BRANE COSMOLOGY AND OBSERVATIONS

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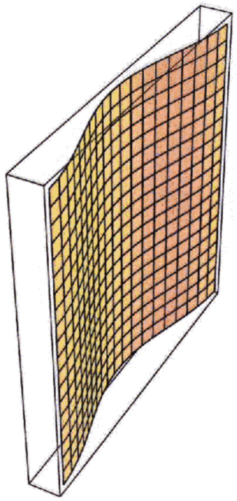
Plan

- Introduction to brane-worlds
- Homogeneous brane cosmology
- Dark radiation
- Brane inflation
- Cosmological perturbations and the CMB

Cosmology in a brane-world

Idea: Confinement of cosmological matter on a submanifold (BRANE) embedded in a higher-dimensional spacetime (BULK)



We restrict our study to self-gravitating branes in a five-dimensional bulk space-time.

In such a brane-universe, the cosmological evolution is modified:

- The first Friedmann equation is modified at high energy,
- The bulk influences the cosmological evolution (Dark radiation)

Homogeneous brane cosmology

- As in standard cosmology, one assumes homogeneity and isotropy along the *three ordinary spatial dimensions*.

  $\Longrightarrow$all quantities depend only on time and on the extra dimension.

  The five-dimensional metric can be written in the form

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)d\vec{x}^2 + dy^2,$$

  with the brane located at $y = 0$.

- One must solve the five-dimensional Einstein equations

$$G_{AB} + \Lambda g_{AB} = \kappa^2 T_{AB}$$

  with the energy-momentum tensor (empty bulk)

$$T_A^B = Diag(-\rho_b(t), P_b(t), P_b(t), P_b(t), 0)\delta(y)$$

- One then finds on the brane ('b' corresponds to $y = 0$) the *modified Friedmann equation*:

$$H_b^2 \equiv \frac{\dot{a}_b^2}{a_b^2} = \frac{\Lambda}{6} + \frac{\kappa^4}{36}\rho_b^2 + \frac{\mathcal{C}}{a_b^4}$$

  where $\mathcal{C}$ is an integration constant.

  The conservation equation $\nabla_\mu T^\mu_\nu = 0$ still holds (for an empty bulk).

- For $\Lambda = 0$ and $\mathcal{C} = 0$ (the bulk is Minkowski),

the cosmological evolution of the brane is determined by the system

$$\frac{\dot{a}_b^2}{a_b^2} = \frac{\kappa^4}{36}\rho_b^2$$
$$\dot{\rho}_b + 3\frac{\dot{a}_b}{a_b}(\rho_b + p_b) = 0.$$

For $w = P/\rho = const$, $\rho_b \sim a_b^{-3(1+w)}$ and therefore

$$a_b(t) \sim t^q, \qquad q = \frac{1}{3(1+w)}.$$

In particular, $a_b(t) \sim t^{1/4}$ for *radiation*, $a_b(t) \sim t^{1/3}$ for *non relativistic matter*.

INCOMPATIBLE with NUCLEOSYNTHESIS !

- Warped geometries [Randall and Sundrum]
  - The bulk space-time is curved. The extra-dimension can become infinite.
  - In an empty bulk with a negative cosmological constant Λ_5 (AdS), put at $y = 0$ a self-gravitating brane with intrinsic tension $\sigma > 0$, satisfying

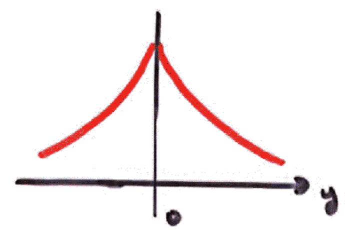
$$\Lambda + \frac{\kappa^4}{6}\sigma^2 = 0.$$

  - Assuming Z_2-symmetry ($y \to -y$), one gets the geometry

$$ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2,$$

with

$$a(y) = e^{-|y|/\ell}, \qquad \ell \equiv \sqrt{-6/\Lambda}$$



  - In this case, the effective 4-dim Planck's mass is given by

$$M_{Pl}^2 = M_{(5)}^3\ell$$

The curvature scale ℓ plays the rôle of a compactification scale and standard gravity is recovered down to scales $\sim \ell$.

