Outline

1. Brief Intro: Stringy spacetime dynamics / math
   (A Non-Mathematician's Apology)

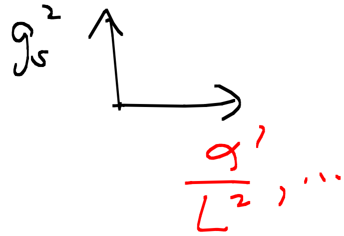
2. The Tachyon at the End of the Universe

   w/ J. McGreevy hep-th/0506130

   + background & extensions

3. Ongoing work on black holes

   w/ Gary cf "New Endpoint for Hawking Evaporation" hep-th/0506166



Classical String effects correct GR and generalize point particle geometry

e.g. Calabi-Yau worldsheet instanton corrections $\rightarrow$ mirror symmetry, topology change, etc. much studied in the math/physics interface

The idea that spacetime not fundamental already clear from string sigma models; realized beautifully in non-perturbative formulations (M.M., AdS/CFT, ...)

Some of the most basic challenges involve classical geometry that is old hat (e.g. Black Holes, FRW cosmology, ...)

A more prosaic example, Riemann surface compactifications, were nailed at large radius by mathematicians in the 19th century. Some of what follows came from asking simply what happens to R.S. target spaces at small radius,

KK scale SUSY breaking

as done in the CY case.

Even if $m_{SUSY} \ll m_s$ in late-time particle physics, $m_{SUSY} \sim m_{KK}$ in BHs & FRW

SUSY is a useful theoretical tool: quantum/stringy corrections are cancelled according to non-renormalization theorems, allowing one to venture into strong coupling regimes.

However, more generic (e.g. SUSY-breaking) effects can smooth out or evade singular strong-coupling limits, yielding greater control

cf Liouville theory

The Tachyon at The End of the Universe

J. McGreevy & E.S. hep-th/0506130

+ work in progress w/ Horowitz

+ 0502021 w/ Adams, Liu, Saltman

Consider a flat FRW solution to GR

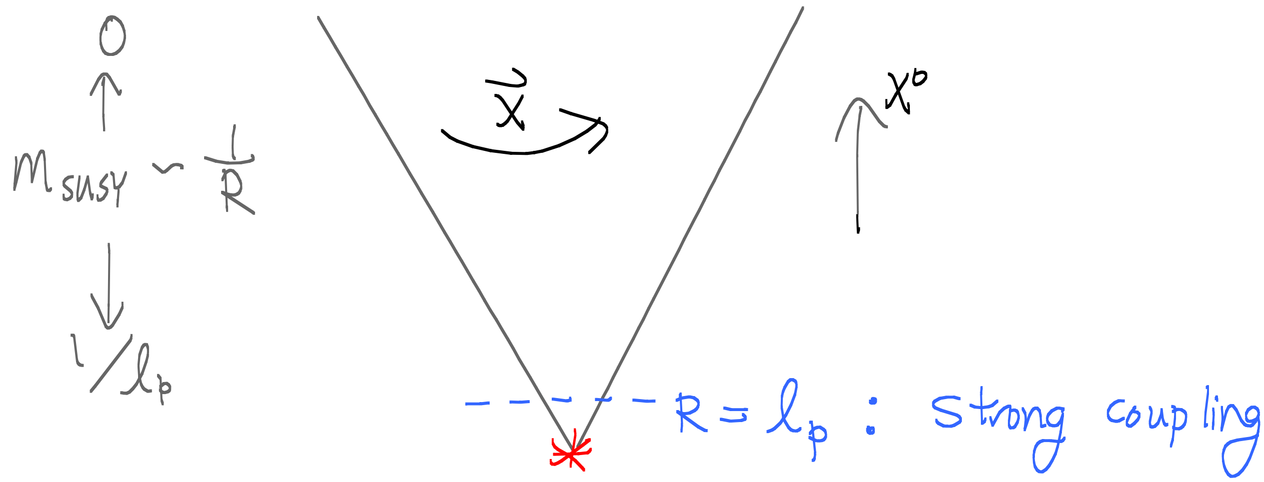
$$ds^2 = -dt^2 + a^2(t)\, d\vec{x}^2 + ds_\perp^2 \qquad \text{with}$$

- at least one circle $\vec{x} = \vec{x} + \vec{L}$ around which spacetime fermions have antiperiodic boundary conditions.

