R. Kallosh

Review of

some recent attempts to understand

Inflation in String Theory

I Early Universe Inflation

* $\ddot{a} > 0$, $\dot{a} > 0$, 10^{-35} sec

** Number of e-foldings $N \geq 60$

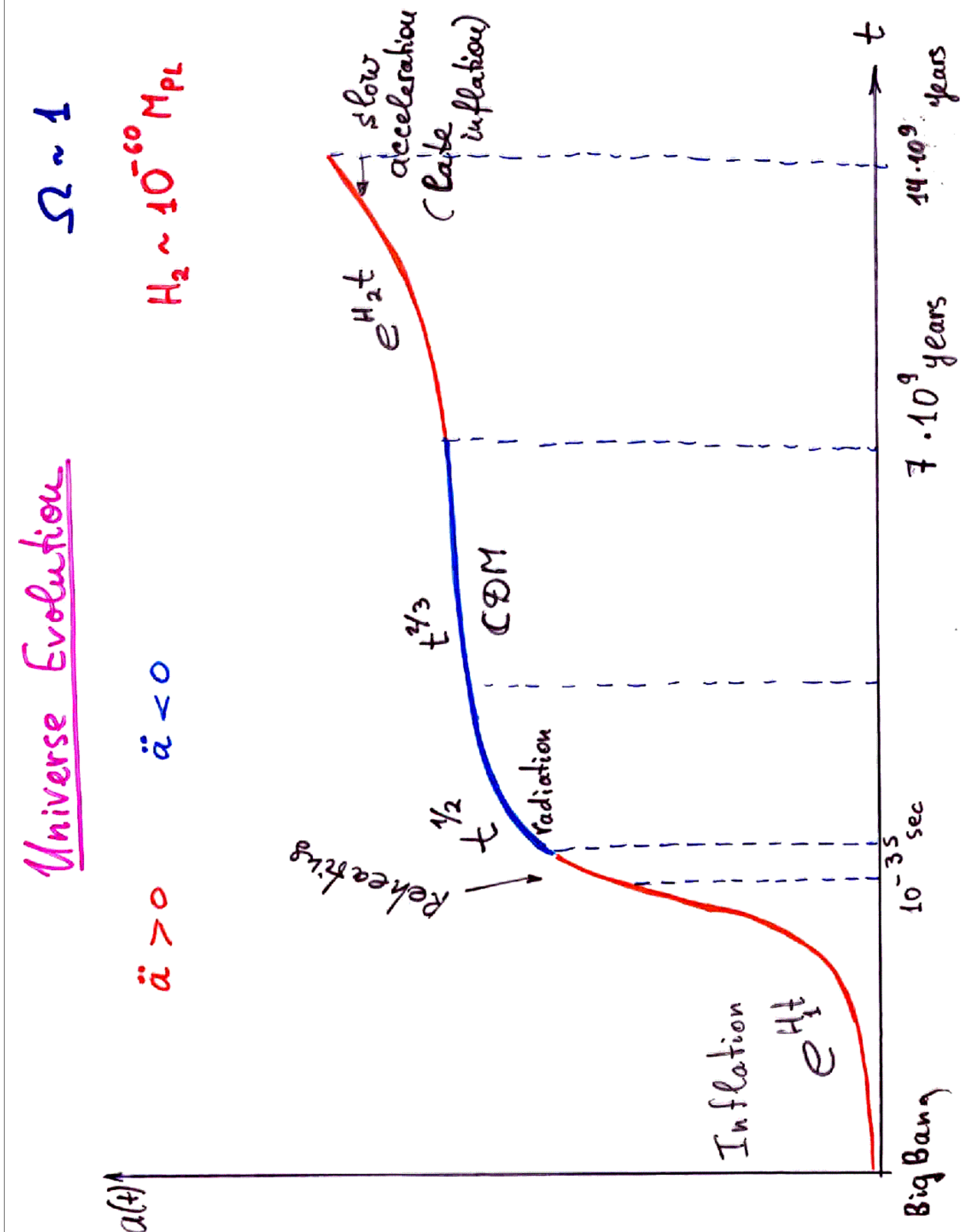
*** Exit to decelleration $\ddot{a} < 0$, $\dot{a} > 0$

II Late time inflation (accelerating universe)