Gravity experiments $\Longrightarrow \ell \lesssim 0.1\mathrm{mm}$.

Viable brane cosmology ?

Use the idea of Randall and Sundrum:

1. Consider a bulk with negative cosmological constant $\Lambda < 0$
2. Assume the brane is endowed with an intrinsic tension σ, so that

$$\rho_b(t) = \sigma + \rho(t),$$

where $\rho(t)$ is the energy density of usual cosmological matter.

THEN

$$H_b^2 = \left(\frac{\Lambda}{6} + \frac{\kappa^4}{36}\sigma^2\right) + \frac{\kappa^4}{18}\sigma\rho + \frac{\kappa^4}{36}\rho^2 + \frac{\mathcal{C}}{a_b^4}$$

and one recovers *approximatively* the usual Friedmann equation if

$\frac{\Lambda}{6} + \frac{\kappa^4}{36}\sigma^2 \simeq 0$ (Randall-Sundrum condition) $\Longrightarrow 8\pi G \equiv \frac{\kappa^4}{6}\sigma$.

Two new features

- A ρ^2 term, which dominates at high energy;
- A radiation-like term, $\mathcal{C}/a_b^4$, usually called dark radiation.

TRANSITION : High energy regime $\rightarrow$ Low energy regime

$\rho \gg \sigma \rightarrow \rho \ll \sigma$

Unconventional cosmology $\rightarrow$ Standard cosmology

- Analytical solutions ($\mathcal{C} = 0$, $w = P/\rho = const$)

$$\rho \sim a^{-q}, \qquad q = 3(1+w)$$

gives

$$a(t) \propto t^{1/q}\left(1 + \frac{q\,t}{2\ell}\right)^{1/q}$$

At time $t \sim \ell$, transition $t^{1/q} \rightarrow t^{2/q}$.

- Non zero effective cosmological constant [redacted] :

$$\frac{\Lambda}{6} + \frac{\kappa^4}{36}\sigma^2 = \frac{\lambda}{3},$$

which gives

$$a(t) \propto \left\{\mathrm{sh}\left(q\sqrt{\lambda/3}\,t\right) + \left[\mathrm{ch}\left(q\sqrt{\lambda/3}\,t\right) - 1\right] / \left(\ell\sqrt{\lambda/3}\right)\right\}^{1/q}$$

- One can also solve explicitly for the bulk metric.

For $\mathcal{C} = 0$,

$$\begin{aligned} a(t,y) &= a_0(t)\left[\mathrm{ch}\mu y - \eta(t)\mathrm{sh}\mu|y|\right] \\ n(t,y) &= \mathrm{ch}\mu y - \left(\eta(t) + H_b^{-1}\dot\eta\right)\mathrm{sh}\mu|y|, \end{aligned}$$

with

$$\mu = \ell^{-1}, \qquad \eta \equiv 1 + \frac{\rho}{\sigma}$$

Constraints on the parameters

The model is characterized by

- the *fundamental mass* M_5 related to κ^2 via

$$\kappa^2 = M_5^{-3}$$

- the AdS lengthscale ℓ, defined by

$$\Lambda = -\frac{6}{\ell^2}$$

M_5 and ℓ are not independent since the four-dimensional Planck mass is given by

$$M_{Pl}^2 = M_5^3 \ell$$

Two constraints:

- Nucleosynthesis: $\sigma^{1/4} > 1$ MeV (and $\sigma = 6/(\kappa^2\ell) = 6M_5^6/M_P^2$)
  $\Longrightarrow M_5 > 10^4$ GeV
- Newton's law: $\ell \lesssim 1$ mm $\Longrightarrow M_5 > 10^8$ GeV, $\sigma^{1/4} > 10^2$ GeV

Another point of view

- The bulk metric is in fact AdS-Schwarzschild. In a static coordinate system

$$ds^2 = -f(R)dT^2 + \frac{dR^2}{f(R)} + R^2 d\Sigma_k^2,$$

with

$$f(R) = k + \frac{R^2}{\ell^2} - \frac{\mathcal{C}}{R^2}, \quad k = 0, \pm 1.$$

- The brane is moving

Given the trajectory $(T(t), R(t))$, the induced metric on the brane is given by

$$ds^2 = -dt^2 + R^2(t) d\Sigma_k^2$$

$R(t)$ is the brane scale factor !

