Modifications of CDM inspired by string theory

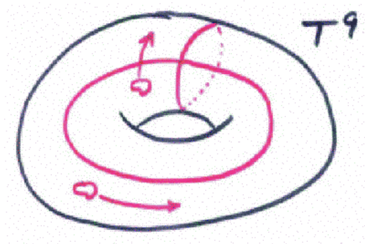
Steven S. Gubser

Some motivations:

1. What is the late-time behavior of the Brandenberger-Vafa scenario?
2. Can the presence of winding and momentum strings stabilize a modulus?
3. Might dark matter be precisely such winding and momentum strings?
4. What are the consequences of a long-range scalar interaction among dark matter particles?

This work grew in part out of discussions with Jim Peebles based on his recent paper with Farrar, astro-ph/0307316, which addresses point 4.

Recall that in *[Brandenberger-Vafa]* the universe supposedly was small and hot at early times: for instance a T^9 at the self-dual radius, with all string modes excited.

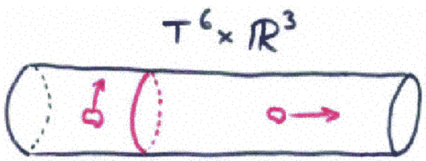


- If three dimensions start to grow (through thermal fluctuations), the strings that wind them annihilate efficiently because strings moving in three spatial dimensions generically intersect.
- If four or more dimensions start to grow, the winding strings do not efficiently annihilate, so they pull these dimensions back in.
- If several dimensions start to shrink instead, we perform T-duality on them, and then they're growing.

Hence three large dimensions is preferred as a late-time behavior!

Development of early-universe scenarios inspired by Brandenberger-Vafa has been extensive: work of Brandenberger and collaborators and of Greene and collaborators.

At late times, still have strings winding and with momentum on the six dimensions that stayed small. Maybe these strings help stabilize the compactification at around the self-dual radius.



Recent numerical analysis *[Brandenberger-Watson]* shows that they do, provided we stick to a homogenous FRW ansatz.

What about inhomogeneities?

Instead of many moduli, consider one scalar, ϕ, coupled to massive particles:

$$S = \int d^4x \sqrt{g} \left[\frac{R}{16\pi G} - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right] - \sum_Q \int_{\gamma_Q} ds\, m_Q(\phi)\,, \tag{1}$$

Treat each species of particles in a hydrodynamic approximation:

$$N_q^i = n_q U_q^i \qquad T_q^{ij} = n_q m_q(\phi) U_q^i U_q^j\,, \tag{2}$$

n_1, n_2 are number densities of winding and momentum strings, and $i, j = 0, 1, 2, 3$. The equations of motion are

$$\begin{aligned} &\nabla_i N_q^i = 0 \qquad \nabla_i T_{qj}^i = -n_q \partial_j m_q \\ &\nabla^2 \phi = \frac{dV}{d\phi} + \sum_q n_q \frac{dm_q}{d\phi} \\ &R_{ij} = 8\pi G \left(T_{ij} - \frac{1}{2} g_{ij} T_k^k \right)\,, \end{aligned} \tag{3}$$

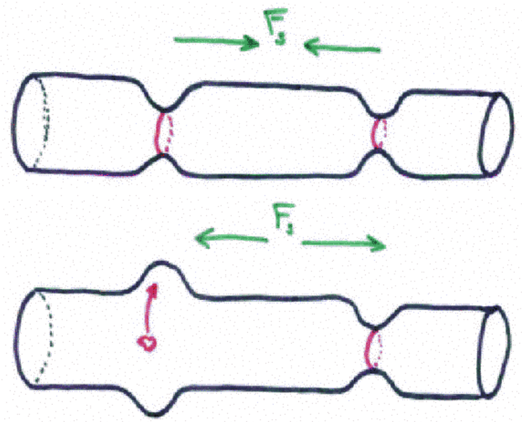
where $T^{ij} = \sum_q T_q^{ij} + T_s^{ij}$.

