Geometric Transitions, Black Rings and Black Hole Microstates

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with Per Kraus (UCLA) and Nick Warner (USC)

hep-th/0505166, 0504142, 0503053, 0408106, 0408186, 0402144

Summary

- Motivation and review
- D-brane physics of three charge supertubes and black rings.
- Geometric transitions and microstate geometries.
- Implications for black hole physics.

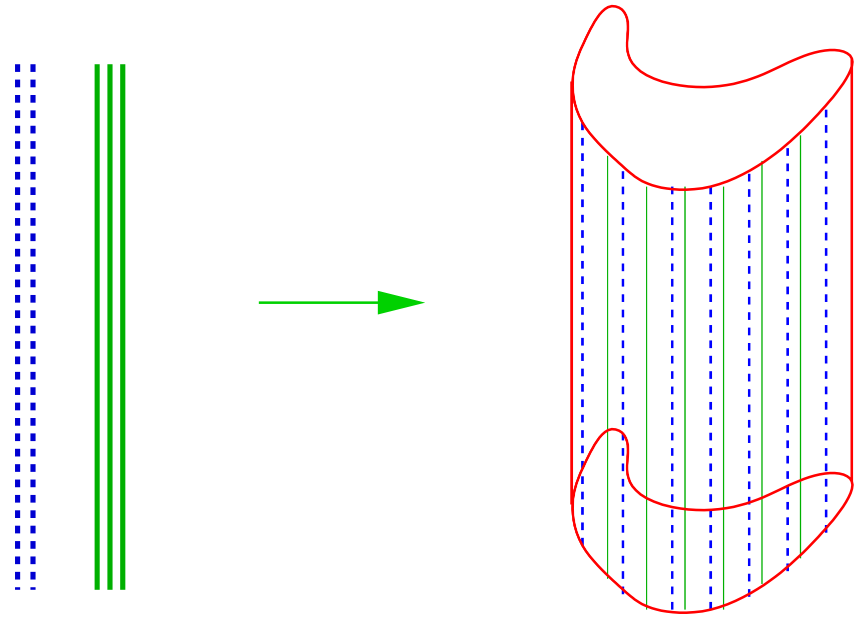
Many other groups working on these issues:

P. Berglund, E. G. Gimon, T. S. Levi hep-th/0505167
S.Giusto, S. D. Mathur, A. Saxena hep-th/0405017, hep-th/0406103, hep-th/0409067
V. Jejjala, O. Madden, S. F. Ross, G. Titchener hep-th/0504181
H. Elvang, R. Emparan, D. Mateos, H. S. Reall hep-th/0407065, hep-th/0408120, hep-th/0504125
R. Emparan, D. Mateos hep-th/0506110
J. P. Gauntlett, J. B. Gutowski hep-th/0408122, hep-th/0408010
D. Gaiotto, A. Strominger, X. Yin hep-th/0503217, hep-th/0504126,
M. Cyrier, M. Guică, D. Mateos, A. Strominger hep-th/0411187
G. T. Horowitz, H. S. Reall hep-th/0411268
P. Kraus, F. Larsen hep-th/0503219, hep-th/0506176
N. Iizuka, M. Shigemori hep-th/0506215
K. Copsey, G. T. Horowitz hep-th/0505278
A. Saxena, G. Potvin, S. Giusto, A. W. Peet hep-th/0509214

Motivation and Review

Two charge supertube:

Mateos and Townsend



D0 + F1 $\longrightarrow$ D2 dipole

8 supercharges

Shape any closed curve

D2 brane does not affect supersymmetry

Different duality frames:

D4 + F1 $\longrightarrow$ D6

D0 + D4 $\longrightarrow$ NS5

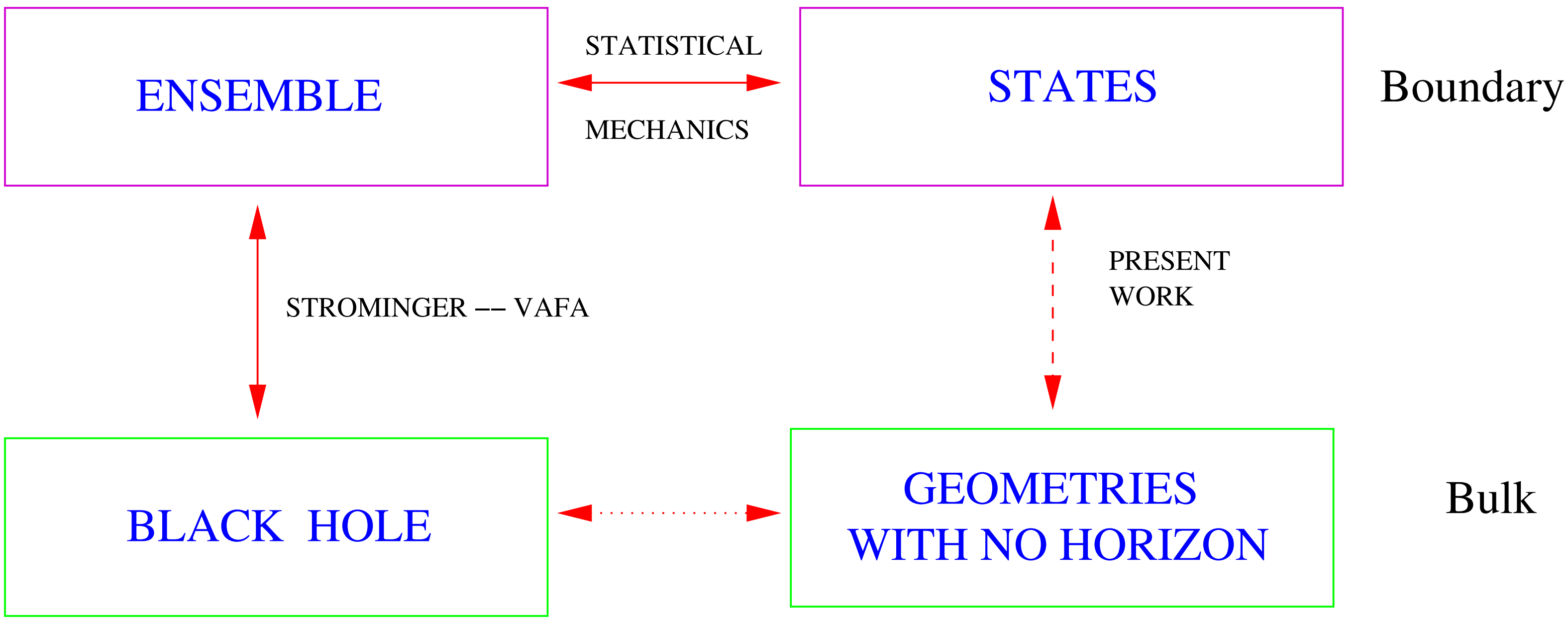
D1 + D5 $\longrightarrow$ KKM

D1 + D5 $\longrightarrow$ KKM   smooth solutions with no horizon

- Dual to microstates of the D1-D5 system Mathur, Lunin, Maldacena, Maoz
- Count $\Rightarrow$ entropy of D1-D5 system ($2\pi\sqrt{2N_1N_5}$) Mathur, Lunin, Marolf, Cabrera-Palmer

- D1-D5 system is not black hole
- D1-D5-P system is black hole (S= $2\pi\sqrt{N_1 N_5 N_P}$) (3 charges) .

Big Question:
Can we construct 3-charge solutions dual to the microstates of the D1-D5-P system ?



