BLACK HOLES IN ASYMPTOTICALLY PLANE WAVE SPACETIMES

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- hep-th/0210234
- hep-th/0211195
- hep-th/0211206

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- hep-th/0306131

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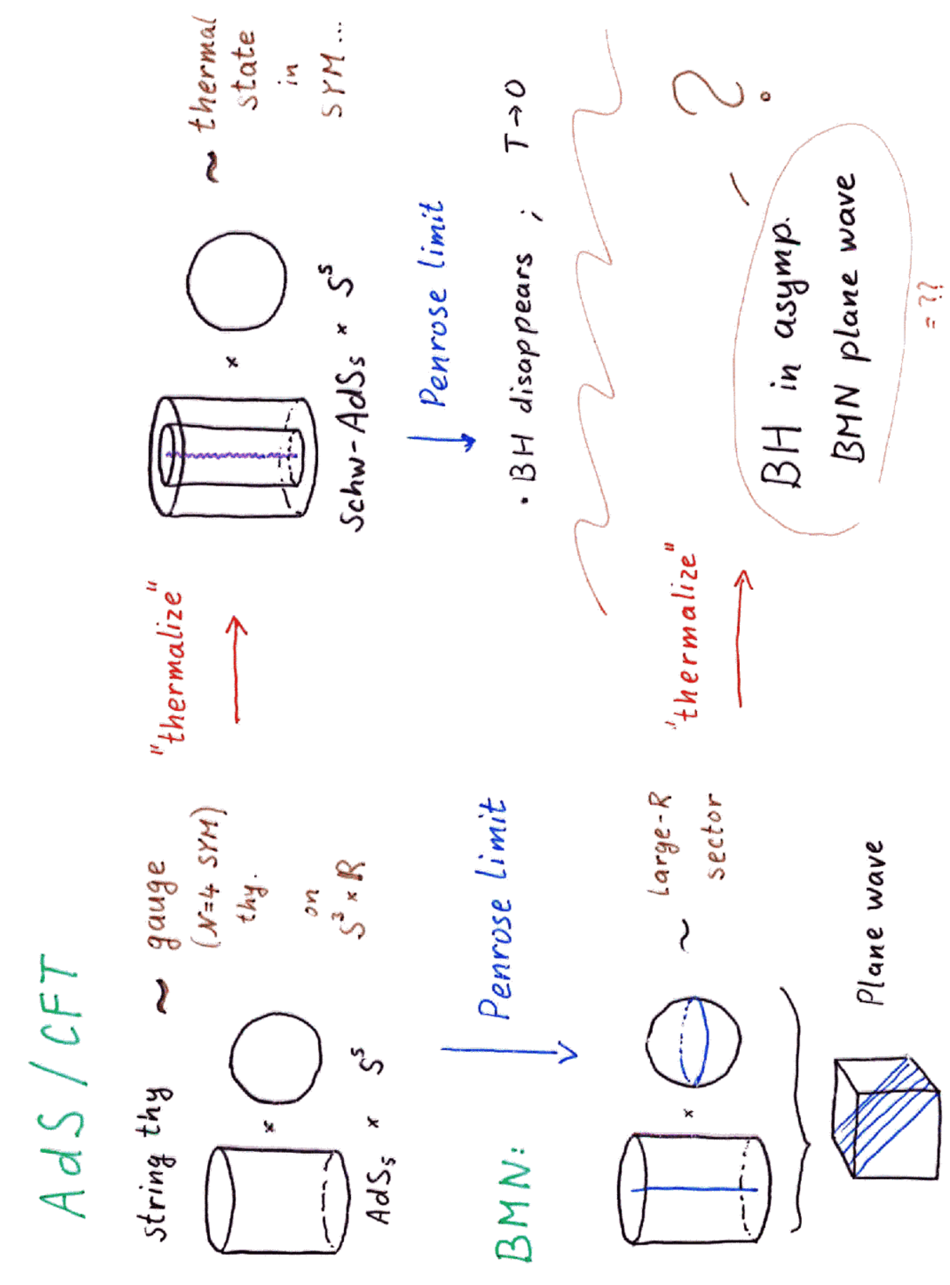
- hep-th/0307257