* $\ddot{a} > 0$, $\dot{a} > 0$, 7-14 billion years

** Number of e-foldings $N \sim 1$ so far

*** Exit? Future of the universe?



Outline

1) KKLT model of dS space + landscape of string theory

2) Warm up: inflation in N=1 d=4 supergravity
F-term, D-term, D-term
D-branes

3) Issues raised in KKLMMT paper
on inflation in string theory

4) Brane inflation models

D-term inflation ≠ D-brane inflation
with some exceptions

de Sitter space and inflation in string theory

No-go theorems: Maldacena & Nunez 2001
Gibbons 1985

Problem: In terms of canonical scalars (moduli) representing dilaton and volume of the compactified space

$$V = e^{\sqrt{2}\,\overset{\text{dilaton}}{\varphi} - \sqrt{6}\,\overset{\text{volume}}{\rho}}\ \tilde{V}(\varphi, \rho) \qquad \text{type IIB}$$

Exponents are too steep, runaway behavior

$$\varphi \to -\infty$$

$$\rho \to +\infty$$

Interpretation: Decompactification

$$4D \Rightarrow 10D$$

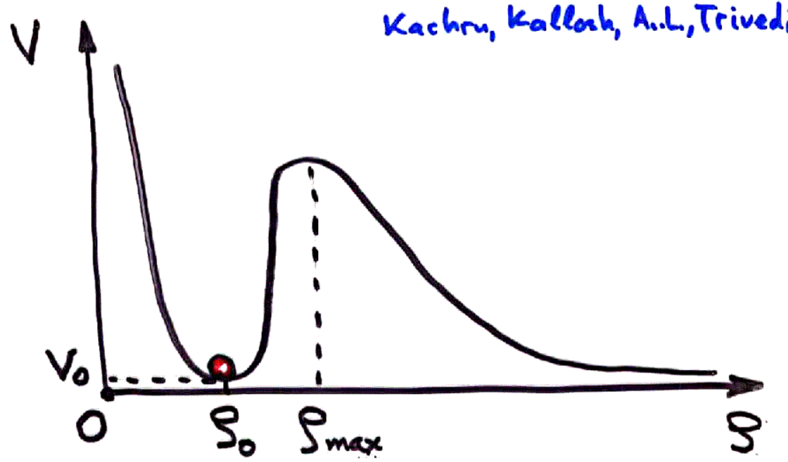
Possible solution

Kachru, Kallosh, A.L., Trivedi
hep-th/0301240

Use IIB compactifications with nontrivial NS and RR fluxes and calculate nonperturbative contributions to the moduli potential in presence of $\overline{D3}$ branes

Result: Dilaton is fixed
Giddings, Kachru, Polchinski 2002

and volume modulus is fixed as well:
Kachru, Kallosh, A.L., Trivedi



dS space in noncritical string theory
Maloney, Silverstein, Strominger hep-th/0205316

KKLT model

IIB string theory → no scale SUGRA

no-scale must be broken to fix the volume, $\rho+\bar{\rho}$

$$V = e^K\left(|DW|^2 - 3|W|^2\right)$$

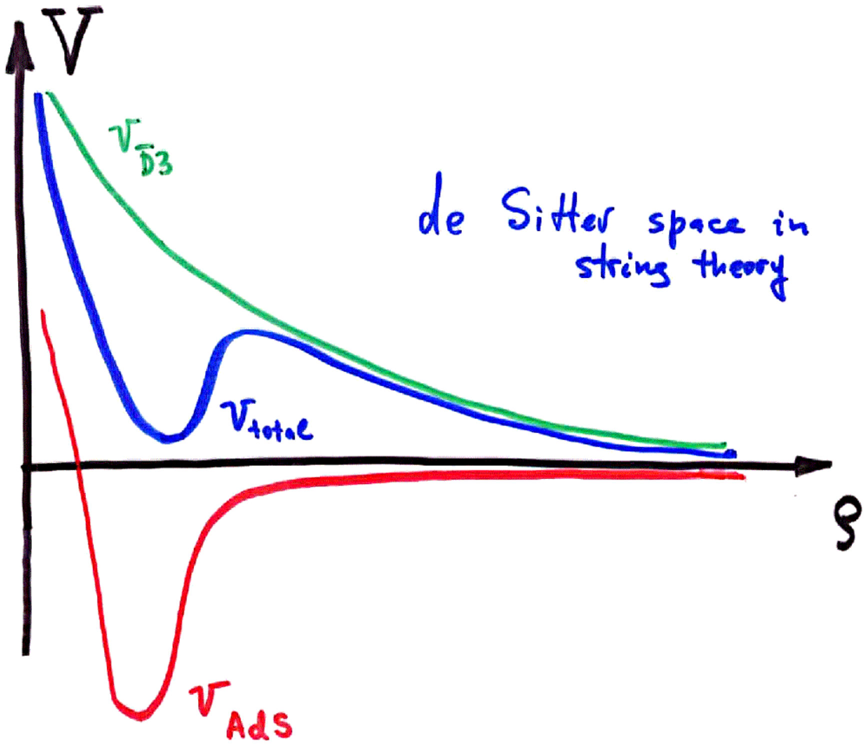
$$K = -3\ln(\rho+\bar{\rho})$$

Non-perturbative string corrections to the superpotential:

$$W(\rho) = \underbrace{W_0}_{\text{from fluxes}} + A\underbrace{e^{-a\rho}}_{\text{instantons, gaugino cond.}}$$