- Junction conditions: $[K_{\mu\nu}] = -\kappa^2 \left(S_{\mu\nu} - (S/3) g_{\mu\nu}\right)$

One finds

$$(ij): \quad \frac{1}{R}\sqrt{f + \dot{R}^2} = \frac{\kappa^2}{6}\rho_b$$

$\Longrightarrow$Modified Friedmann eq

$$\frac{\dot{R}^2}{R^2} = \frac{\kappa^4}{36}\rho_b^2 - \frac{1}{\ell^2} + \frac{\mathcal{C}}{R^4}$$

$$(tt): \quad \dot{\rho}_b + 3H(\rho_b + P_b) = 0.$$

$\Longrightarrow$Standard conservation law

Production of bulk gravitons by the brane

So far, the bulk has been assumed to be *strictly empty*. However, the fluctuations of brane matter generate bulk gravitational waves.

- Bulk gravitons are produced in the process

$$\psi + \bar{\psi} \to G$$

- In the Randall-Sundrum framework, this interaction is governed by the action

$$S_{int} = \kappa \int d^4x\, \tau^{\mu\nu} h_{\mu\nu}(x, y=0),$$

where $h_{\mu\nu}(x,y)$ are the metric fluctuations defined as

$$ds^2 = \left(e^{-2\mu|y|}\eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x,\, y)\right) dx^\mu dx^\nu + dy^2\ ,$$

- The metric fluctuations can be decomposed into "Kaluza-Klein" modes:

$$h_{\mu\nu}(x,\, y) = \int dm\, u_m(y) \phi^{(m)}_{\mu\nu}(x),$$

- For $\psi(p_1) + \bar{\psi}(p_2) \to G(m)$, the averaged square amplitude is given by ($s = (p_1+p_2)^2$)

$$\sum |\mathcal{M}|^2 = \kappa^2 |u_m(0)|^2 A\, \frac{s^2}{8}$$

with

$$A_s = \frac{2}{3}, \quad A_f = 1, \quad A_v = 4.$$

Substituting in the Boltzmann equation,

$$\frac{d\rho}{dt} + 3H(\rho + p) = -\int dm \int \frac{d^3 p_m}{(2\pi)^3} \mathbf{C}_m,$$

with the collision term

$$\mathbf{C}[f] = \frac{1}{2}\int \frac{d^3p_1}{(2\pi)^3\, 2E_1}\, \frac{d^3p_2}{(2\pi)^3\, 2E_2} \sum |\mathcal{M}|^2\, f_1\, f_2\, (2\pi)^4\, \delta^{(4)}\left(p_1 + p_2 - p_m\right)\ ,$$

one finds

$$\dot{\rho} + 4H\rho = -\frac{315}{512\,\pi^3}\, \hat{g}(T)\, \kappa^2\, T^8,$$

with

$$\hat{g}(T) = (2/3)g_s + 4g_v + g_f.$$

Using $\rho = (\pi^2/30) g_* T^4$, one gets

$$\dot{\rho} + 4H\rho = -2\mathcal{F}$$

with

$$\mathcal{F} = \alpha\kappa^2\rho^2, \qquad \alpha = \left(\frac{70875}{256\pi^7}\right) \frac{\hat{g}}{g_*^2} \simeq 0.09\, \frac{\hat{g}}{g_*^2}$$

Radiating brane

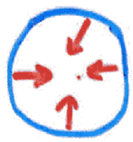
[D.L. + L. Sorbo + M. Rodriguez-Martinez, PRL (2002)]

- To model the system Bulk + Radiating Brane, one can use a generalized five-dimensional Vaidya metric

$$ds^2 = -f(R,v)dv^2 + 2dRdv + R^2 d\mathbf{x}^2,$$

with

$$f(R,v) = \mu^2 R^2 - \frac{\mathcal{C}(v)}{R^2}.$$



describing an *ingoing* radiation flow. Here, v is a null coordinate.

