DE SITTER HOLOGRAPHY AND THE CMB

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MOTIVATION

WANT TH'Y OF VERY EARLY UNIVERSE WHICH RESOLVES / CIRCUMVENTS INITIAL SINGULARITY

STRATEGY HERE: DS / CFT MAY MAP EARLY UNIVERSE TO SYSTEM WITH NO GRAVITY

$\longrightarrow$ HOLOGRAPHY

STROMINGER

VISION:

QUASI-DE SITTER IN 4D SPACETIME
(INFLATION)

HOLOGRAPHY $\updownarrow$

QFT IN 3 EUCLIDEAN DIM'S

THE CMB

STANDARD STORY

INFLATIONARY EPOCH IS NEARLY DE SITTER

⇝ CMB IS NEARLY SCALE INVARIANT

INTERPRETATION

∃ SCALE INVARIANT FIXED PT AND CMB IS DESCRIBED BY PERTURBATION TH'Y AROUND IT

STRONG PROPOSAL (HOLOGRAPHY)

INFLATIONARY FIXED PT IS IN UNIVERSALITY CLASS OF 3D EUKLIDEAN QFT NEAR RG FIXED PT.

WEAKER PROPOSAL

GRAVITY IN IR IS GOVERNED BY UNIV. CLASSES. THESE ARE USEFUL FOR DESCRIBING CMB (& DARK ENERGY)

OUTLINE OF TALK

DEFINE HOLOGRAPHIC MAP IN ANALOGY WITH ADS / CFT

COMMENTS:
1) NO MICROSCOPICS ("EFFECTIVE FIELD TH'Y")
2) COMPUTATIONAL SCHEME DIFFERENT FROM "DS/CFT" MALDACENA
3) CLOSE RELATION TO STANDARD INFLATIONARY COSMOLOGY

APPLICATIONS TO COSMOLOGY

1) RECOVER STANDARD PREDICTIONS FOR CMB

2) DISCUSS POSSIBLE HOLOGRAPHIC INTERPRETATION

H-J THEORY

NATURAL SET-UP FOR SEMI-CLASSICAL TREATMENT (STANDARD IN COSMOLOGY)

EXAMPLE: CLASSICAL MECHANICS

$$\underbrace{S(q,t)}_{\text{FCT OF } t,\, q(t)} = \underbrace{\int^t dt'\, L(q,\dot q,t')}_{\text{EVALUATED ON-SHELL} \to \text{ENFORCE E.O.M}}$$

$$\delta_q S = \int dt' \left(\frac{\partial L}{\partial q} - \partial_t \frac{\partial L}{\partial \dot q}\right)\delta q + \frac{\delta L}{\delta \dot q}\delta q \Big|^t$$

$$\Rightarrow \quad \frac{\partial S}{\partial q} = \frac{\partial L}{\partial \dot q} = P.$$

SEMICLASSICAL WAVEFCT

$$\Psi \sim e^{iS(q,t)}$$

$$\Rightarrow \quad \hat p = -i\frac{\partial}{\partial q} \text{ ACTS AS } \frac{\partial S}{\partial q}$$

HAMILTONIAN

$$\frac{\partial S}{\partial t} + H\left(q, \frac{\partial S}{\partial q}\right) \Leftrightarrow \text{SCHRÖDINGER EQUATION}$$

INFLATION

ϕ IS SPECTATOR SCALAR FIELD IN DE SITTER

$$ds^2 = a(\tau)^2(-d\tau^2 + d\vec x^2)$$

$$a(\tau) = -\frac{1}{H\tau} \qquad \text{NOTE } \tau < 0$$

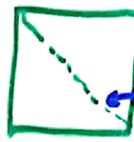
$\tau \to 0_-$ IS FUTURE

$$S(\phi,\tau) = \int \mathcal{L}\, d^3x\, d\tau = \int d^4x \sqrt{g}\left[\tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\right]$$

BNDY CONDITION @ $\tau = -\infty$

ACTION FINITE (INTEGRAL CONVERGENT)

PHYSICS: EUKLIDEAN CONTINUATION REGULAR



REGULAR @ "HORIZON"

