Cosmic Galois Group (ac + M. Marcolli)

Conjectured by Cartier (La folle journée, multiple zeta values)

$$\mathcal{H}_c = \mathcal{U}(\mathcal{F}(3,5,7,\cdots)_\bullet)^\vee$$

U **universal symmetry group** of renormalizable theories.

$$U \longrightarrow \mathrm{Difg}(\mathcal{T}) \xrightarrow{\rho} \mathrm{Diff}$$

$$\mathcal{H} = \mathcal{U}(\mathcal{F}(1,2,3,\cdots)_\bullet)^\vee$$

Motivic Galois group (mixed Tate motives) of the scheme S_4 of 4-cyclotomic integers

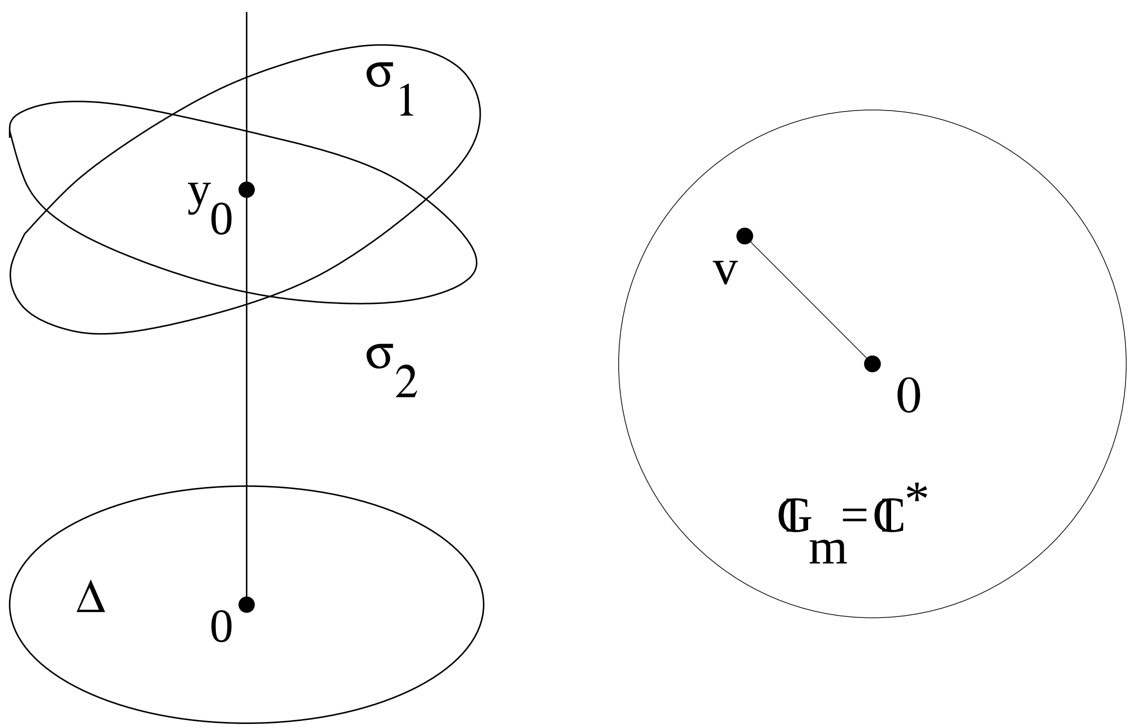
$$U^* \sim G_{\mathcal{M}_T}(\mathcal{O}), \quad \mathcal{O} = \mathbb{Z}[i][\frac{1}{2}]$$

Renormalization Group $\subset U$

$$e = \sum_1^\infty e_{-n}, \quad \mathbf{rg} : \mathbb{G}_a \to U$$

Space B : $\mathrm{Dim}_{\mathbb{C}}\ B = 2$

Complex Dimensions $\times$ **Normalization**



Irregular Singularities, Ramis.