pp-wave

$\equiv$ spacetime w/ covariantly-constant, null Killing field

$$ds^2 = -2\,du\,dv - F(u,x^i)\,du^2 + dx^i dx^i$$

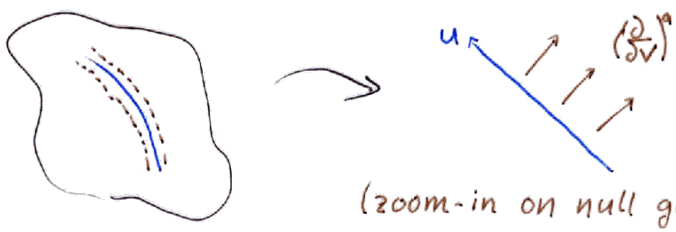
- all curvature invariants vanish

  $\Rightarrow$ exact soln. to classical string theory

plane wave

$$ds^2 = -2\,du\,dv - f_{ij}(u)\,x^i x^j\,du^2 + dx^i dx^i$$

- has extra planar symmetry
- arises as Penrose limit:



(zoom-in on null geod.)

$\Rightarrow$ encodes 1-d slice of "parent" ST

"BMN" plane wave

$f_{ij} = \mu^2 \delta_{ij}$ (maximally symmetric)

Matter content of plane waves

$$ds^2 = -2\,du\,dv - \underbrace{f_{ij}(u)\,x^i x^j}_{\text{pp-waves: } F(u,x^i)}\,du^2 + dx^i dx^i$$

$$R_{uu} = \frac{1}{2}\nabla_x^2 F(u,x^i) = \mathrm{tr}\, f_{ij}(u) \sim T_{uu}$$

- vacuum plane waves $\mathcal{V}_d$:

  $\mathrm{tr}\, f_{ij} = 0$ $\Rightarrow$ can't be spherically symmetric

- maximally symmetric plane waves $\mathcal{P}_d$:

  $f_{ij} = \mu^2 \delta_{ij}$ $\Rightarrow$ $R_{uu} = d\cdot\mu^2 \neq 0$ $\Rightarrow$ can't be vacuum...

In 10-D IIB SUGRA:

$\exists$ family of solutions w/ metric $\mathcal{P}_{10}$ supported by combination of RR 3-form + RR 5-form.

- generic member: $U(2) \times U(2)$ isom. gp.
- just RR 5-form $\leftarrow$ Penrose limit of $AdS_5 \times S^5$

  $SO(4) \times SO(4)$; maximally supersymmetric

- just RR 3-form $\xleftrightarrow{S}$ $\mathcal{P}_{10}$ w/ NS-NS B-field

  $U(4)$ isom. gp.

Overview:

Q1: Can pp-waves admit horizons?

– No: cov. const. null K.F. $\Rightarrow \nexists$ horizon (hep-th/0210234)

– confirmed by causal structure has interesting properties (hepth/0211195)

Q2: Can vacuum soln's w/ null K.F. admit horizons?

– No; can solve explicitly $\rightarrow$ new soln's (hep-th/0211206)

Q3: Can non-vac. soln's w/ null K.F. admit horizons?

– Yes; can construct from asymp. flat SUSY soln's w/ horizons

$\rightarrow$ asymp. vac. plane wave $\times$ flat (hep-th/0211206)

Q4: Can we obtain BHs w/ BMN asymptotics?

