Is Cosmic Acceleration Telling us Something about Gravity?

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Outline

- How does the data constrain the dark energy equation of state?
- How strange might the phenomenon driving acceleration be (w_Q<-1)?
- How are these ideas constrained by the coupling to gravity?
- Can changing gravity yield new approaches to cosmic acceleration.
- Conclusions.

Acknowledgements

Most of the talk is based on

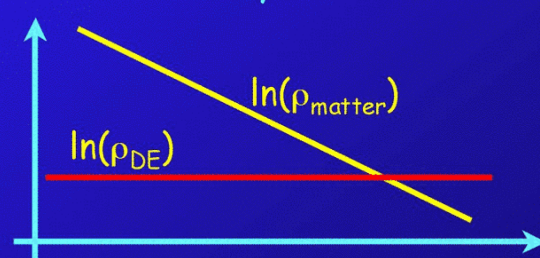
- *"The State of the Dark Energy Equation of State"*, Alessandro Melchiorri, Laura Mersini, Carolina Odman & M.T., **Phys.Rev.D68:043509,2003 [astro-ph/0211522]**
- *"Can the Dark Energy Equation of State Parameter w be Less than -1?"*, Sean M. Carroll, Mark Hoffman & M.T., **Phys.Rev.D68:023509,2003 [astro-ph/0301273]**
- *"Is Cosmic Speed-Up Due to New Gravitational Physics?"*, Sean M. Carroll, Vikram Duvvuri, M.T. and Michael S. Turner, **[astro-ph/0306438]**

Our Problems

Really two problems associated with dark energy.

1. Its magnitude - we need to show why dark energy is the (ridiculous, as we'll soon see) size it is today.
2. The coincidence problem.

$$\frac{\rho_{\rm matter}}{\rho_{\rm DE}} \propto a^{-\alpha} \quad \text{with} \quad \alpha > 1$$



So even if dark energy wasn't important in the past, it will dominate in the future. Why do we observe it at a time when it is comparable to matter - why now?

Remember, we could have lived easily with no dark energy.

What Might Explain These?

Theoretical Description

Evolution of the universe governed by Einstein eqns

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3M_P^2}\rho$$

The Friedmann equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3M_P^2}(\rho + 3p)$$

The "acceleration" equation

Parameterize different types of matter by equations of state: $p_i = w_i \rho_i$

When evolution dominated by type i, obtain

$$a(t) \propto t^{2/3(1+w_i)} \qquad \rho(a) \propto a^{3(1+w_i)} \qquad (w_i \neq -1)$$

Cosmic Accelelation

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3M_P^2}(\rho + 3p)$$

So, accelerating expansion means $p < -\rho/3$ or

$$\boxed{w_Q < -1/3}$$

$$a(t) \propto t^{2/3(1+w_i)} \qquad \rho(a) \propto a^{3(1+w_i)}$$

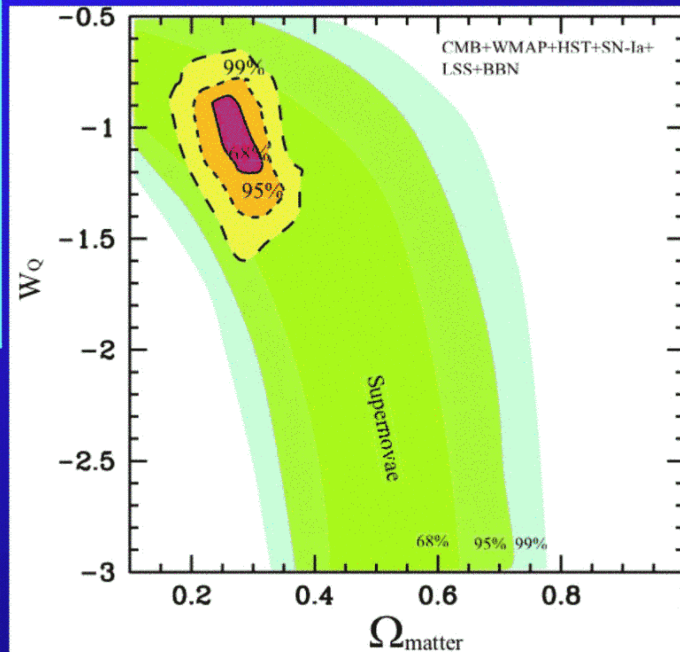
Three Broad Possibilities

	$-1 < w < -1/3$	$w = -1$	$w < -1$
Evolution of Energy Density	Dilutes slower than any matter	Stays absolutely constant (Λ)	Increases with the expansion!!
Evolution of Scale Factor	Power-law quintessence	Exponential expansion	Infinite value in a finite time!!

Data on w_Q

Basically measuring (luminosity) distance as fn of redshift.

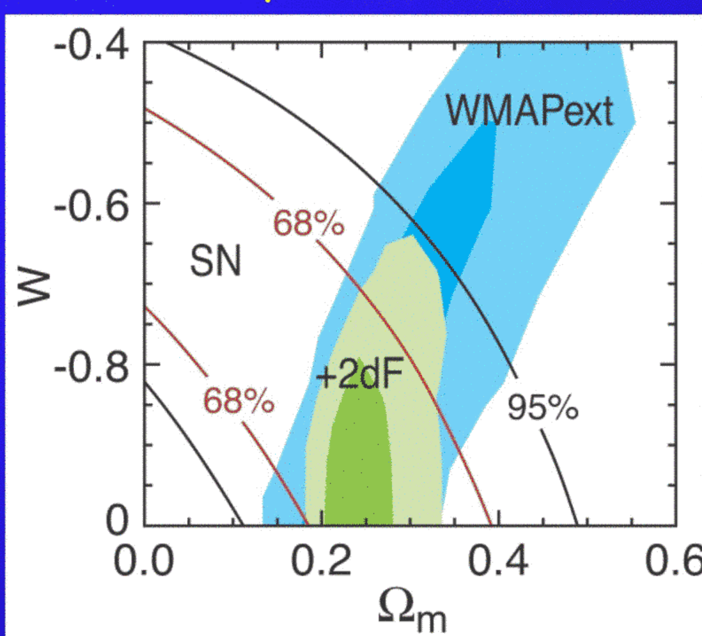
CMB+HST	$-1.65 < w_Q < -0.54$ $0.19 < \Omega_m < 0.43$
CMB+HST+BBN	$-1.61 < w_Q < -0.57$ $0.20 < \Omega_m < 0.42$
CMB+HST+BBN +SNIa	$-1.45 < w_Q < -0.74$ $0.21 < \Omega_m < 0.36$
CMB+HST+BBN +SNIa+2dF	$-1.38 < w_Q < -0.82$ $0.22 < \Omega_m < 0.35$