If true

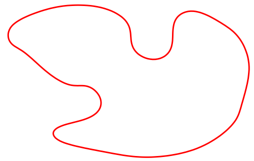
Thermodynamics $\Rightarrow$ Statistical Mechanics

Resolve Information Paradox, derive Holography, etc.

Three-charge supertubes

D0 + F1 $\rightarrow$ D2
D4 + F1 $\rightarrow$ D6 $\qquad \Rightarrow \qquad$ D0 + F1 + D4 $\rightarrow$ D2 D6 NS5
D0 + D4 $\rightarrow$ NS5

- Three charges and three dipole charges
- Born Infeld description for D0 + F1 + D4 $\rightarrow$ D2 D6 Bena, Kraus
- Arbitrary shape



- Huge number of configurations 7 functions

As gravity gets stronger, size of microstates and size of black hole increase at the same rate:

$$r^2_{\text{tube}} \sim g_s \frac{J^2}{N^2}$$

$$r^2_{\text{Black Hole}} \sim g_s \frac{N^3 - J^2}{N^2}$$

Three-Charge Supergravity Solutions

Maximal angular momentum of BPS 3-charge black hole:

$$J_{12} = J_{34} \leq \sqrt{N_1 N_5 N_p}$$

Very large families of solutions with $J > \sqrt{N_1 N_5 N_p}$

Conjectured existence of BPS black rings Bena, Kraus

Theorems ...

$U(1) \times U(1)$ found Elvang, Emparan, Mateos, Reall; Bena, Warner; Gauntlett, Gutowski

- Want solutions for generic brane configuration
- Usual techniques do not work
- Drive to USC

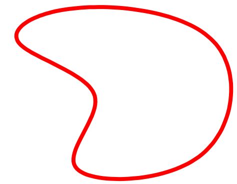
Key Idea: dipole charges do not affect supersymmetries.
Use Killing spinors to find solutions.

Three-Charge Supergravity Solutions

M2	0	1	2					
M2	0			3	4			
M2	0					5	6	
M5	0			3	4	5	6	θ
M5	0	1	2			5	6	θ
M5	0	1	2	3	4			θ

T^6 $\mathbb{R}^4$

CLOSED CURVE



$$ds^2 = Z_1^{-2/3} Z_2^{-2/3} Z_3^{-2/3} (dt + \vec{k})^2 + Z_1^{1/3} Z_2^{1/3} Z_3^{1/3} dx^2_{\mathbb{R}^4} + ds^2_{T^6}$$

$$F_{120i} = \partial_i Z_1^{-1} \qquad F_{340i} = \partial_i Z_2^{-1} \qquad F_{560i} = \partial_i Z_3^{-1} \qquad \text{electric}$$

$$F_{12ij} = G^1_{ij} \qquad F_{56ij} = G^2_{ij} \qquad F_{56ij} = G^3_{ij} \qquad \text{magnetic}$$

Solution depends on G^1 G^2 G^3 Z_1 Z_2 Z_3 $\vec{k}$

The solution has 4 layers:

- Base $\mathbb{R}^4$ (Hyper-Kähler 4D space)
- Dipole field strengths G^1, G^2, G^3 – selfdual

$$*G^I = G^I$$

- Warp factors Z_1, Z_2, Z_3

$$d * dZ_1 = G^2 \wedge G^3$$

- Rotation vector $\vec{k}$

$$d\vec{k} + *d\vec{k} = G^1 Z_1 + G^2 Z_2 + G^3 Z_3$$

System is linear when solved in this order ! Bena, Warner

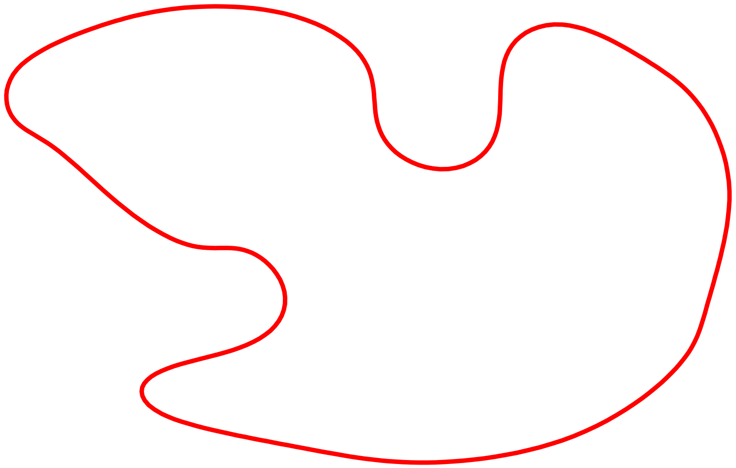
Also found in 5D sugra work Gauntlett, Gutowski, Hull, Pakis, Reall

Constructing Three-Charge Solutions

$$*G^I = G^I$$

$$d * dZ_1 = G^2 \wedge G^3$$

$$d\vec{k} + *d\vec{k} = G^1 Z_1 + G^2 Z_2 + G^3 Z_3$$



- Choose M5 dipole profile:
- Find selfdual G^I

Constructing Three-Charge Solutions

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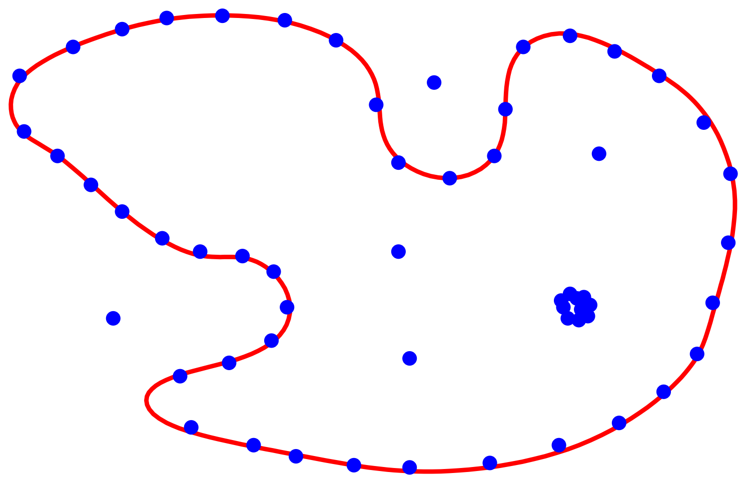


- Choose M5 dipole profile:
- Find selfdual G^I
- Sprinkle M2 brane charges. Find Z_I
- Find $\vec{k}$

Electromagnetism in $\mathbb{R}^4$ Can write down implicitly any solution

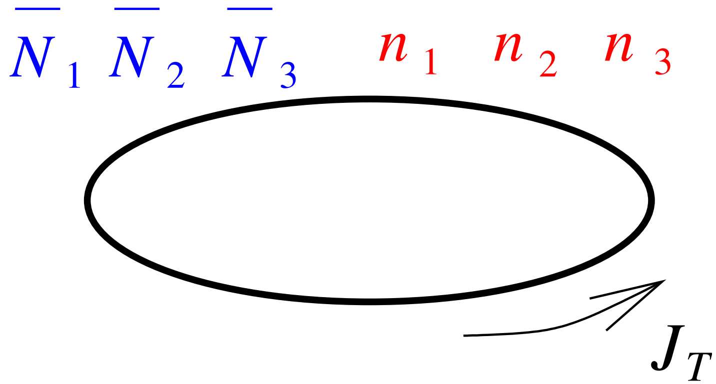
$U(1) \times U(1)$ easiest to construct explicitly.