- If $\mathcal{C}(v)$ is constant, one recovers the AdS-Schwarzschild metric

$$ds^2 = -f(R)dT^2 + \frac{dR^2}{f(R)} + R^2 d\mathbf{x}^2,$$

with the coordinate change $T = v - \int dR/f(R)$.

- The generalized Vaidya's metric is a solution of the five-dimensional Einstein's equations with

$$T_{AB} = \mathcal{F}\, k_A k_B, \qquad k_A k^A = 0, \quad (k_A u^A = 1).$$

The gravitons are radial !

The junction conditions give

- the modified Friedmann equation:

$$\frac{\dot{R}^2}{R^2} = \frac{\kappa^4}{36}\rho_b^2 - \frac{f}{R^2} = \frac{\kappa^4}{36}\rho_b^2 - \mu^2 + \frac{\mathcal{C}}{R^4}$$

- the non-conservation equation

$$\dot{\rho} + 3H(\rho + P) = -2\mathcal{F}.$$

Einstein's equations imply that, *on the brane*, the evolution of the Weyl parameter is given by

$$\dot{\mathcal{C}} = \frac{2}{3}\kappa^2 \mathcal{F} a^4 \left(\frac{\kappa^2}{6}\rho_b - H\right).$$

Inserting $\mathcal{F} = \alpha\kappa^2\rho^2$, one gets a *closed* system of first-order differential equations for the evolution of the variables $(a(t), \rho(t), \mathcal{C}(t))$.

- High energy regime:

$$a \sim t^{1/4(1+3\alpha)}, \quad \mathcal{C} \sim t^{1/(1+3\alpha)},$$

- Low energy regime:

$$a \sim t^{1/2}, \quad \mathcal{C} \sim \text{const}.$$

The production of bulk gravitons is negligible.

General framework

- The bulk is not empty but filled with a gas of gravitons, with energy-momentum tensor

$$\mathcal{T}_{AB} = \int d^5p\, \delta\left(p_M p^M\right) \sqrt{-g}\, f\, p_A p_B,$$

- From the 5D Einstein equations,

$$R_{AB} - \frac{1}{2} g_{AB} R + \Lambda_5 g_{AB} = \kappa^2 \left[\mathcal{T}_{AB} + S_{AB}\delta(y)\right] ,$$

one can derive the effective 4D Einstein equations [Shiromizu, Maeda, Sasaki, PRD (2000)]

$${}^{(4)}G_{\mu\nu} = \kappa_4^2(\tau_{\mu\nu} + \tau_{\mu\nu}^{(\pi)} + \tau_{\mu\nu}^{(W)} + \tau_{\mu\nu}^{(B)}),$$

with $\kappa_4^2 = \kappa^2 \mu$ and

$$\begin{aligned}
\kappa_4^2 \tau_{\mu\nu}^{(\pi)} &= -\frac{\kappa^2}{24}\left[6\tau_{\mu\alpha}\tau_\nu{}^\alpha - 2\tau\tau_{\mu\nu} - h_{\mu\nu}(3\tau_{\alpha\beta}\tau^{\alpha\beta} - \tau^2)\right] , \\
\kappa_4^2 \tau_{\mu\nu}^{(W)} &= -{}^{(5)}C_{ABCD}\, n^A\, n^B\, h_\mu^B\, h_\nu^D , \\
\kappa_4^2 \tau_{\mu\nu}^{(B)} &= \frac{2\kappa^2}{3}\left[\mathcal{T}_{AB} h^A{}_\mu h^B{}_\nu + h_{\mu\nu}\left(\mathcal{T}_{AB} n^A n^B - \frac{1}{4}\mathcal{T}^A{}_A\right)\right] ,
\end{aligned}$$