[From: Melchiorri, Mersini, Odman and M.T. (2002)]

Restricting to $w_Q > -1$

(this means imposing the null energy condition, for example, more about this soon)

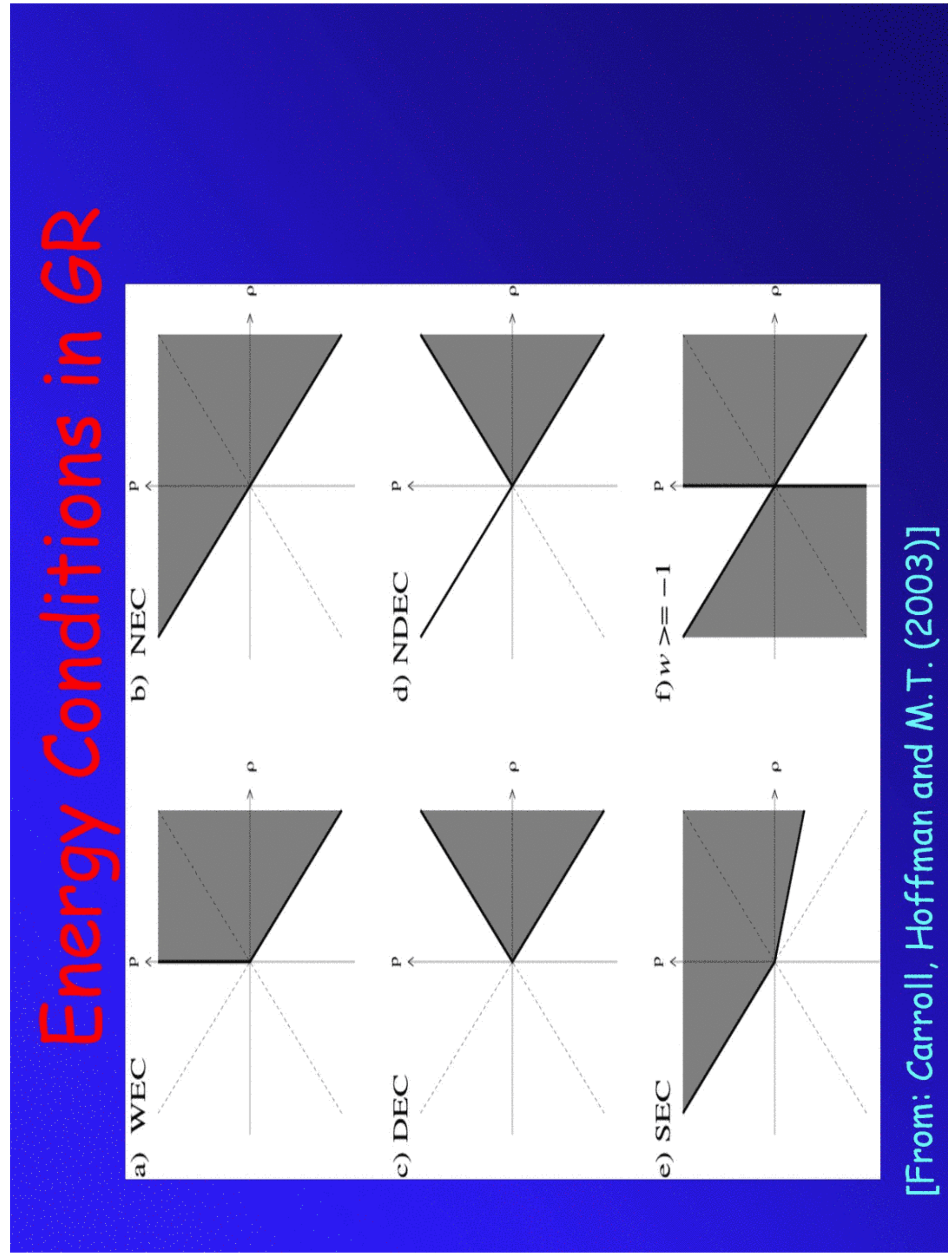


This analysis gives $w_Q < -0.78$

But as we've seen, Taking the data at face value leads to negative values allowed also

[From: Spergel et al. WMAP paper (2003)]


So, does it make sense
to consider values of
w_Q that are less than -1?


Energy Conditions in GR
a) WEC
b) NEC
c) DEC
d) NDEC
e) SEC
f) $w >= -1$
[From: Carroll, Hoffman and M.T. (2003)]

$w_Q < -1$?

What are theorists to make of the possibility of $w_Q < -1$ matter?

- Violates NEC (with positive energy): $T_{\mu\nu}N^\mu N^\nu < 0 \Rightarrow \rho_i + p_i < 0$
- One expects instabilities
- Are they there and are they fatal/irrelevant now in the universe?