Universal Singular Frame

$$\gamma_U(z,v) = \mathsf{T}\mathbf{e}^{-\frac{\mathbf{1}}{\mathbf{z}}\int_{\mathbf{0}}^{\mathbf{v}} \mathbf{u}^{\mathbf{Y}}(\mathbf{e})\,\frac{\mathbf{du}}{\mathbf{u}}} \in U$$

$$\gamma_U(z,v) = \sum_{n\geq 0} \sum_{k_j>0}$$

$$\frac{e(-k_1)e(-k_2)\cdots e(-k_n)}{k_1\,(k_1+k_2)\cdots(k_1+k_2+\cdots+k_n)}\,v^{\sum k_j}\,z^{-n}$$

Same coefficients as in

Local Index Formula in NCG (ac + hm)

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$$\varphi_n(a^0, \ldots, a^n) =$$

$$\sum_k b_{n,k} ⨍ a^0 [D, a^1]^{(k_1)} \ldots [D, a^n]^{(k_n)} \, |D|^{-n-2|k|}$$

$$T^{(k+1)} = \frac{D^2 T^{(k)} - T^{(k)} D^2}{k+1}$$

$$b_{n,k} = (-1)^{|k|} \sqrt{2i} \, \Gamma \left(|k| + n/2\right)$$

$$((k_1 + 1) \ldots (k_1 + k_2 + \ldots + k_n + n))^{-1}$$

$$|k| = k_1 + \ldots + k_n$$

Space X	**Algebra** $\mathcal{A}$
Real variable x^μ	**Self-adjoint** T
Set of values	**Spectrum of** T
Infinitesimal of order α	**Compact** ϵ $\mu_n(\epsilon) = O(n^{-\alpha})$
Integral of infinitesimal	$⨍\, \epsilon =$ **Coefficient of** $\log(\Lambda)$ **in** $\mathrm{Tr}_\Lambda(\epsilon)$
Line element $\sqrt{g_{\mu\nu}\, dx^\mu dx^\nu}$	**ds = Fermion propagator**

Meter → Wave length (Krypton (1967) spectrum of 86Kr then Caesium (1984) hyperfine levels of C133)

Spectral Triple $(\mathcal{A}, \mathcal{H}, D)\,, \quad ds = 1/D$

Geodesic equation	$\frac{d\psi(t)}{dt} = i\,\lvert D\rvert\,\psi(t)$
Geodesic Flow	$e^{i\,t\,\lvert D\rvert}$
Geodesic distance	$d(x,y) = \mathrm{Sup}\,\{\lvert f(x) - f(y)\rvert$ $f \in \mathcal{A}\,,\ \lVert [D,f] \rVert \leq 1\}$
Volume form	$⨍ f\,\lvert ds\rvert^{n}$
Einstein action	$⨍ f\,\lvert ds\rvert^{n-2}$

Dim-Reg

The spaces X_z of dimension z (ac + mm)

t'Hooft-Veltman and Breitenlohner-Maison prescription = taking the product of the standard geometry of (Euclidean) space-time by a very specific spectral triple X_z of dimension $z \in \mathbb{C}$, $\text{Re}(z) > 0$

$$\mathcal{H}'' = \mathcal{H} \otimes \mathcal{H}' , \quad D'' = D \otimes 1 + \gamma_5 \otimes D' .$$

Dimension spectrum of X_z is reduced to the complex number z.

Spectral triple whose $D' = D_z$ fulfills

$$\text{Trace}(e^{-\lambda D^2}) = \pi^{z/2} \lambda^{-z/2} , \quad \forall \lambda \in \mathbb{R}_+^* .$$

Morita equivalence → Gauge potentials

$$\mathcal{B} = \mathrm{End}_{\mathcal{A}}(\mathcal{E})$$

Spectral triple $(\mathcal{A}, \mathcal{H}, D) \to (\mathcal{B}, \mathcal{H}', D')$

$\mathcal{H}' = \mathcal{E} \otimes_{\mathcal{A}} \mathcal{H}$, $\quad D' = 1 \otimes D + \nabla \otimes 1$.

