Entropy & Holography for (A)dS Flux Compactifications

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KKLT
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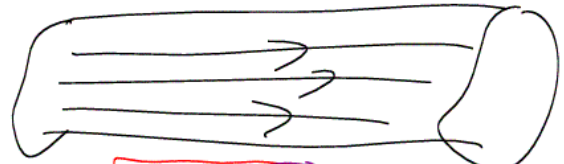
Flux compactifications



X

$\boxed{\text{(A)dS}}$ → stringy models of dark energy

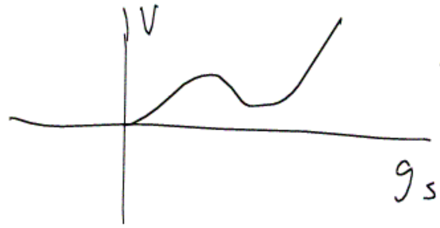
$$X = \frac{CY}{\Omega}\Big|_{flux} \quad \text{or} \quad \left(\frac{T^D}{\Omega, \Gamma}\right)\Big|_{flux}$$

cf S. Giddings talk

(Acharya) (GKP) KKLT — ES, MSS

* Low-energy SUSY, 4d; exhibit warping (RS); LED-compatible
* non-SUSY; noncritical; nongeometrical → non-LED

Moduli fixed. e.g. dilaton



$$\Lambda \sim \frac{g_s^{\frac{4}{d-2}}}{l_d^2}\left(a - b g_s + c g_s^2\right)$$

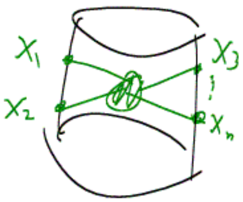
H_{NS}^2, D-$\bar{D}$ · orientifolds · F_{RR}

* Question: what is the "holographic dual" (if any) of these flux compactifications?

– Not obtained by near horizon limit of brane system

AdS case:

- Expect a CFT dual since the AdS/CFT dictionary ...



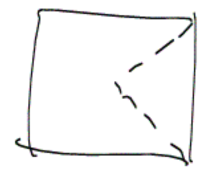
$$\leftrightarrow \langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_n) \rangle$$

Feynman diagrams in AdS_d

conformally invariant QFT_{d-1} Greens functions

... applies without reference to a larger system from which the AdS/CFT arises via a near horizon limit

dS case:



Gibbons - Hawking horizon entropy

$$S = \frac{L_{dS}^{d-2}}{4\, l_d^{d-2}}$$

suggests a microphysical stat mech origin which may arise from a holographic dual

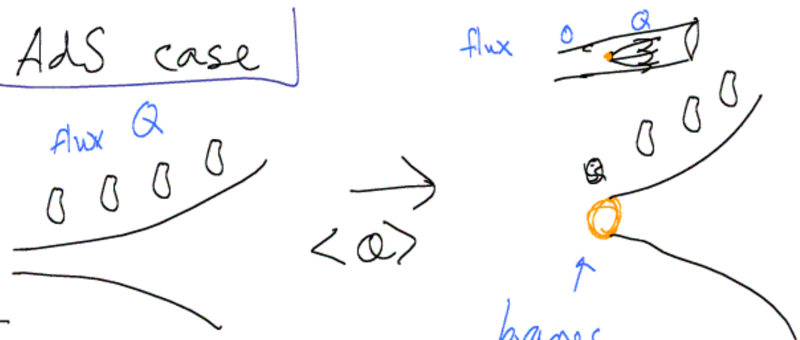
cf previous ideas: dS/CFT, Strominger...; novel Q. grav. Banks, Fischler, Paban...; cf F. Larsen talk

In cosmology, when we ask about the "nature of dark energy" we usually refer to its eqn of state and corresponding scalar field dynamics. Maybe we should consider also the G-H entropy etc.

So how do we find the putative duals?