Don't expect an instability just for a single field uncoupled to others. To investigate an instability have to consider a coupled system

A Toy Model (uncoupled)

Flip the kinetic terms for a real scalar field

$$L_\phi = -\frac{1}{2}(\partial_\mu \phi)\partial^\nu \phi - V(\phi) \quad \Rightarrow \quad w_\phi \equiv \frac{p_\phi}{\rho_\phi} = -\left[\frac{\dot{\phi}^2 + 2V(\phi))}{-\dot{\phi}^2 + 2V(\phi))}\right]$$

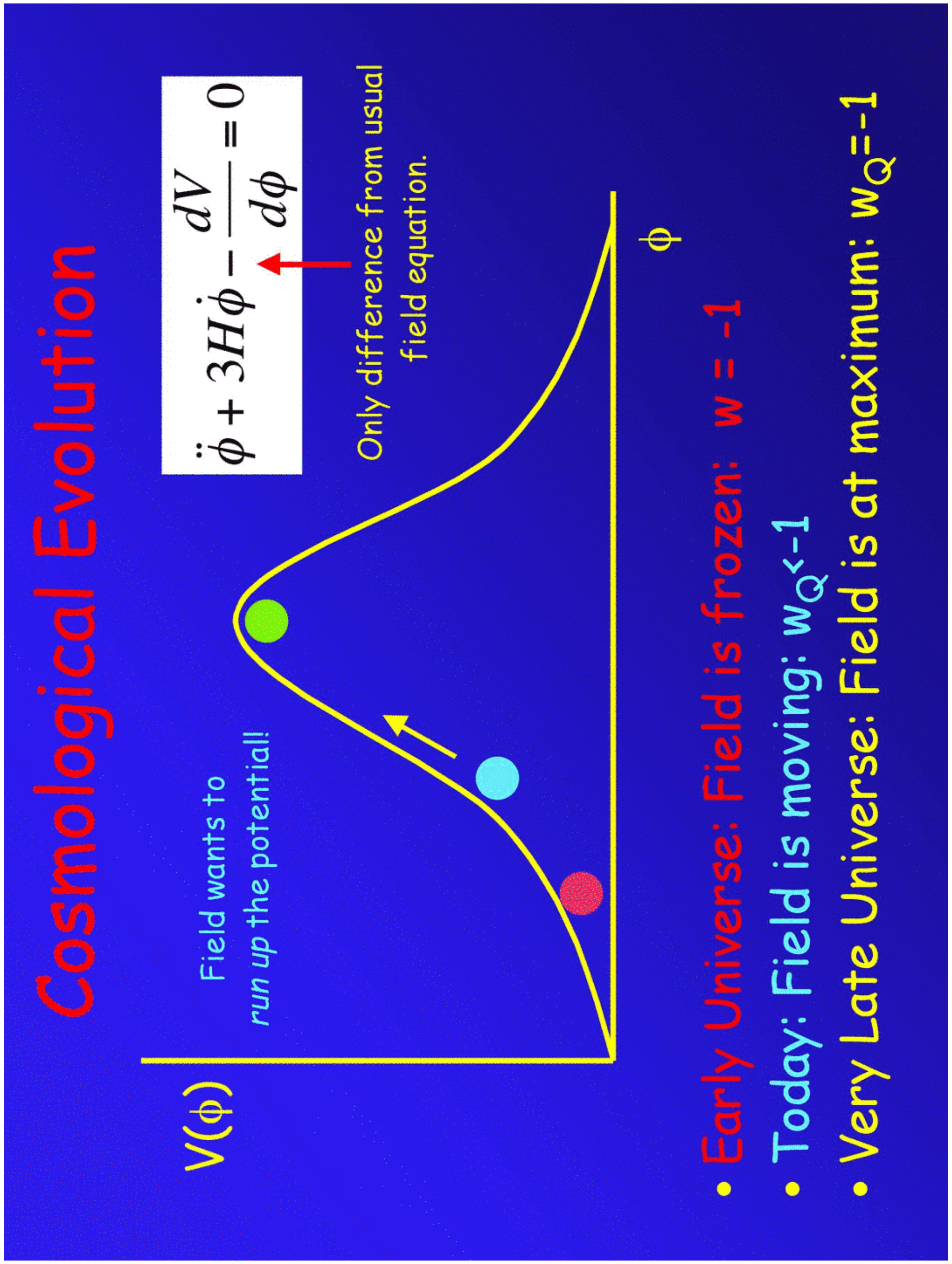
Caldwell; Schulz & White; Carroll, Hoffman & M.T.
Related to k-essence work of
Armendariz-Picon, Mukhanov & Steinhardt