Black Rings and Three Charge Supertubes

M5 dipole charges n_1, n_2, n_3

M2 charges $\bar{N}_1, \bar{N}_2, \bar{N}_3$

Rotation in plane of ring J_T



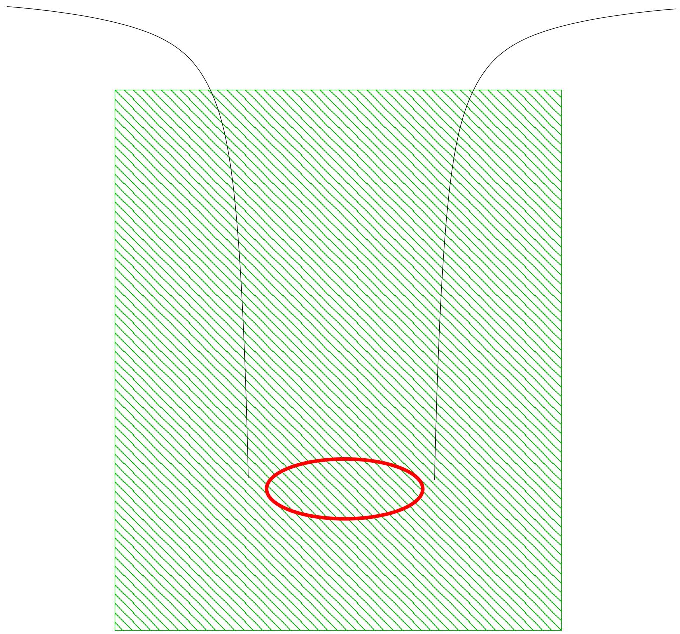
$$S = \pi\sqrt{2n_1n_2\bar{N}_1\bar{N}_2 + 2n_1n_3\bar{N}_1\bar{N}_3 + 2n_2n_3\bar{N}_2\bar{N}_3 - n_1^2\bar{N}_1^2 - n_2^2\bar{N}_2^2 - n_3^2\bar{N}_3^2 - 4n_1n_2n_3J_T}$$

Charges:

$$\begin{aligned} N_1 &= \bar{N}_1 + n_2n_3 \\ N_2 &= \bar{N}_2 + n_1n_3 \\ N_3 &= \bar{N}_3 + n_1n_1 \\ J_\psi &= J_T + J_B \\ J_\phi &= J_B \end{aligned}$$

Two Microscopic Descriptions:

- Take near-horizon limit. Solution asymptotically $AdS_3 \times S^3 \times T^4$. Ring described in D1-D5-P CFT. Bena and Kraus
- Take near-ring limit. Black Ring $\rightarrow$ Black String. 4D black hole CFT.



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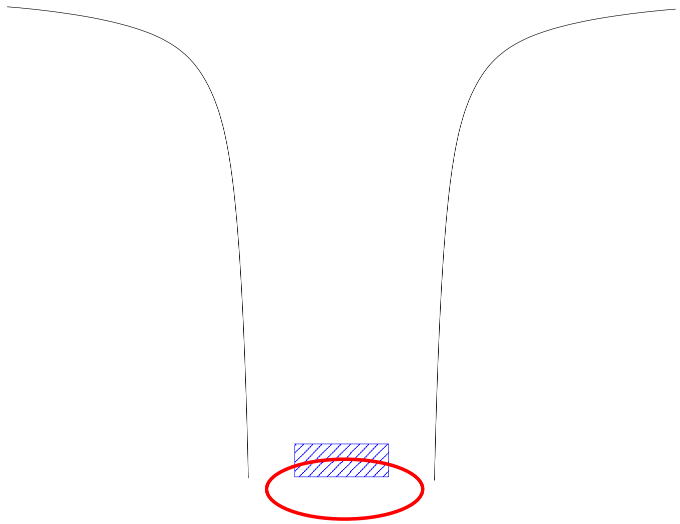
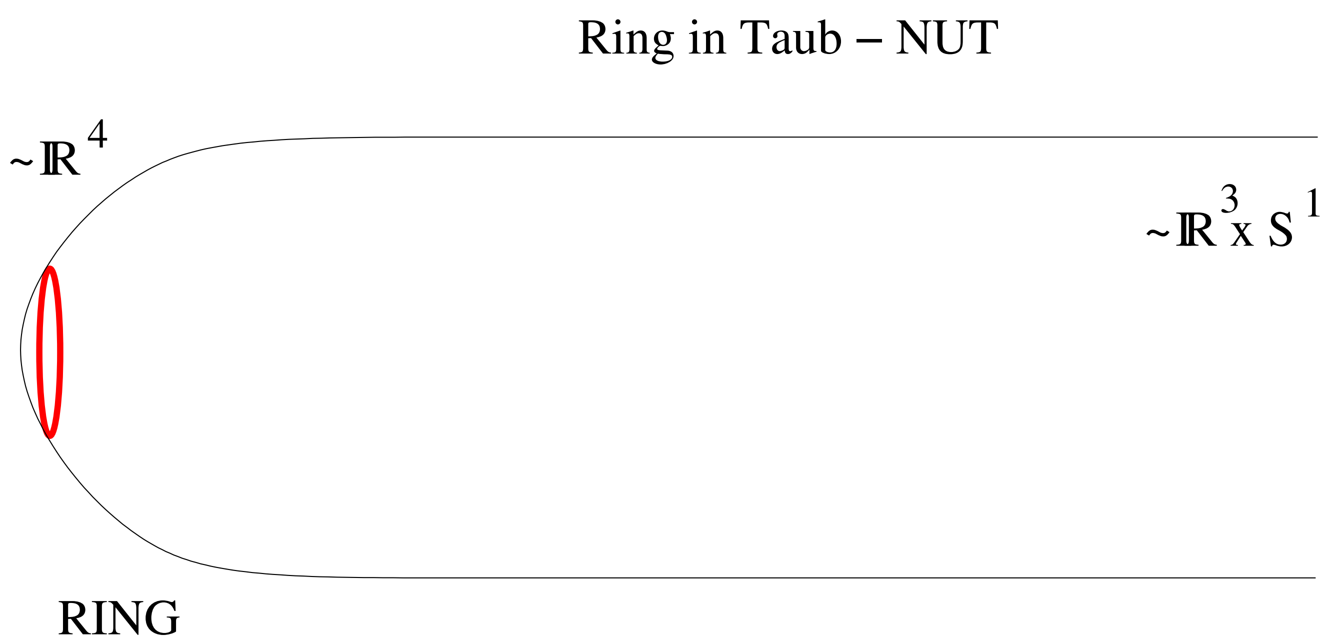