Basic Strategy: The degrees of freedom of the system become much more manifest when we deform it to its Coulomb Branch:

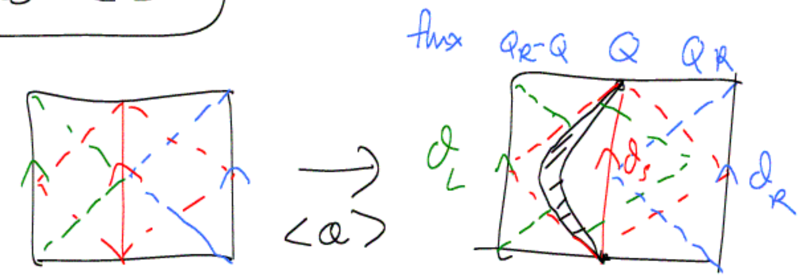
• AdS case



horizon of Poincare patch (IR from CFT point of view)

branes sourcing same flux Q contain n_B degrees of freedom

• dS case



Branes contain n_B degrees of freedom

Concrete questions:

1. Does n_B account for the expected entropy?
2. What is the field content and dynamics of the candidate duals, & does it match the gravity side?
3. What is the duality dictionary (if any) in the dS case?

Let's start with 1, on which we have the most concrete results so far. Then I'll discuss some aspects of 2 & 3, still mostly in progress.

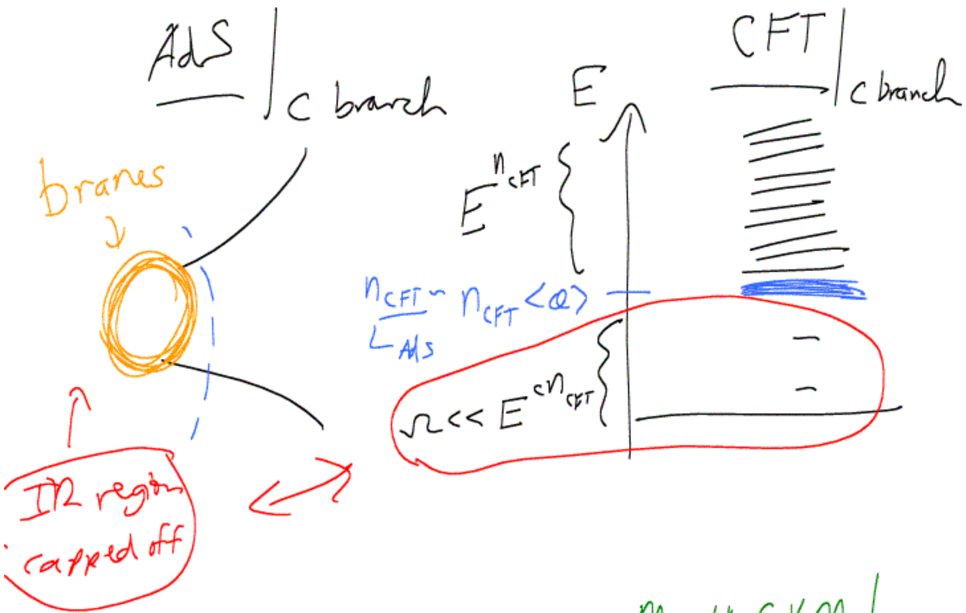
AdS case:

Bekenstein-Hawking entropy in the context of AdS/CFT (Susskind/Witten)

AdS_d	CFT_{d-1}
$A \sim \frac{L_{AdS}^{d-2}}{\delta^{d-2}}$ fits BH of mass $\boxed{\frac{n_{CFT}}{L_{AdS}\,\delta}}$	$0 \le E \le \boxed{\frac{1}{\delta}\frac{1}{L_{AdS}}\, n_{CFT}}$
IR cutoff	UV cutoff
$S \sim \frac{A}{l_d^{d-2}}$	$S \sim n_{CFT}$ ← # of d.o.f. of CFT
e.g. $AdS_5 \times S^5$	$\mathcal{N}=4$ SYM
$S \sim \frac{[(g_s N)^{\frac{1}{4}} l_s]^8}{g_s^2 l_s^8} = N^2$	$n_{CFT} = N^2$ (even in strong coupling regime)

$*$ n_{CFT} degrees of freedom per L_{AdS}^{d-2} area $=$ 1 per Planck area

We can apply this analysis out on the Coulomb branch (Poincare patch version: flat (d-1)-dimensional slices)



$AdS_5 \times S^5 |_{c\ branch}$

$n_B \sim N^2$ (# of d.o.f. on branes)