Connection $\nabla : \mathcal{E} \to \mathcal{E} \otimes_{\mathcal{A}} \Omega^1_D$

$\Omega^1_D \subset \mathcal{L}(\mathcal{H})$ is the $\mathcal{A}$–bimodule $\{a_i[D, b_i]\}$.

$$\nabla \xi \, a = (\nabla \xi) \, a + \xi \otimes da$$

for $da = [D, a]$.

$$\mathcal{E} = \mathcal{A}\,, \quad \mathcal{B} = \mathrm{End}_{\mathcal{A}}(\mathcal{E}) = \mathcal{A}$$

$$D \to D + A \qquad A = \Sigma \, a_i[D, b_i] \ , \ a_i, b_i \in \mathcal{A}\,.$$

Evanescent gauge potentials

Graded case $[D, a]_- := D\, a - (-1)^{\deg(a)} a\, D$

$$\theta(a + b\,\gamma) = a - b\,\gamma\,, \quad \theta \in \mathrm{Aut}(\tilde{\mathcal{A}})$$

$$\bar{D} = D \otimes 1\,, \quad \hat{D} = \gamma \otimes D'\,, \quad D'' = \bar{D} + \hat{D}\,.$$

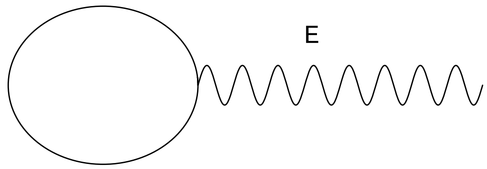
$$B = [D'', \gamma]_- = 2\,\gamma\,\hat{D}\,.$$

$$\mathcal{L}_{\mathrm{fermions}} = \langle \eta\,,\, D'' \xi \rangle\,, \quad \gamma_u(\xi) = u\,\xi\,,$$

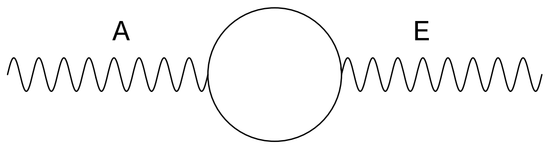
$$\gamma_u(\eta) = \theta(u)\,\eta\,.$$

$$\delta\,\mathcal{L}_{\mathrm{fermions}} = < \eta,\, (i\,[D, \omega]\,\gamma + i\,\omega\,B)\,\xi >$$

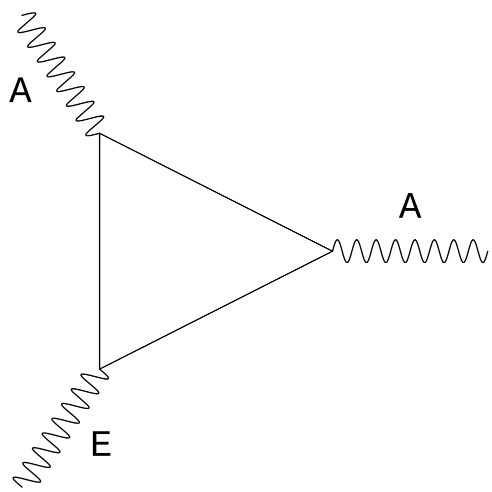
Anomalies → Local Index



Tadpole



Self-energy graph



Triangle graph

Fermions $\rightarrow$ Geometry

Fermions	$\psi \in \mathcal{H}$
Chirality	**Grading** γ
Internal symmetries	**Int**$(\mathcal{A})$ $f \longrightarrow u\, f\, u^*$
Gauge Bosons	**Internal Fluctuations**
Bosonic Action	**Spectral Action**