The volume is fixed at AdS minimum with unbroken supersymmetry

$$V_{AdS} = -3e^K|W|^2 < 0$$



de Sitter space in string theory

Add to the picture a $\bar{D}3$ brane at the tip of the resolved conifold

New contribution

$$\Delta V = \frac{D}{\rho^3}$$

Explicit breaking of supersymmetry

de Sitter space is metastable

KKLT model may describe late inflation $w=-1$

Burgess, R.K., Quevedo work in progress

An alternative to KKLT with spontaneous breaking of supersymmetry

Replace $\bar{D}3$ by fluxes of gauge fields within the D7 brane

The uplifting of AdS to dS

$$\Delta V = \frac{D}{\rho^x}$$

can be understood as a field-dependent D-term

in dS vacuum D-flatness condition is broken spontaneously!

New features in landscape

↓ Susskind, Giddings, Douglas

NYT

$N=1$ supergravity in $d=4$

Effective superstring theory

Kähler potential $K(z,\bar{z})$

Superpotential $W(z)$ holomorphic

Kinetic term function for vector multiplet $f_{\alpha\beta}(z)$ holomorphic

Most general potential

$$V_F = e^K\left(|DW|^2 - 3|W|^2\right) \sim |F|^2$$

$$V_D = \frac{1}{2}\mathrm{Re}\, f^{\alpha\beta}(z) D_\alpha D_\beta \geq 0 \sim |D|^2$$

$$-\frac{1}{4}\mathrm{Re}\, f^{\alpha\beta}(z)\, F_{\mu\nu\,\alpha} F_\beta^{\mu\nu} + \frac{i}{4}\mathrm{Im}\, f_\alpha^{\beta}(z)\, F_{\mu\nu\alpha} \tilde{F}_\beta^{\mu\nu}$$

Chiral superfield auxiliary field $F = P_1 + i P_2$

Vector — " — " — " — $D = P_3$

Supergravity and Cosmology $\frac{1}{W}$ issue

R.K., Kofman, Linde, Van Proeyen hep-th/0006179

COSMOLOGICAL MODEL

Hybrid inflation

Linde 1991

- Long slow-roll near de Sitter inflationary stage
- Change of regime near the bifurcation point of the potential
- End of inflation: waterfall stage towards a ground state, tachyon condensation and reheating

F-term hybrid inflation

Copeland, Liddle, Lyth, Stewart, Wands
Drali, Shafi, Schafer 94

$$K = \frac{|S|^2 + |\Phi_+|^2 + |\Phi_-|^2}{M_p^2} \qquad W = \sqrt{2}\, g\, S\left(\Phi_+\Phi_- - \frac{\xi_+}{2}\right)$$

$$V_F = 2g^2 e^{\frac{|S|^2}{M_p^2}}\left[\left|\frac{\partial W}{\partial S}\right|^2\left(\left(1+\frac{|S|^2}{M_p^2}\right)^2 - 3\,\frac{|S|^2}{M_p^2}\right) + \dots\right]$$

$$V_F = V_0\left[1 + \cancel{\frac{|S|^2}{M_p^2}} + 2\cancel{\frac{|S|^2}{M_p^2}} - 3\cancel{\frac{|S|^2}{M_p^2}} + \frac{|S|^4}{2M_p^4} + \dots\right]$$

S is an inflaton

$m_S^2 \sim H^2$ kills inflation

Fine-tuning K, W

$m_S^2 \sim 0$

D-term inflation

Binetruy, Dvali
Halyo 96

$$V_{F+D} = e^K\left[|DW|^2 - 3|W|^2\right] + \frac{1}{2}\,\mathrm{Re}\, f(z)\, D^2$$

$f(z) = 1$ or $f(z)$ may depend on dilaton, volume moduli but not on inflaton!

No problem $m^2 \sim H^2$

No need for fine-tuning!