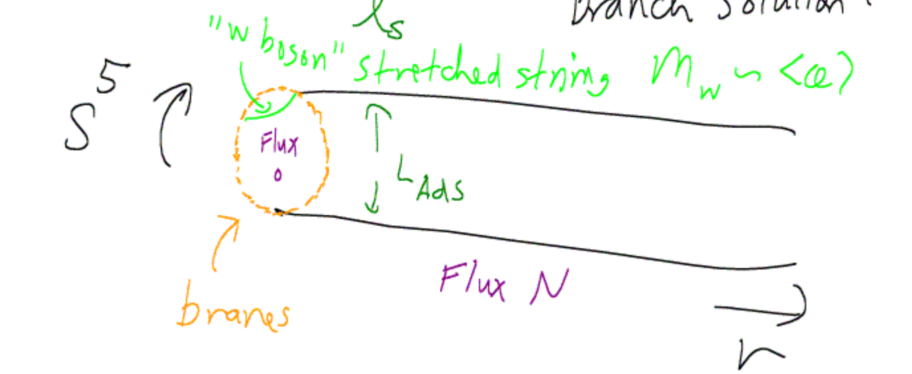
$\mathcal{N}=4$ SYM $|_{C\ branch}$

$n_{CFT} \sim N^2$

* brane degrees of freedom at mass scale of VEV account for of order the full BH entropy

Energetics:

$\langle\alpha\rangle = \frac{L_{AdS}}{l_s^2}$ from KLT Coulomb branch solution:



We are interested in counting the degrees of freedom on the branes at energy $\langle\alpha\rangle$. These are the (elementary, electric) degrees of freedom from the "W boson" stretched strings.

$n_B \sim N^2 \sim n_{CFT}$ ✓

Can the brane degrees of freedom account for the entropy in the more generic flux compactifications?

Yes – will shortly do the counting

First – heuristic arguments & issues:

Yes: Trade the flux for branes in the IR region. The Solution will again get capped off in the IR, and only cross over from $\Omega(E) \ll E^{c n_{CFT}}$

to $\Omega(E) \sim E^{c n_{CFT}}$

at the scale $E_{c\,brane} \sim n_{CFT} \langle \phi \rangle$ associated with the branes and their degrees of freedom

But there is a naive puzzle:

Q units of flux translates into Q branes and therefore "$n_B = Q^2$ degrees of freedom"

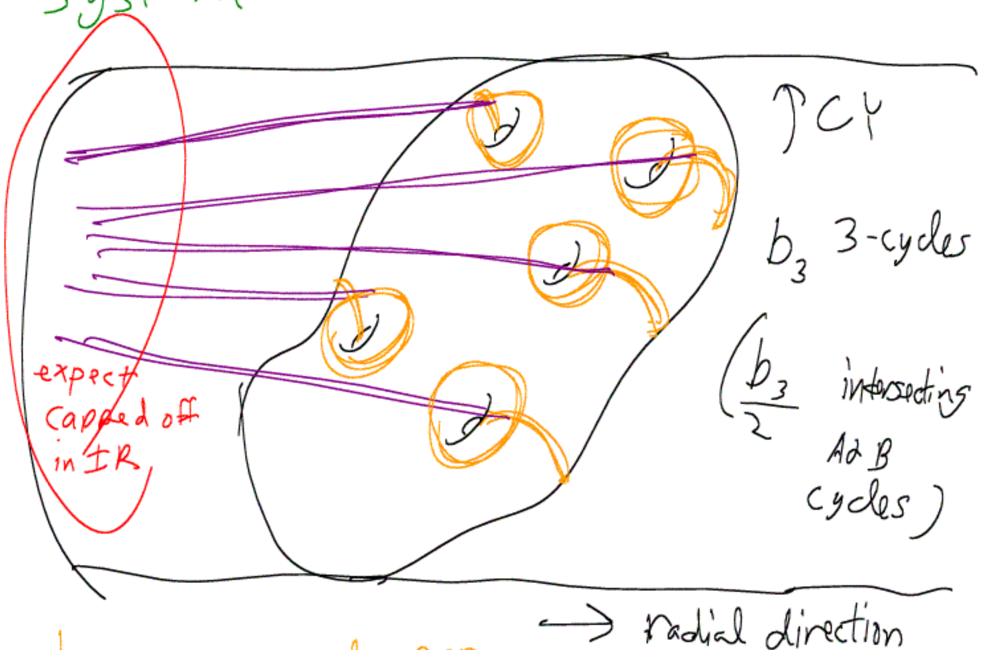
But by the Bousso-Polchinski mechanism, we can get

$$S \sim \frac{Q^{2b_3}}{b_3!} \gg Q^2$$

if $L_{(A)ds} \gg l_s$.

