Running Spectral Index, Inflation & Gravity Waves — an astrophysicist's perspective

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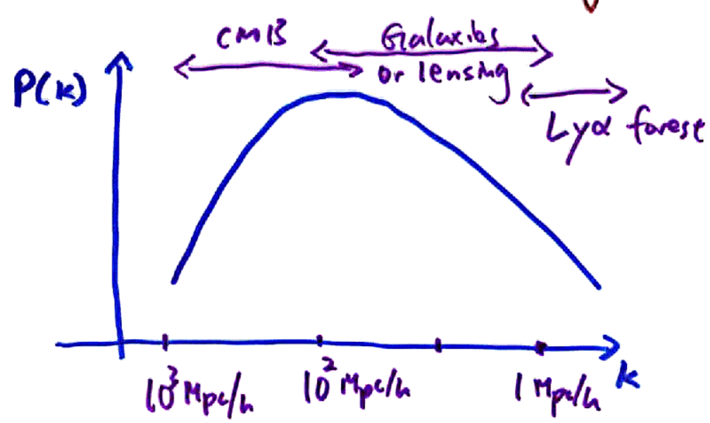
Outline

- Is there evidence for a running spectral index?
  - a discussion on the Lyman-alpha forest.
- Is $\lambda\phi^4$ inflation potential ruled out?
  - a discussion on the number of efolds.
- Gravity waves

Useful things to keep in mind:

1. CMB is not the whole story!



To measure P(k) accurately, A large range in scales is naturally useful.

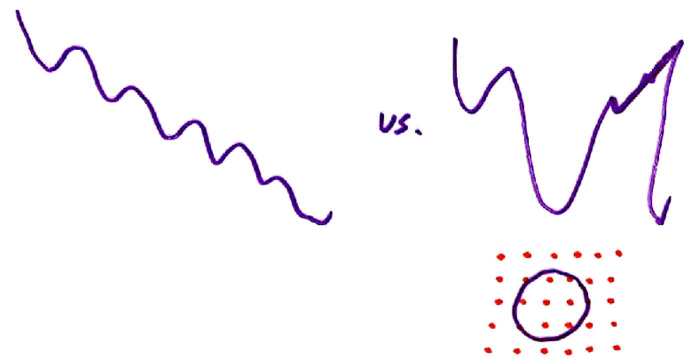
note: 1 pc = 3 light yr

$10^3 \sim e^7$

Useful things to keep in mind:

2. Your favorite theory might have large scale structure implications.

e.g. A.



B. Proposals of modifying gravity on large scales

c.f. Sean's talk

Running or not?

Primordial $P(k) \propto k^{n_s - 1}$

$$(n_s(k)-1) = (n_s(k_0)-1) + \frac{dn_s}{d\ln k}\ln\left(\frac{k}{k_0}\right)$$

From WMAP (Spergel et al.):

$k_0 = 0.05 \text{ Mpc}^{-1}$

	n_s	$dn_s/d\ln k$
WMAP	0.93 ± 0.07	-0.047 ± 0.04
WMAP+ & 2dF galaxies	$0.93^{+0.04}_{-0.05}$	$-0.031^{+0.023}_{-0.025}$
WMAP+ & 2dF & Lyα forest	0.93 ± 0.03	$-0.031^{+0.016}_{-0.017}$

Cautionary note:

- When running is large, the whole expansion is suspect:

  i.e. one would like: Liddle & Leach.

$$n_s(k_0) - 1 \gg \frac{dn_s}{d\ln k}\ln\frac{k}{k_0} \quad \& \text{ so on}$$

$0.07 \qquad -0.031? \quad 7$

- Or see it this way:

$$n_s - 1 \sim O\left(\left(\frac{V'}{V}\right)^2, \frac{V''}{V}\right) \qquad M_{Pl} = 1$$

$$\frac{dn_s}{d\ln k} \sim O\left(\frac{V'}{V} \times \frac{V'''}{V}\right)$$

<u>usual</u> assumption: $\frac{dn_s}{d\ln k} \sim (n_s - 1)^2$

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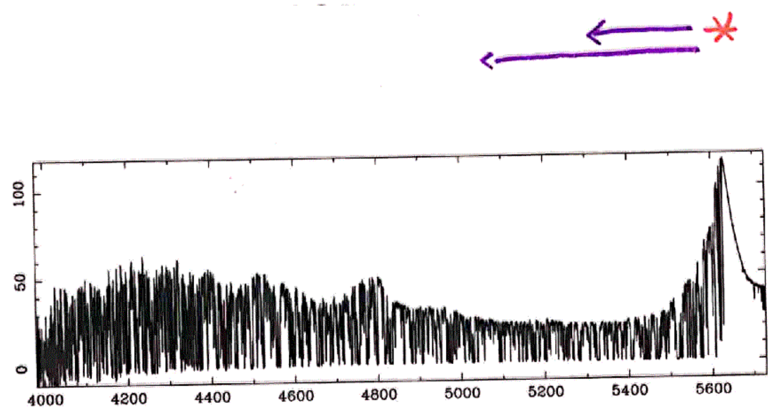
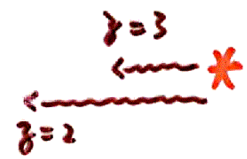