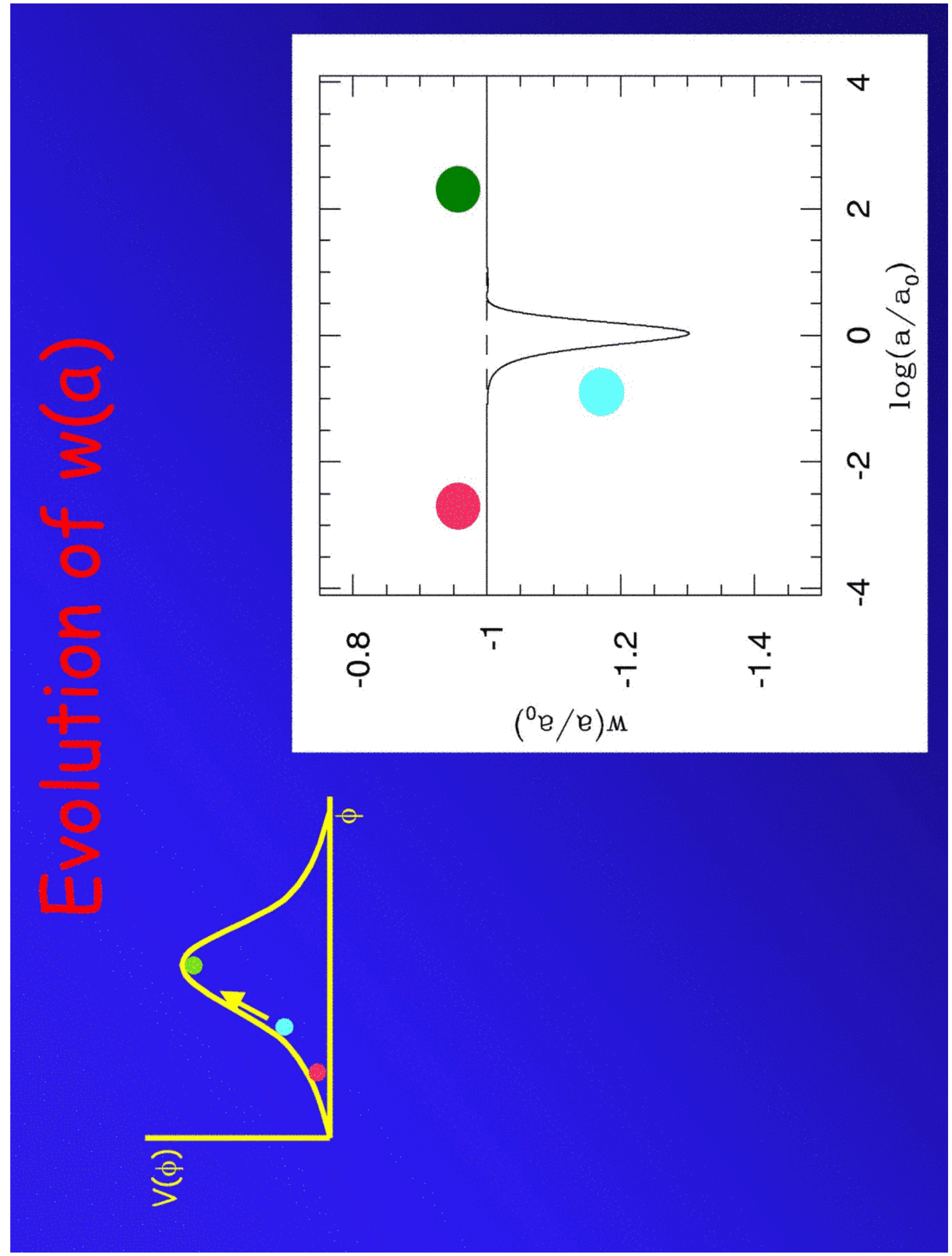
So $w_\phi < -1$ if we keep $V(\phi) > 0$

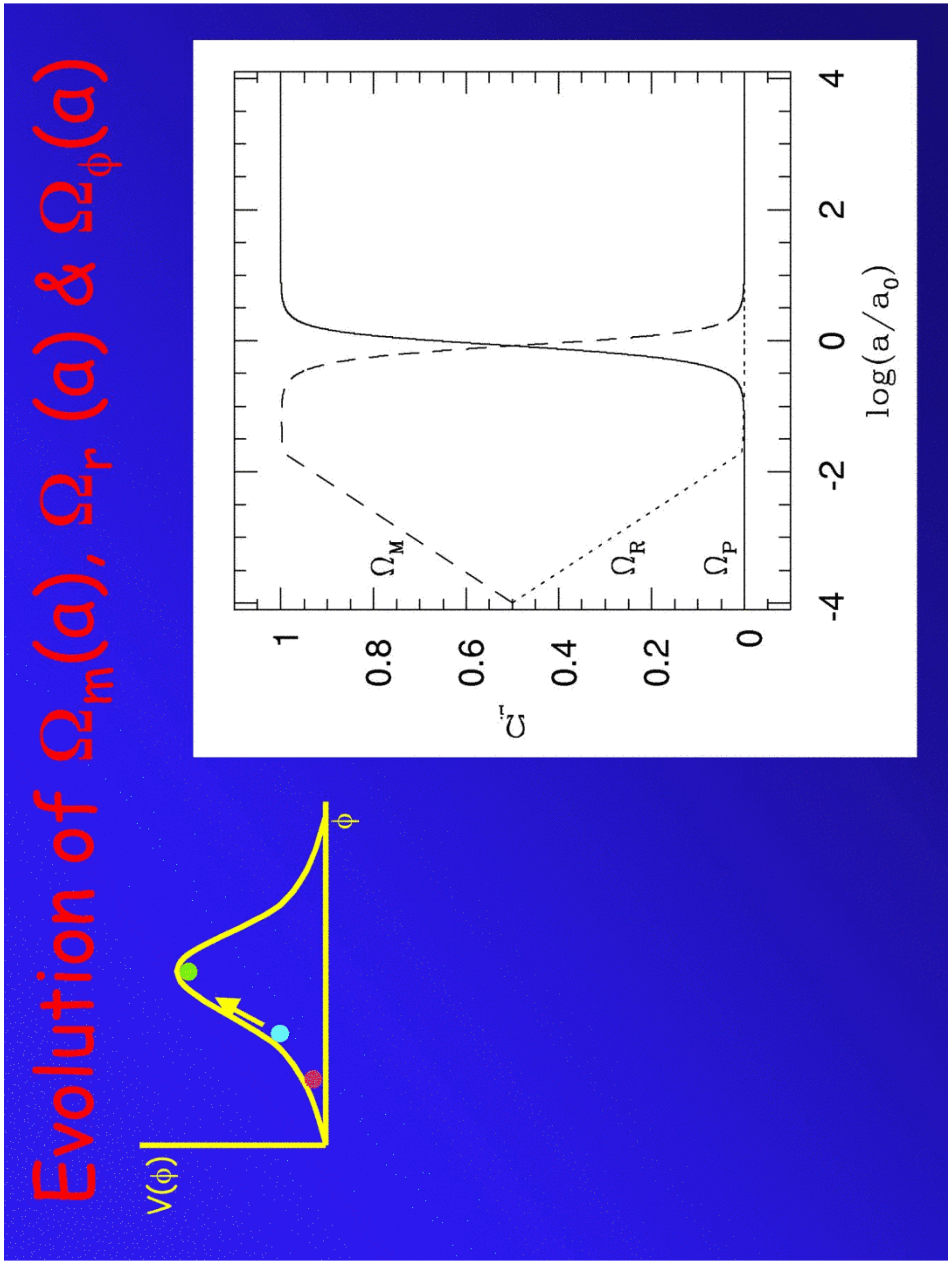
Also note - for $w < -1$, dark energy density *increases* as universe expands.

