Cosmic Censorship Violation in String Theory

G.H, T. Hertog, K. Maeda

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Outline

1) What is cosmic censorship
2) Violation for some field theories in AdS
3) Violation in string theory
4) Surprise
5) Implications

The singularity theorems of G.R. do not prove black holes form.

Gravitational collapse could produce naked singularities, i.e., one visible to distant observers

Cosmic Censorship

(Penrose 1969)

Generic smooth initial data cannot form naked singularities

We are still very far from a proof of this

Attempts to violate CC in asympt. flat spacetimes

1) Use pressureless dust (but also is singular without gravity)

2) Spherically symmetric scalar field (but need to fine tune initial data)

It is much easier to violate CC in AdS:

a) Black holes are harder to form

b) Singularities are easier to form

a) Black hole in AdS

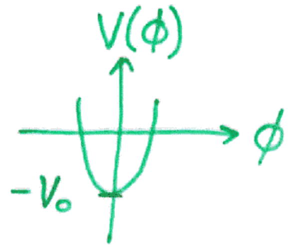
$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega$$

$$f(r) = 1 - \frac{2M}{r} + \frac{r^2}{\ell^2}$$

A large black hole of size $R_s > \ell$ requires a mass $M \propto R_s^3$

Much larger than Schwarzschild: $M \propto R_s$

b) Homogeneous scalar field in potential $V(\phi) < 0$



$$ds^2 = -dt^2 + a^2(t)\,d\sigma^2$$

($K = -1$ FRW metric)

If $\phi = 0$ initially $(t=0)$ then $\phi(t) = 0$, $a(t) \propto \cos(\sqrt{V_0}\,t)$

This is just AdS: $a = 0$ is a coordinate singularity.

If $\phi = \varepsilon \neq 0$ initially $a = 0$ is a curvature singularity

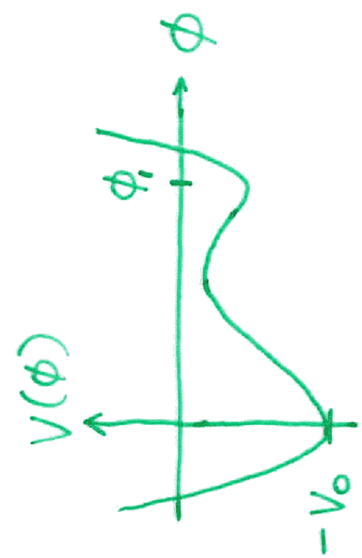
Positive energy theorem:

The total energy of all nonsingular initial data is positive and vanishes iff metric is AdS everywhere

More precise claim:

There exists $V(\phi)$ such that positive energy theorem holds but cosmic censorship is violated