ANALOGY W. ADS/CFT

PAST HORIZON ~ ORIGIN OF EAdS

$$\Psi(\phi,t) \sim e^{iS} \longleftrightarrow Z(\phi)$$

INFRARED DIVERGENCES

EXAMPLE: S_{HJ} FOR MASSIVE SCALAR

$$S(\phi,\tau) = \int_{-\infty}^{\tau} d^3x\, d\tau \sqrt{-g}\left(\tfrac{1}{2}\left(\tfrac{\varphi'}{a}\right)^2 - m^2\varphi^2\right)$$

$$= \int d^3x\, \tfrac{1}{2} a^2 \varphi\varphi' \Big|_{-\infty}^{\tau} + \underbrace{e.o.m.}_{0}$$

E.O.M. $\Rightarrow$ $\varphi(\tau) = c_- \tau^{\lambda_-} + c_+ \tau^{\lambda_+}$

$$\lambda_\pm = \tfrac{3}{2}\left(1 \pm \sqrt{1 - \left(\tfrac{2m}{3H}\right)^2}\right)$$

S REGULARITY @ $\tau_i = -\infty$ $\Rightarrow$ λ_- SOL ONLY

$$S_{HJ} = \int d^3x\, \frac{1}{2H^2}\, c_-^2 \lambda_-\, \tau^{-3+2\lambda_-} \underset{\tau\to 0_-}{\longrightarrow} \infty$$

$\Rightarrow$ H-J FUNCTIONAL DIVERGES AT LATE TIMES ~ LONG WAVELENGTH

$\Rightarrow$ INFRARED DIVERGENCE

LOCAL DIVERGENCES: GENERAL

WANT TO FIND GENERAL EXPRESSION FOR DIVERGENCES.

HAMILTONIAN CONSTRAINT $\Rightarrow$ H-J EQUATION

$$\left(\frac{1}{\sqrt{g}}\, g^{ij} \frac{\delta S}{\delta g_{ij}}\right)^2 - 2\frac{1}{\sqrt{g}}\frac{\delta S}{\delta g^{ij}} \frac{1}{\sqrt{g}}\frac{\delta S}{\delta g_{ij}} - \frac{1}{2}\left(\frac{\delta S}{\delta\varphi}\right)^2$$

$$= V - \tfrac{1}{2}R + \tfrac{1}{2}\vec{D}\phi\,\vec{D}\phi$$

GENERAL CASE (INCLUDING GRAVITY)

ANSATZ: DERIVATIVE EXPANSION

$$S = \int d^3x\left[U(\varphi) + M(\varphi)\vec{D}\varphi\vec{D}\varphi + \Phi(\varphi)R + \dots\right]$$

$$\Rightarrow \quad V(\varphi) + \tfrac{1}{2}(\partial_\varphi U)^2 - \tfrac{3}{4}U(\varphi)^2 = 0$$

+ EQ'S FOR $M(\varphi)$, $\Phi(\varphi)$

$$\Rightarrow \quad U(\varphi) = -2H(\varphi)$$

SLOWLY EVOLVING φ

$\rightarrow$ EVOLVING H

$\rightarrow$ DIV DEPENDS ON POSITION OF BNDY

COMMENTS

1) $U(\varphi)$, $M(\varphi)$, $\Phi(\varphi)$ ARE ALL DIVERGENCES (HIGHER ORDERS IN DERIVATIVE EXPANSION FINITE AS $z\to 0$)

2) FUNCTIONAL DEPENDENCE ON φ IS GENERAL – NOT LIMITED TO HOMOGENEOUS BACKGROUNDS.

RENORMALIZED H-J FUNCTIONAL

GENERAL DIVERGENCES

$$S_{loc} = \int_{z_0} d^3x \left[U(\varphi) + M(\varphi)\vec{\partial}\varphi\vec{\partial}\varphi + \Phi(\varphi) R \right]$$

$\Rightarrow$

$$S_{ren} = S - S_{loc}$$

IS FINITE. INTERPRETATIONS:

1) S_{ren} IS ACTION RELATIVE TO (DIVERGENT) BACKGROUND ACTION

$\rightarrow$ H-J FUNCTIONAL OF FLUCTUATIONS

2) IN LOCAL FIELD TH'Y WE CAN (MUST) SUBTRACT LOCAL COUNTERTERMS TO CANCEL UV DIVERGENCES

IF INFLATION DUAL TO LOCAL 3D QFT

THE IR DIV'S OF GRAVITY ~ UV DIV'S OF QFT.

$\rightsquigarrow$ HOLOGRAPHY.

