} \sim \frac{1}{c_s^2} \sim \gamma^2$$

- The shape of non-Gaussianities vanishes in the squeeze triangle limit $k_3 \ll k_1, k_2$, as required by Maldacena's consistency relation:

$$F(k_1, k_2, k_3) k_1^3 k_3^3 \sim n_s - 1$$

This contradicts that the non-Gaussianities are large, unless the shape vanishes in the squeeze limit.

- The shape of non-Gaussianities for DBI inflation




$$|f_{NL}| \leq 300$$

- Peak at the equilateral triangle limit and vanishes in the squeeze limit.

- If non-Gaussianities of this shape is measured, gives interesting constraint on $m^2\phi^2$ term and in turn 4-cycles of CY.

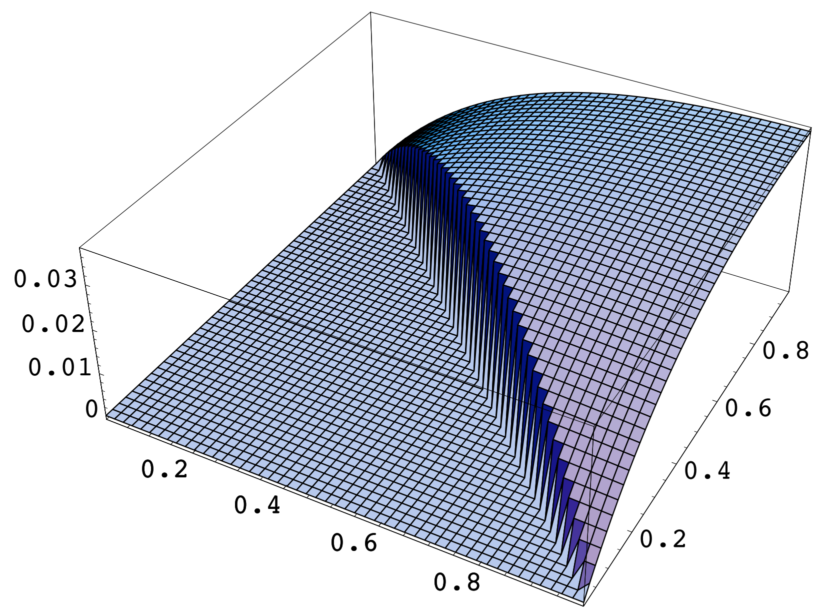
[Baumann, Dymarsky, Klebanov, Maldacena, McAllister, and Murugan]

Also: [Berg, Haack, Kors]

# More Shapes

Not realized in D-brane inflation. Similar to the DBI inflation but with an opposite sign.

$$\mathcal{A}_\lambda \;=\; \left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right)\frac{3k_1^2 k_2^2 k_3^2}{2(k_1+k_2+k_3)^3}$$



# Confronting Data

$$\frac{\ddot{a}}{a} = H^2(1 - \epsilon_D)$$

$$\epsilon_D \equiv \frac{2M_p^2}{\gamma}\left(\frac{H'(\phi)}{H(\phi)}\right)^2$$

$$\eta_D \equiv \frac{2M_p^2}{\gamma}\left(\frac{H''(\phi)}{H(\phi)}\right)$$

$$\kappa_D \equiv \frac{2M_p^2}{\gamma}\left(\frac{H'}{H}\frac{\gamma'}{\gamma}\right)$$

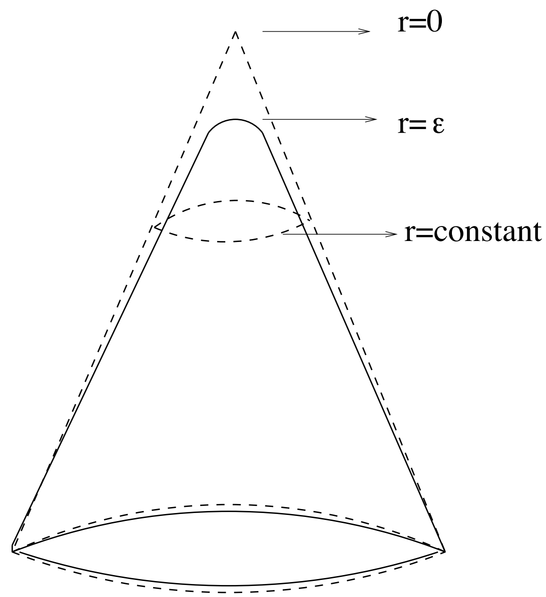
$$r = \frac{16\epsilon_D}{\gamma}$$

$$f_{NL} \leq 0.3\gamma^2$$

$$n_s - 1 \simeq (1 + \epsilon_D + \kappa_D)(-4\epsilon_D + 2\eta_D - 2\kappa_D)\,.$$

If r saturates the observational bound, non-Gaussianity is small.