Fig. 1.— High resolution (FWHM ≈ 6.6 kms^{-1}) spectrum of the z_{em} = 3.62 QSO 1422+23 (V = 16.5), taken with the Keck HIRES (signal-to-noise ratio ~ 150 per resolution element, exposure time 25000 s). Data from Womble et al (1996).

Rauch & Sargent

Lyman α absorption Physics

- $n=1 \longrightarrow n=2$ transition
  1216 Å



- $d\tau = n_{HI}\, \sigma_\alpha\, dl$

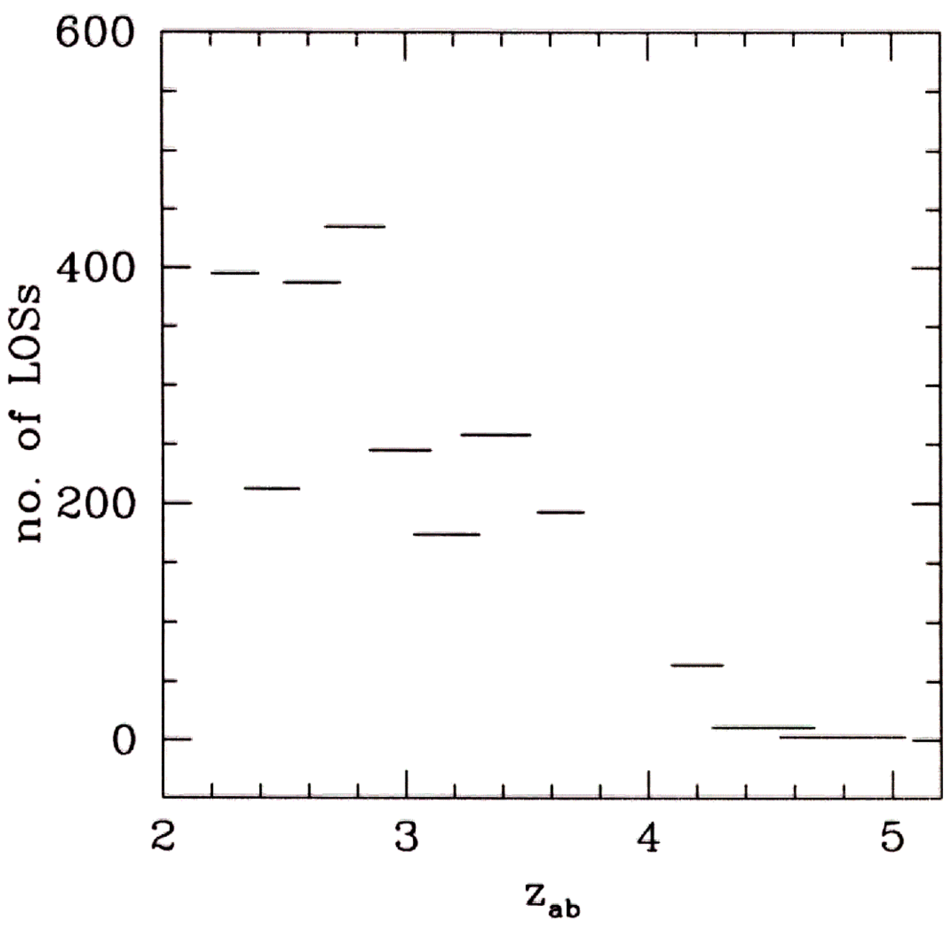
  $e^{-\tau}$ = probability of transmission
  ∝ observed flux