- $v(t) \equiv \dot{a}(t)|\vec{L}| \ll 1$ at time t_s at which the circle size is string scale $R \equiv |\vec{L}|\, a(t_s) = l_s$

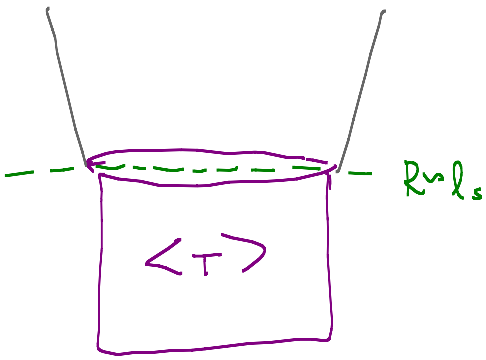
(obtained by sufficiently weak matter source)

This has a spacelike big bang singularity in the past in the GR solution:

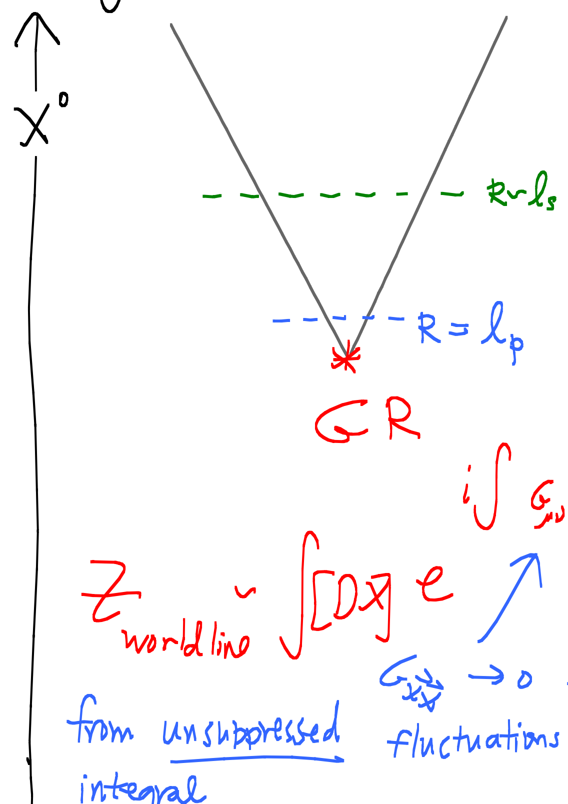


with a level of SUSY breaking appropriate to that in early universe cosmology (& inside black holes)

In string theory, a winding tachyon appears and condenses at the string scale



String theory



GR

$$Z_{\text{worldsheet}} \sim \int [DX] \exp\left\{ i \iint \left[G_{\mu\nu} \partial X^\mu \partial X^\nu - e^{-\kappa X^0} V_T \right] \right\}$$

tachyon background suppresses fluctuations in path integral

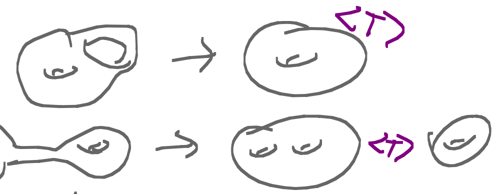
$$Z_{\text{worldline}} \sim \int [DX]\, e^{i \int G_{\mu\nu} \dot{X}^\mu \dot{X}^\nu}\, \Pi\, \theta$$

$G_{xx} \to 0$ singularity from unsuppressed fluctuations in path integral

<u>Remark</u>: The problem of closed string tachyon condensation, often motivated by the question of the vacuum structure of string theory, is crucial to a basic question about gravity (spacelike singularity resolution).

We have seen winding tachyon condensation address questions of spacetime dynamics also in

- timelike conical singularities 

w/ Adams Polchinski ...
- topology change & baby universes 

w/ Adams, Liu, McGreevy, Saltman
- Closed timelike curves Costa et al Null singularities Berkooz et al
- Endpoint of Hawking decay for black strings Horowitz, Ross

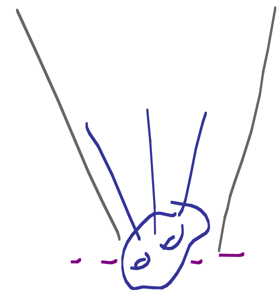
Because the tachyon background

(semiclassically $\left(\int d^2\sigma\, \mu\, e^{-2\kappa X^0}\, \hat{T}\right)_r$ deformation of the worldsheet action)

damps contributions to the worldsheet path integral, it provides a possibility of <u>perturbatively</u> curing the singularity.