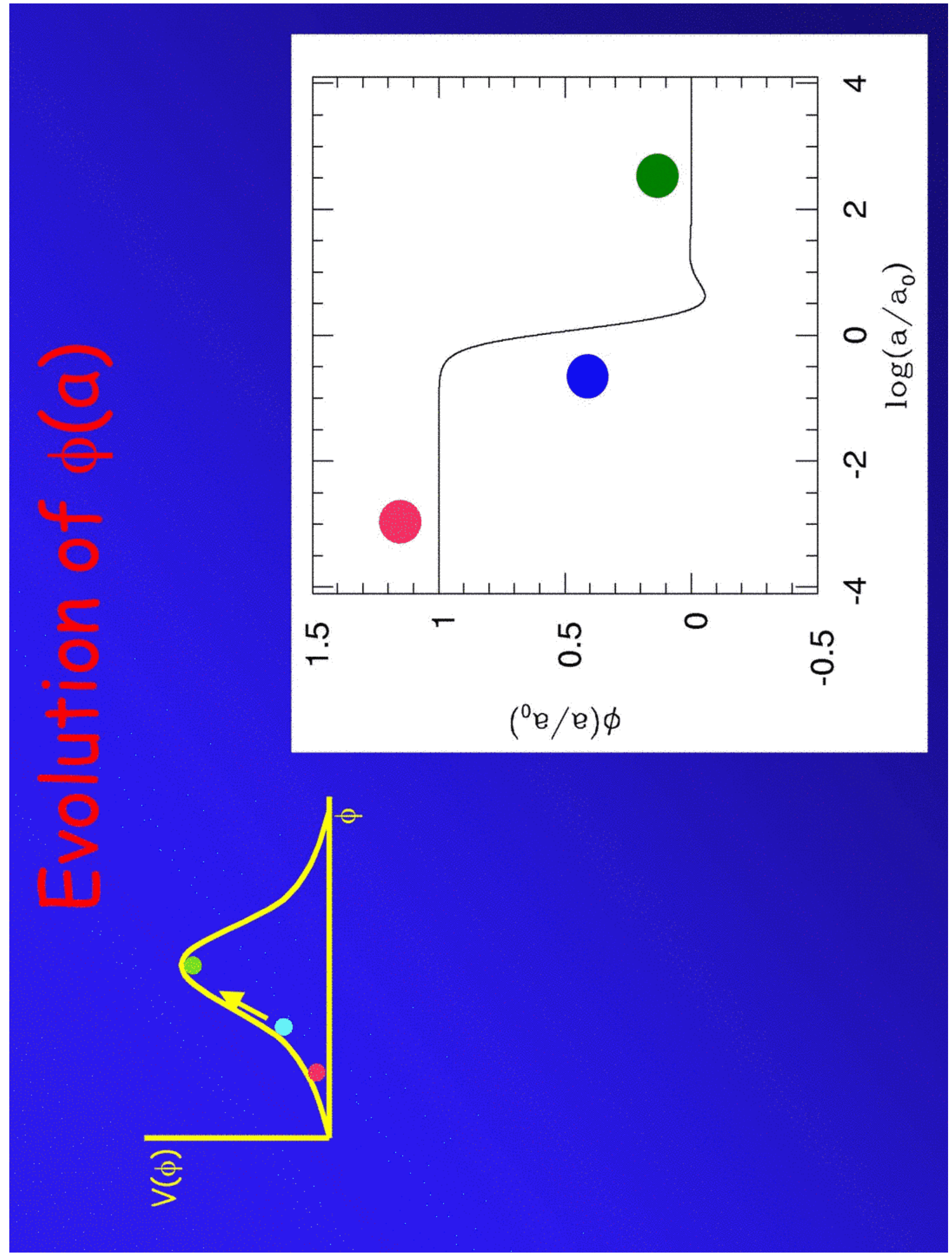
To analyze properly need a model for $V(\phi)$:

$$V(\phi) = V_0 e^{-\phi^2/M_P^2}$$ is convenient


Cosmological Evolution
$\ddot{\phi} + 3H\dot{\phi} - \frac{dV}{d\phi} = 0$
Only difference from usual field equation.
Field wants to run up the potential!
V(φ)
φ
• Early Universe: Field is frozen: w = -1
• Today: Field is moving: w_Q<-1
• Very Late Universe: Field is at maximum: w_Q=-1


Evolution of w(a)
V(φ)
φ
-0.8
-1
-1.2
-1.4
$w(a/a_0)$
-4
-2
0
2
4
$\log(a/a_0)$


Evolution of $\Omega_m(a)$, $\Omega_r(a)$ & $\Omega_\phi(a)$
$V(\phi)$
ϕ
Ω_i
Ω_M
Ω_R
Ω_P
$\log(a/a_0)$


Evolution of $\phi(a)$
$V(\phi)$
ϕ
$\phi(a/a_0)$
$\log(a/a_0)$

A Coupled System

A Lagrangian for a toy model:

$$L = \frac{1}{2}(\partial_\mu \phi)\partial^\nu \phi - \frac{1}{2}m^2\phi^2 - \left[\frac{1}{2}(\partial_\mu \psi)\partial^\nu \psi - \frac{1}{2}m^2\psi^2\right] - \lambda\phi^2\psi^2$$