Propagator $= ds \to$ **Fermionic Action**

Bosonic Action = Spectral Action $(ac)^2$

$N(\Lambda) = \#$ eigenvalues of D in $[-\Lambda, \Lambda]$.

$$N(\Lambda) = \langle N(\Lambda) \rangle + N_{\text{osc}}(\Lambda)$$

$$\langle N(\Lambda) \rangle = S_\Lambda(D) = \sum_{k \in S} \frac{\Lambda^k}{k} ⨍ |ds|^k + \zeta_D(0)\,,$$

$$\zeta_D(s) = \text{Trace}(|D|^{-s})$$

Minimally coupled Standard Model

$$\mathcal{L}_E + \mathcal{L}_G + \mathcal{L}_{GH} + \mathcal{L}_H + \mathcal{L}_{Gf} + \mathcal{L}_{Hf}$$

Spectral Action

$$S = \int d^4x \sqrt{g}\ (1/2\kappa_0^2\, R - \mu_0^2 (H^* H)$$

$$+ a_0\, C_{\mu\nu\rho\sigma}\, C^{\mu\nu\rho\sigma} + \ b_0\, R^2 + c_0\, {}^*R^*R + d_0\, R;_\mu{}^\mu$$

$$+\ e_0 + 1/4\, G^i_{\mu\nu}\, G^{\mu\nu i} + 1/4\, F^\alpha_{\mu\nu}\, F^{\mu\nu\alpha}$$

$$+ 1/4\, B_{\mu\nu}\, B^{\mu\nu} + |D_\mu\, H|^2 - \xi_0\, R|H|^2 + \lambda_0 (H^* H)^2)$$

Standard Model

$$X = M \times F$$

$$\mathcal{A} = \mathcal{A}_M \otimes \mathcal{A}_F \ , \ \mathcal{H} = \mathcal{H}_M \otimes \mathcal{H}_F \ ,$$

$$D = D_M \otimes 1 + \gamma_5 \otimes D_F$$

$$\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C})$$

$$\mathcal{H}_F = Q \oplus L \oplus \bar{Q} \oplus \bar{L}$$

$$Q = \begin{pmatrix} u_L & u_R \\ d_L & d_R \end{pmatrix} , \quad L = \begin{pmatrix} \nu_L & ? \\ e_L & e_R \end{pmatrix}$$

Action of $\mathcal{A}$ in $\mathcal{H}_F$

$$a = (\lambda, q, m) \in \mathcal{A}\,, \quad q = \begin{pmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{pmatrix}$$

$$a\, u_R = \lambda\, u_R \qquad a\, u_L = \alpha\, u_L - \overline{\beta}\, d_L$$

$$a\, d_R = \overline{\lambda}\, d_R \qquad a\, d_L = \beta\, u_L + \overline{\alpha}\, d_L\,.$$

$$a\,\overline{f} = \lambda\,\overline{f} \quad \text{if } f \text{ is a lepton}$$

$$a\,\overline{f} = m\,\overline{f} \quad \text{if } f \text{ is a quark}$$

$$\gamma(f_R) = f_R\ ,\ \gamma(f_L) = -f_L$$

Finite Space

$$D_F = \begin{pmatrix} Y & 0 \\ 0 & \bar{Y} \end{pmatrix}$$

$$Y = Y_q \otimes 1_3 \oplus Y_\ell$$

$$Y_q = \begin{pmatrix} 0 & 0 & M_u & 0 \\ 0 & 0 & 0 & M_d \\ M_u^* & 0 & 0 & 0 \\ 0 & M_d^* & 0 & 0 \end{pmatrix}$$

$$Y_\ell = \begin{pmatrix} 0 & 0 & M_e \\ 0 & 0 & 0 \\ M_e^* & 0 & 0 \end{pmatrix}$$