GRAVITATIONAL BACKREACTION

PROPER SYSTEM TO CONSIDER: SCALAR + GRAVITY

PERTURBED METRIC (LONGITUDINAL GAUGE)

$$ds^2 = a(\tau)^2\left(-(1+2\Phi)d\tau^2 + (1+2\Psi)\,d\vec{x}^2\right)$$

$\Rightarrow$ 3 SCALAR FIELDS $\Phi, \Psi, \delta\varphi$

2 CONSTRAINTS (EOM OF GAUGEFIXED METRIC COMPONENTS $g_{\tau i}$, $g_{ij}\ (i \neq j)$)

$\Rightarrow$ ONE NET EFFECTIVE SCALAR.

$$\zeta = \delta\varphi - \frac{\varphi'}{\mathcal{H}}\Psi$$

($\exists$ NICE GEOMETRIC INTERPRETATION)
MUKHANOV

THIS SCALAR IS FIELD OF REAL INTEREST

THE WAVE EQUATION

LINEARIZING EINSTEIN EQUATIONS (HARD WORK) GIVE E.O.M.

$$\zeta'' + 2\left(\mathcal{H} - \frac{\varphi''}{\varphi'}\right)\zeta' - \vec{\nabla}^2\zeta + 2\left(\mathcal{H}' - \mathcal{H}\frac{\varphi''}{\varphi'}\right)\zeta = 0$$

DIFFICULT TO SOLVE FOR GENERAL $V(\varphi)$ (WHICH ENTERS THROUGH BACKGROUND $\mathcal{H}, \varphi$)

SLOW ROLL INFLATION

$$\epsilon = \frac{1}{2}\left(\frac{\partial_\varphi V}{V}\right)^2 \ll 1$$

$$\eta = \frac{\partial_\varphi^2 V}{V} \ll 1$$

THEN

$$\zeta_k = |\tau|^{\frac{1}{2}+\eta-6}\, \mathcal{H}_\nu(|k\tau|)$$

$$\nu = \tfrac{1}{2} - \eta + 3\epsilon$$

HANKEL FCT.

THE QUADRATIC ACTION

THE H-J FUNCTIONAL IS THE ON-SHELL ACTION LESS COUNTER TERMS

$$S_{ren} = \int \underset{\frac{1}{2}R^{(4)} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)}{\underset{\parallel}{\mathcal{L}}} - S_{loc}$$

QUADRATIC FLUCTUATIONS

$$= \int \mathcal{L}_{quad} - S^{quad}_{loc}$$

USE E.O.M. INTEGRATE BY PARTS

$$= -2\int d^3k \left(\frac{\mathcal{H}}{\varphi'}\right)^{-2} \frac{ik^3}{H^2}(k\tau_0)^{6\epsilon-2\eta} \varrho_k \varrho_{-k}$$

THIS IS RENORMALIZED ACTION WRITTEN IN TERMS OF GAUGE-INVARIANT VARIABLES

THE FLUCTUATION SPECTRUM

$$\Psi[\varrho] = e^{iS_{ren}[\varrho]}$$

$$\Rightarrow \langle \varrho_{\vec{k}} \varrho_{-\vec{k}} \rangle = \int D\varrho\, \varrho_{\vec{k}} \varrho_{-\vec{k}} |\Psi[\varrho]|^2$$

$$= \left(\frac{H}{\dot{\phi}}\right)^2 \frac{H^2}{4k^3}(k\tau_0)^{2\eta-6\epsilon}$$

POWERSPECTRUM

$$P_\varrho = 4\pi\left(\frac{k}{2\pi}\right)^3 \langle \varrho_{\vec{k}} \varrho_{-\vec{k}} \rangle = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2 (k\tau_0)^{2\eta-6\epsilon}$$

SPECTRAL INDEX

$$P_\varrho \sim k^{n_s-1} \Rightarrow \quad n_s - 1 = 2\eta - 6\epsilon$$

AMPLITUDE ENHANCEMENT

$$\frac{P_S}{P_T} = \left(\frac{H}{\dot{\phi}}\right)^2 = \frac{1}{\epsilon} \gg 1$$

CUT OFF (IN) DEPENDENCE

$$P_Q = \underbrace{\left(\frac{H}{\dot\phi}\right)^2 \left(\frac{H}{2\pi}\right)^2}_{\text{BACKGROUND DATA}} (k\tau_0)^{2\eta - 6\epsilon}$$

CUT OFF

BACKGROUND DATA SATISFIES FRW EQUATION ⇒ MILDLY DEPENDENT ON TIME

SLOW ROLL

⇒ ALL τ_0 DEPENDENCE CANCELS

REMOVING CUTOFF GIVES FINITE RESULT.