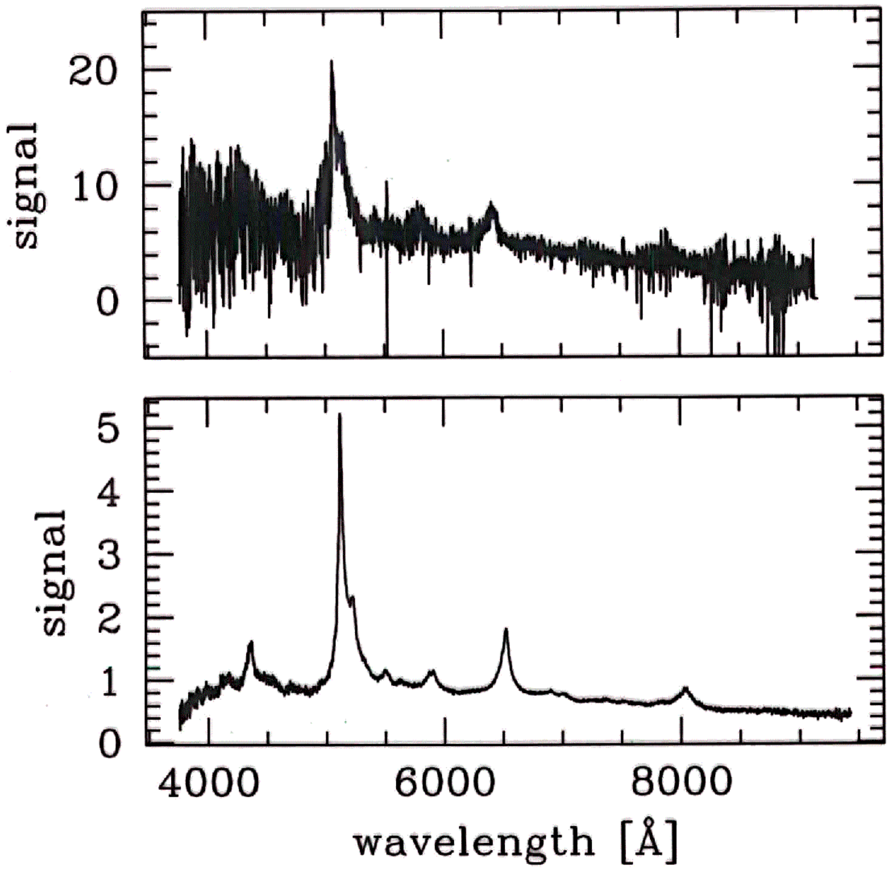
- $\tau \sim \rho^2$

- The program:
  1. measure $P(k)$ of $e^{-\tau}$
  2. infer $P(k)$ of ρ.

Analysis of Lyman α forest from SDSS

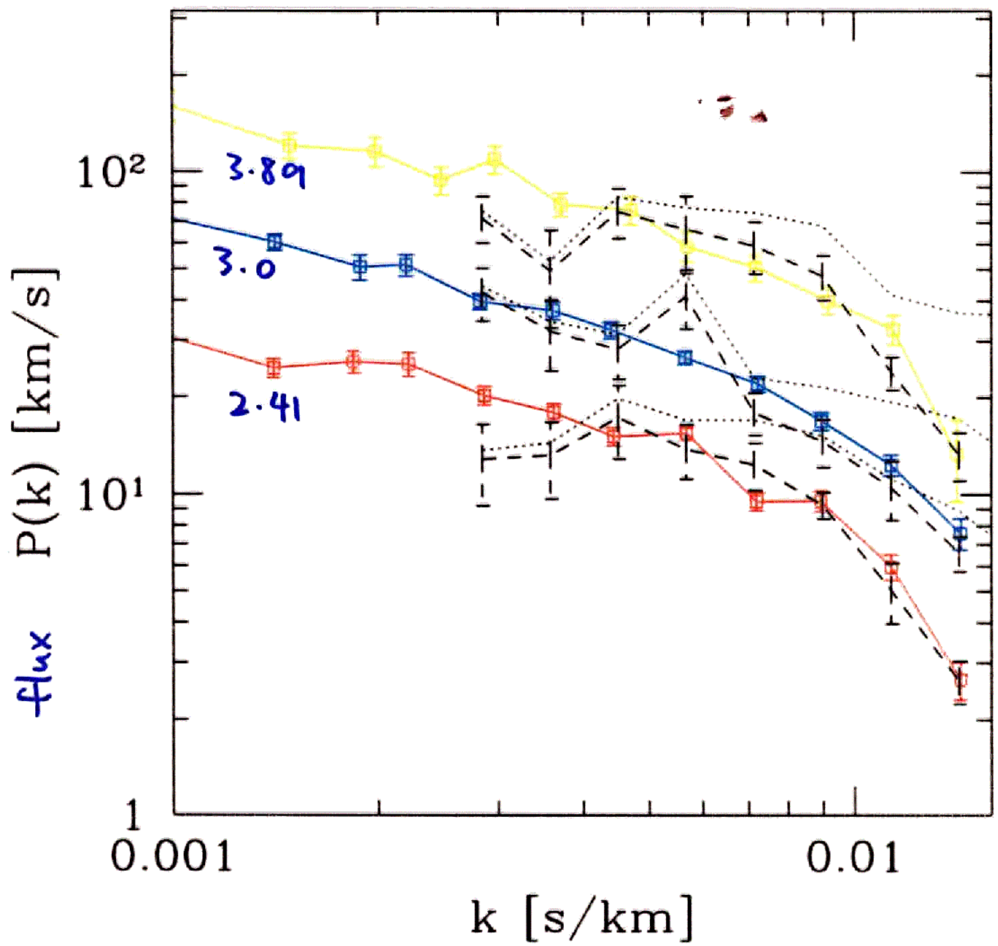
Abazajian
Bernardi
Burles
Cen
Dodelson
Frieman
Hui
Lidz
• McDonald
Schlegel
• Seljak
Stebbins
Subarno
Weinberg
York