This leads to

- the first Friedmann equation

$$H^2 = \frac{\kappa_4^2}{3}\left[\left(1 + \frac{\rho}{2\sigma}\right)\rho + \rho^{(W)} + \rho^{(B)}\right] ,$$

- the non-conservation equation for brane matter

$$\dot\rho + 3\,H\,\left(\rho + p\right) = 2\,\mathcal{T}_{RS}\, n^R\, u^S < 0$$

- the non-conservation equation for "dark radiation" $\rho_{\rm D} = \rho^{(B)} + \rho^{(W)}$:

$$\dot\rho_{\rm D} + 4H\rho_{\rm D} = -2\left(1 + \frac{\rho}{\sigma}\right)\mathcal{T}_{AB} u^A n^B - 2H\ell\,\mathcal{T}_{AB} n^A n^B .$$

  - the flux term *increases* the amount of dark radiation
  - the extra-dimensional pressure term *decreases* the amount of dark radiation.

Gravitons in AdS

[A. Hebecker + J. March-Russel NPB (2001); D.L. + L. Sorbo, hep-th/0306281]

Consider AdS as background

$$ds^2 = -f(r)\, dT^2 + \frac{dr^2}{f(r)} + r^2\, d\mathbf{x}^2, \quad f(r) = \mu^2 r^2.$$

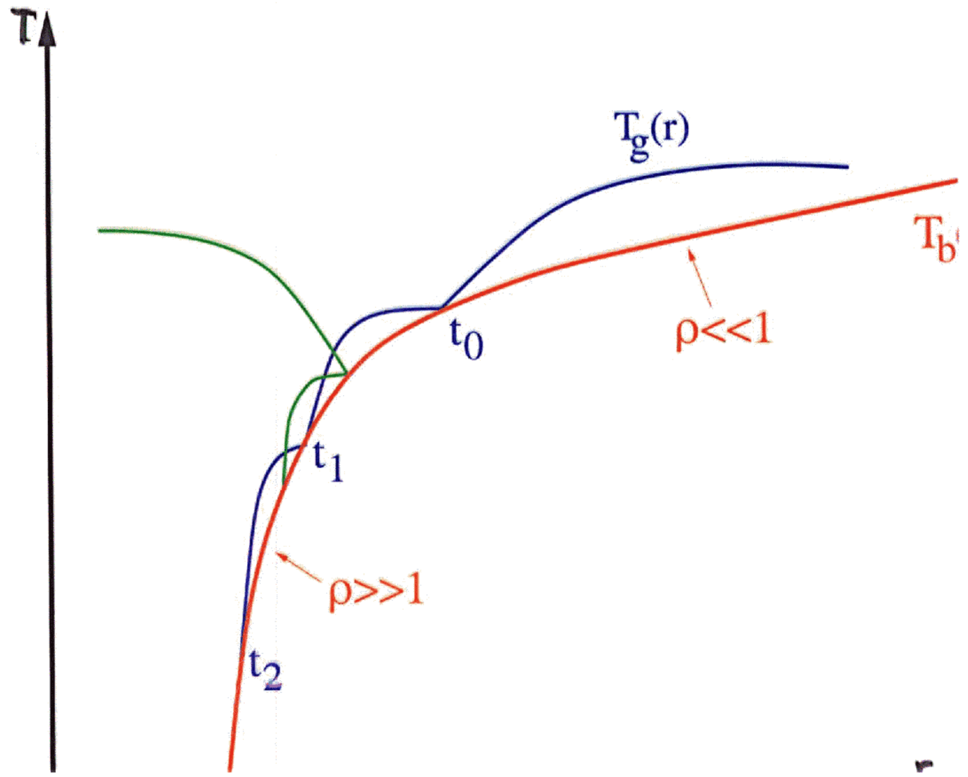
With dimensionless quantities ($\tilde{H} = H/\mu$, $\tilde{\rho} = \rho/\sigma$), the Friedmann equation reads $\tilde{H}^2 = 2\tilde{\rho} + \tilde{\rho}^2$, and the brane trajectory is given by