Fayet-Iliopoulos terms

$$D = |\Phi_+|^2 - |\Phi_-|^2 - \xi_3$$

$$K = \frac{|S|^2 + |\Phi_+|^2 + |\Phi_-|^2}{M_p^2} \qquad W = \lambda S \Phi_+ \Phi_-$$

Inflation at $W \sim DW \sim 0$

$D^2 \neq 0$

P-term Hybrid Inflation

R. Kallosh and A. Linde hep-th/0306058

- Unification of F- and D-term models of Brane Inflation with $\lambda = \sqrt{2} g$

- Gravity coupling: new parameter,

$$0 \leq f \leq 1$$

D-term $\rightarrow f = 0$, F-term $\rightarrow f = 1$

- Cosmological applications

Controllable running of the spectral index:

$$\frac{dn_s}{dk} \sim f$$

Suppression of cosmic strings

$$n_s = 1$$

THE POTENTIAL IN P-TERM INFLATION

Salam-Strathdee-Fayet N=2 susy gauge model

Auxiliary field triplet $\vec{P}$, FI triplet $\vec{\xi}$

$$\vec{P} = -g[(\Phi^\dagger \vec{\sigma} \Phi) - \vec{\xi}]$$

Potential

$$V^P = 2g^2 \left[\Phi^\dagger \Phi |\mathbf{S}|^2 + \left(\frac{1}{4} \Phi^\dagger \vec{\sigma} \Phi - \frac{\vec{\xi}}{g} \right)^2 \right]$$

F and D

$$2g^2 \left(|S\Phi_+|^2 + |S\Phi_-|^2 + |\Phi_+\Phi_- - \frac{\xi_1 + i\xi_2}{2}|^2 \right) +$$

$$\frac{g^2}{2} (|\Phi_+|^2 - |\Phi_-|^2 - \xi_3)^2 \}.$$

Add 1-loop gauge corrections and coupling to gravity

$$V = \frac{g^2\xi^2}{2}\left(1 + \frac{g^2}{8\pi^2}\ln\frac{|S^2|}{|S_c^2|} + f\frac{|S|^4}{2M_p^4} + \dots\right),$$

$$f = \frac{\xi_1^2 + \xi_2^2}{\vec{\xi}^2}, \qquad 0 \le f \le 1,$$

D-term inflation, $f = 0$, minimal running of n_s

F-term inflation, $f = 1$, maximal running of n_s
(A. L. and A. Riotto, 1997)

P-term inflation model interpolates between these two cases

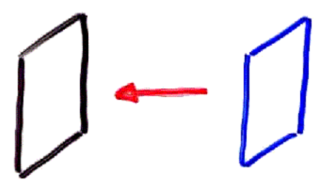
Choice of $\xi_1, \xi_2, \xi_3 \Rightarrow$ Fluxes on a brane

Towards Inflation in String Theory

Kachru, R.K., Linde, Maldacena, McAllister, Trivedi hep-th/0308055

An investigation of inflation based on KKLT

Brane inflation (Dvali, Tye 1998 .....)
Review: Quevedo 2002



Branes and/or antibranes in 6d internal space of finite volume

The issue of volume stabilization was not addressed in previous work $\Longrightarrow$
"naive brane inflation models"

Inflaton $\longleftrightarrow$ distance between branes, ℓ
Linear size of internal space, L

Two main points

1) Even in "naive brane-anti-brane inflation models" there was a problem to make small slow roll parameter

$$\eta = \frac{V''}{V} M_p^2 \ll 1$$

it was necessary to have distance between $\bar{D}3 - D3$ larger than the scale of CY

$$\ell \gg L$$

which does not make sense

We studied $\bar{D}3 - D3$ model in warped geometry (Klebanov-Strassler)

$$\eta_{war} = \eta\, e^{-\frac{8\pi K}{3 g_s M}} \ll 1 \qquad \text{GKP}$$

K, M are the background fluxes

easy to make η very small

2) A general argument why ALL "naive brane inflation models" may not work with stabilization of volume was suggested