0355-51788-050

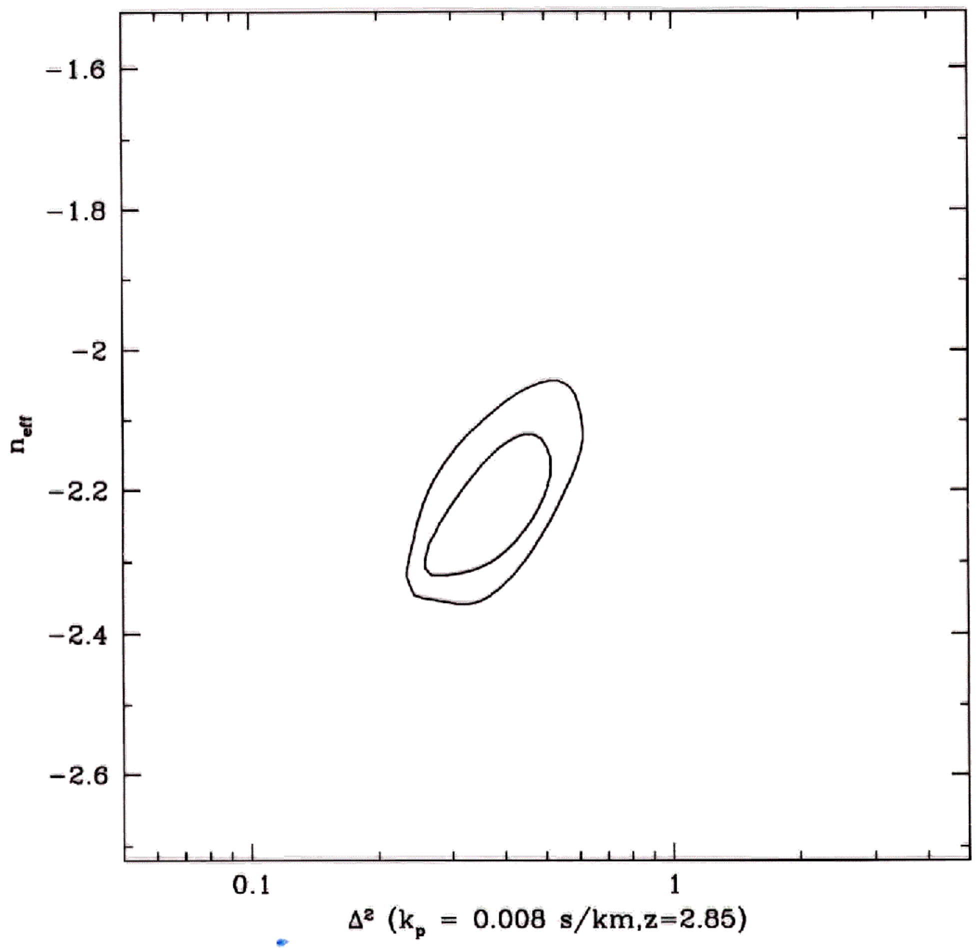
composite at z=3



SDSS vs. Keck

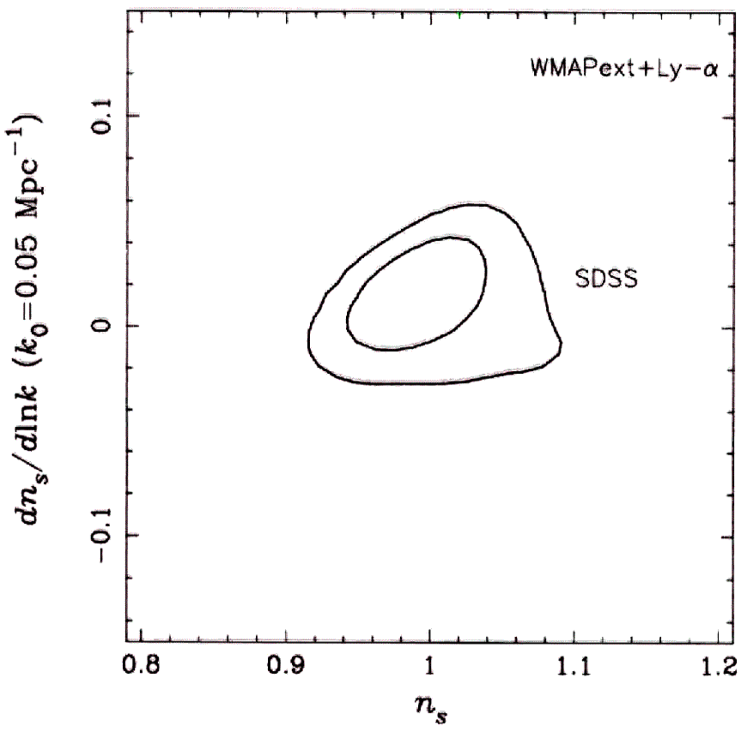
(McDonald et al.)