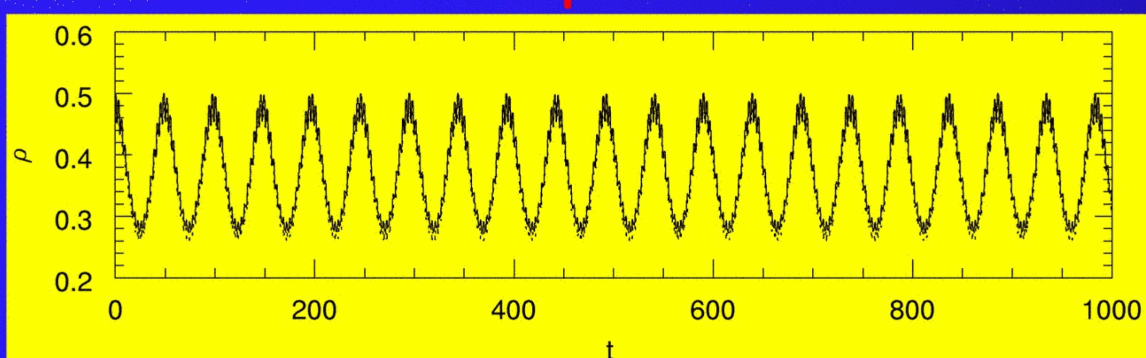
Analyze behavior for different values of λ and different initial conditions $\{\phi(0), \partial_t\phi(0), \psi(0), \partial_t\psi(0)\}$

Plot energy in each field, approximately

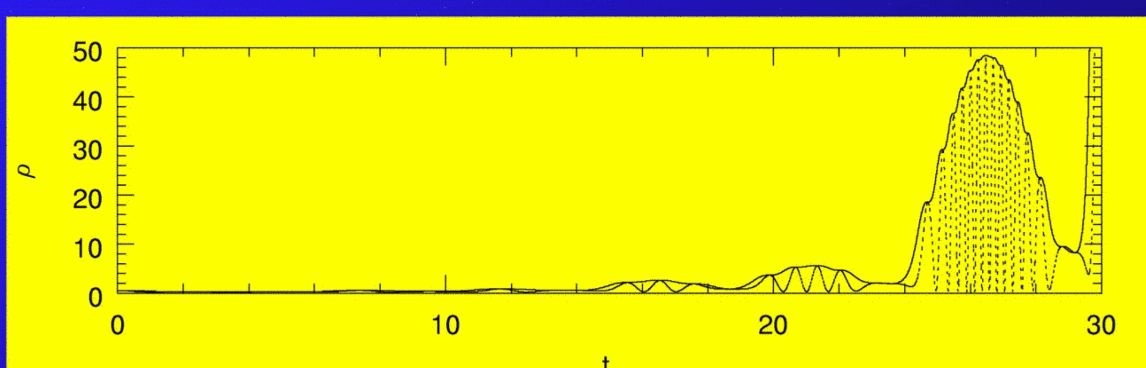
$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2$$

$$\rho_\phi = -\left(\frac{1}{2}\dot{\psi}^2 + \frac{1}{2}m^2\psi^2\right)$$

Some Sample Results



λ = 0.2



λ = 2.0

Doing Better - a Quantum Analysis

- Would like to ask about the decay rates of phantom particles to regular ones.
- Could forbid couplings to most other particles…
- …but everything couples to gravity.
- So study decay rate to gravitons.
- It'll be useful first to say something about phantom kinematics

Phantom Kinematics

- Notation: ordinary particles ψ, phantoms ϕ
- Reaction involving phantoms allowed if the equivalent reaction switching phantoms from left right and vice versa would conventionally occur.

Simple Example

$\psi_1 \to \psi_2 + \phi$ allowed if $\psi_1 + \phi \to \psi_2$ normally allowed

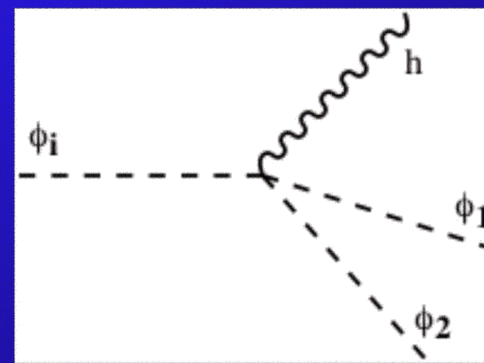
But this can happen even if mass of ψ_2 is greater than sum of masses of ψ_1 and ϕ.

So ordinary particles can decay into *heavier* particles plus phantoms (negative energy!)

The Most Inevitable Decay

Ask me later for details, but the phantom decay involving least number of particles is:

$$\phi_i \to \underset{\text{graviton}}{h} + \phi_1 + \phi_2$$



Investigate this in our toy model with potential

$$V(\phi) = V_0 e^{-\phi^2 / M_P^2}$$

expanded around some background $\phi_0 \sim M_{Pl}$

The Interaction Lagrangian

Need this to first order in h and third order in ϕ

$$L_I \sim \frac{1}{M_{Pl}}(M_{Pl}h)\frac{1}{3!}V'''(\phi_0)\phi^3$$
$$\equiv \lambda_{\text{eff}}(M_{Pl}h)\phi^3$$

But note that

$$\lambda_{\text{eff}} \sim 2\phi_0 \frac{V(\phi_0)}{M_{Pl}^5} \sim \frac{V(\phi_0)}{M_{Pl}^4} \sim 10^{-120}$$

Because of *entirely independent* cosmological constraints!

Phantom Decay Rate

$$\Gamma \sim \frac{1}{m_\phi}\int \frac{d^3p_h}{(2\pi)^3 2E_h}\frac{d^3p_{\phi_1}}{(2\pi)^3 2E_{\phi_1}}\frac{d^3p_{\phi_2}}{(2\pi)^3 2E_{\phi_2}}|M|^2(2\pi)^4\delta^{(4)}\left(p_{\phi_i}-p_{\phi_1}-p_{\phi_2}-p_h\right)$$

At tree level $|M| \sim \lambda_{\rm eff}$

Assume isotropy and approximate E~p (masses small)

$$\Gamma \sim \frac{\lambda_{\rm eff}^2}{m_\phi}\int |p_h| dp_h \int |p_{\phi_1}| dp_{\phi_1} \int |p_{\phi_2}| dp_{\phi_2}\,\delta^{(4)}\left(p_{\phi_i}-p_{\phi_1}-p_{\phi_2}-p_h\right)$$

If upper limits on integrals are ∞ then decay rate is infinite!! - Highly unstable!!