(BUT LINEAR PERT TH'Y AND/OR SLOW ROLL APX MAY NOT BE VALID THROUGHOUT)

RUNNING (PHYSICAL) CUTOFF $\tau_0 = \frac{1}{aH}$ (HORIZON CROSSING) MAY BE MORE APPRIATE (SUM'S LARGE LOG'S)

PHYSICAL OBSERVABLES

FOUNDATIONAL PROBLEM IN QUANTUM DE SITTER:

WHAT ARE PHYSICAL OBSERVABLES?

E.G. 

X'S SPACELIKE SEPARATED

⇒ CORRELATORS META-OBSERVABLES

WITTEN

END OF INFLATION

⇒ MODES REENTER HORIZON

⇒ BECOME PHYSICALLY OBSERVABLE

"WE ARE METAOBSERVERS"

DANIELSSON

PRECISE CONSTRUCTION (IE DETAILS OF REHEATING ETC) DOES NOT SEEM IMPORTANT.

THE HOLOGRAPHIC RG EQUATION

S_{HJ} SOLVES H-J EQ ; S_{loc} SOLVES IT TOO

$\Rightarrow$ EQ FOR $S_{REN} = S - S_{loc}$

$$\left(2 g_{ij} \frac{\delta}{\delta g_{ij}} + \beta \frac{\delta}{\delta \varphi}\right) S_{ren}(g_{ij}, \varphi) = 0$$

$$\beta \equiv -2 \frac{\partial_\varphi U}{U} = -\frac{\dot{\phi}}{H}$$

QUADRATIC ORDER

$$\left(a \frac{\partial}{\partial a} + \beta_0 \frac{\partial}{\partial \varphi} + 2\gamma\right) S^{quad}(a, \varphi) = 0$$

$$\gamma = \partial_\varphi \beta = 2\epsilon - \eta$$

$$\beta_0^2 = 2\epsilon$$

SPECTRAL INDEX

$$n_s = 1 - \beta_0^2 - 2\gamma = 1 + 2\eta - 6\epsilon$$

SCALING OF CORRELATORS FROM CALLAN-SYMANZIK EQUATION !

HOLOGRAPHIC INTERPRETATION

COSMOLOGICAL EVOLUTION ~ RG FLOW

STROMINGER

AS TIME MOVES FORWARD ENTROPY INCREASES

$\longrightarrow$ MOTION TOWARDS UV

("INTEGRATING IN" D.O.F.)

SO MOTION BACK TOWARDS INFLATIONARY EPOCH $\rightarrow$ MOTION TOWARDS IR

SO WE ARE NOT SURPRISED THERE IS A SCALE INVARIANT FIXED PT IN OUR PAST !

PREDICTION FIXED PT A STABLE ATTRACTOR

$\Rightarrow$ $(k z)^{n_s - 1} \to 0 \quad z \to -\infty$

$\Rightarrow$ SPECTRUM RED $\quad n_s < 1$

(LOG GROWTH AT LARGE SCALES)

A TOY MODEL

VISION: COMPUTE $n_s, n_T, \ldots$ FROM 3D DUAL RATHER THAN $V(\varphi)$

TOY MODEL

$$S = \int d^3x \left[+ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{6!} g \varphi^6 \right]$$

CLASSICALLY SCALEINVARIANT BUT φ^6 IRRELEVANT IN IR_p → JUST WHAT WE WANT
DUE TO QUANTUM

$$\left(\frac{\partial}{\partial \ln r} + \beta \frac{\partial}{\partial g} - 2\lambda \right) \langle \varphi^6 \varphi^6 \rangle = 0$$

$$\lambda = \frac{5}{3} \frac{g}{16\pi^2} > 0$$

$$\Rightarrow \quad G_{\varphi^6} \sim \left(\frac{\mu}{k} \right)^{2\lambda} \sim (kz)^{-2\lambda}$$

$$\Rightarrow \quad n_s = 1 - 2\lambda$$

SUMMARY

DEVELOPED TH'Y OF CMB IN ANALOGY WITH ADS/CFT. SUGGESTIVE

FOCUS ON IR DIV'S, FULLY CHARACTERIZED. INTERPRETATION? UNIVERSALITY?

FUTURE DIRECTIONS

COMPLETE RG ANALYSIS
– HIGHER PT FCTS, BEYOND SLOWROLL
IN PROGRESS

CONNECTION TO STRING TH'Y ?