$$P_{1D}(k_z) = \int P_{3D}(k_z, \vec{k}_\perp) \frac{d^2 k_\perp}{(2\pi)^2}$$



$$\Delta^2 = \frac{4\pi k^3 P}{(2\pi)^3}$$

P = mass power spectrum (linear)

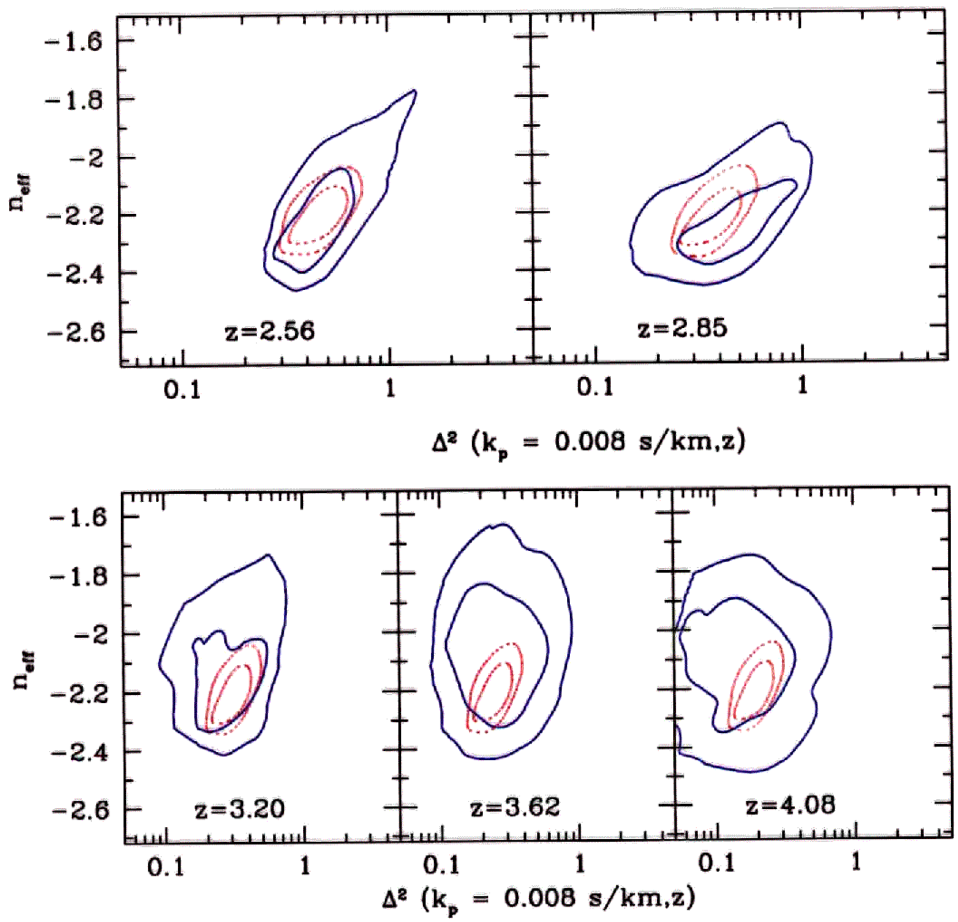


Abazajian, Dodelson et al.

Complications to keep in mind.

- The program of $P_{flux} \longrightarrow P_{mass}$ involves several "nuissance" parameters. e.g. T, α, J
- The ionizing background might introduce additional fluctuations.

  - need for consistency checks
    e.g. - linear growth
    - hierachy of correlations
    $\xi_3 \sim \xi_2^2$
    $\xi_N \sim \xi_2^{N-1}$



Lidz et al.