An Effective Theory

- Can't treat as fundamental theory - so treat as effective theory, valid up to a cutoff Λ
- Crude approximation - just cutoff integrals at Λ

$$\Gamma \sim \lambda_{\rm eff}^2 \frac{\Lambda^2}{m_\phi}$$ So that $$H_0\tau \sim 10^{120}\left(\frac{M_{Pl}}{\Lambda}\right)^2$$

So, lifetime greater than age of universe if

$$\Lambda < 10^{60} M_{Pl}$$ Extremely weak constraint!

(Un)fortunately, there are other decay channels

Other Couplings

Since we're dealing with an effective theory, must include all possible nonrenormalizable interactions

So consider $L = \frac{\beta}{M_{\rm Pl}\Lambda}\phi\left(M_{\rm Pl}h^{\mu\nu}\right)\partial_\mu\phi\partial_\nu\phi$

Very similar decay calculation yields $\Lambda < 10^{-3}\ \text{eV}$

Even if we impose discrete symmetries to forbid some terms, e.g. $\phi \to \phi + \varepsilon$, we can still show that the above bound holds.

Modifying Gravity

Since we're on the subject of abandoning sacred principles…

Quintessence requires incredible fine-tuning, so much so that one really is considering incredibly unnatural matter to put on the RHS of Einstein's equations

Maybe we should look at the LHS - or more properly, the gravitational action.

GDP Braneworlds

(Dvali, Gabadadze, Parrati)

One way to do this is to include both a 5d *and* a 4d Einstein term in the action for a 3-brane in a 4+1 dimensional flat spacetime. Claim is that such a term will be induced anyway.

$$S = M^3 \int d^5X \sqrt{-G} R_{(5)} + M_P^2 \int d^4x \sqrt{-g} R$$

Results in a theory that looks like Einstein theory (4d) at *short* distances, and shows 5d deviations at *large* distances

Gravity ``leaking off the brane" at large distances might mimic dark energy (Deffayet, Dvali, Gabadadze)

New Gravitational Physics

Carroll, Duvvuri, M.T. & Turner, astro-ph/0306438

Consider modifying the Einstein-Hilbert action

$$S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left(R - \frac{\mu^{2(n+1)}}{R^n} \right) + \int d^4x \sqrt{-g} L_M$$

(I'll focus on n=1 for most of this)

Field equation (n=1):

(Recall, there are similar ways to get inflation)

$$\left(1 + \frac{\mu^4}{R^2}\right) R_{\mu\nu} - \frac{1}{2}\left(1 - \frac{\mu^4}{R^2}\right) R g_{\mu\nu} + \mu^4 \left[g_{\mu\nu} \nabla_\alpha \nabla^\alpha - \nabla_{(\mu} \nabla_{\nu)} \right] \left(\frac{1}{R^2} \right) = \frac{T_{\mu\nu}^M}{M_P^2}$$

With, for cosmology

$$T_{\mu\nu}^M = (\rho_M + P_M) U_\mu U_\nu + P_M g_{\mu\nu}$$

The Matter Frame

Can see immediately constant curvature vacuum solutions are de Sitter and anti-de Sitter $R = \pm\sqrt{3}\mu^2$

Seems encouraging for dark energy, but - dS in unstable, with decay time $\tau \sim \mu^{-1}$ (Easy to see soon)

But cosmological evolution hard to see in this frame. E.g. Friedmann equation becomes

$$\dot{H} - \frac{\mu^4}{12\left(\dot{H}+2H^2\right)^3}\left(2H\ddot{H} + 15H^2\dot{H} + 2\dot{H}^2 + 6H^4\right) = \frac{\rho_M}{M_P^2}$$

Fortunately can transform to an Einstein frame

The Einstein Frame

Make a conformal transformation $\tilde{g}_{\mu\nu} = p(\phi)g_{\mu\nu}$

$$p(\phi) \equiv \exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_P}\right) \equiv 1 + \frac{\mu^4}{R^2}$$

$$\tilde{T}^M_{\mu\nu} = \left(\tilde{\rho}_M + \tilde{P}_M\right)\tilde{U}_\mu\tilde{U}_\nu + \tilde{P}_M\tilde{g}_{\mu\nu}$$

$$d\tilde{t} \equiv \sqrt{p}dt$$
$$\tilde{a}(t) \equiv \sqrt{p}a(t)$$
$$\tilde{U}_a \equiv \sqrt{p}U_a$$
$$\tilde{\rho}_M \equiv p^{-2}\rho_M$$
$$\tilde{P}_M \equiv p^{-2}P_M$$

In the ``tilde'd" frame, this becomes a theory of Einstein gravity, minimally coupled to a scalar field, with a potential and which is nonminimally coupled to matter.

Einstein-Frame Dynamics

$$\tilde{H}^2 = \frac{1}{3M_P^2}\left(\rho_\phi + \tilde{\rho}_M\right)$$

$$\phi'' + 3\tilde{H}\phi' + \frac{dV}{d\phi} - \frac{(1-3w)}{M_P\sqrt{6}}\tilde{\rho}_M = 0$$

$$\rho_\phi = \frac{1}{2}\phi'^2 + V(\phi)$$

$$\tilde{\rho}_M = \frac{C}{\tilde{a}^{3(1+w)}}\exp\left[-\frac{(1-3w)}{\sqrt{6}}\frac{\phi}{M_P}\right]$$

$$V(\phi) = \mu^2 M_P^2 \frac{\sqrt{p-1}}{p^2}$$

$$V(\phi) \sim \mu^2 M_P^2 \exp\left(-\sqrt{\frac{3}{2}}\frac{\phi}{M_P}\right)$$

Some features of quintessence

Solutions-I

Clear in the matter frame - no Minkowski solution (can also be seen in the Einstein frame - c.f. usual case of a scalar coupled to gravity)