# Warped Deformed Conifold




$$\sum_{i=1}^{4} z_i^2 = \varepsilon^2$$

$$ds_{10}^2 = h^{-1/2}(\tau) dx_n dx_n + h^{1/2}(\tau) ds_6^2$$

$$ds_6^2 = \frac{1}{2}\varepsilon^{4/3} K(\tau)\Bigg[\frac{1}{3K^3(\tau)}(d\tau^2 + (g^5)^2) + \cosh^2\left(\frac{\tau}{2}\right)[(g^3)^2 + (g^4)^2] + \sinh^2\left(\frac{\tau}{2}\right)[(g^1)^2 + (g^2)^2]\Bigg] ,$$

where

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3}\sinh\tau} .$$

$$h(\tau) = \alpha \frac{2^{2/3}}{4} I(\tau) = (g_s M \alpha')^2 2^{2/3} \varepsilon^{-8/3} I(\tau) ,$$

$$I(\tau) \equiv \int_\tau^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3} .$$

# DBI ultra-relativistic region

$$f_{NL} \simeq \left(\frac{m}{M_p}\right)^2 \left(\frac{M_p}{m_s h_A}\right)^4 \simeq 10^{-12} \frac{1}{(G\mu_s)^2}$$

$$\frac{m_s}{M_p} > 10^{-2} \qquad N_A \sim 10^{14}$$

$$\frac{m}{M_p} \simeq 10^{-6} \qquad h_A \sim 10^{-1} - 10^{-2}$$

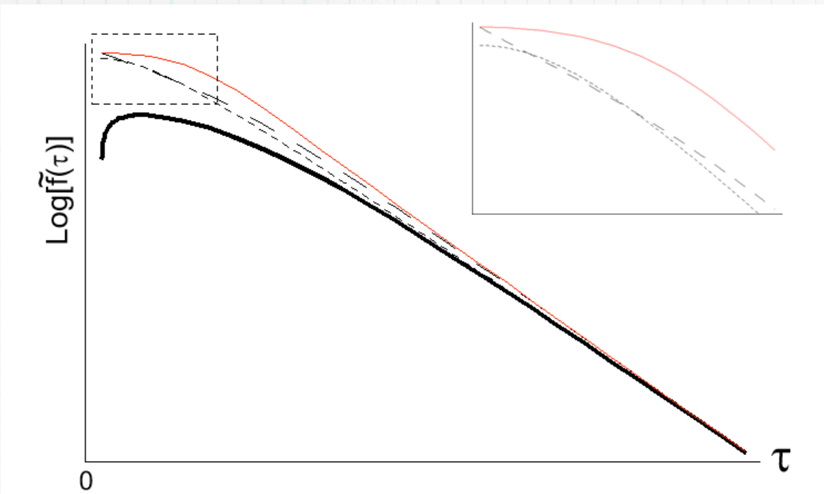
To fit a KS-like throat inside the bulk : $\frac{m_s}{M_p} \sim 10^{-12}$