String theory version of $m^2 \sim H^2$ problem

Any source of inflationary energy in GKP setting

$$\frac{C}{r^\alpha} \qquad \alpha > 0$$

$$\alpha \sim 1$$

$$r \sim R^4$$

$$ds_{10}^2 = R^{-6} \tilde{g}_{\mu\nu} dx^\mu dx^\nu + R^2 g_{a b} dy^a dy^b$$

$$\boxed{2r = \rho + \bar{\rho} - \varphi\bar{\varphi}}$$ inflaton

assumption

$r \sim R^4 \to$ volume modulus

KKLT breaking of "no-scale"

$$V \sim \frac{C[\rho]}{(\rho+\bar{\rho}-\varphi\bar{\varphi})^{\alpha}}$$

If ρ is stabilized as in KKLT

$$V \sim V_0 \left(1 + |\Phi|^2 + \dots\right)$$

where Φ is a canonically normalized inflaton field

$$m^2 \sim H^2$$

We need an analog of fine-tuning in F-term supergravity

Specific example

$$W[\rho, \varphi]$$

ρ and φ dependent superpotential

Problem and its possible resolution

We may extend the KKLT model to include the dependence on fields $\varphi, \bar{\varphi}$ describing D-brane position

$$K = -3 \log(\rho + \bar{\rho} - \varphi\bar{\varphi})$$

$$W = W_0 + A e^{-a\rho}(1 + \delta\varphi^2)$$

The calculation of the supergravity potential V_F shows that

$$m^2 = 2H^2 - \frac{2}{3}|V_{AdS}|(\beta - 2\beta^2)$$

$$\beta = -\frac{\delta}{a}$$

Subexponential dependence of W on φ may save the day

Stringy model ?

Can we avoid fine-tuning altogether?

The source of the problem was e^K contribution to the F-term

Our model is similar to F-term hybrid inflation, which requirs fine-tuning

It is known, however, that there is no e^K contribution in D-term hybrid inflation Binetruy, Dvali 96

A similar model may exist in string theory: D4/D6 and D3/D7 inflation Herdeiro, Hirano, Kallosh 2001
Dasgupta, Herdeiro, Hirano, Kallosh 2002

Everything might work without any fine-tuning if FI terms are possible in string theory

Recent SB discussion Burgess, Kachru, R.K., Quevedo, Maldacena

To embed "naive brane inflation models" into full string theory with volume stabilization one has to resolve the following issue

Take GKP setting with the fixed dilaton as a background for D7

$$\int d^8\xi \sqrt{g_8}\, F_{\mu\nu} F_{\lambda\delta} g^{\mu\lambda} g^{\nu\delta} \longrightarrow$$

in 4d Einstein frame $\quad \frac{1}{g^2} \sim r \sim R^4$

However "the volume" inflaton

$$r = \tfrac{1}{2}(\rho + \bar\rho - \varphi\bar\varphi)$$

is not a real part of the holomorphic superfield !

Contradiction with supersymmetry

$$\tfrac{1}{2} \operatorname{Re} f(z)\, F_{\mu\nu}^2$$

for any Dp brane $\quad p \neq 3$

Interpretation of energy from fluxes on branes in IIB with fixed dilaton

$$\frac{F_{ab}^2}{r^\alpha} \quad \text{with inflaton dependent } r$$

is inconsistent with GKP + supersym.

A better understanding of

VOLUME MODULUS

in presence of D_p branes with $p \neq 3$ is required to promote brane inflation models into string theory

What is the correct dependence on inflaton (distance between branes)?

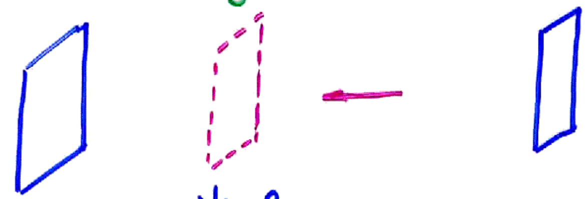
What is the correct ansatz for the metric?

How to avoid contradiction with susy?

Before these problems are resolved, we may study "naive brane inflation models"

D-brane inflation

Exit, reheating



critical distance

$D_p\bar{D}_p$, D_pD_p at angles	$D_p\ D_{p+n}$
	D_4D_6 , D_3D_7
Tachyon condensation	Tachyon potential is calculable
Sen's conjecture e^{-T}	
Min at infinite distance	Min at finite distance
Kofman, Linde problems with tachyon reheating	
Exit and reheating difficult	No problem with exit and reheating
	D-term inflation or F-term inflation

D3/D7 COSMOLOGICAL MODEL
AND M THEORY

Dasgupta, Herdeiro, Hirano, R. K.