$$\frac{dr}{dT} = \frac{\dot{r}}{\dot{T}} = r^2 \frac{\tilde{H}}{\sqrt{1+\tilde{H}^2}} = r^2 \frac{\sqrt{2\tilde{\rho}+\tilde{\rho}^2}}{1+\tilde{\rho}} = r^2 \frac{\sqrt{2\,\tilde{\rho}_i\, r^4 + \tilde{\rho}_i^2}}{r^4 + \tilde{\rho}_i}$$

The graviton trajectories can be computed explicitly [R. Caldwell, D.L., PLB (2001)]

$$T - T_* = -\frac{1}{\mathcal{V}}\left(\frac{1}{r} - \frac{1}{r_*}\right).$$

The gravitons are emitted by the brane and then propagate in the bulk



At any given point of the brane trajectory, the bulk gravitons are of two types:

- the gravitons that are just being emitted by the brane, whose distribution is given by

$$f_{(em)}(m,\, \mathbf{p}) = \frac{\hat{g}}{2^{10}\,\pi^5}\,\kappa^2\, m^3\, e^{-\sqrt{\mathbf{p}^2+m^2}/T}\,.$$

  so that

$$\mathcal{T}_{un}^{(em)} = -\int dm\, d^3\mathbf{p}\, \frac{m}{2}\, f_{(em)} = -\frac{315}{2^{10}\,\pi^3}\,\hat{g}\,\kappa^2\, T^8$$

$$\mathcal{T}_{nn}^{(em)} = \int dm\, d^3\mathbf{p}\, \frac{m^2}{2E}\, f_{(em)} = \frac{3}{4\,\pi^4}\,\hat{g}\,\kappa^2\, T^8$$

- the gravitons that are bouncing off the brane.

  Their distribution is evaluated by taking into account

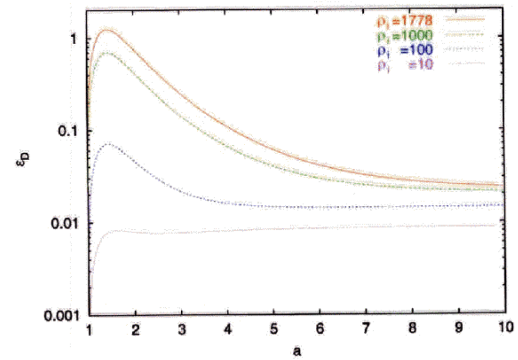
  - the free propagation in the bulk (Liouville equation);
  - the reflexions on the brane.

  The gravitons bouncing off the brane contribute only to the pressure term (cancellation in the flux term).

In summary

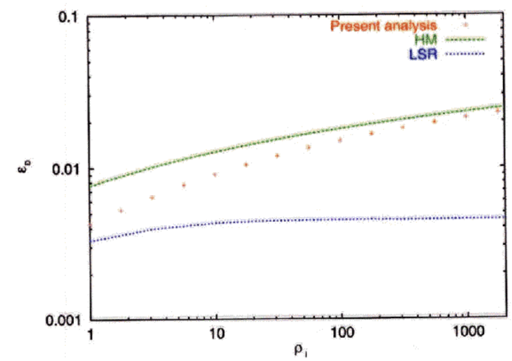
$$\dot{\rho}_D + 4H\rho_D = -2\,(1+\rho)\,\mathcal{T}_{un}^{(em)} - 2H\mathcal{T}_{nn}^{(em)} - 4H\mathcal{T}_{nn}^{(in)}.$$

- Evolution of the "dark radiation" $\epsilon_D \equiv \rho_D/\rho_r$



Late in the low energy regime, ρ_D behaves like radiation, i.e. the production of bulk gravitons becomes negligible.