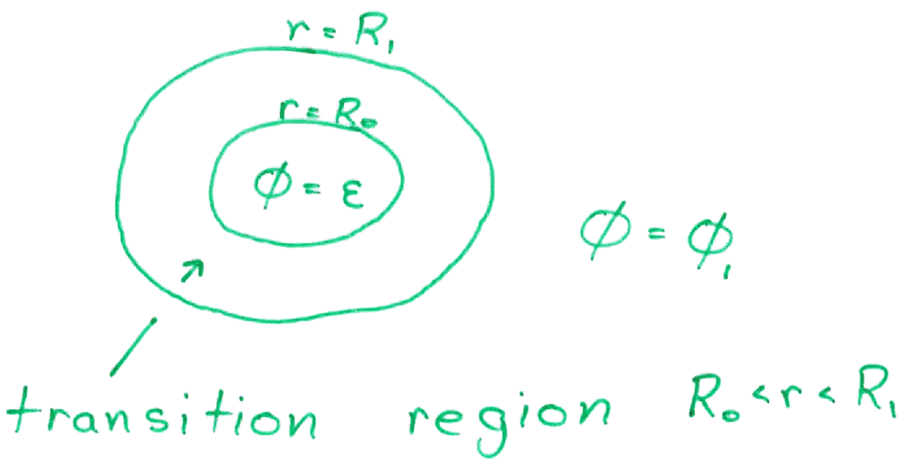
Consider



Require $\phi \to \phi_i$ asymptotically

Positive energy theorem holds if barrier is high enough

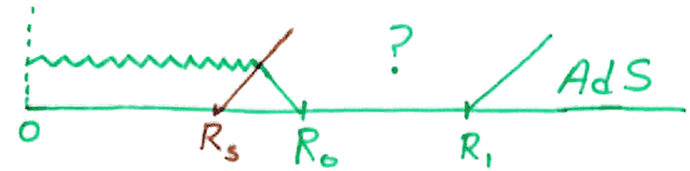
Suppose initially $\phi(r)$ is



and $\dot{\phi} = 0$. Then region $r < R_0$ becomes singular in time $T \sim V_0^{-1/2}$

Can this be hidden inside a black hole?

Evolution:



The black hole area theorem still holds even with $V < 0$, so if black hole forms

$$R_{BH} > R_s$$

Is there enough mass to form black hole of size R_s?

For certain $\phi(r)$:

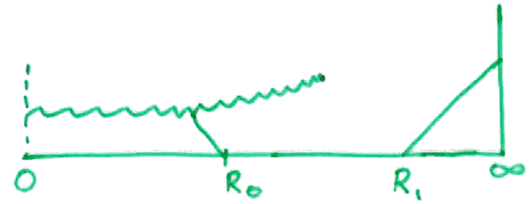
No! Not even close

In extreme case

$M \propto R_1$, but $M_{BH} \propto R_1^3$

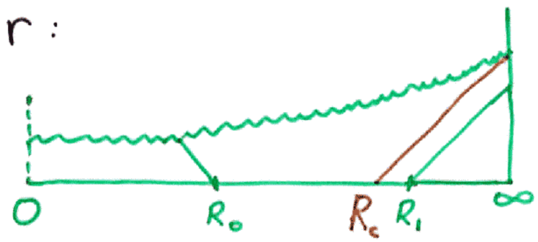
Small changes in initial data doesn't change conclusion ⇒ get open set of solutions which produce naked singularities

Either:



Endpt of singularity is naked

Or:



Much worse than a naked singularity. Can probably be ruled out.

Open Questions

1) Can we violate cosmic censorship with potentials coming from string theory?

2) Is cosmic censorship also violated for asymptotically flat spacetimes?

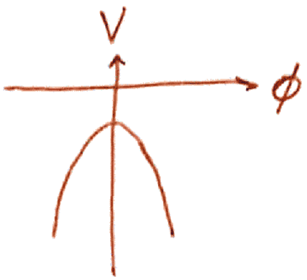
1) Yes!

Low energy limit of string theory with $AdS_5 \times S^5$ bdy conditions is $D=5$, $N=8$ SUGRA

This theory has scalars with $m^2 < 0$. Breitenlohner & Freedman showed AdS_d is stable provided

$$m^2 \geq -\frac{(d-1)^2}{4} \quad \text{(BF bound)}$$

Consider field which saturates the BF bound



Initial data:

ϕ = constant $\neq 0$ inside large sphere

$\phi \to 0$ at infinity

At what rate should $\phi \to 0$?

In AdS_5, solutions to $\nabla^2\phi - m^2\phi = 0$ with time dependence $e^{-i\omega t}$ all fall off like $\frac{1}{r^{\lambda_\pm}}$ where

$$\lambda_\pm = 2 \pm \sqrt{4+m^2}$$

BF bound, $m^2 = -4$, has $1/r^2$ and $\ln r / r^2$

So initial data is

$$\phi(r) = \frac{A}{R_0^2} \quad (r \leq R_0)$$

$$\phi(r) = \frac{A}{r^2} \quad (r > R_0)$$

The total ADM mass is negative: $M \propto -A^2$

How can this be?