We will verify this via systematic computations of perturbative amplitudes, applying methods of Liouville theory

cf Strominger, Takayanagi, ...



$\langle T \rangle$

will find <u>limited</u> support of amplitudes in X^0 direction:

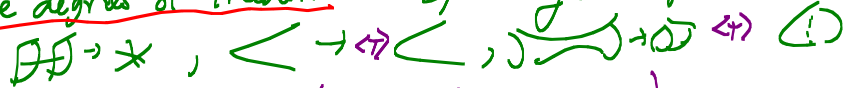
$$\Delta X^0 = \frac{-\ln(\mu/\mu_*)}{\kappa}$$

$\ll$ time to would-be singularity

There are many heuristic arguments suggesting that

$\langle T \rangle \Rightarrow$ "nothing" phase lifting closed strings

- In ~~matter~~ sector, tachyon vertex operator is relevant
  $\rightarrow$ lose degrees of freedom: cf spatially localized cases



- Worldline QFT analogue (a.k.a. minisuperspace) is an exponentially increasing mass (Strominger ...)

$$S_{\text{worldline}} = \int d\tau \left(-(\partial_\tau X^0)^2 + (\partial_\tau \vec{X})^2 - (m_0^2 + \mu^2 e^{-2\kappa X^0}) \right)$$

- There is some large $\leftrightarrow$ small radius correlation between ~~tachyonic~~ systems and those with Witten "bubble of nothing" decays

Our goal in part is to upgrade these arguments to the full perturbative string theory (including worldsheet dynamics & gravity)

Woody Allen: "Eternal Nothingness is fine as long as you're dressed for it."

$$ds^2 = -dt^2 + v^2 t^2 d\vec{\theta}^2 + \dots$$

From the time X^0_s that the smallest circle is string-scale, the time to the singularity is

$$(\Delta X^0)_{sing} = \frac{l_s}{v}$$

This is a difficult problem, so we will procrastinate (tune $v \ll 1$ so that the tachyon effects kick in in time) :

To illustrate tachyon effect, consider formal path integral for 1-loop amplitude

$$Z_1^{(\mu)} = \int [DX] \, e^{-\int d^2\sigma \left(-(\partial X^0)^2 + \underline{v^2} X^{0\,2} (\partial \vec{X})^2 + \mu e^{-\kappa X^0} \hat{T}(\vec{X})\right)}$$

mildly varying $v \ll 1$

$X^0 = X^0_0 +$ fourier modes

* Integrate first over X^0_0

(Gupta Trivedi Wise, Gaoliang Li, Bershadsky Klebanov, ..)

$$\frac{\partial Z_1}{\partial \mu} = \int [DX'] \underbrace{dX^0_0 \left(-\int e^{-\kappa X^0} \hat{T}\right)}_{\frac{d(e^{-\kappa X^0_0})}{\kappa} C} e^{-S_{kinetic}} \, e^{\underbrace{-\mu \int e^{-\kappa X^0} \hat{T}}_{\equiv C \mu e^{-\kappa X^0_0}}}$$

Now setting $y = e^{-KX^0_0}$, the 0-mode integral is of the form $\int_\infty^0 dy\, e^{-\mu c y} = \frac{1}{-\mu c}(-1) = \frac{1}{\mu c}$

yielding
$$Z_1 = \underbrace{\left(\int [DX'] e^{-S_{kin}}\right)}_{\text{Free nonzero mode path integral}} \underbrace{\left(-\frac{1}{K}\ln\frac{\mu}{\mu_*}\right)}_{\substack{\text{finite result} \\ \text{where in flat} \\ \text{space would sit} \\ \delta(0) \text{ from } \infty \\ \text{range of } X^0_0}}$$

$v \ll 1 \Rightarrow$

$X^0_* \gg -\frac{1}{K}\ln\frac{\mu}{\mu_*}$

$\mu_* = e^{KX^0_*}$

$$-\frac{1}{K}\left(\ln\mu - \ln\mu_*\right)$$
$$= \underbrace{-\frac{\ln\mu}{K}}_{\text{finite cutoff in past}} + X^0_*$$

This effect kicks in at the string scale

$l_s \gg l_{planck} \qquad (g_s \ll 1)$

so black hole production avoided.

(cf Liu, Moore, Seiberg, Horowitz/Polchinski, Lawrence Fabinger/McGreevy)

In particular, the SUSY-breaking choice of boundary conditions which led to $\langle T \rangle$ improves control: generic fluctuations can help smooth out the physics.