"So the branes account for the entropy only if we distribute their degrees of freedom more finely than n_B per AdS area (n_B per string area works) or if $L_{AdS} \sim l_s$ (correspondence point)"

In fact n_B can be $\gg Q^2$ via string junctions and webs in the branes coming from the KKLT system:



branes: wrapped D5Bs & NS5Bs → b_3 sets of (p,q) 5Bs

- Stretched 3-branes:

Gauss' law:

$$\underbrace{\frac{1}{2(2\pi)^4 g'^2}\int H\wedge F}_{\substack{|||\\ L\sim\chi}} = \underbrace{\frac{1}{4}}_{\substack{\int\\ \chi}}\left(N_{O3}-\cancel{N_{\overline{O3}}}\right) - N_{D3} + \cancel{N_{\overline{D3}}}$$

for $N_3=0$ in the flux vacuum

requires $N_3 \sim \chi \sim L$ D3-branes in the region with no flux (i.e. the IR region where we traded the flux for branes). These 3-branes end on the 5-branes wrapped on 3-cycles.

We want to know the number of degrees of freedom n_B on the branes which are elementary at mass scale $\frac{L_{CY}}{l_s^2} \sim \frac{b_3}{l_s}$ (taking CY cycles to be of order l_s because we want to study the $L_{CY} \ll L_{(A)ds}$ highly-(Bousso/Polchinski)-tuned situation.)

Junction with 1 endpoint on each bunch of branes: mass $\sim b_3/l_s$

3Bs: $\frac{L}{(b_3/2)}$ ways to end per bunch, $\frac{b_3}{2}$ bunches

5Bs: $N_5 \sim \frac{1}{2}\sum_{i=1}^{b_3}(|Q_i| + |N_i|)$ $\quad Q_i \sim N_i \sim \frac{N_5}{b_3}$

$$L \sim N_{D3} \sim \sum_{i=1}^{b_3}(b_i Q_i^2 + a_i N_i^2) \sim b_3\left(\frac{N_5}{b_3}\right)^2$$

(flux kinetic energy cancels against orientifold tensions in no-scale (GKP) solutions entering KKLT)

$N_5 \sim \sqrt{b_3}\, L^{\frac{1}{2}}$

$\frac{N_5}{b_3}$ ways to end per bunch of 5Bs, b_3 bunches

$$\Rightarrow n_B \sim n_3\, n_5$$

$$= \left(\frac{L}{b_3}\right)^{\frac{b_3}{2}} \left(\frac{L^{\frac{1}{2}}}{\sqrt{b_3}}\right)^{b_3} = \left(\frac{L}{b_3}\right)^{b_3}$$

drop "$O(1)$" factors

Now using $L \sim \sum(Q_i^2 + N_i^2)$ $\sim (\text{radius})^2$ in flux space, this agrees with the entropy predicted by the Bousso-Polchinski tuning (also with the more systematic treatment of Ashok & Douglas)

Basic idea:

$$\Lambda \sim \frac{g_s^4}{\ell_4^2}\left[\sum_{i=1}^{b_3} \frac{a_i N_i^2}{g_s^2} + c_i Q_i^2 - \frac{L(N,Q)}{g_s}\right]$$

moduli-dependent and thus Q_i, N_i-dependent

If the flux quantum #s become too large we lose control. Suppose we bound it via the constraint

$$\sum_i \left(\gamma_i N_i^2 + \gamma_i Q_i^2\right) < R_{max}^2$$

with R_{max} chosen so that no moduli take extreme values, and with γ_i and γ_i of order 1. Then if each choice of flux corresponds to $\mathcal{O}(1)$ vacua, there are $N_{vac} \sim \frac{R_{max}^{2b_3}}{b_3!}$ vacua

For nonextreme values of the moduli, $a_i \sim c_i \sim \mathcal{O}(1)$. So in the GKP vacuum $L \sim R^2 < R_{max}^2 \equiv L_{max}$

So rewrite

$$N_{vac} \sim \frac{L_{max}^{b_3}}{b_3!} \sim \left(\frac{L_{max}}{b_3}\right)^{b_3}$$

Ashok & Douglas: integration over moduli yields same scaling

*G. Moore: why integrate? The vacua are dense

If the vacua are distributed roughly uniformly between $\Lambda \sim -\frac{1}{\ell_4^2}$ and $+\frac{1}{\ell_4^2}$, this predicts $\Lambda_{min} \sim \frac{1}{\ell_4^2 N_{vac}}$

or an entropy as large as $S \sim N_{vac} \sim \left(\frac{L}{b_3}\right)^{b_3}$

This prediction is saturated by the above junction count as we saw.