4D Black Hole CFT

To establish this description put ring in Taub-NUT and change moduli

Elvang, Emparan, Mateos, Reall; Bena, Kraus, Warner; Gaiotto, Strominger, Yin



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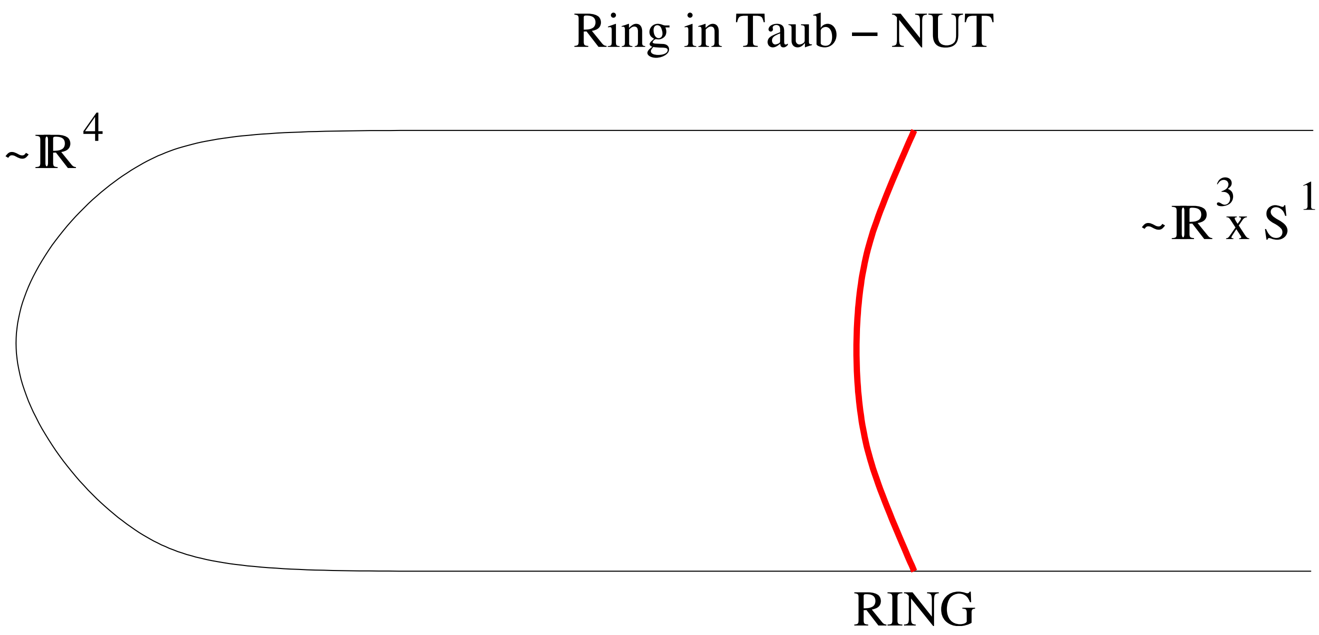


- Microscopic description 4D black hole: $\bar{N}_1\ \bar{N}_2\ \bar{N}_3\ n_1\ n_2\ n_3\ J_T$

  Bena, Kraus, Warner; Gaiotto, Strominger, Yin; Cyrier, Guică, Mateos, Strominger

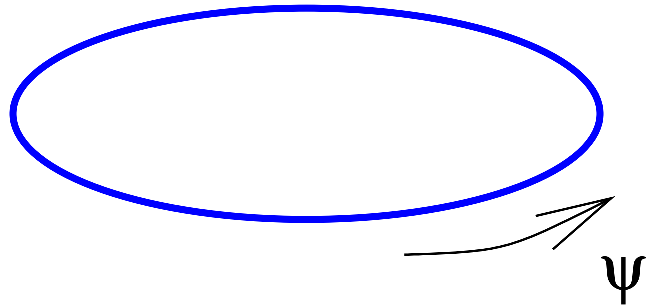
- $E_{7(7)}$ quartic invariant

  $S = \pi\sqrt{2n_1n_2\bar{N}_1\bar{N}_2 + 2n_1n_3\bar{N}_1\bar{N}_3 + 2n_2n_3\bar{N}_2\bar{N}_3 - n_1^2\bar{N}_1^2 - n_2^2\bar{N}_2^2 - n_3^2\bar{N}_3^2 - 4n_1n_2n_3J_T}$

- Microscopic charges $\neq$ charges measured at infinity.

  Similar to Klebanov - Tseytlin, Klebanov - Strassler.

Looking for Microstates



$S > 0$ black ring

$S = 0$ candidate for microstate

$S < 0$ CTC's, unphysical

- Near-horizon metric is $AdS_3 \times S^2 \times T^6$
- Horizon curvature $\sim \dfrac{1}{(n_1\ n_2\ n_3)^{\frac{1}{3}}}$

Naive microstate solution ($S = 0$) is singular.

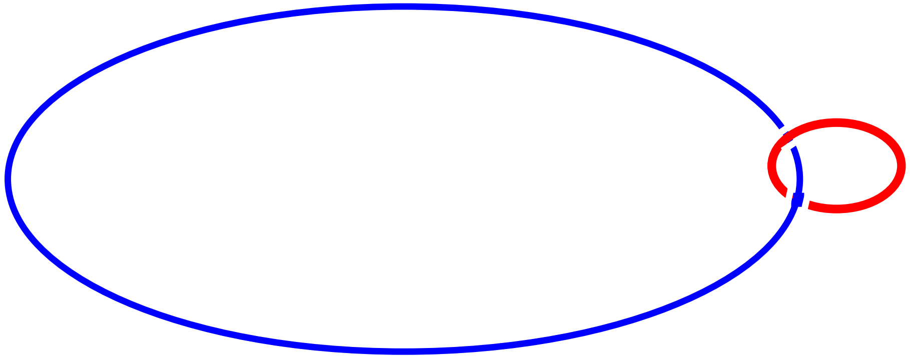
Compactified AdS_3, zero size S^1.

Resolve singularity:

Geometric transitions

S=0 4D black hole

What is a geometric transition ?



Start with branes wrapping cycle

Dual cycle gives brane charge

Turn on gravity:

- Branes shrink cycle to zero size.
- The dual cycle becomes large.

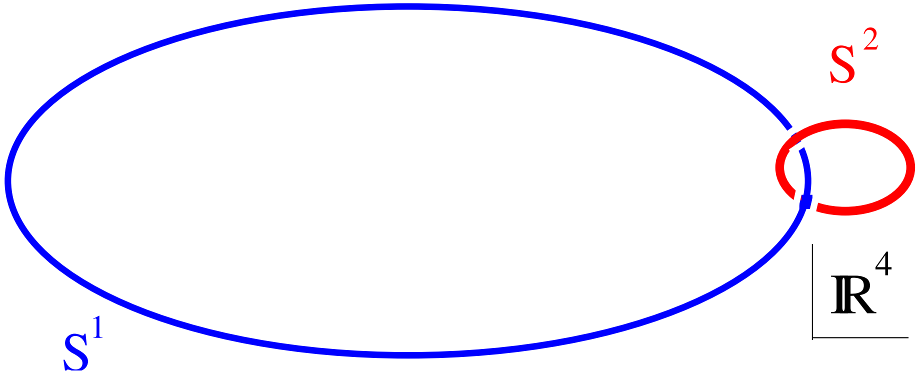
Resulting solution has different topology and no brane sources.