QFT interlude:

Because this effect arose from the behavior of X^0: (albeit in the full string path integral), it is interesting to understand how much of this physics appears at the level of QFT   cf Strominger, Schomerus

Scalar φ with X^0-dependent mass $m^2 = \mu^2 e^{-2\kappa X^0} + m_0^2$

worldline action

$$S_{wl} = \int d\tau \left\{ -(\partial_\tau X^0)^2 + (\partial_\tau \vec{X})^2 - m^2(X^0) \right\}$$

Time dependent Q.M.: generically, even if start in ground state, will end up in excited state

Heisenberg picture field $\varphi = u_{\vec k}(t)\, a^\dagger_{\vec k} + u^*_{\vec k}(t)\, a_{\vec k}$

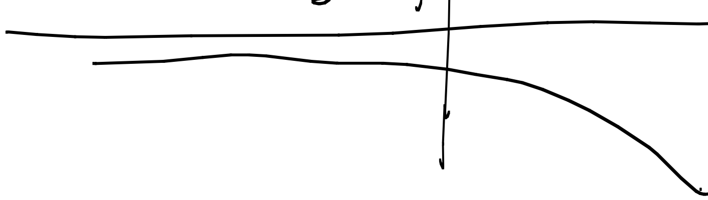
satisfies Heisenberg equation of motion

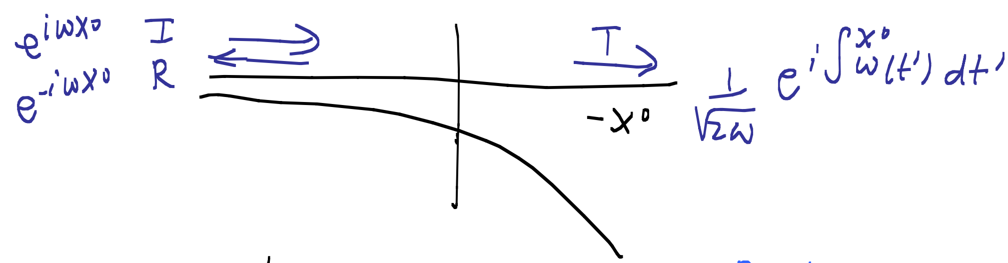
$$-\ddot u - \left(\vec k^2 + m^2(X^0)\right) u = 0$$

In the worldline description, this is the Hamiltonian constraint for on-shell vertex operators

$$\mathcal{H} = \frac{\partial^2}{\partial X^{0\,2}} + \left(\vec k^2 + m^2(X^0)\right)$$

Effective Schrodinger problem with potential $-m^2(X^0)$





transmission/reflection problem yields Bogoliubov coefficients

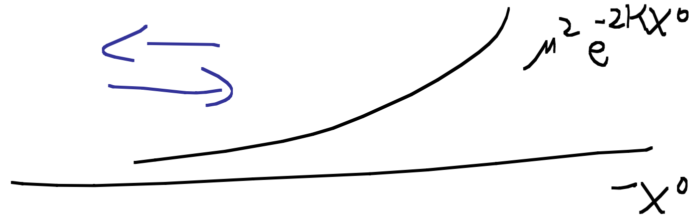
$$\alpha e^{i\omega x^0} + \beta e^{-i\omega x^0} \to \frac{1}{\sqrt{2\omega}} e^{i\int^{x^0} \omega(t')dt'}$$

$$|\alpha|^2 - |\beta|^2 = 1 \qquad \langle in | a^\dagger_{out} a_{out} | in \rangle = |\beta|^2$$

Can read off α, β from Bessel ftn sol'ns.

Easier trick: $\left\{ \begin{array}{c} \mu^2 \to -\mu^2 \\ \delta_2 \\ X^0 \to iX^0, \; K \to iK \end{array} \right\}$ continuation from positive exponential potential

$$-\frac{\partial^2}{\partial X^{0\,2}} - \mu^2 e^{-2KX^0}$$



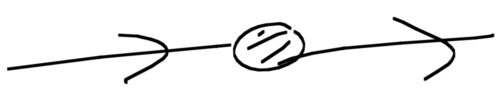
$$\frac{R(\omega)}{I(\omega)} \propto \mu^{\frac{2i\omega}{\kappa}} \; ; \quad \left|\frac{R(\omega)}{I(\omega)}\right| = 1$$

$$\mu^2 \to e^{-i\pi}\mu^2 \Rightarrow \left|\frac{\beta}{\alpha}\right| = e^{-\frac{\pi\omega}{\kappa}}$$