It is really a lower bound on both sides.

In fact there are more vacua to be expected in the KKLT system than have been considered so far - including dS solutions without the antibranes

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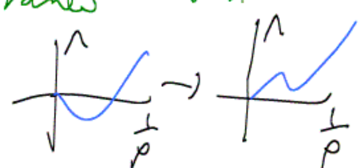
At the no-scale level,

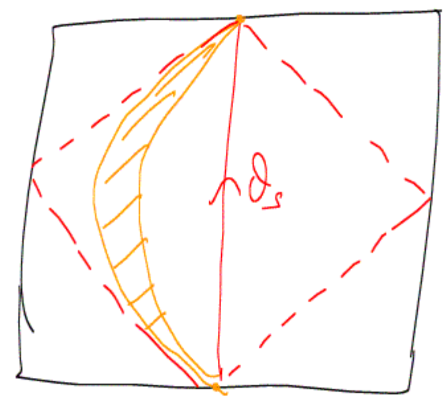
$$\Lambda_{ns} \sim e^{K} \sum_{\substack{\text{complex}\\ \text{structure}\\ \text{\& dilaton}}} |D_i W|^2$$

KKLT took the GKP solution $D_i W = 0$ and later added antibranes which contribute $\Delta\Lambda_{\overline{D3}} \sim \frac{1}{(\rho-\bar{\rho})^3}$



If we consider a $\Lambda > 0$ solution of $\partial_i \Lambda_{ns} = 0$, it scales with ρ and g_s like $\Delta\Lambda_{\overline{D3}}$. So we can avoid the antibrane with such solutions.

Although I explained it in the AdS case, the result also applies to the Gibbons-Hawking entropy on the branes as seen by a static observer in dS!



O_s sees redshift of string scale as go toward horizon, so the junction states on the branes become excited in the canonical ensemble at ~ cutoff temp ~ $\frac{1}{l_s}$.

On to question (2): What is the content and dynamics of the dual theory?

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in progress

On the Coulomb branch we see

$$\prod_{i=1}^{b_3} U(J_i) \times \prod_{i=1}^{b_{3/2}} U(Q_A N_B - N_A Q_B)$$

with bifundamentals from strings
multifundamentals from junctions

Couplings: Kinetic terms, Induced CS terms on classical moduli space,
. . . potential on Coulomb branch

We are still working out the details of the duals. There are many concrete aspects to check.

RG flow: gravity side moduli fixing $\{ D_I W = 0 \}$

$\Updownarrow$

QFT side $\{ \beta_I = 0 \}$

$I \leftrightarrow$ {complex structure moduli & dilaton}

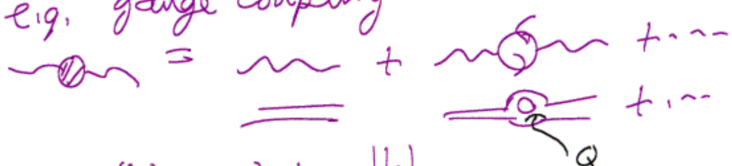
$\Updownarrow$

$$I \leftrightarrow \left\{ \frac{V_{C_I}}{g_{5B}^2}, \ \frac{1}{g_{3B}^2} \right\}$$

In particular, we should see a discretuum of 3d CFTs for at least the AdS case

3d CFTs are plentiful and have the right general structure in the perturbative regime of 3d gauge theory, via a large-N_f expansion (Appelquist et al, cf 4d Banks-Zaks fixed pts which also extended into the strong coupling regime)

e.g. gauge coupling



$$\Gamma^{(2)} \sim k^2 + g|k|$$

$$\Rightarrow \underset{\text{renormalized coupling}}{\bar{g}} \sim \frac{g}{k}\,\frac{1}{1+Q\frac{g}{|k|}} \xrightarrow{k\to 0} \frac{1}{Q}$$

driven strong in IR by bare dimension; with enough matter $N_f \gtrsim N_c$ $\quad \beta_{tree}+\beta_{1\text{-loop}}=0$