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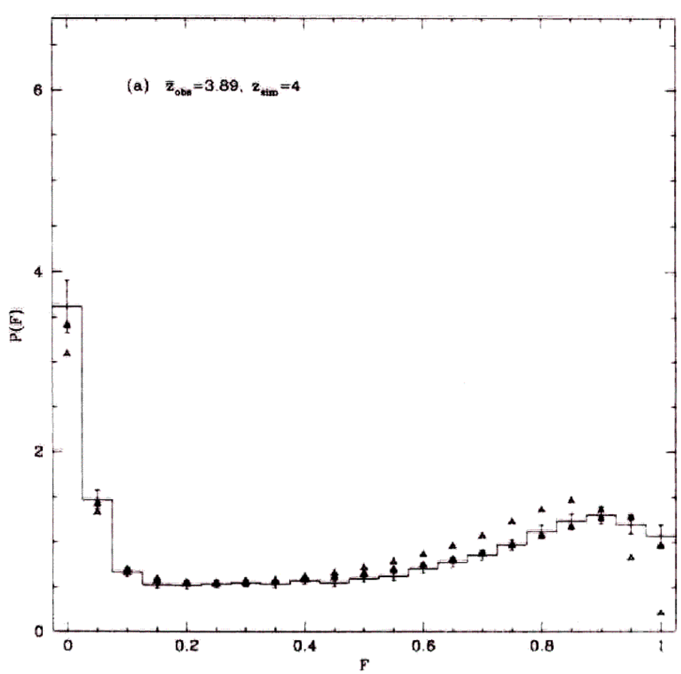


Fig. 4.— The PDF of F for the observations (*histograms*) and for the simulation with the continuum fitting approximation (*filled points*) and without it (*open points*). The small number of points outside the displayed range of F are included in the outermost bins. Errors bars were generated by bootstrap resampling. The numerical simulation has $\bar{F}$ fixed to agree with the observations. (a) shows $\bar{z} = 3.89$, (b) shows $\bar{z} = 3.00$, and (c) shows $\bar{z} = 2.41$.

McDonald et al. 99

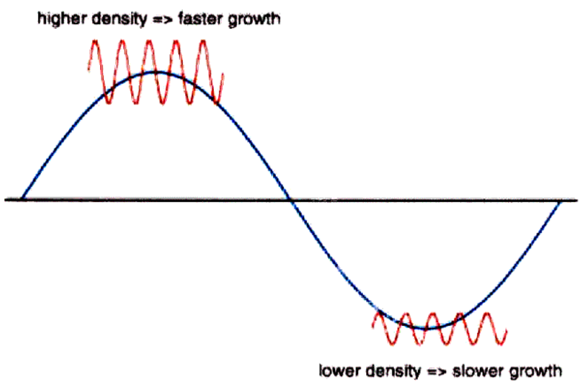


Fig. 1.— The fluctuating density field has both long and short wavelength modes. The short wavelength modes in an overdense region caused by a long wavelength mode effectively evolve as if they are in a universe with a higher mean density, hence they evolve faster. The oposite is true for short wavelength modes in an underdense region. This effect creates a correlation between the small scale power and the large scale density fluctuations.

Zaldarriaga. Seljak. LH

Is $\lambda\phi^4$ inflation potential ruled out?
(chaotic inflation)

- Peiris et al. ruled out ϕ^4 from WMAP data by setting $N=50$ ($k=0.002\ \mathrm{Mpc}^{-1}$)
- Barger et al. found that ϕ^4 can be made consistent with data by taking N large enough. (See also Kinney et al.)

Some backgd.

- Scalar $P \sim H^2/\epsilon$ $\epsilon \sim \left(\frac{V'}{V}\right)^2$

  Tensor $P \sim H^2$

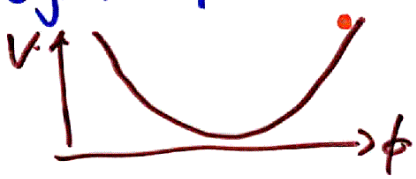
  $n_s - 1 \sim O(\epsilon, \frac{V''}{V})$

  $n_T \sim \epsilon$

  Tensor/Scalar $\equiv r \sim \epsilon$

- Observations tell us $n_s - 1$ & r are small.

- e.g. for ϕ^4 chaotic inflation:



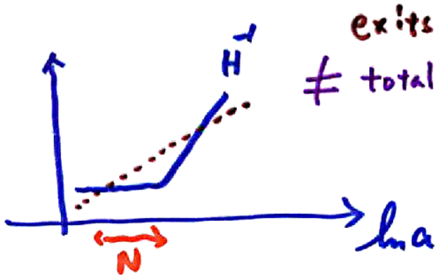
$\epsilon \sim \frac{1}{\phi^2}$

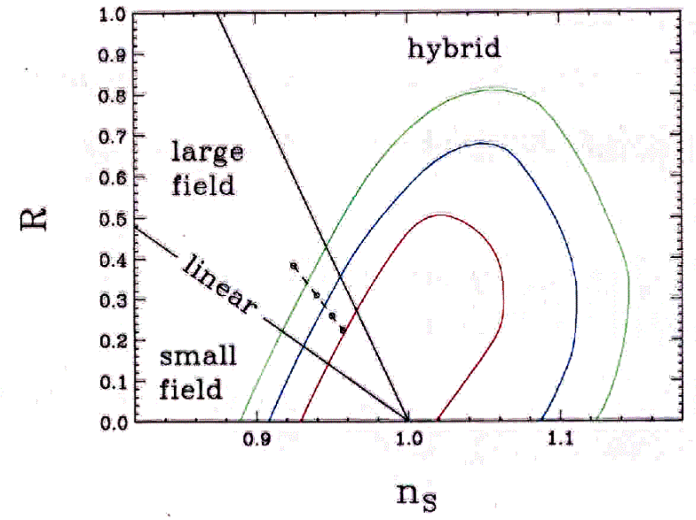
$\uparrow \phi \Longleftrightarrow \downarrow (n_s - 1)$ & r.