The geometric transition of the supertube

M5 branes wrap S^1 in the $\mathbb{R}^4$ base

Dual cycle S^2

$$\int_{S^2} F_{12ij} = n_1 \qquad \int_{S^2} F_{34ij} = n_2$$



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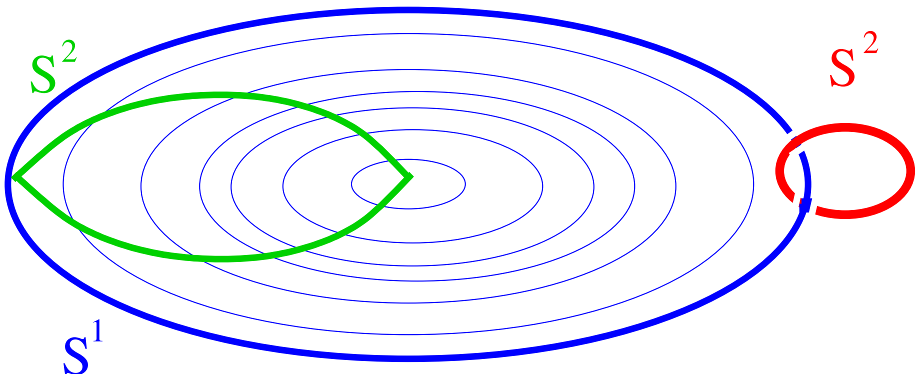


After the transition:

$S^2 \to$ large $\qquad S^1 \to 0$

Fibered $S^1 \to$ large S^2

NEW BASE with nontrivial S^2 , S^2 and no brane sources

Can we find this base ?

Hyper-Kähler —- For generic supertube not enough information

$U(1) \times U(1)$ *invariant microstates*

HyperKähler + $U(1) \times U(1)$ $\Rightarrow$ Gibbons-Hawking Gibbons, Ruback

$$
\begin{aligned}
ds^2 &= V\left(dx_1^2 + dx_2^2 + dx_3^2\right) + V^{-1}(d\psi + \vec{A})^2 \\
\nabla \times \vec{A} &= \nabla V
\end{aligned}
$$

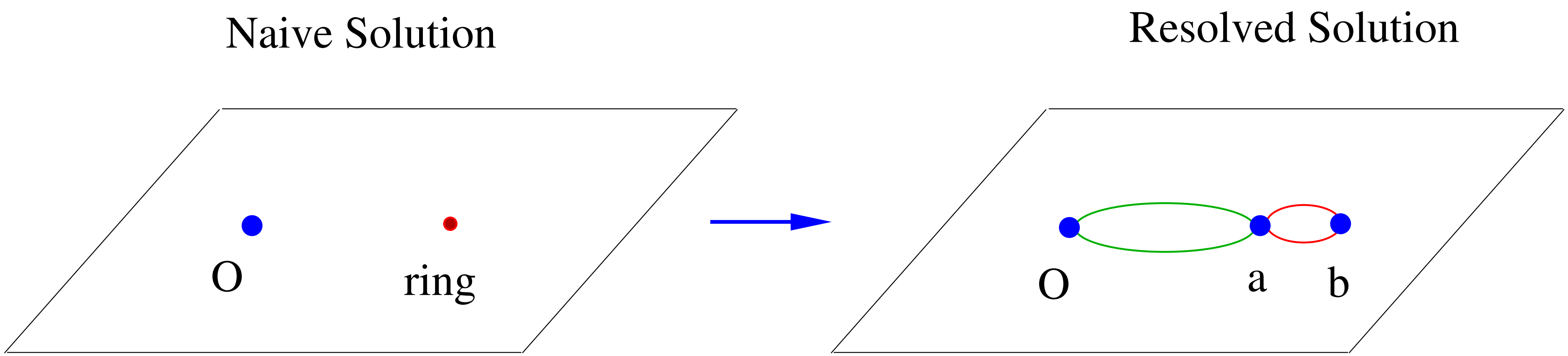
$V = \dfrac{1}{r}$ $\qquad \mathbb{R}^4$

$V = 1 + \dfrac{1}{r}$ $\qquad$ Taub-NUT

- Nontrivial S^2, $S^2 \rightarrow V$ has 3 centers
- Asymptotically $\mathbb{R}^4$ + integer charges $\Rightarrow$

$$V = \frac{1}{r} - \frac{Q}{r_a} + \frac{Q}{r_b} \qquad\qquad Q \in \mathbb{Z}$$

$$V = \frac{1}{r} - \frac{Q}{r_a} + \frac{Q}{r_b}$$



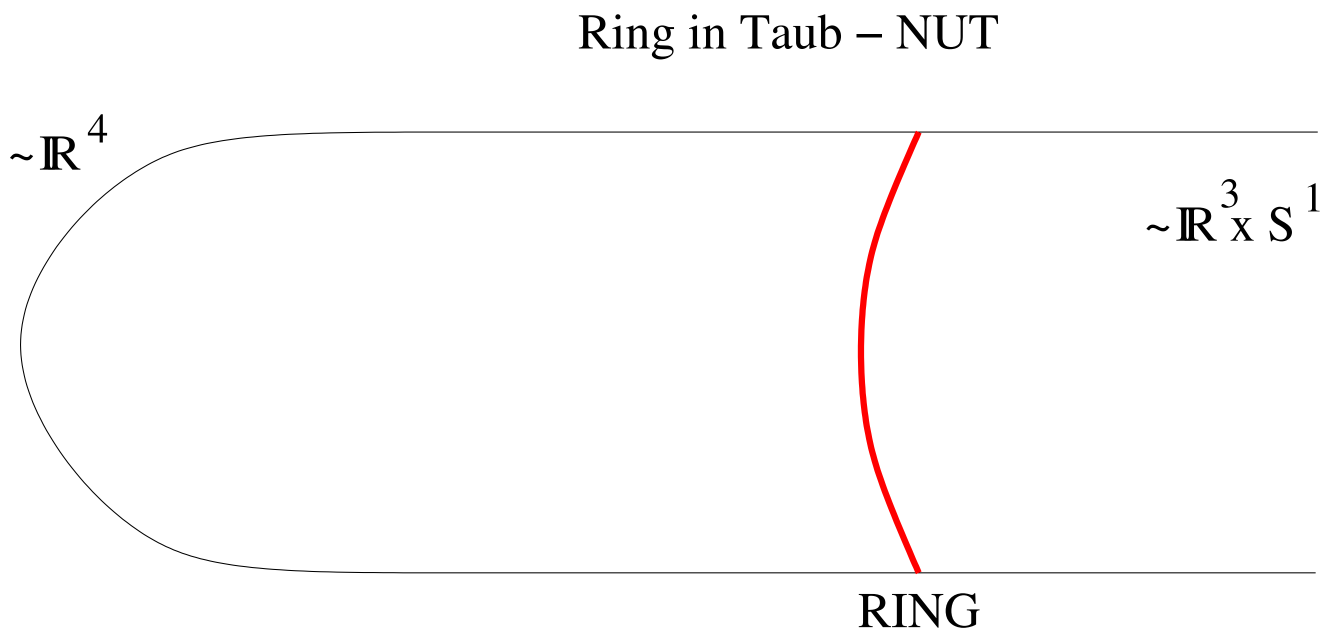
$$\int_{a-b} F_{12ij} = n_1 \qquad \int_{O-a} F_{34ij} = f_2$$

$N_3 = n_1 f_2 + f_1 n_2$ M2 charge dissolved in fluxes

- Resembles naive solution away from $a - b$ bubble
- Small $a - b$ bubble $\rightarrow$ brane description
- Similar to LLM Lin, Lunin, Maldacena