The scaling $\bar{g} \sim \frac{1}{Q}$ correlates nicely with a similar scaling from the gravity side at large RR flux quantum #Q : g_s dependence

$$\Lambda \sim \frac{g_s^4}{l_4^2}\left(\frac{aN^2}{g_s} - bQN + cQ^2\right)$$

$$\Lambda' = 0 \Rightarrow g_s \sim \frac{N}{Q}$$

If scale up RR flux, the perturbative analysis on D-branes improves, and get $g_s \sim \frac{1}{Q}$ on D-branes, $\bar{g} \sim g_s$ so scaling agrees $\bar{g} \sim g_s \sim \frac{1}{Q}$

Going back to the strongly coupled case:

Dualities: Monodromies of CY 3-cycles

$\updownarrow$?

dualities of QFTs

In general we need to understand monodromy-covariance of our putative dual theories ...

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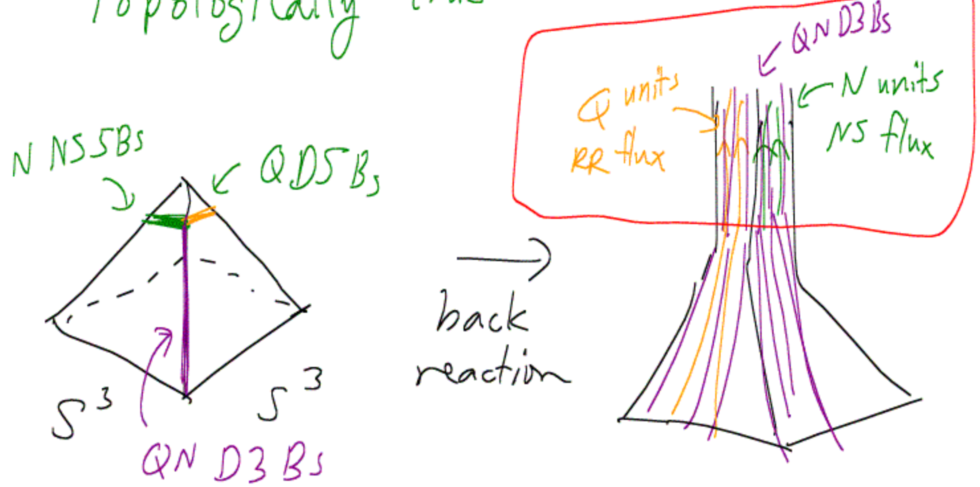
* what is the meaning (if any) of the Gauss' law constraint

$\int H \wedge F \sim N_{D3} - N_{\overline{D3}} + N_{D3} - N_{\overline{D3}}$ for the set of CFT_3 duals?

A simpler somewhat related problem:

$\exists$ $\boxed{S^3_Q \times S^3_N \times AdS_4}$ solution to the equations of motion with $N_3 \sim QN$ AdS_4-filling D3s

Topologically this can come from



↑ Field content again visible on branes, still analyzing the theory ...

③ What can we say about the duality dictionary (if any) in the dS case?

Again use Coulomb branch deformation (and above entropy agreement) for clues.

We have worked out the geometry & thermodynamic & causal structure of the Coulomb branch

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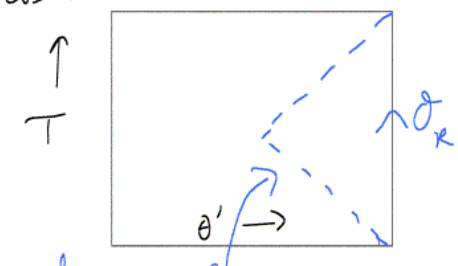
i) DS causal structure, thermodynamics

$$ds^2 = \frac{1}{\cos^2 T}\left(-dT^2 + d\theta'^2 + \sin^2\theta' \, d\Omega_{d-2}^2\right)$$