We'll discuss inhomogeneities by linearizing (3) around an FRW background:

$$\begin{aligned} ds^2 &= a(\tau)^2 \left[-d\tau^2 + (\delta_{\alpha\beta} - h_{\alpha\beta}) dx^\alpha dx^\beta \right] \\ n_q &= \bar{n}_q (1 + \delta_q) \,, \end{aligned} \qquad (4)$$

where we suppose $\phi = 0$ in the background: so ϕ is a first order quantity. [Note: must assume $V'(0) = 0$; I'll also take $V(0) = 0$.]

Scalar-mediated forces cause like particles to attract and unlike particles to repel:



If these forces are comparable to gravity, structure formation might happen differently than in the CDM model.

Two combinations of perturbations evolve simply: the total contrast,

$$\begin{aligned} \ddot{\delta}_m + \frac{\dot{a}}{a} \dot{\delta}_m - \frac{3}{2} \frac{\dot{a}^2}{a^2} \delta_m &= 0 \\ \delta_m \equiv \sum_q \Omega_q \delta_q \,, \end{aligned} \qquad (5)$$

and the difference of contrasts:

$$\begin{cases} \ddot{\Delta} + \frac{\dot{a}}{a} \dot{\Delta} = \frac{\zeta_1}{2} \partial_\alpha \partial_\alpha \varphi \\ \ddot{\varphi} + 2 \frac{\dot{a}}{a} \dot{\varphi} - \partial_\alpha \partial_\alpha \varphi + \frac{a^2}{M_{Pl}^2} \frac{d^2 V}{d\varphi^2} \varphi \\ \qquad + 3\zeta_2 \frac{\dot{a}^2}{a^2} \varphi = -3 \frac{\dot{a}^2}{a^2} \Delta \end{cases} \qquad (6)$$

$$\Delta \equiv \sum_q \Omega_q \frac{d \log m_q}{d\varphi} \delta_q \qquad \varphi = \phi / M_{Pl}$$

where we've defined

$$\begin{aligned} \zeta_1 \equiv 2 \sum_q \Omega_q \left(\frac{d \log m_q}{d\varphi} \right)^2 \qquad \zeta_2 = \sum_q \Omega_q \frac{1}{m_q} \frac{d^2 m_q}{d\varphi^2} \\ \Omega_q = \frac{\rho_q}{\rho_{\text{tot}}} = \frac{\rho_q a^4}{3 M_{Pl}^2 \dot{a}^2} \,. \end{aligned} \qquad (7)$$

At late times, $\partial_\alpha \partial_\alpha \varphi$ dominates in the scalar eom, so we get

$$\begin{aligned} \ddot{\delta}_m + \frac{\dot a}{a}\dot\delta_m - \frac{3}{2}\frac{\dot a^2}{a^2}\delta_m &= 0 \\ \ddot\Delta + \frac{\dot a}{a}\dot\Delta - \frac{3}{2}\zeta_1 \frac{\dot a^2}{a^2}\Delta &= 0\,. \end{aligned} \qquad (8)$$

In matter-dominated epoch, $a(\tau) = (\tau/\tau_0)^2$, and

$$\delta_m \sim \tau^2 \qquad \Delta \sim \tau^{2\gamma} \qquad \gamma = \frac{-1+\sqrt{1+24\zeta_1}}{4}\,. \qquad (9)$$

If $\zeta_1 = 1$ then $\gamma = 1$.

[Aside: replacing scalar interactions by gauge interactions leads to $\zeta_1 < 0$: decaying / oscillatory Δ]

Suppose

$$\Omega_1 = \Omega_2 = 1/2$$

$$\left.\begin{aligned} m_1 &= m_2 \\ \frac{dm_1}{d\phi} &= -\frac{dm_2}{d\phi} \end{aligned}\right\} \qquad \text{at } \phi = 0\,. \qquad (10)$$

Then we have

$$\zeta_1 = \frac{F_s}{F_g}\,. \qquad (11)$$

So it's natural to have $\zeta_1 \sim O(1)$ in a string theory setup.