Comparison to $S = 0$ 4D black hole

Singularity of $S = 0$ black ring resolved by nucleation of $+, -$ GH pair



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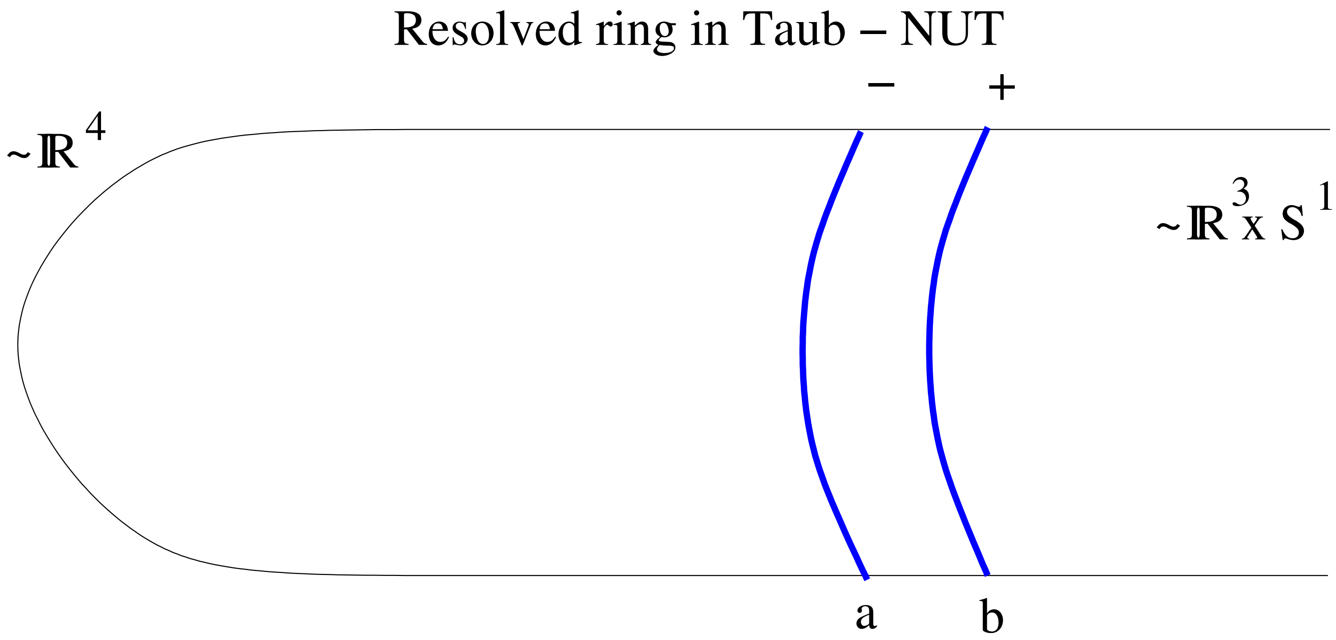


Nucleation of GH pair $\Longleftrightarrow$ splitting of 4D BH in two stacks of branes

D1-D5-KKM-P system is 4D BH. $S = 0$ when $P \to 0$.

CFT analysis of D1-D5-KKM system: Bena, Kraus

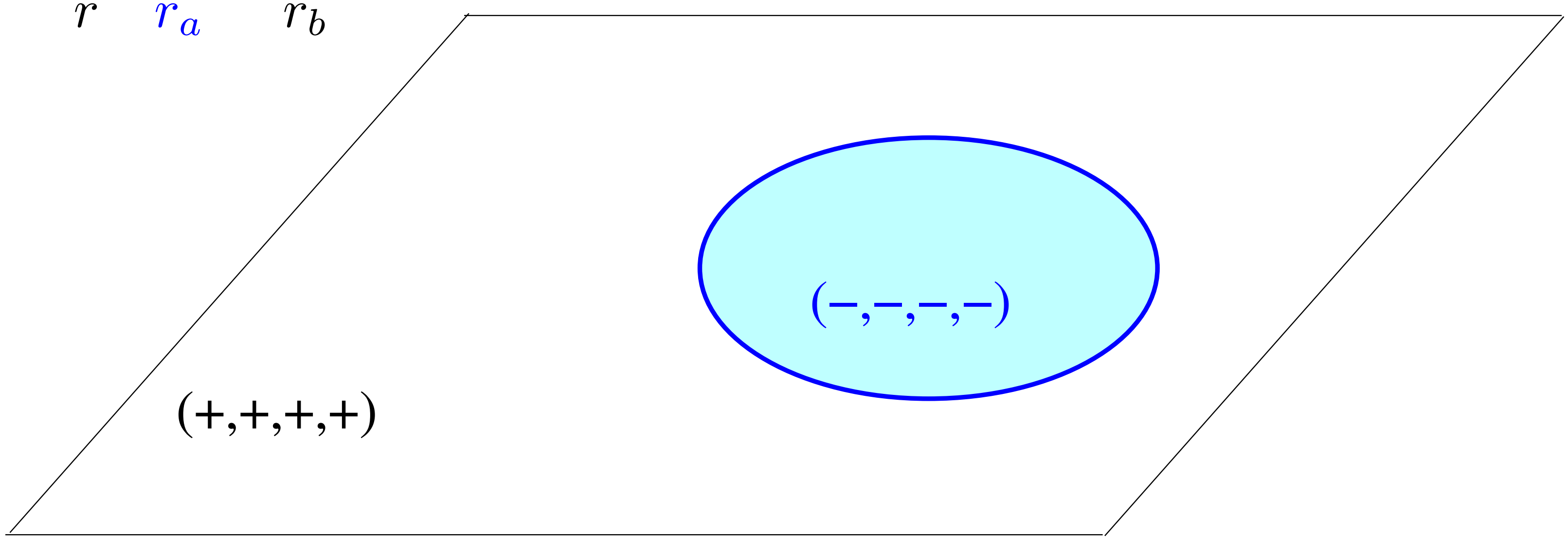
$S = 0$ 4D BH resolved by splitting D1-D5 from KKM

Resolution mechanism is the same !

The New Base

$$ds^2 = V\left(dx_1^2 + dx_2^2 + dx_3^2\right) + V^{-1}(d\psi + \vec{A})^2$$

$$V = \frac{1}{r} - \frac{Q}{r_a} + \frac{Q}{r_b}$$

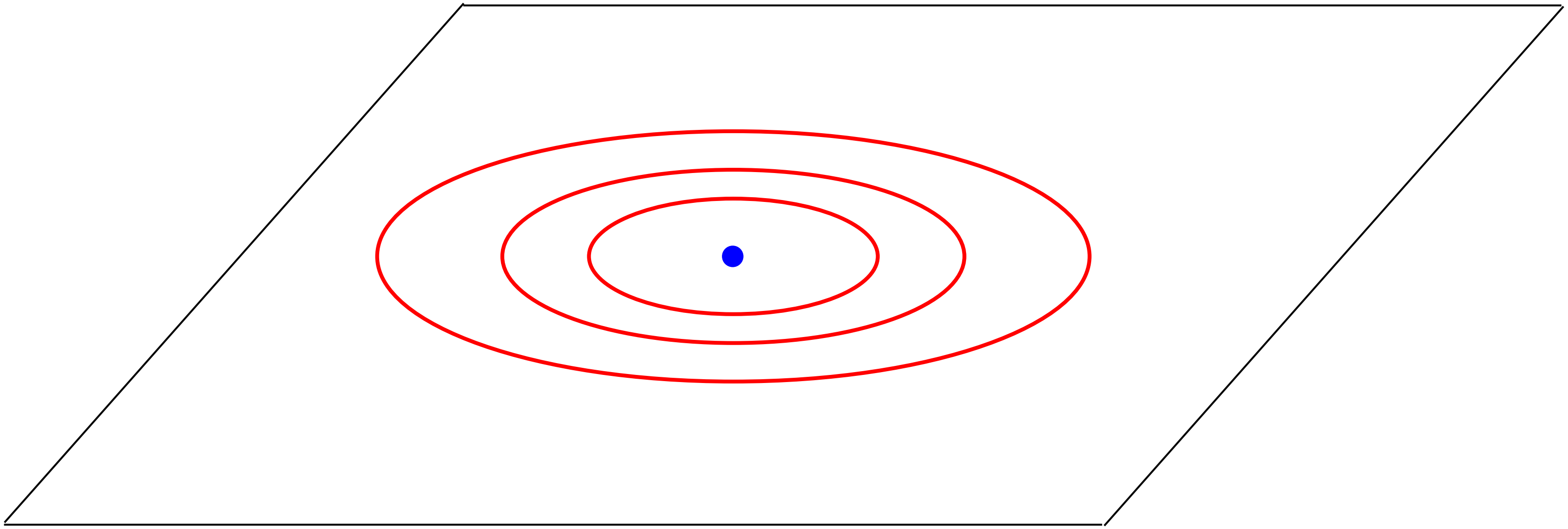


- Signature of base changes from $(+,+,+,+)$ to $(-,-,-,-)$
- Z_i blow up and change sign at interface:

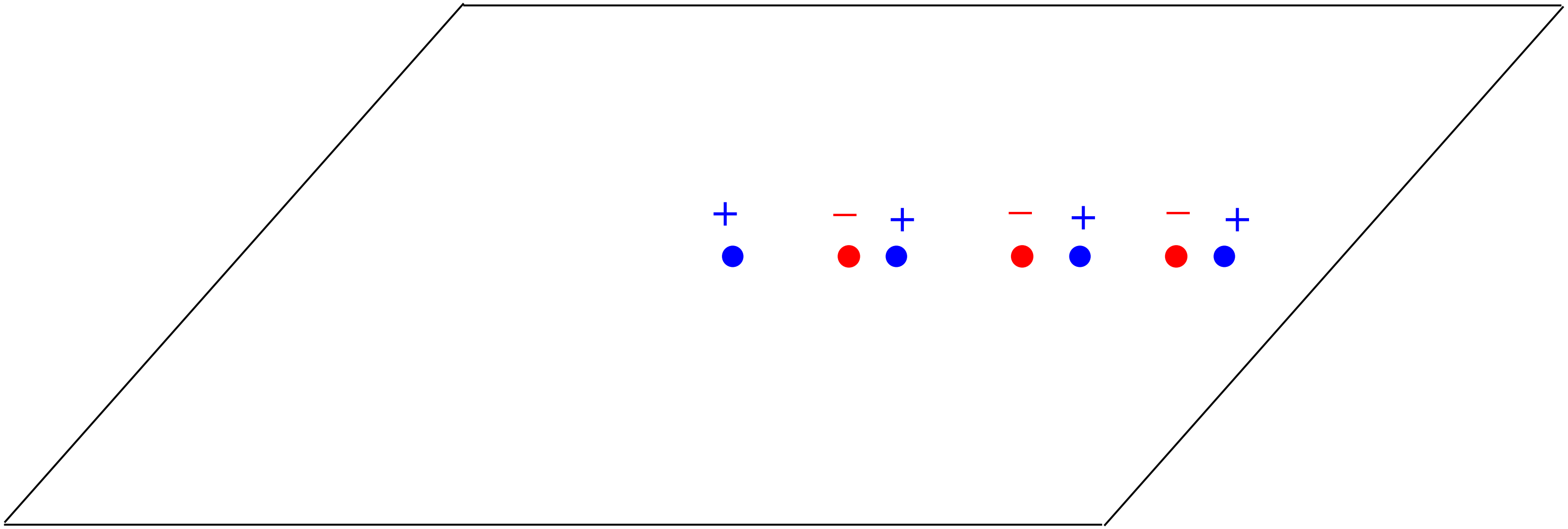
$$d * d\, Z_1 = G^2 \wedge G^3 \qquad \Rightarrow \qquad Z_i \sim \frac{1}{V}(...)$$