	0	1	2	3	4	5	6	7	8	9
$D3$	×	×	×	×						
$D7$	×	×	×	×			×	×	×	×
$\mathcal{F} = dA - B$							×	×	×	×

D7 worldvolume: $\mathcal{F}_{67} = \tan\theta$ $\quad$ $\mathcal{F}_{89} = \tan\theta'$

$\mathcal{F}^- \neq 0$: spontaneously broken susy

D7-D3 STRINGS SPECTRUM

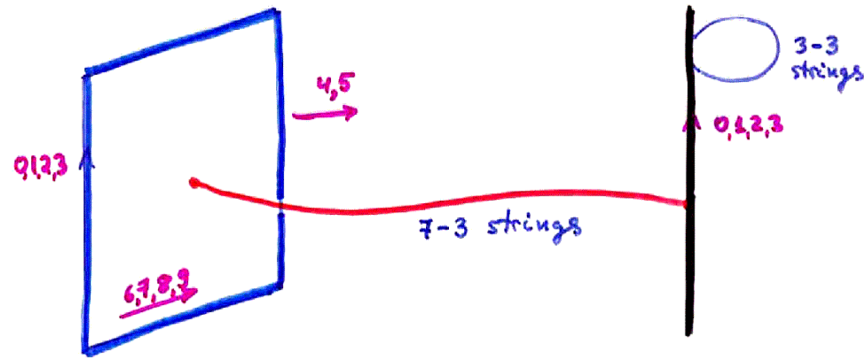
Joe's Big Book of String

$$M^2_{\pm} = \frac{d^2}{(\pi\alpha')^2} \pm \frac{\theta - \theta'}{2\pi\alpha'} \qquad M^2_{\psi} = \frac{d^2}{(\pi\alpha')^2}$$

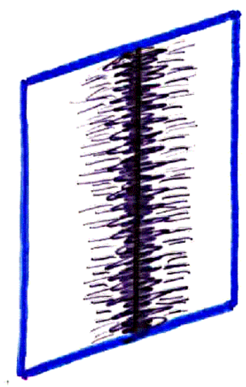
$$d^2 = (X^4)^2 + (X^5)^2$$

$d^2_{critical} \sim \mathcal{F}^-$

Slow-roll inflation near de Sitter stage : susy broken



Tachyon condensation at d critical



D3/D7 bound state

Absolute ground state

susy unbroken

N=2 susy, 1 vector, 1 hyper with mass splitting ⟶ UNIQUE POTENTIAL

THE POTENTIAL IN P-TERM INFLATION

Salam-Strathdee-Fayet model

$$V = \frac{g^2}{2}\left[\Phi^\dagger\Phi|\varphi|^2 + \left(\frac{1}{2}\Phi^\dagger\sigma^r\Phi - \frac{\xi^r}{g}\right)^2\right]$$

- Unstable Non-supersymmetric Vacuum
  Inflation when coupled to gravity
  De Sitter valley, Coulomb phase   D3 far from D7

$$P^r = \xi^r \qquad |\varphi| >> 0 \qquad V = \frac{1}{2}\vec{\xi}^2$$

- Stable Supersymmetric vacuum, Higgs phase

$$P^r = 0 \qquad \varphi = 0 \qquad V = 0$$

Known to String Theorists also as D-flatness condition and ADHM equations in instanton construction

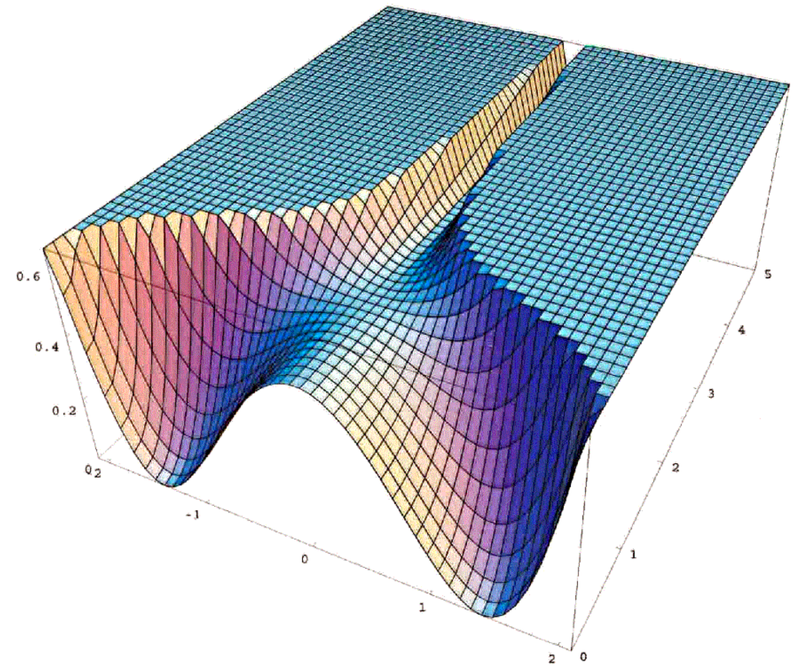
$$P^r = \frac{g}{2}\Phi^\dagger\sigma^r\Phi - \xi^r = 0$$