– ?; hard to construct directly

YES ... (hep-th/0306131)

$\exists$ asymp. (maximally sym) plane wave black strings

metrically BMN, but different matter support

Generalized defn. of a black hole

- in asymp. flat ST, $\mathcal{H} \equiv \partial I^{-}[\mathcal{I}^{+}]$

BH = region of spacetime invisible to asymptotic observers

$\Rightarrow$ define BH = region of ST invisible to observers who are "arbitrarily far"

- generally murky
- but makes sense in pp-waves
- trivial to get arb. far along null K.F. $\left(\frac{\partial}{\partial v}\right)^a$

Def. (non-existence of BH)

$\nexists$ horizon $\Longleftrightarrow$ $\forall\ p_0 = (u_0, v_0, x_0^i)$ and arb. large $v_\infty, x_\infty^i \to \infty$,

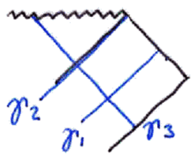
$\exists$ causal curve $\gamma: p_0 \to p_1 = (u_1, v_\infty, x_\infty^i)$

(in fact, will show: $u_1 = u_0 + \varepsilon$)

No black holes in plane waves

- can't retain horizon in taking a Penrose limit

e.g.



($\gamma_1, \gamma_2 \to$ flat ; γ_3 retains singularity)

- plane wave:

$$ds^2 = -2\,du\,dv - f_{ij}(u)\,x^i x^j\,du^2 + dx^i dx^i$$

is invariant under (const.) rescaling

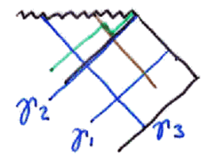
$$\begin{cases} x^i \to \lambda\, x^i \\ v \to \lambda^2\, v \end{cases}$$

$\Rightarrow$ no special $\{x^i, v\}$ to support a horizon.

No black holes in plane waves

- can't retain horizon in taking a Penrose limit

e.g.



throw in shell later

$\Rightarrow$ at γ_3 apparent horizon $\neq$ event horizon ...

($\gamma_1, \gamma_2 \to$ flat ; γ_3 retains singularity)

- plane wave:

$$ds^2 = -2\,du\,dv - f_{ij}(u)\,x^i x^j\,du^2 + dx^i dx^i$$

is invariant under (const.) rescaling

$$\begin{cases} x^i \to \lambda\, x^i \\ v \to \lambda^2\, v \end{cases}$$

$\Rightarrow$ no special $\{x^i, v\}$ to support a horizon.

No horizons in pp-waves

$$ds_{pp}^2 = -2\,du\,dv - F(u,r,\Omega)\,du^2 + dr^2 + r^2 d\Omega^2$$

(causal $\gamma \Leftrightarrow -2\dot{u}\dot{v} - F\dot{u}^2 + \dot{r}^2 + r^2\dot{\Omega}^2 \le 0$)

- can show $\nexists$ horizon by constructing explicit curve from any p_0 to arb. large v & r

e.g. $\begin{cases} \dot{\Omega} = 0 \\ \dot{r}^2 = \alpha F \\ 2\dot{v} = (\alpha-1) F \end{cases}$ $\Rightarrow ds^2 = 0$; α s.t. integrate $\dot{r}$ to large r...

- more elegant proof (simplified version) :
  - suppose $F(u,r,\Omega) \geq 0$ everywhere
  - then $ds_{pp}^2 \leq ds_{flat}^2$
  - $\Rightarrow$ causal γ in $ds_{flat}^2 \Rightarrow \gamma$ causal in ds_{pp}^2
  - flat ST sym. $\rightsquigarrow$ start from origin:

    for $\dot{\Omega} = 0$, $\Delta u = \varepsilon$; want $-2\varepsilon v + r^2 \le 0$

    $\therefore$ can pick arb. large r & let $v \geq \frac{r^2}{2\varepsilon}$

$\Rightarrow$ in ds_{pp}^2, $\exists$ causal γ reaching arb. large r & v from any pt.

(Q.E.D.)