- Dependence of ϵ_D on the initial energy density ρ_i



$$\rho_i \lesssim M_5^4 \Longrightarrow \rho_i/\sigma \lesssim (M_P/M_5)^2 \simeq 10^{20}(M_5/10^8\mathrm{GeV})^{-2}$$

Constraints from nucleosynthesis

- The nucleosynthesis scenario constrains the number of *additional relativistic degrees of freedom*, usually expressed in terms of ΔN_ν.
- The relation between ΔN_ν and ϵ_D is given by

$$\epsilon_D = \frac{7}{43}\left(\frac{g_*}{g_*^{\rm nucl}}\right)^{1/3}\Delta N_\nu,$$

where $g_*^{\rm nucl} = 10.75$ is the number of degrees of freedom at nucleosynthesis (in fact before the electron-positron annihilation).

Assuming $g_* = 106.75$ (standard model), this gives

$$\epsilon_D \simeq 0.35\Delta N_\nu.$$

- The constraint

$$\Delta N_\nu \lesssim 0.2$$

implies

$$\epsilon_D \equiv \frac{\rho_D}{\rho_r} \lesssim 0.03\left(\frac{g_*}{g_*^{\rm nucl}}\right)^{1/3}$$

[$\epsilon_D \lesssim 0.07$ with d.o.f. of the standard model].

Inflation in the brane

- Bulk inflation: inflation in the brane induced by a bulk scalar field.
- Brane inflation: 4D scalar field confined on the brane
  - Modified Friedmann equation $\Longrightarrow$slow-roll conditions are changed (inflation with steeper potentials than usual GR)
  - Scale-invariant spectra for scalar and tensor modes:

$$\mathcal{P}_S \propto \frac{V^3}{m_P^6 V'^2}\left(1+\frac{V}{2\sigma}\right)^3$$

$$\mathcal{P}_T \propto \frac{V}{m_P^4}\mathcal{F}^2(H\ell)$$

with $\mathcal{F} \simeq 1$ for $H\ell \ll 1$ (low energy) and $\mathcal{F} \simeq (3/2)H\ell$ for $H\ell \gg 1$ (high energy).

At high energy, i.e. $V \gg \sigma$, the two amplitudes are enhanced but the tensor/scalar ratio is suppressed:

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_S} \propto r_{4D}\left(\frac{V}{\sigma}\right)^{-1}, \qquad V \gg \sigma.$$

Perturbations in brane cosmology

- After considering the *homogenous* evolution, one must study deviations from homogeneity and isotropy

  $\Longrightarrow$The theory of cosmological perturbations must be revisited

- Direct link with cosmological observations, in particular

  - large scale structure

  - anisotropies of the Cosmic Microwave Background.

- Question: does brane cosmology predict deviations from the usual picture ?

- The full problem requires a 5-dimensional analysis.

Two difficulties:

- the evolution of (metric) perturbations is governed by partial differential equations which are not separable

- there is potential information outside our brane-Universe, in the bulk. One must specify the boundary conditions.

Description from the brane point of view

- Effective Einstein equations on the brane:

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = \kappa_4^2 \tau_{\mu\nu} + \kappa^2 \Pi_{\mu\nu} - E_{\mu\nu},$$

- The metric with linear scalar perturbations reads (in the longitudinal gauge)

$$ds^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)\delta_{ij}dx^i dx^j$$

- Projected Weyl tensor $E_{\mu\nu}$ $\Longrightarrow$effective energy-momentum tensor for a "Weyl fluid":

$$-\kappa_4^{-2} E_{\mu\nu} = \begin{bmatrix} -(\rho_D + \delta\rho_D) & a\delta q_{D|j} \\ -a^{-1}\delta q_D{}^{|i} & (P_D + \delta P_D)\delta^i_j + \delta\pi_D{}^i{}_j \end{bmatrix}.$$

with

$$E^\mu_\mu = 0, \qquad \nabla_\mu E^\mu_\nu = \kappa^4 \nabla_\mu \Pi^\mu_\nu .$$

- The equations governing the cosmological perturbations on the brane are similar to the standard ones with *two types of corrections*:

  – Modified background-dependent coefficients : negligible when $\rho \ll \sigma$.