Consider strings on a compact K_6 with some π_1.

$$\begin{aligned} &\frac{1}{2\kappa_{10}^2}\int d^4x \int_K d^6y \sqrt{G_{10}} e^{-2\Phi_{10}} \left[R_{10} + 4(\partial\Phi_{10})^2\right] \\ &\quad = \frac{1}{2\kappa_4^2}\int d^4x \sqrt{G_4} e^{-\Phi_4}\left[R_4 + (\partial\Phi_4)^2 + \frac{1}{4}\partial_i G_{uv}\partial^i G^{uv}\right] \\ &\quad = \frac{1}{16\pi G}\int d^4x\sqrt{g}\left[R - \frac{1}{2}(\partial\Phi_4)^2 + \frac{1}{4}\partial_i G_{uv}\partial^i G^{uv}\right] \\ &e^{-\Phi_4} = \frac{\text{Vol } K}{(2\pi\sqrt{\alpha'})^6} e^{-2\Phi_{10}} \qquad g_{ij} = e^{-\Phi_4} G_{ij} \\ &2\kappa_{10}^2 = \frac{(2\pi\sqrt{\alpha'})^8}{2\pi} g_s^2 \qquad 2\kappa_4^2 = 16\pi G = 2\pi\alpha' g_s^2\,, \end{aligned} \qquad (12)$$

G_{MN} is the string frame metric; g_{ij} is the 4-d Einstein frame metric. Let's assume K_6 is a square T^6 at the self-dual radius, $R = \sqrt{\alpha'}$. Put $G_{99} = e^{2\phi/M_{Pl}}$ (still M_{Pl} is 4-d Planck mass) and assume other S^1's stay of constant size. Then

$$\begin{aligned} &S = \int d^4x \sqrt{g}\left[\frac{1}{16\pi G}R - \frac{1}{2}(\partial\phi)^2 + \ldots\right] \\ &m_1\sqrt{\alpha'} = e^{\phi/M_{Pl}} \qquad m_2\sqrt{\alpha'} = e^{-\phi/M_{Pl}} \\ &\zeta_1 = 2 \qquad \zeta_2 = 1\,. \end{aligned} \qquad (13)$$

Values of ζ_1, ζ_2 don't depend on $\langle\phi\rangle$ (but Ω_1, Ω_2 do).

$\Delta \sim \tau^4$ seems clearly untenable: bound structures form *way* sooner than in CDM model. Simplest way to fix this is assume a third species of dark matter with $\Omega_3 \geq 1/2$: then $\zeta_1 \leq 1$.

Punchline: Scalar forces are only *slightly* too strong in simplest string theory construction.

Alternatively: String theory with a free modulus sits close to the border of measurable deviations from CDM model.

How do we make a testable model?

When a Fourier mode crosses into the horizon, it's plausible to assume that $\Delta_H \approx \delta_{m,H} \approx 2 \times 10^{-5}$; but thereafter,

$$\frac{\Delta}{\Delta_H} \approx \left(\frac{\delta_m}{\delta_{m,H}}\right)^\gamma \qquad \Delta \approx \frac{\delta^\gamma}{\delta_H^{\gamma-1}}, \tag{14}$$

If $\zeta_1 > 1$, then (Δ, φ) bound structures form before (δ_m, g_{ij}) structures: exit from linear regime is at

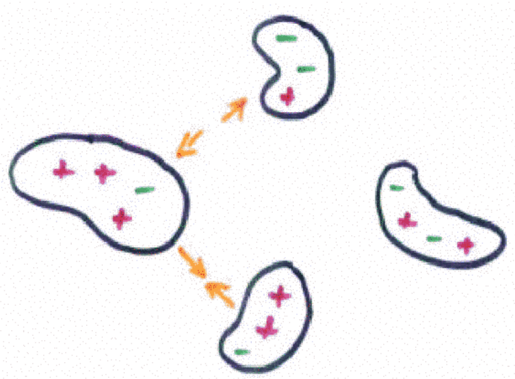
$$(1+z)_{nl,\Delta} = (1+z)_{nl,\delta_m} \cdot (2 \times 10^{-5})^{-1+1/\gamma}. \tag{15}$$

These structures are highly "charged"—mostly m_1's or else mostly m_2's.