D7/D3 bound state

Hybrid Inflation

A.L. 91

Easier to implement in supergravity and brane cosmology



Cosmological potential with Fayet-Iliopoulos term

De Sitter valley is classically flat; it is lifted by the one-loop correction corresponding to the one-loop potential between D4-D6. In this figure the valley is along the $|\Phi_3|$ axis; the orthogonal direction is a line passing through the origin of the complex Φ_2 plane and we have put $|\Phi_1| = 0$. The bifurcation point corresponds to $|\Phi_3| = \sqrt{\xi/g}$, $\Phi_2 = 0$. The absolute minimum is at $\Phi_3 = 0$, $\Phi_2 = \sqrt{2\xi/g}$.

$\boxed{\text{D3-D7}}$

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HYPERMULTIPLET MASS SPLITTING

controlled by spontaneous susy breaking

$$\mathrm{STr}\, M^2 = 0$$

$$M_\pm^2 = g^2|\varphi|^2 \pm g\xi \,, \qquad M_\psi = g|\varphi|$$

Bifurcation point: hyper becomes massless,

$$M_-^2 = 0 \quad \Leftrightarrow \quad |\varphi|_c^2 = \frac{\xi}{g}$$

At $\varphi \geq \varphi_c$, de Sitter minimum

At $\varphi \leq \varphi_c$, de Sitter maximum

Beyond the critical point, scalars become tachyonic. The system is unstable and the waterfall stage of the potential, mixed Coulomb-Higgs phase, leads it to a ground state, Higgsed phase.

Instability and κ-symmetry of the worldvolume

With FI parameter D3/D7 system at the distance from each other is unstable.

D7-brane worldvolume action in $D3$-brane background

$$-T_7 \left(\int d^8\sigma e^{-\phi} \sqrt{-\det(g+\mathcal{F})} - \int \sum \mathcal{A}_{p+1} \wedge e^{\mathcal{F}} \right)$$

We turn on the worldvolume gauge field

$$F_{67} = \xi \qquad F_{89} = \xi'$$

Effective potential

$$\mathrm{BI}: \quad T_7 \mathcal{V}_3 \int d^4\sigma \left[\sqrt{(1+H^{-1}\xi^2)(1+H^{-1}\xi'^2)} \right]$$

$$\mathrm{WZ}: \quad T_7 \mathcal{V}_3 \int d^4\sigma \left[-H^{-1}\xi\xi' \right]$$

$$H = 1 + Q/r^4 \qquad r^2 = d^2 + (\vec{\sigma})^2$$

When $\xi = \xi'$, the force between the D3 and D7 vanishes.

$$\mathcal{F}^- = 0$$

For arbitrary but small values of the parameters ξ, ξ' and large distances d the finite, non-constant part of the potential becomes

$$V \simeq \pi^2 T_7 \mathcal{V}_3 Q (\mathcal{F}^-)^2 \ln d^2$$

d^2 is interpreted as an inflaton field φ^2

$(\mathcal{F}^-)^2$ is interpreted as FI terms ξ^2

Same for D7 brane background with $B^- \neq 0$ probed by D3 brane

In Coulomb branch D3/D7 system has broken supersymmetry and slow-roll inflation is possible due to

$$\mathcal{F}^- \neq 0 \, .$$

THE BIG QUESTION is: how D3/D7 system can become supersymmetric with $\mathcal{F}^- \neq 0$ in the Higgs branch? The solution is:

D3/D7 SUPERSYMMETRIC BOUND STATE

The necessary condition for the bound state to be supersymmetric is that the rotation factor a of κ-symmetry is σ-independent.

$$\frac{\partial a(\mathcal{F})}{\partial \sigma} = 0 \qquad a = \frac{1}{2} Y_{ij} \Gamma^{ij} \otimes \sigma_3$$

$$Y = Y(\mathcal{F})$$

A 2-form Y is a known non-linear function of $\mathcal{F}$

κ-SYMMETRY AND DEFORMATION

deformed vector field strength $\hat{F}$

$$\boxed{\hat{F} = Y(F - B) - Y(-B)}$$

D3/D7 bound state is supersymmetric iff

$$exp\Big(-\frac{1}{2}\sigma_3 \otimes \Gamma^{ij} Y^-_{ij}(B)\Big) i\sigma_2 \otimes \Gamma_{01236789}\,\epsilon = \epsilon$$

$$(1 - \Gamma_{6789})\epsilon = 0$$

$$\boxed{\hat{F}^-(\sigma) = 0}$$

Non-linear supersymmetric instanton in 6,7,8,9 space of D7

$$B^- = \frac{1}{2}(B_{67} - B_{89}) \neq 0$$

Supersymmetry with FI terms present, ADHM equations follow! D3/D7 cosmological model justified: Coulomb branch, tachyon condensation, Higgs branch