- Alternatively, think of $n_s - 1 = -3/N$, $r = 1/N$

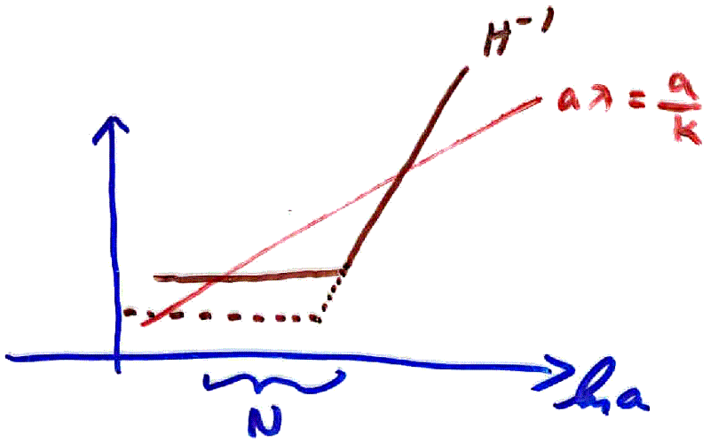
  where N = # of efolds before end of inflation at which mode exits horizon (for $k = 0.002\ \mathrm{Mpc}^{-1}$, say)

  $\neq$ total # of efolds of inflation.





Barger, Lee, Marfatia 03



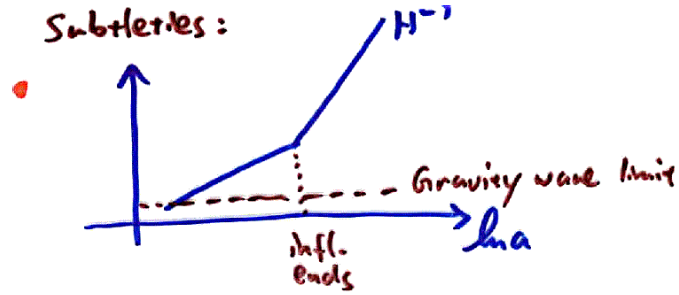
Limits on gravity wave ($\sim \frac{H}{M_{pl.}}$)
$H < 3\times10^{14}$ GeV

$\Rightarrow \quad N \lesssim 60$

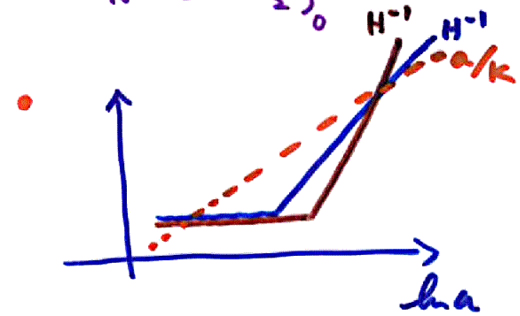
(for ϕ^4: $N < 62$)

Liddle & Leach 03
Dodelson, LH 03

Subtleties:

- 

$$N < 60 + \frac{1}{2}\int_0^N \epsilon(N')\, dN' + \ln\left(\frac{0.002\ \mathrm{Mpc}^{-1}}{k}\right)$$

- 

Bound weakens if ρ redshifts faster than radiation after inflation.

- Complications in reheating, change in d.o.f. late entropy production, etc strengthens the bound.

Interesting thought: we have some observational handle on the 'desert' between nucleosynthesis & inflation.

Upshot for ϕ^4:

Barger et al.: N has to > 60 to be consistent with data. (35)

Kinney et al.: N has to be > 66 to be consistent (35)

i.e. some tension but perhaps not firmly ruled out.

Lesson: Data are now good enough for us to care about N of a few.