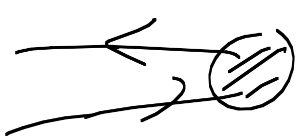
$$\Rightarrow \quad |\beta|^2 = \frac{1}{e^{\frac{\pi}{\kappa}(2\omega)} - 1}$$

Thermal distribution of pairs of particles (pure state)

$|\alpha|^2 - |\beta|^2 = 1$



$$\langle e^{ipa} e^{-ipa} \rangle$$



$$\langle e^{ipa} e^{ipa} \rangle$$

We will find this thermal spectrum also in string theory, using similar tricks. First, one more point in the QFT case: S-matrix fails for arbitrary states, as if time stopped:

$$-m^2(X^0) = -\mu^2 e^{-2kX^0} \to -\infty$$ so fast that the worldline Hamiltonian is not self-adjoint on the full ~~set~~ of states of the system

$$\psi_{\omega_0} \to \frac{1}{\sqrt{2\omega(t)}}\, e^{\pm i\int^{-X^0} \omega(t')dt'}$$ become independent of $\omega_0^2 = \vec{k}^2 + m_0^2$ at large $|X^0|$

$$(\psi_1, \hat{H}_{we}\psi_2) - (\hat{H}\psi_1, \psi_2) = (E_1 - E_2)(\psi_1, \psi_2) \neq 0$$

The system behaves as if it is in a finite time box

$$\int dt \; \psi^*_{\omega_0}(t)\, \psi_{\omega_0'}(t) \Big|_{\omega_0 \to \omega_0'} \propto \underline{\frac{1}{K}}_{\text{finite}}$$

$$\left(\psi \propto \frac{1}{\sqrt{2\omega}} \propto e^{-\frac{K(x-x_0)}{2}} \right)$$

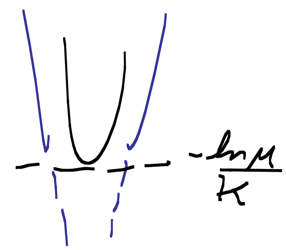
even though it was officially formulated in Minkowski space, which ordinarily affords a square S-matrix

This leads to failure of naive LSZ prescription for S-matrix unless restrict states to linear combinations of (at a linearized level)

$$|\nu\rangle = e^{e^{i\nu} a^\dagger_{out} a^\dagger_{out}} |in\rangle$$

in terms of a phase ν. Our previous choice is

$$|in\rangle = \int d\nu \, |\nu\rangle$$



Armed with this background, now go back to string computations. (Analogy not sufficient: e.g. $\langle T \rangle$ lifts graviton as well as QFT & higher string modes)

Work in superstring in critical dimension in conformal gauge, e.g. heterotic:

Formal Path Integral in Lorentzian signature

$$G(\{V_n\}) = \int [d\mathbb{X}][d\Psi_-][d(\text{ghosts})]\, d(\text{moduli})\, e^{iS} \prod_n \left(i\int d\sigma\, d\tau\, V_n(\mathbb{X})\right)$$

$$\mathbb{X}^\mu = X^\mu + \theta^+ \psi_+^\mu \qquad \Psi_-^a = \psi_-^a + \theta^+ F^a$$

where S = semiclassical action $= \int d\sigma\, d\tau\, d\theta^+ \Big\{ D_{\theta^+}\mathbb{X}^\mu \partial_- \mathbb{X}^\nu G_{\mu\nu}(\mathbb{X})$

$$+ \Psi_- D_{\theta^+} \Psi_- - \mu\, \underbrace{\Psi_- :e^{-k\mathbb{X}^0} \cos(w\tilde{\theta}):}_{\text{winding tachyon semiclassically}} + (\text{ghost}) + (\text{dilaton}) \Big\}$$

Timelike Liouville

and the other ingredients are:

vertex operators $\quad V_{\vec{k},n} \simeq \int d\theta^{+}\, e^{i k\cdot \mathbb{X}}\, \hat{V}_n$

semiclassically

dilaton $\quad \Phi = \Phi_0 \simeq -\infty$

semiclassically.

* As in Liouville field theory, the path integral generates automatically the appropriate corrections to the semiclassical quantities.