New solutions

- globally null K.F. (+ rotational symmetry ...)

$$ds^2 = \frac{1}{H(r)}\left[-2\,du\,dv + F(r)\,du^2\right] + G(r)\left[dr^2 + r^2 d\Omega_{d-3}^2\right]$$

- can solve vacuum Einstein eq:
  - pp-waves
  - asymp. flat new solution : $(d>4)$

$$H(r) = c_1 \left(\frac{r^{d-4} - a}{r^{d-4} + a}\right)^{\sqrt{\frac{2(d-3)}{(d-2)}}}$$

$$G(r) = c_2 \left(\frac{H(r)^2}{H'(r) \cdot r^{d-3}}\right)^{\frac{2}{d-4}}$$

$$F(r) = c_3 + c_4 \ln H(r)$$

- Properties of new soln. :
  - singular ($\text{Weyl}^2 \to \infty$ as $r^{d-4} \to |a|$)
  - $\nexists$ horizon
  - asymp. flat

Garfinkle-Vachaspati construction

= solution-generating technique

(cf. hep-th/9612248)

$$(g_{\mu\nu}, \Phi) \rightsquigarrow (\bar{g}_{\mu\nu}, \bar{\Phi})$$

w/ same curvature invt's

If $\exists$ vect. field k^μ s.t.

- $k_\mu k^\mu = 0$ ← null
- $\nabla_{(\mu} k_{\nu)} = 0$ ← Killing
- $\nabla_{[\mu} k_{\nu]} = k_{[\mu} \nabla_{\nu]} S$ ← hypersurf. orthog.

  ↑ scalar fn.
- $\mathcal{L}_k \Phi = 0$ ← approp. sym for fields ...

then E. eq. is linear in deformations

$$\rightsquigarrow \quad \bar{g}_{\mu\nu} = g_{\mu\nu} + e^S \Psi \, k_\mu k_\nu$$

$\forall \; \Psi$ s.t. $k^\mu \nabla_\mu \Psi = 0$

$\nabla^2 \Psi = 0$

Asymp. plane wave vacuum solns:

- Can apply G-V construction to new soln's:

$$ds^2 = \frac{1}{H(r)} \left[-2\, du\, dv + F(r)\, du^2 \right] + G(r) \left[dr^2 + r^2 d\Omega^2 \right] + \frac{1}{H(r)} \Psi(u, r, \Omega)\, du^2$$

w/ $\nabla^2 \Psi = 0$; asymp.-flat ST harmonic fn.

$\Rightarrow$ for asymp. (vac.) plane wave,

$$\Psi \to r^2 \, Y_2(\Omega)$$

- Vacuum soln. w/ plane wave asymptotics
- breaks rotational symmetry
- still $\nexists$ horizon
- $\exists$ naked singularity (Weyl$^2 \to \infty$)

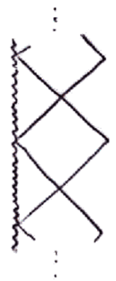
$\therefore$ Vac. E. eq. + null K.F. $\Rightarrow \nexists$ horizon.

+ spher. sym

(or ~~sph. sym~~ s.t. asymp plane wave)

G·V - deformed SUSY solns

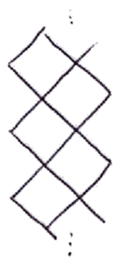
- start w/ SUSY soln $\Rightarrow$ globally null K.F.
- apply G·V to generate plane wave asymptopics :



(near - horizon) G-V :

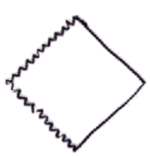
M2 : (11-d) sing. $\to AdS_4 \times S^7 \longrightarrow \mathcal{V}_{10} \times \mathbb{R}^1$

(10-d vac. plane wave)



M5 : (11-d) $AdS_7 \times S^4 \longrightarrow \mathcal{V}_7 \times \mathbb{R}^4$

D3 : (10-d) $AdS_5 \times S^5 \longrightarrow \mathcal{V}_8 \times \mathbb{R}^2$



D·p : (10-d) $p<6,\ \neq 3$ sing. $\longrightarrow \mathcal{V}_{11-p} \times \mathbb{R}^{p-1}$

- singularity / near-horizon structure preserved.