It is tempting to speculate

In the context of Instantons on Non-commutative $\mathbf{R}^4$ Nekrasov-Schwarz, 1998 FI terms are necessary to make the instantons non-singular.

In the context of Dirac-Born-Infeld non-linear instantons Seiberg-Witten, 1999 FI terms are necessary to have a finite non-vanishing instanton number.

This speculation, if taken seriously, would give an explanation of the non-vanishing effective cosmological constant (in the early universe and today): it may be needed to remove certain instanton singularities.

Current work in progress

with Stanford students Hsu, Prokushkin

How to generalize the D_4 D_6 Brane construction at angles

D_3 D_7 with fluxes

to include more general case with controllable spectral index?

Previous case only one angle in 6-7 plane

We add 6-8 and 6-9

only $F_{67} - F_{89}$

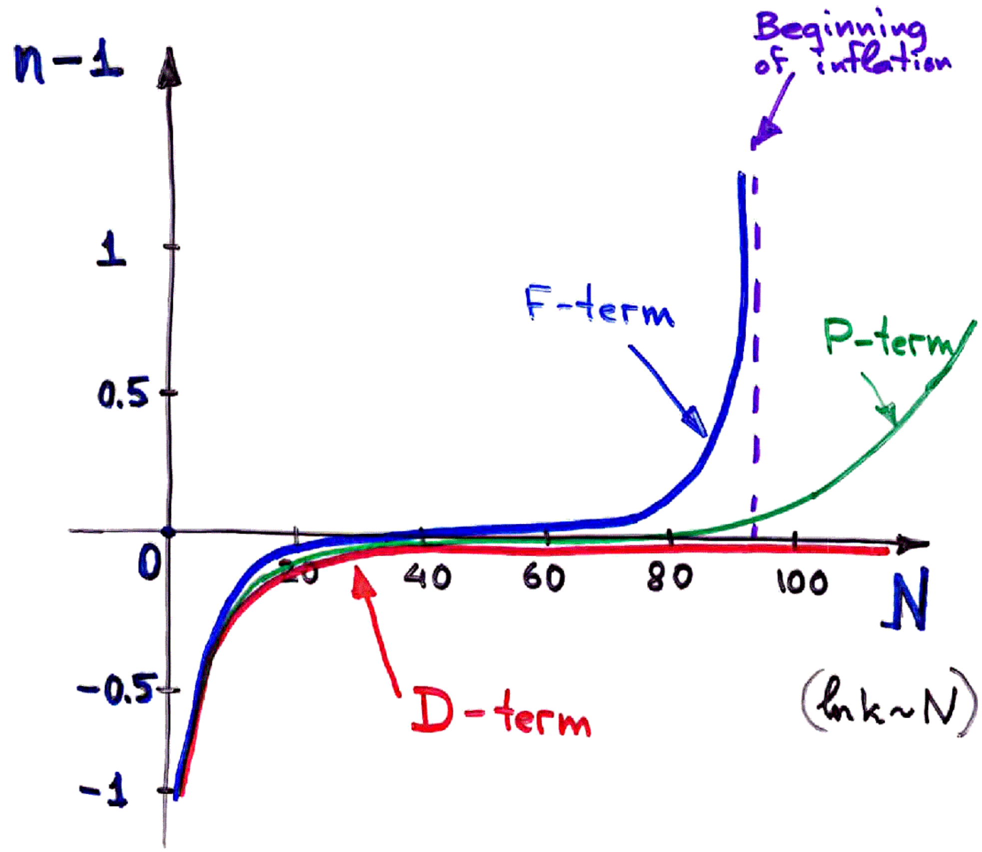
We add $F_{68} - F_{79}$

$F_{69} - F_{78}$

If observations (Hui, Seljak) will lead to $\frac{dn}{dk} \sim 0$ $\longrightarrow$ D-term model

if $\frac{dn}{dk} \neq 0$ $\longrightarrow$ mix with F-term

Fluxes on branes $\longleftrightarrow$ Observations



Spectral index n in F-term, D-term and P-term inflation