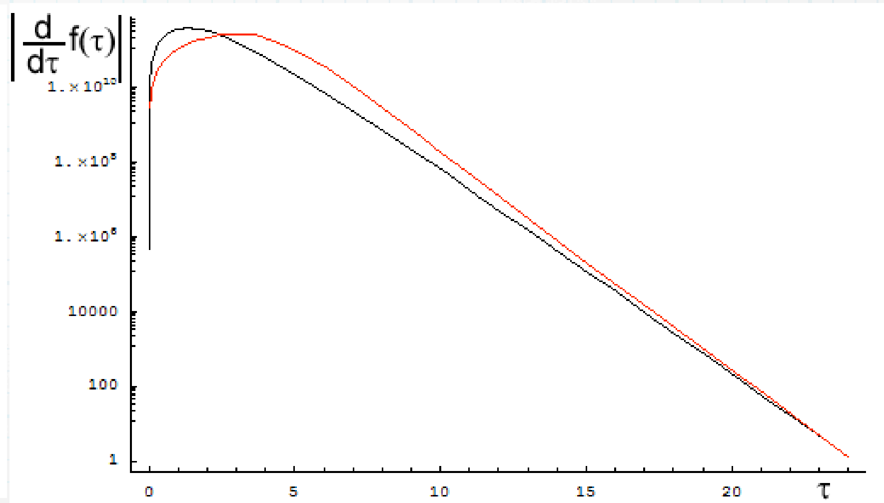
M. Alishahiha, E. Silverstein and D. Tong, hep-th/0404084
S. Kecskemeti, J. Maiden, G. Shiu, B. Underwood, hep-th/0605189

Need a long narrow throat :

- other warped throats?
- $Z_p$-orbifold the KS-like throat?

# Red or blue tilt in DBI ?



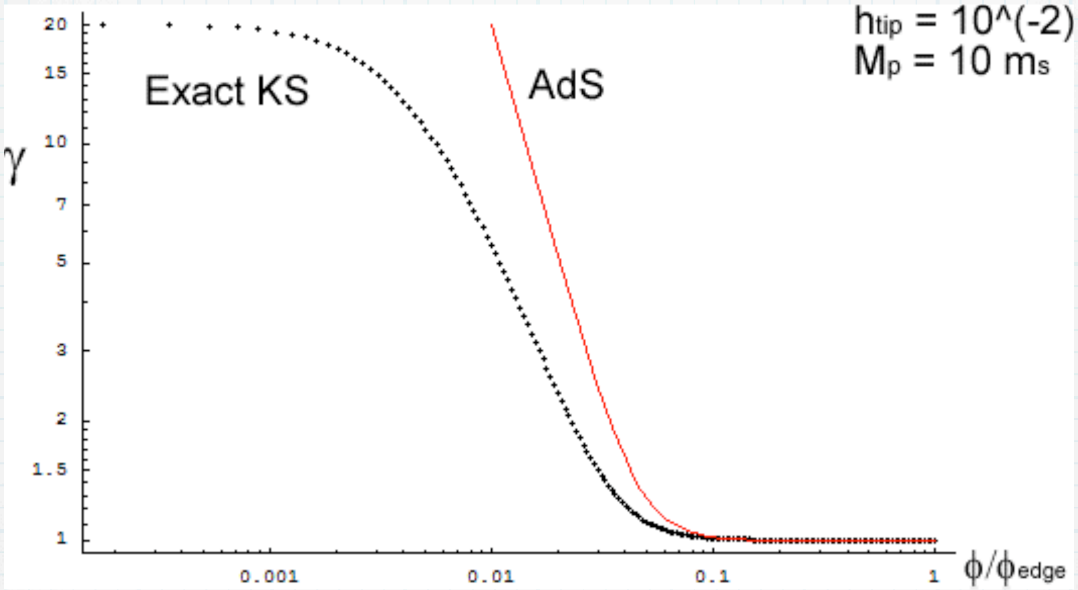

$$h^4(\phi) \simeq \frac{(\phi^2 + b)^2}{\lambda}$$ Red tilt

KS throat ?