Actually, the real lesson:

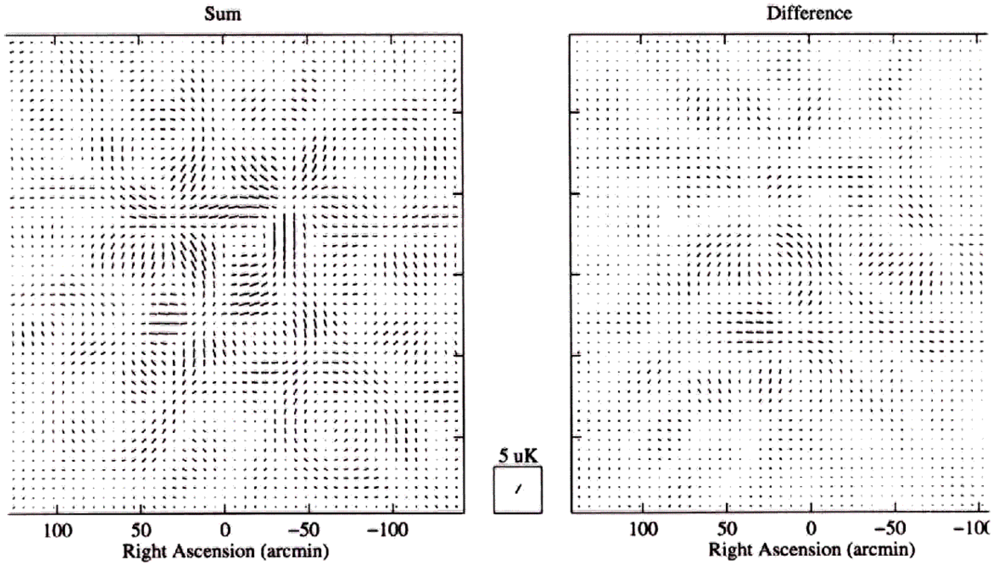
we need better models to rule out (or confirm)!

useful things to keep in mind:

3. Gravity waves are more robustly predicted. (H/M_{pl})
what is the energy scale of your favorite model?

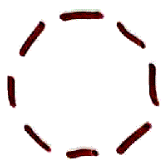
In particular, standard models predict
$$\text{tensor/scalar} = -\frac{n_T}{2}$$
known as the consistency relation.
(see review of Copeland et al.)

Short distance physics might break it.
Brandenberger & Martin
Niemeyer & Kempf
Danielsson
Easther, Greene, Kinney, Shiu
Kaloper, Kleban, Lawrence, Shenker
LH, Kinney.



DASI 2002

Work in progress on
Detection of gravity waves.
(with Jun Zhang)

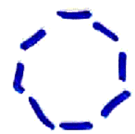


"gradient"
E mode
scalar fluc.

"curl"
B mode
vector or tensor fluc.

Kamionkowski, Kosowsky, Stebbins
Zaldarriaga, Seljak.

Gravitational lensing takes a polarization "vector" & displaces it stochastically.

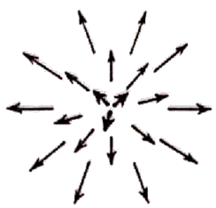
i.e.  pure E ⟶ mixture of E & B modes. 

Lensing introduces a fundamental limit to detection of gravity waves:

$E_{infl.} > 3 \times 10^{15}$ GeV

Knox & Song
Cooray et al.

- Singular Core
- Defect Lines Can't End
- Nematic is Greek for Thread



Also Hedgehog Point Defect: Rare

- The Blue Phases: Introduction
- What Are Liquid Crystals?
- Order Parameter Fields
- Defects in Nematic Liquids
- Homotopy Theory and Defects
- Parallel Transport and Energy
- Twisted Parallelism: Chiral Nematics
- Chiral Nematic Phases: Helical and Blue
- Blue Phases: Networks of Defect Lines
- Defect Lines and Frustration: Curved Space

This research was paid for by **THE US GOVERNMENT** by the NSF.

J. Sethna 95

Topological charges in a polarization/shear map

$-\frac{1}{2}$

$\frac{1}{2}$

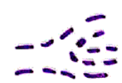
c.f. Thumb:

Charge $= \frac{1}{2\pi}$ (net rotation of polar as you traverse a closed loop counterclockwise)

Incremental counter-clockwise rot. > 0
" clockwise " < 0

Can combine: 

$\frac{1}{2} + \frac{1}{2} = 1$

Can cancel: 

$-\frac{1}{2} + \frac{1}{2} = 0$

Dolgov et al. 99
Vachaspati & Lue 03
LH Zhang 03

- # density of charges tells us about $C^E + C^B$ prelensing

$$\sim 0.2 \, \frac{\int d^2\ell \, \ell^2 C_\ell^{E+B}}{\int d^2\ell \, C_\ell^{E+B}}$$

- If over-sample, |charges| > $\frac{1}{2}$ unlikely

- Information is limited ∵
  Smoothing & lensing don't commute
  (except when $\ell_{smooth} \gtrsim 1000$)

- An experiment with $f_{sky} \sim 0.8$,
  $\Delta_p \sim 30$ μK-arcmin.
  can measure # to 10^{-3}.

- Counting charges is related to genus.
  But genus doesn't add info. unless
  $\Delta T / \Delta p$ non-Gaussian.

- More generally, one-pt. moments
  $\langle Q^n U^m \rangle$ invariant under lensing.