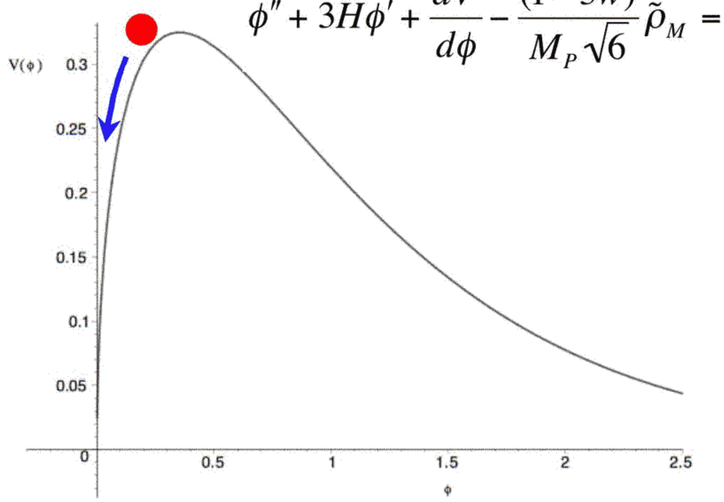
Solve equations in Einstein frame and transform back to physical frame.

1. ϕ sits at maximum. Solution is de Sitter in both frames, but is unstable $\tau \sim \mu^{-1}$ Nevertheless, if had a good reason to start there, dS acceleration might survive to today.

Solutions-II

2. ϕ begins to the left of the maximum, with insufficient velocity to get over the hump
   N.B. matter helps!

$$\phi'' + 3\tilde{H}\phi' + \frac{dV}{d\phi} - \frac{(1-3w)}{M_P\sqrt{6}}\tilde{\rho}_M = 0$$

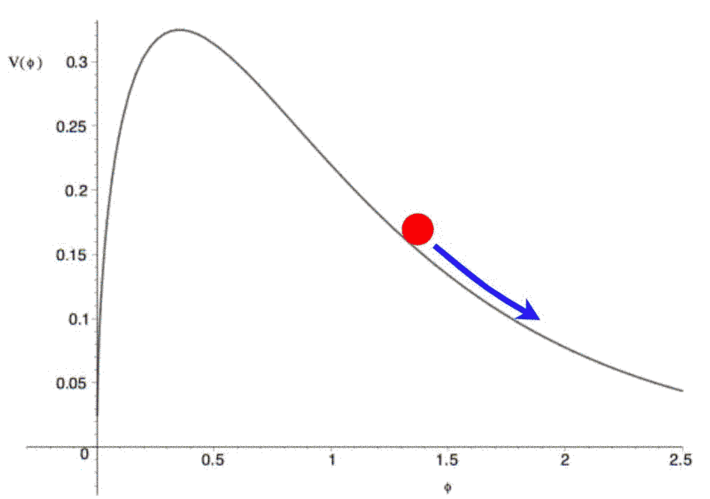


Yields a future curvature singularity in physical frame - ***not*** a Minkowski vacuum!

Not helpful cosmologically

Solutions-III

3. ϕ gets over the maximum (N.B. this can happen even if starts slightly to left of maximum with zero velocity!)



Then, ϕ rolls down potential and asymptotic soln is easy to find…

An Accelerating Universe!

Asymptotically $V(\phi) \sim \mu^2 M_P^2 \exp\left(-\sqrt{\frac{3}{2}\frac{\phi}{M_P}}\right)$ So can solve to get

$\tilde{a}(\tilde{t}) \propto \tilde{t}^{4/3}$ in the Einstein frame, which yields

$a(t) \propto t^2$ in the matter frame

This is power-law acceleration! Recall, w is always inferred from the expansion, so this is *like* having an instantaneous equation of state parameter

$$w_{\rm eff} = -\frac{2}{3}$$

Facing the Data

Remember data yields $-1.45 < w_{eff} < -0.74$
but I was using n=1 for illustrative purposes.
More generally

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\left(R - \frac{\mu^{2(n+1)}}{R^n}\right) + \int d^4x\sqrt{-g}L_M$$

Analysis is very similar for n>1, with similar potential in Einstein frame. Yields, in matter frame

$a(t) \propto t^q$ with $q = \frac{(2n+1)(n+1)}{(n+2)}$

Again, this is like $w_{\rm eff} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)}$

Easily fits data for many n (approaches dS for $n \to \infty$)

Problems with the Simplest Models

Easy to see model has problems agreeing with GR on scales smaller than cosmology. Can map theory to

$$S \propto \int d^4x \sqrt{-g}\left[\Phi R - \frac{\omega}{\Phi}\partial_\mu \Phi \partial^\mu \Phi - U(\Phi)\right]$$

i.e., a Brans-Dicke theory, with a potential that we may ignore, with $\omega=0$

But solar system measurements constrain $\omega>3500$

However, more complicated models seem to work OK

Phantom Conclusions

- If quantum decay into gravitons is to occur with a rate longer than age of universe then effective $w_Q < -1$ theory must be valid only below 10^{-3} eV in most optimistic case
- The onus is squarely on phantom model builders to show how any specific proposal avoids rapid vacuum decay.
- Either way, if $w_Q < -1$, implications for fundamental physics are profound..

Modified Gravity Status

- Have demonstrated that cosmic acceleration may arise from the gravitational sector.
- Simplest model fails solar system tests, but more complicated models seem to work OK

Much more work in progress

- perturbations,
- z-dep of w,
- dependence on initial conditions
- gravitational waves,
- more general modifications of the action
- and a lot more.

Some Questions

- There are claims that one can get ``phantom" behavior from string theory. Does this make sense?
- Does it make sense to think of purely gravitational effects as responsible for cosmic acceleration?
- Does string theory give us any guidance in this?
- What does it mean to have a theory in which corrections only appear at low energy?
- Any connection to the idea of UV/IR relationships?
- …


- Thank You -