AdS_5 is supersymmetric

1) What about positive energy theorem?

2) What about AdS/CFT?

Original BF argument applied to test fields in AdS. If $m^2 \geq -\frac{(d-1)^2}{4}$

then $E = \int (\ )^2 dV + \oint_\infty$

Surface term vanishes if $\phi \to 0$ faster than $1/r^2$

BF said one should use "improved" $T_{\mu\nu}$, like adding $\beta R \phi^2$ to lagrangian

Not present in $N=8$ SUGRA

If you add surface term to action:

$$S = -\int (\nabla\phi)^2 + m^2\phi^2$$

$$\to \tilde{S} = \int \phi\nabla^2\phi - m^2\phi^2$$

Then energy is always positive

Full positive energy thm

Let $\gamma^i \hat{\nabla}_i \varepsilon = 0$

Then $\underbrace{\oint_\infty \bar{\varepsilon}\hat{\nabla}\varepsilon}_{} = \int |\hat{\nabla}\varepsilon|^2 dV$

Equals ADM energy only if $\phi \to 0$ faster than $\frac{1}{r^2}$

Conclusion:

Positive energy theorem requires boundary conditions stronger than finite mass.

$M < 0$ solutions exist

A modified energy is always positive $\Rightarrow$

AdS is stable

CFT hamiltonian should be equated with this modified energy

Evolution

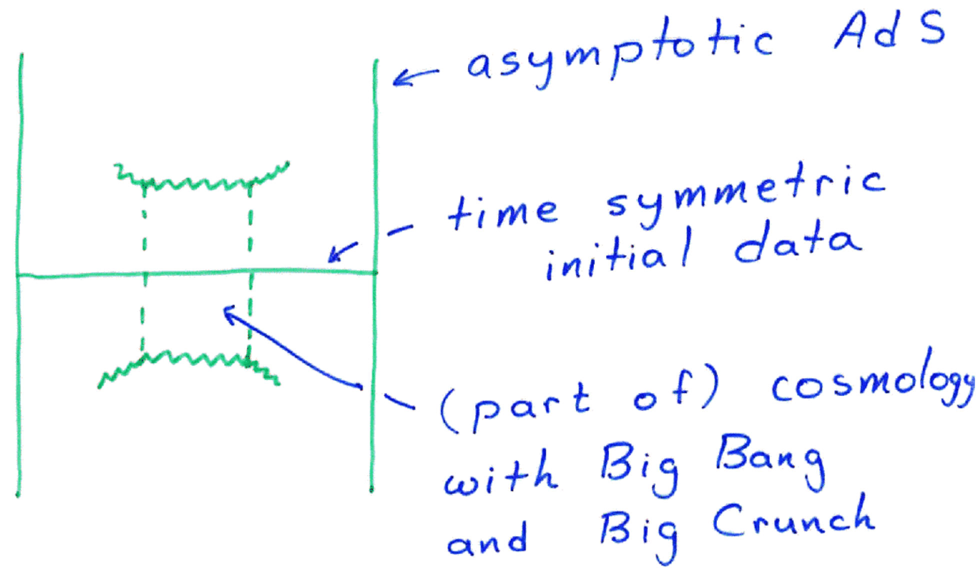
Recall $\phi(r) = \frac{A}{R_0^2} \quad (r \leq R_0)$

$= \frac{A}{r^2} \quad (r > R_0)$

Central region collapses to a singularity. This must be naked since black holes have positive M

Generic violation of cosmic censorship in string theory

Implications



What is dual SYM description?

Operators dual to scalars saturating the BF bound are

$$\mathrm{Tr}\left(X^i X^j - \tfrac{1}{6}\delta^{ij} X^2\right)$$

No reason for SYM evolution to stop.

Obtain:

1) String theory resolution of naked singularities

2) Well defined framework for studying if there is a "bounce" at cosmological singularities