Minkowski $\longrightarrow$ plane wave

- by Garfinkle - Vachaspati :

$$ds^2 = -2\,du\,dv + dx^i dx^i$$

add a term : $\underbrace{F(u,x^i)}\,du^2$ s.t. harmonic $\Rightarrow$ vacuum pp-wave / $\mathcal{V}_d$

- by "Null Melvin Twist" :

$$ds^2 = -dt^2 + dy^2 + dr^2 + r^2 d\Omega^2$$

$$\rightsquigarrow ds^2 = -(1+\beta^2 r^2)\,dt^2 - 2\beta^2 r^2\,dt\,dy + (1-\beta^2 r^2)\,dy^2 + dr^2 + r^2 d\Omega^2$$

(change vars:)

$$= -2\,du\,dv - \beta^2 r^2\,du^2 + dr^2 + r^2 d\Omega^2$$

$\Rightarrow$ maximally symmetric plane wave $\mathcal{P}_{10}$

w/ NS-NS B-field:

$$B = \frac{\beta r^2}{2}\,(dt + dy) \wedge \sigma$$

"Null Melvin Twist"

1. start w/ SUGRA soln. w/ transl. sym. $\frac{\partial}{\partial y}$
2. boost along y by γ
3. T-dualize along y
4. twist rotation of S^7 along y by α
5. T-dualize along y
6. boost along y by $-\gamma$
7. take double-scaling limit:

$$\left.\begin{array}{l}\gamma \to \infty \\ \alpha \to 0\end{array}\right\} \text{ w/ } \beta = \tfrac{1}{2}\alpha e^{\gamma} \text{ fixed}$$

for $Schw_9 \times \mathbb{R}^1$

$$ds_{str}^2 = -\left(1-\frac{M}{r^6}\right)dt^2 + \left(1-\frac{M}{r^6}\right)^{-1}dr^2 + r^2 d\Omega_7^2 + dy^2 \qquad (IIB)$$

$\rightsquigarrow$ black string w/ momentum $P_y = M \sinh\gamma \cosh\gamma$

$\rightsquigarrow$ IIA soln. w/ fund. string charge $Q_{F1} = P_y$

$\rightarrow$ 1-form σ : $d\Omega_7^2 \to d\Omega_7^2 + \alpha\,\sigma\,dy + \alpha^2 dy^2$

$\rightsquigarrow$ IIB b.s. w/ mom. P_y (@ origin of Melvin universe)

$\rightarrow$ cancels net momentum ...

Twist

$$\sigma \to \sigma + 2\alpha\, dy$$

σ is a symmetric 1-form on S^7

$$\begin{array}{ccccccc} & & & S^7 & & & \\ & S^3 & & & & S^3 & \\ \text{from} \quad S^1 & & S^1 & & S^1 & & S^1 \end{array}$$

for $d\Omega_7^2 = d\chi^2 + \frac{1}{4}\cos^2\chi\, d\Omega_3^2 + \frac{1}{4}\sin^2\chi\, d\tilde{\Omega}_3^2$

w/ $d\Omega_3^2 = d\theta^2 + d\psi^2 + d\varphi^2 + 2\cos\theta\, d\psi\, d\varphi$

$$\sigma \equiv \cos^2\chi\,(\cos\theta\, d\psi + d\varphi) + \sin^2\chi\,(\cos\tilde{\theta}\, d\tilde{\psi} + d\tilde{\varphi})$$

$$\therefore \quad d\Omega_7^2 \to d\Omega_7^2 + \alpha\,\sigma\, dy + \alpha^2 dy^2$$

More generally:

$S^7 \to \mathbb{R}^8 : \{x^i, \; i=1,\dots,8\}$

$$(x_1 + i x_2) \to e^{i\alpha v_1 y}(x_1 + i x_2)$$
$$(x_3 + i x_4) \to e^{i\alpha v_2 y}(x_3 + i x_4)$$
$$\vdots \qquad \text{(etc.)}$$

$$\Rightarrow \frac{r^2 \sigma_v}{2} = v_1(x_1\, dx_2 - x_2\, dx_1) + v_2(x_3\, dx_4 - x_4\, dx_3) + \dots$$