$$h^4(\phi) \simeq \frac{\phi^4}{\lambda}$$ A small blue tilt

cut off at $\phi_A$

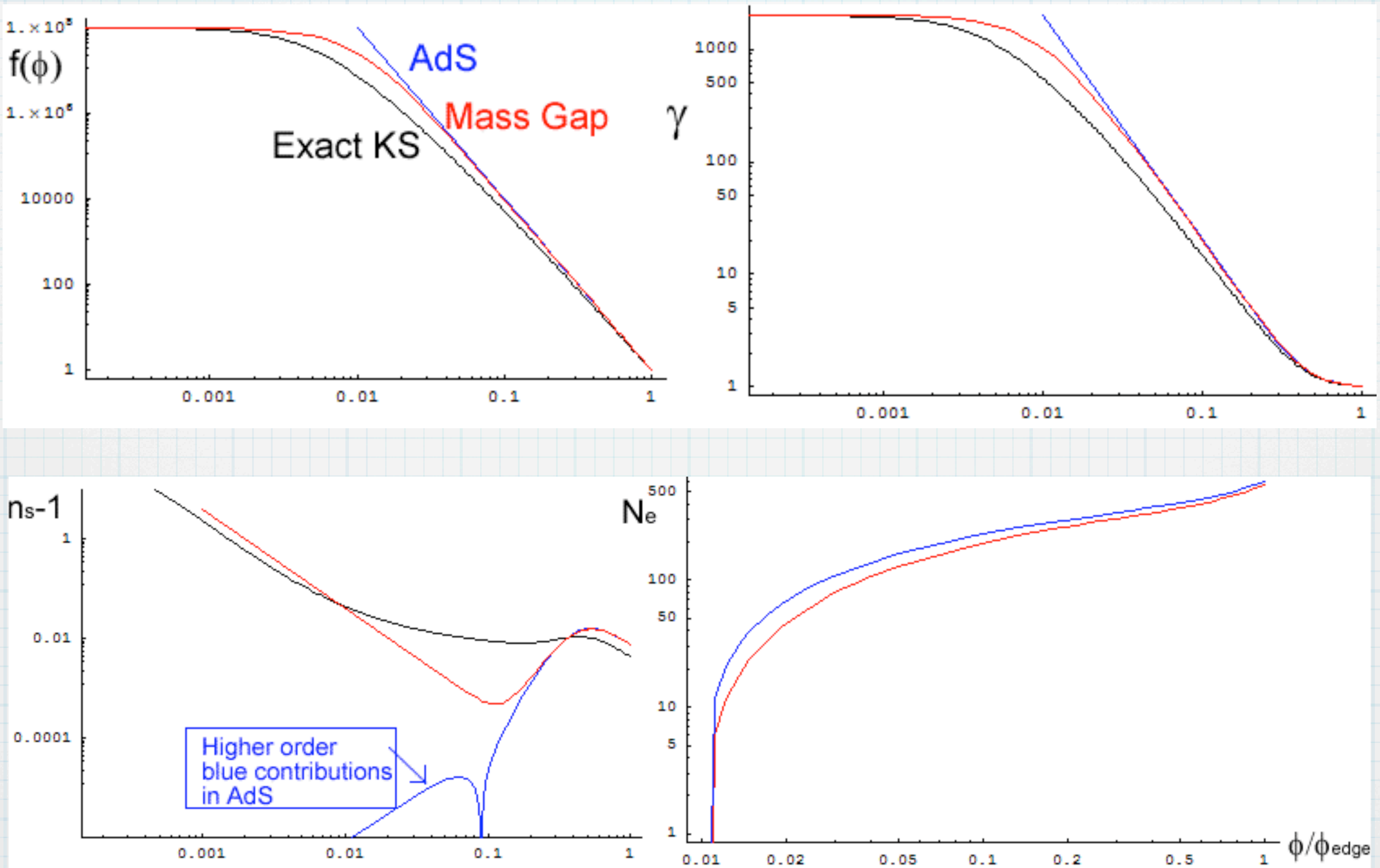
# Red or blue tilt in DBI ?




$$n_s - 1 = \frac{2M_p^2}{\gamma}\left[-4\left(\frac{H'}{H}\right)^2 + 2\frac{H''}{H} + 2\frac{H'}{H}\left|\frac{\gamma'}{\gamma}\right|\right]$$

red (small) blue

# Tip from the Sky ?




Bret Underwood

# Red or blue tilt in DBI-KS ?

$$n_s - 1 = \frac{2M_p^2}{\gamma}\left[-4\left(\frac{H'}{H}\right)^2 + 2\frac{H''}{H} + 2\frac{H'}{H}\left|\frac{\gamma'}{\gamma}\right|\right]$$

red blue

For example, if $h_{tip} \geq 10^{-2}$ and $M_s \sim 10^{-2} M_P$

red tilt dominates for KS throat

# Summary

- Brane inflation is robust: number of e-foldings, reheating, ...
- Interesting signatures: can lead to large tensor-scalar ratio r, or large non-Gaussianities, cosmic strings ...
- Data probe warped geometry.

[c.f. talks of Giddings, Hebecker]

**Large influx of data from Cosmology + LHC !**