If $\zeta_1 < 1$, then when δ_m bound structures form, they will have a small (random) charge imbalance: setting $\delta_m = 1$ in (14), get

$$\Delta = (2 \times 10^{-5})^{1-\gamma}. \tag{16}$$

Bottom-up approach to structure formation in non-linear regime will be affected by these charge imbalances: structures merge preferentially with like charged structures.



Too bad this is in non-linear regime...

Some concerns:

1. We neglected gauge interactions. But they average out and screen, so no big deal.
2. T^6 compactification is excessively naive.
3. Branes can wrap compact dimensions, too.
4. How did we produce the right number density of these super-heavy dark matter particles?
5. Shouldn't ϕ have some couplings to the visible sector? Are they a problem?
6. Shouldn't ϕ pick up a mass through quantum effects?

I find 6 particularly troubling: if SUSY prevents a mass, then after SUSY breaking, $m_\phi \sim m_{\text{gravitino}} \gtrsim 0.1\,\text{eV}$. Perhaps this is part and parcel with cosmological constant problem.

Let's expand our scope a bit:

If we grant the possibility of a very light scalar, what might its effects be in late time cosmology, particularly as motivated by considerations of string theory?

Quintessence is one answer (that is, coherent motion of ϕ leads to w between -1 and 0).

Dark matter interactions are another, and then naturalness leads us at once to super-heavy dark matter—else we need extremely small coupling.

If scalars don't roll, coupling to the visible sector can be controlled:

in $\mathcal{N}=1$, $d=4$ supergravity, scalar is part of chiral superfield Φ.

$$K(\Phi,\Phi^\dagger) = |\Phi|^2 + \ldots$$
$$gH_uQ_LU_R \to \exp\left(\frac{K}{2M_{Pl}^2}\right) gH_uQ_LU_R \tag{17}$$

Quark masses and hence proton mass the depend on the scalar:

$$m_u = \bar{m}_u + \epsilon_u \frac{\phi^2}{M_{Pl}^2} \qquad m_p = \bar{m}_p + \epsilon_p \frac{\phi^2}{M_{Pl}^2}, \tag{18}$$

where $\epsilon_u \sim m_u \sim 5\,\mathrm{MeV}$ and $\bar{m}_p$ is of similar size if QCD scale doesn't depend on ϕ (plausible). This *quadratic* dependence doesn't spoil tests of Equivalence Principle.

For instance, suppose we have some local $\langle\phi\rangle \neq 0$.

$$m_p = \bar{m}_p + 2\epsilon_p \frac{\langle\phi\rangle\delta\phi}{M_{Pl}^2}$$
$$F_s = \frac{\epsilon_p^2 \langle\phi\rangle^2 / M_{Pl}^4}{\pi r^2} \tag{19}$$
$$\xi \equiv \frac{F_s}{F_g} = 8\frac{\epsilon_p^2}{m_p^2}\frac{\langle\phi\rangle^2}{M_{Pl}^2} = 8\frac{\epsilon_p^2}{m_p^2}\langle\varphi\rangle^2 .$$

Universality of free fall would be violated by isotope dependence of F_s. Experimental bounds are something like $\xi \lesssim 10^{-12}$, which translates to $\langle\varphi\rangle \lesssim 10^{-4}$.

Similarly obtain a bound on $\langle\nabla\varphi\rangle$. Neither bound seems particularly restrictive.

This is in contrast with situation for a rolling scalar field: dimension 5 couplings, e.g. $\frac{g}{M_{Pl}}\phi H\bar{\psi}\psi$, contradict tests of Equivalence Principle. Why are these couplings absent now?

Generating a tiny mass dynamically

- If $m_\phi \gtrsim 1\,\mathrm{pc}^{-1}$, then scalar forces don't seem likely to have played a role in structure formation.
- Conservative particle physics view is to give up at this point
- Since we don't know how to break SUSY, propose to consider mass generation effects for ϕ that depends on presence of other particles.