"Plane wave black string"

- start w/ $Schw_9 \times \mathbb{R}^1$

$$ds_{str}^2 = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_7^2 + dy^2$$
$$\phi = 0, \quad B = 0.$$

- perform Null Melvin Twist:

$$\boxed{\begin{aligned} ds_{str}^2 = &- \frac{f(r)\,(1+\beta^2 r^2)}{k(r)}\, dt^2 - \frac{\beta^2 r^2 f(r)}{k(r)} \cdot 2\, dt\, dy \\ &+ \left(1 - \frac{\beta^2 r^2}{k(r)}\right) dy^2 + \frac{dr^2}{f(r)} \\ &+ r^2 d\Omega_7^2 - \frac{\beta^2 r^4 (1-f(r))}{k(r)} \cdot \frac{\sigma^2}{4} \end{aligned}}$$

$$e^\phi = \frac{1}{\sqrt{k(r)}}, \qquad B = \frac{\beta r^2}{2k(r)}\,(f(r)\, dt + dy) \wedge \sigma$$

w/ $f(r) = 1 - \frac{M}{r^6}$, $\quad k(r) = 1 + \frac{\beta^2 M}{r^4}$

Properties of PWBS:

Limits:

- $\beta = 0 \Rightarrow k = 1 \Rightarrow$ black string
- $M = 0 \Rightarrow f = k = 1 \Rightarrow$ plane wave $\mathcal{P}_{10}$

for general $M > 0$, $\beta > 0$:

- as $r \to \infty \Rightarrow f, k \to 1 \rightarrow$ plane wave $\mathcal{P}_{10}$
- $\exists$ a regular horizon at $r_+ = M^{1/6}$ $(\forall \beta)$

$\Rightarrow$ asymptotically (in transverse directions)

$\mathcal{P}_{10}$ plane wave black string

↳ but not BMN matter...

- $\exists$ curvature singularity at $r = 0$

  (causally: spacelike)

- PWBS spacetime is stably causal

  (non-compact y)

? : Causal structure ...

PWBS Thermodynamics

- Entropy:

  $S \sim A \rightarrow$ preserved under Null Melvin Twist

  $$\frac{A_H}{L} = M^{7/6}\,\Omega_7 \quad \rightarrow \text{indep. of } \beta$$

- Temperature: (normalization ambig.)

  using $\kappa^2 = -\frac{1}{2}(\nabla^a \xi^b)(\nabla_a \xi^b)$ w/ $\xi^a = \left(\frac{\partial}{\partial t}\right)^a$

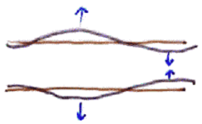
  $$T_H = \frac{3}{2\pi} M^{-1/6} \quad \rightarrow \text{indep. of } \beta$$

~ Energy: M ~ "mass" of black string in asymp. plane wave geometry ...

PW analog of ADM ?

$\Rightarrow$ PWBS thermodynamics is indep. of β !

Puzzle: is the PW black string Gregory-Laflamme unstable?



Generalizations

- start w/ different transl. inv. black strings;
  Null Melvin Twist generates plane wave asymp.
  but preserves nature of soln...
  - → rotating PWBS
  - → charged PWBS
  - → PWBS w/ general twists

⤳ 13-parameter family of solutions w/ horizons + $\mathcal{P}_{10}$ asymptopia

- generate asymp. $\mathcal{P}_d$ black strings ...
- compactify y ⟶ Gödel black holes
- topologically nontrivial soln's ...