Inner Fluctuations $\to$ Gauge Bosons

$$A = \sum a_i[D, a_i'] \quad a_i, a_i' \in \mathcal{A}$$

$$\sum a_i[\gamma_5 \otimes D_F, a_i'] \ \to \ \textbf{Higgs Fields}$$

$$\begin{pmatrix} 0 & X \\ X' & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} M_u\,\varphi_1 & M_u\,\varphi_2 \\ -M_d\,\overline{\varphi}_2 & M_d\,\overline{\varphi}_1 \end{pmatrix}$$

$$X' = \begin{pmatrix} M_u^*\,\varphi_1' & M_d^*\,\varphi_2' \\ -M_u^*\,\overline{\varphi}_2' & M_d^*\,\overline{\varphi}_1' \end{pmatrix}$$

$\sum a_i[D_M \otimes 1, a_i'] \;\rightarrow\;$ **Gauge Fields**

$a_i = (\lambda_i, q_i, m_i)$, $a_i' = (\lambda_i', q_i', m_i')$

$U(1)$ gauge field $\Lambda = \Sigma\, \lambda_i\, d\, \lambda_i'$

$SU(2)$ gauge field $Q = \Sigma\, q_i\, d\, q_i'$

$U(3)$ gauge field $V = \Sigma\, m_i\, d\, m_i'$.

Hypercharges

$$D \mapsto \tilde{D} = D + A + JAJ^{-1} \qquad A = A^*$$

$$\text{trace } V = \Lambda$$

$$V = V' + \frac{1}{3}\,\Lambda$$

V' is an $SU(3)$ gauge potential.

$$\begin{pmatrix} \frac{4}{3}\Lambda + V' & 0 & 0 & 0 \\ 0 & -\frac{2}{3}\Lambda + V' & 0 & 0 \\ 0 & 0 & Q_{11} + \frac{1}{3}\Lambda + V' & Q_{12} \\ 0 & 0 & Q_{21} & Q_{22} + \frac{1}{3}\Lambda + V' \end{pmatrix}$$

$$\begin{pmatrix} -2\Lambda & 0 & 0 \\ 0 & Q_{11} - \Lambda & Q_{12} \\ 0 & Q_{21} & Q_{22} - \Lambda \end{pmatrix}$$

Open Questions

1) beyond SM

Massive neutrinos

$$\mathcal{A}_F = \mathbb{C} \oplus \mathbb{H} \oplus M_3(\mathbb{C}) \to \mathbb{C} \oplus \mathbb{H}_L \oplus \mathbb{H}_R \oplus M_3(\mathbb{C})$$

Left-Right symmetric

$$\mathcal{H}_F \to Q \oplus L \oplus \bar{Q} \oplus \bar{L}$$

$$Q = \begin{pmatrix} u_L & u_R \\ d_L & d_R \end{pmatrix}, \quad L = \begin{pmatrix} \nu_L & \nu_R \\ e_L & e_R \end{pmatrix}$$

gives the **spinors** for an extension of the group Spin_4.

Extension by quantum group at j, $j^3 = 1$.

2) q-Gravity

Spectral observables (Diff-invariance)

$$\langle \sigma \rangle = \mathcal{N} \int \sigma(D,\psi) e^{-S_\Lambda(D) - \langle \bar{\psi}, D\,\psi \rangle}\, D[D]\, D[\psi]\, D[\bar{\psi}] \to$$

Matrix Model

Constraints

Polynomial Equation of degree = dimension

$$U^*\,[D,U] = 1 \to \text{geometry of } S^1$$

$$\sum a_0\,[D,a_1] \cdots [D,a_4] = \gamma \to$$

spherical manifolds (ac + mdv)

Unimodular gravity

3) The field of "running physical constants"

$$g(\mu)$$

Quantum physics is not done over field $\mathbb{C}$ but over a field of functions : the "constants" which in fact depend upon the energy level μ.

Dressing of Geometry

QFT corrections to ds which is now "running"