First example:

$$\ddot{\varphi} + 2\frac{\dot{a}}{a}\dot{\varphi} - \partial_\alpha\partial_\alpha\varphi + \frac{a^2}{M_{Pl}^2}\frac{d^2V}{d\varphi^2}\varphi + \boxed{3\zeta_2\frac{\dot{a}^2}{a^2}}\varphi = -3\frac{\dot{a}^2}{a^2}\Delta \qquad (20)$$

$$m_{\text{eff}}^2 = \frac{d^2V}{d\phi^2} + 3\zeta_2 H^2$$

Second example *[Peebles-Farrar]*: consider adding a field Ψ whose mass comes *only* from $\langle\phi\rangle \neq 0$:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \bar{\Psi}i\partial\!\!\!/\Psi - y\phi\bar{\Psi}\Psi\,. \qquad (21)$$

If we have a density ρ_f of Ψ quanta with typical energy E_{typ}, then

$$e^{i\theta}y\phi\bar{\Psi}\gamma^0\Psi = E_{\text{typ}}\bar{\Psi}\Psi \qquad (22)$$

for some phase θ. But $\bar{\Psi}\gamma^0\Psi = n_f$. Scalar eom (without further fields) is thus

$$0 = \nabla^2\phi - y\bar{\Psi}\Psi \approx \nabla^2\phi - \frac{y^2 n_f}{E_{\text{typ}}}\phi\,, \qquad (23)$$

and scalar screening length is

$$r_s^2 = \frac{E_{\text{typ}}}{y^2 n_f} = \frac{E_{\text{typ}}^2}{y^2\rho_f}\,. \qquad (24)$$

This is smaller than the Hubble length:

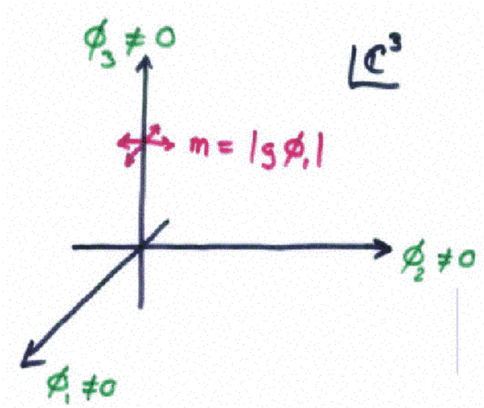
$$r_s^2\frac{\dot{a}^2}{a^4} = \frac{1}{3y^2\Omega_f}\frac{E_{\text{typ}}^2}{M_{Pl}^2}\,, \qquad (25)$$

but still huge.

This mechanism hasn't a prayer unless we prevent $m_\phi^2|\phi|^2$.

Spontaneously broken global symmetry won't do— then couplings to dark matter will be quite different from previously considered.

Unbroken supersymmetry will do nicely. Consider for example $W = g\Phi_1\Phi_2\Phi_3$: an elaboration of (21).



Key is that along flat directions, e.g. $\phi_1 = \phi_2 = 0$, masses of the Φ_1, Φ_2 quanta are $|g\phi_3|$.

This model is easily embedded in string theory: 3 intersecting D6-branes, or M-theory near a conical singularity over $SU(3)/U(1)^2$. Leaving blow-up modulus unfixed is inoffensive (except after SUSY breaking).

Some conclusions:

- Rolling scalars are a problem even *before* SUSY breaking because of dimension 5 couplings to visible sector.
- Dimension 6 couplings to visible sector need not visibly spoil Equivalence Principle tests.
- Astrophysically light scalars are unnatural after SUSY, but let's think about them anyway because they're the only force besides gravity that modifies structure formation in an interesting way.
- If $F_s/F_g \sim 1$, we may indeed get interesting deviations from CDM, but in non-linear regime.
- Super-massive dark matter (e.g. winding and momentum strings) give $F_s/F_g \sim 1$; they could stabilize moduli and provide an interesting alternative to CDM.
- Screening effect based on light fields can easily be embedded in string theory, and it allows us further freedom to tweak CDM.
- Epoch of precision cosmology is coming, so it's worth poking at every aspect of theory.