- Full metric is smooth

N Supertubes - The Naive Configuration

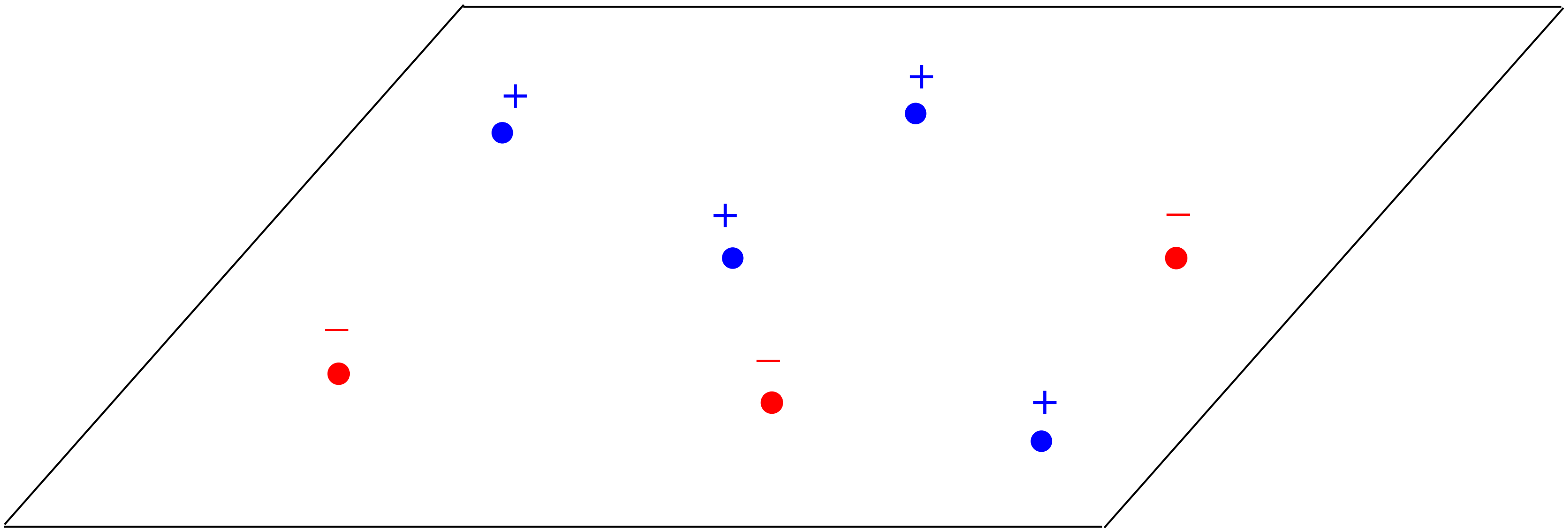


N Supertubes - The Resolved Solution



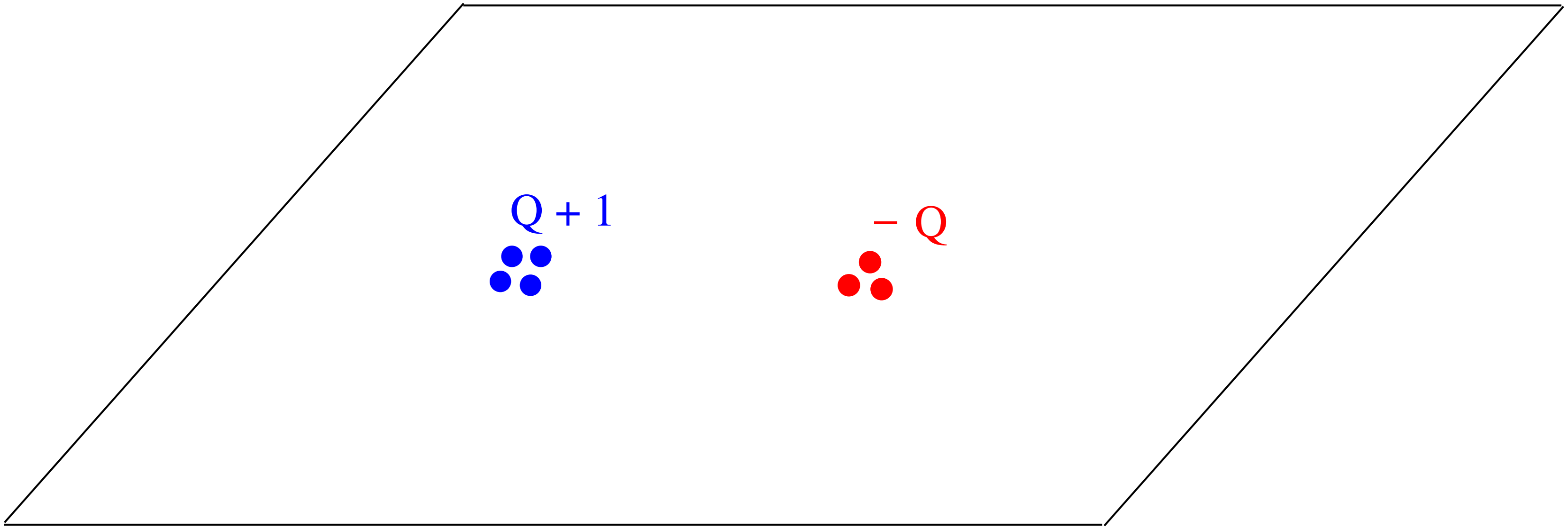
- Each supertube resolved by nucleation of $+$, $-$ GH pair

N Supertubes - Microstates with GH Base



- Each supertube resolved by nucleation of $+$, $-$ GH pair
- GH centers can move
- Smooth solutions with Gibbons-Hawking base, and arbitrary distribution of $+$ and $-$ centers Bena, Warner; Berglund, Gimon, Levi

Other 3-charge Microstates *Giusto, Mathur, Saxena*



- Novel extremal limit of 3-charge non-extremal 5D BH
- $V = \dfrac{Q+1}{r} - \dfrac{Q}{r_a}$
- Special case of bubbling solution

3 ways to get D1-D5-P microstates

Geometric transitions

S=0 4D black hole

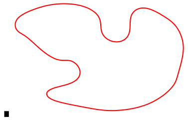
Extremal limits of 5D BH

Very nontrivial agreement

More general solutions

Supertubes can have arbitrary shapes and M2 densities Bena, Kraus, Warner

$S = 0$ configurations given by 6 functions:

4 : shape 

3 : M2 densities

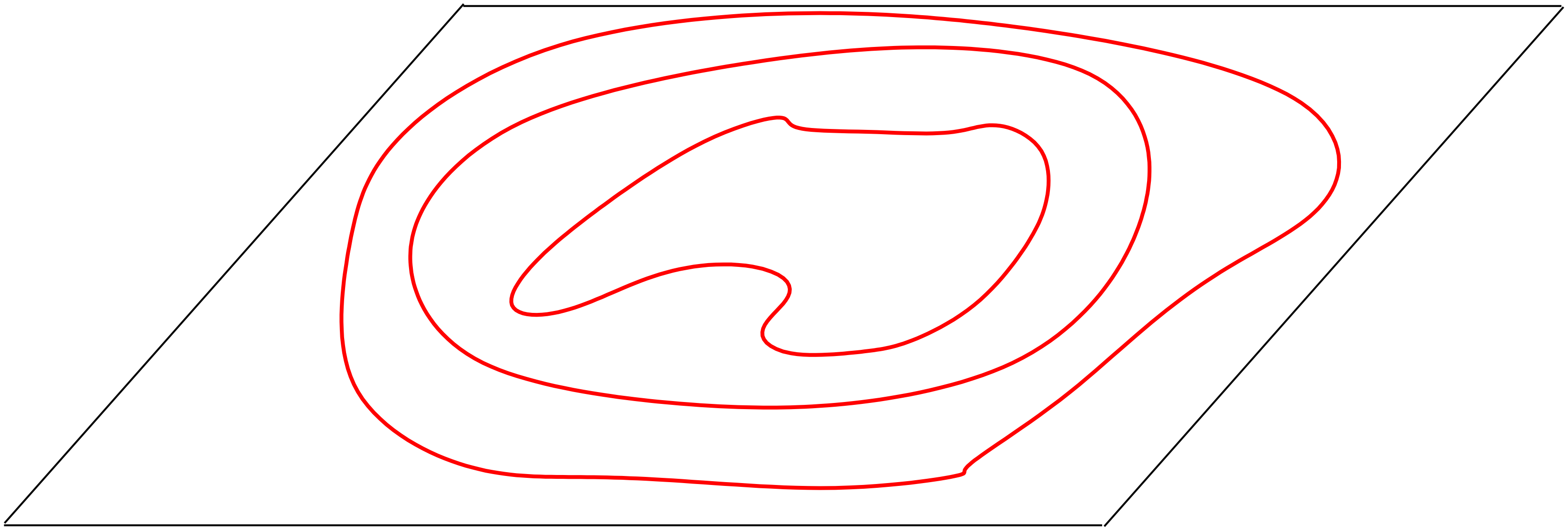
-1 : $S = 0$

Geometric transition $\Rightarrow$

New base: Hyper-Kähler (SUSY) + asymptotically $\mathbb{R}^4$

6 functions worth of Hyper-Kähler geometries

More general solutions - N supertubes



Geometric transition $\Rightarrow$

$6N$ functions worth of asymptotically $\mathbb{R}^4$ Hyper-Kähler geometries with $(-,-,-,-)$ signature

Huge number of geometries dual to D1-D5-P states

Might as well be enough to acount for entropy

3 Possibilities for Black Hole Physics of D1-D5-P system

1. D1-D5-P states dual to black hole do not have individual bulk duals. AdS-CFT only relates partition functions, not states.

- Some D1-D5-P states do have bulk duals. Distinction is unnatural.
- Other systems (LLM, Giant Gravitons, D1-D5, Polchinski-Strassler, D4 $\rightarrow$ NS5) do have one bulk state for each boundary state.

2. Each boundary state has bulk dual. Generic bulk microstate very similar to BH.

- Each microstate has horizon, entropy.
- Microstates do not have unitary physics.

3. Each boundary state has bulk dual. Generic bulk microstate has no horizon, and is LARGE (horizon size) Mathur

- Hard to obtain using collapsing shells
- Nontrivial check: size of microstate solution grows with g_s like BH horizon

Which of the these versions of black hole physics is correct ?

Summary

- D-brane physics behind existence of black rings and supertubes
- Supergravity solutions for arbitrary shapes
- Geometric transitions $\Rightarrow$
  Microstates of D1-D5-P system correspond to asymptotically $\mathbb{R}^4$ Hyper-Kähler geometries with patches of $(-,-,-,-)$ signature
- 6N functions worth of geometries

A few things I would like to know

- Classification of Hyper-Kähler spaces with changing signature.
- Find CFT microstates dual to bubbling solutions. What are the features of generic microstates (long effective strings).
- Which of the three versions of black hole physics is correct ?