Rotating PWBS:

- start w/ rotating BH $\times \mathbb{R}^1$ in IIB SUGRA:

$$ds^2_{str} = -dt^2 + (1-f(r))\left(dt + \frac{\ell}{2}\sigma\right)^2 + dy^2 + \frac{dr^2}{h(r)} + r^2 d\Omega_7^2$$

- perform Null Melvin Twist:

$$ds^2_{str} = -\frac{f(r) + \beta^2 r^2 h(r)}{k(r)}\, dt^2 - \frac{\beta^2 r^2 h(r)}{k(r)} \cdot 2\, dt\, dy + \left(\frac{1-\beta^2 r^2 h(r)}{k(r)}\right) dy^2 + \frac{dr^2}{h(r)} + r^2 d\Omega_7^2 - \frac{M}{r^2 k(r)}\left(\beta^2 - \frac{\ell^2}{r^4}\right)\frac{\sigma^2}{4} - \frac{M\ell}{r^6 k(r)}\,\sigma\, dt$$

$$e^{\Phi} = \frac{1}{\sqrt{k(r)}} \;; \quad B = \frac{\beta r^2}{2 k(r)}\left(h(r)\, dt\wedge\sigma + \left(1 + \frac{M\ell^2}{r^8}\right) dy\wedge\sigma + \frac{2M\ell}{r^8}\, dt\wedge dy\right)$$

w/ $f(r) = 1 - \frac{M}{r^6}$, $k(r) = 1 + \frac{M\beta^2}{r^4}$,

+ $h(r) = 1 - \frac{M}{r^6} + \frac{M\ell^2}{r^8}$

PWBS in other dimensions

Apply Null Melvin Twist to black p-brane

= effectively $d = 11-p$ - dim. black string

$$ds_{str}^2 = -f_d(r)\, dt^2 + \frac{dr^2}{f_d(r)} + r^2 d\Omega_{d-3}^2 + \sum_{i=1}^{10-d} dz_i^2$$

w/ $f_d(r) = 1 - \frac{M}{r^{d-4}}$

N.M.T. $\rightsquigarrow$ $\mathcal{P}_d$ asymptopia:

$$ds_{str}^2 = -\frac{f_d(r)\,(1+\beta^2 r^2)}{k_d(r)}\, dt^2 - \frac{\beta^2 r^2 f_d(r)}{k_d(r)} \cdot 2\, dt\, dy + \left(1 - \frac{\beta^2 r^2}{k_d(r)}\right) dy^2 + \frac{dr^2}{f_d(r)} + r^2 d\Omega_{d-3}^2 - \frac{r^4 \beta^2 (1 - f_d(r))}{k_d(r)} \frac{\sigma_d^2}{4} + \sum_{i=1}^{10-d} dz_i^2$$

w/ $k_d(r) = 1 + \frac{\beta^2 M}{r^{d-6}}$

$\Rightarrow$ $d=6$: special ... (not quite $\mathcal{P}_6$ asymptotics)

SUMMARY

- covariantly const. null Killing field (pp-waves)
  - $\Rightarrow$ $\nexists$ horizons
- globally null K.F. + vacuum GR :
  - $\Rightarrow$ $\nexists$ horizons
- globally null K.F. + matter (SUSY)
  - $\Rightarrow$ $\exists$ horizons (extremal BHs)
  - Garfinkle-Vachaspati $\rightsquigarrow$ $\mathcal{V}_d \times \mathbb{R}^n$
  - (asymp. vacuum plane wave ...)
- break null symmetry
  - $\Rightarrow$ $\exists$ horizons
  - Null Melvin Twist $\rightsquigarrow$ $\mathcal{P}_{10}$ "PWBS"
  - (maximally symmetric plane wave)
  - - but not quite BMN asymptotics ...

Future directions

- Find plane wave black strings w/ genuine BMN asymptotics
  ↳ i.e. supported by $F_{(5)}$
- Find plane wave black holes
- Proceed w/ building BMN dictionary...
  - translate into gauge thy. side
  - use to unravel QG ...
- Understand causal structure of PWBS solutions
  ↳ role of causal structure in string theory