  – Additional terms due to the Weyl fluid E^ν_μ

CMB anisotropies

- It is useful to define the (gauge-invariant quantity)
$$\zeta = \Phi + \frac{\delta\rho}{3(\rho+P)},$$
which is *conserved*, $\zeta = \zeta_*$, *on large lengthscales* ($k \ll aH$) for *adiabatic* perturbations.

- One can also define ζ_{tot}, that includes the "dark radiation" perturbations, and whose evolution can be solved explicitly for large scales :
$$\zeta_{tot} = \zeta_* + \frac{\rho_r}{3(\rho+P)(1+\rho/\sigma)}\delta\mathcal{C}_* \qquad (\mathcal{C}=0)$$

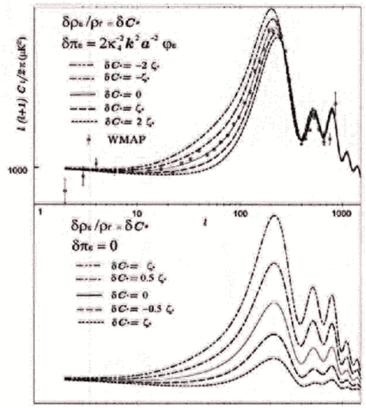
- Einstein's equations relate ζ_{tot} to the metric perturbations Φ and Ψ but now
$$\Phi + \Psi = -\kappa_4^2 a^2 \delta\pi_{\rm D} \neq 0.$$

- The Sachs-Wolfe effect can be expressed as
$$\left(\frac{\delta T}{T}\right)_{SW} = (\zeta_r + \Psi - \Phi) + \int_{\eta_{ls}}^{\eta_0} d\eta\, \partial_\eta (\Psi - \Phi)\,.$$

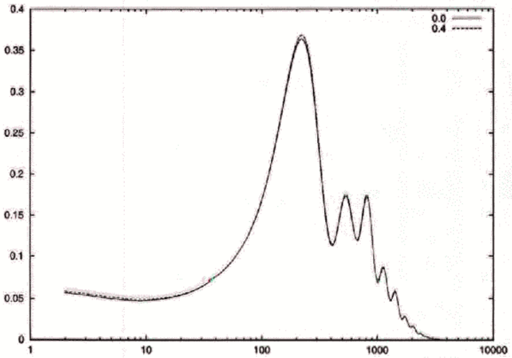
- Direct Sachs-Wolfe effect
$$\zeta_r + \Psi - \Phi = -\frac{1}{5}\zeta_* - \frac{2}{3}\frac{\rho_r}{\rho}\delta\mathcal{C}_* - \kappa_4^2 a^2 \delta\pi_{\rm D} + \frac{2\kappa_4^2}{a^{5/2}}\int_0^a \delta\pi_{\rm D}\, a^{7/2} da\,.$$

Conclusion: the CMB signature depends on the anisotropic stress of the Weyl fluid ⟹one needs to study the evolution of the perturbations in the bulk.

Two recent studies in the low-energy limit for a two-brane system:



K. Koyama
astro-ph/0303108



Rhodes, van de Bruck, Brax & Davis, astro-ph/0306343

Conclusions

- One can test the braneworld idea in
  1. Modification of Newton's law
  2. Signatures in colliders
  3. Cosmology

  Cosmology is the most indirect way but might be the only one (if M_* is not low enough)
- Randall-Sundrum cosmology is a *simple* toy model to study brane effects in cosmology.
- Homogeneous cosmology

  Two new features:
  - ρ^2 term in the generalized Friedmann equation
  - "Dark radiation" (constrained by nucleosynthesis)
- Cosmological perturbations
  - predictions for a single-brane model not yet obtained
  - some recent results in low-energy two-brane models