dS Penrose diagram



static causal patch

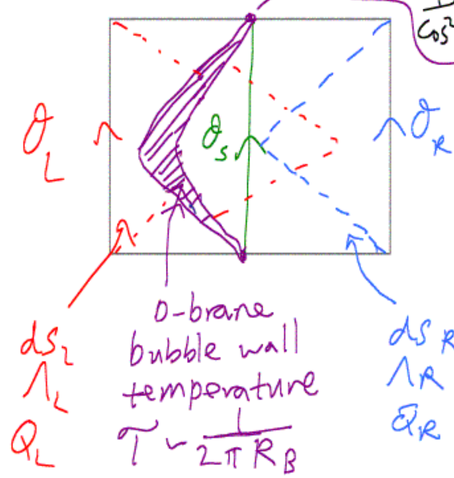
hot horizon, area $A \sim R_{dS}^{d-2}$

suggest entropy $S \sim \frac{R_{dS}^{d-2}}{l_d^{d-2}}$

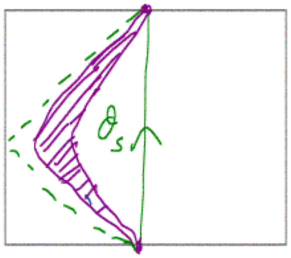
$\downarrow \langle Q \rangle$

identification $\frac{1}{\cos^2 T_L} = 1 + \frac{R_R^2}{R_L^2}\left(\frac{1}{\cos^2 T_R} - 1\right)$

DS Penrose diagram

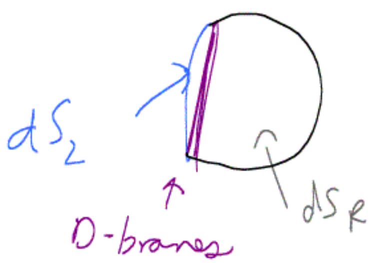


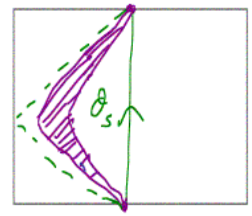
* (2) For $\mathcal{O}_s$:



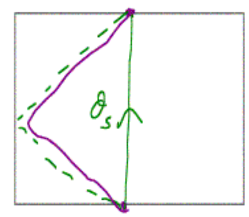
$\mathcal{O}_s$ is an observer staying a fixed proper distance from the branes. $\mathcal{O}_s$ has a static (time-independent) coordinate system covering its causal patch.

Spatial slice:





* As we deform the system back to dS_R, the D-branes approach (a patch of) the horizon for all time



Suggests open string theory (or its low energy QFT limit) living on the horizon

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As we saw above, we can saturate the expected entropy on our branes even for $L_{dS} \gg L_{CY} \gtrsim l_s$

Questions: 1) "Doesn't the dual have to live at the boundary like in AdS/CFT?"

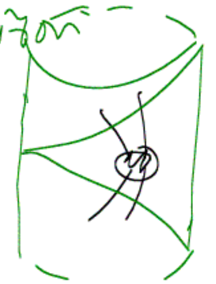
Answer: not necessarily

- the dual doesn't live anywhere - it is dual to the whole space.

So the question is how observables are defined - do they have to be injected in from the boundary?

Answer: not necessarily

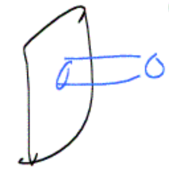
- Even in AdS/CFT, in the (geodesically incomplete) Poincare patch, off-shell excitations can enter the horizon -



How can you define the observables at the horizon - it fluctuates!

Answer: the fluctuations of the horizon are reflected in the off-shell graviton modes.

It is well-known how D-branes can inject off-shell closed string modes into the bulk of spacetime, at a finite point in spacetime consistent with worldsheet BRST invariance



$$\frac{1}{L_0+\bar{L}_0}|B\rangle = |C\rangle$$

$$Q_B|C\rangle \neq 0$$

So if the horizon acts like a D-brane, with open string (or QFT) degrees of freedom on it, as suggested by the D-Sitter Coulomb branch deformation, then it could well inject the requisite off-shell gravitons in a similar way to the way ordinary D-branes do.

This "horizon holography" picture if correct may not be mutually exclusive with a global dS/CFT or eternal inflation metaobserver type of picture (again as in AdS/CFT).

cf Sachs et al

→ Question: Spacelike D-Sitter branes?

I think the flux models and related brane systems are likely to guide us toward the right dictionary.