Wick rotation

$$r \equiv e^{i\gamma} r_\gamma \quad \vec{X} \equiv e^{i\gamma}\vec{X} \quad \mu \equiv e^{-i\gamma}\mu_\gamma \quad \vec{k} = e^{-i\gamma}\vec{k}_\gamma \quad Q_\gamma = \gamma\left(\frac{2}{\pi}\right) Q$$

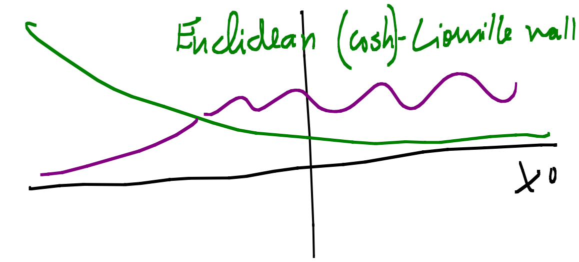
with $\gamma \to \frac{\pi}{2}$

produces a convergent definition of the path integral $\to$ $\int (DX_E) \cdots e^{-S_E}$

* positive potential from tachyon suppresses contributions to amplitudes

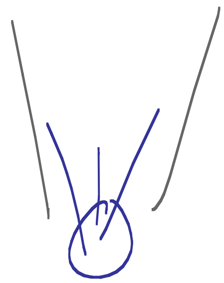
$$S_E^{(Het)} = \text{positive kinetic terms} + \text{fermions} + \mu_E^2 e^{-2\kappa X^0} \cosh^2 w \Omega_E^n$$

☆ See Strominger, Takayanagi, Schomerus... for a more conventional rotation ($X^0 \to iX^0$ $\kappa \to i\kappa \cdots$) which yields same results.



To start, focus on the "Euclidean" or "Hartle-Hawking" state: no excitations above $\langle T \rangle$ in the past. More on the status of other vacua later.

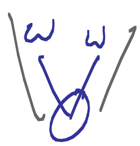
Interpretation of Amplitudes: (cf Polyakov '92)



Correlation functions of bulk vertex operators give components of the state of closed strings

(cf Gutperle, Strominger, Takayanagi, ...)

e.g. 2-point function: gives the Bogoliubov coefficients:



$$|\Psi\rangle = \mathcal{N} e^{\frac{\beta}{2\gamma} a^{+2}} |0\rangle$$

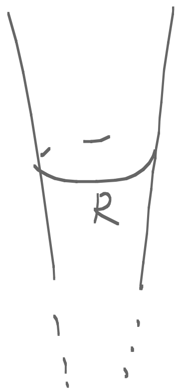
$$\rightarrow \frac{\beta}{\gamma} = \frac{\langle\Psi| a^{+2} |0\rangle}{\langle\Psi|0\rangle} = \langle \int V_\omega \int V_\omega \rangle \leftrightarrow$$

related to reflection amplitude in Euclidean continuation

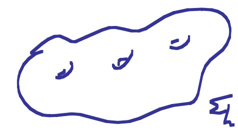
(Mixing of positive & negative frequency modes $e^{i\omega t} \rightarrow \gamma e^{i\omega t} + \beta e^{-i\omega t}$)

(Mixing of positive & negative momentum modes)

Let us start with the vacuum amplitudes



we chose velocity $v = \frac{dR}{dX^0} \ll 1$



so that starting at string scale, the time to the GR-predicted singularity is

$$\Delta X^0_* \sim \frac{l_s}{v} .$$

Whereas flat space vacuum amplitudes are extensive in spacetime,

$*$

$$Z_{flat} = \delta(0)\, \hat{Z}$$

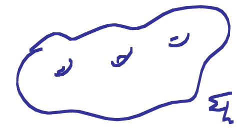
↖ volume of time from X^0_0 zero-mode integral

ours will only have support for

$$\left(\Delta X^0 \sim \frac{-\ln \mu/\mu_*}{\kappa}\right) \ll \left(\Delta X^0_* \sim \frac{l_s}{v}\right)$$

In our case the vacuum amplitudes are

$$Z_h = \int [dX_E][d(\ldots)]\, e^{-\int_{\Sigma_h} d\sigma_E^2\, d\theta^+ \left(\mathcal{L}^{(T=0)} + \mu_E e^{-\kappa X^0} \hat{T}\right)}$$



Split $X^0 = X^0_0 + \hat{X}^0(\sigma, \tau_E)$ and calculate

X^0_0: 0-mode

$$\frac{\partial Z_h}{\partial \mu_E} = \int [d\vec{X}][d(\ldots)][d\hat{X}^0]\, \underbrace{dX^0_0\, e^{-\kappa X^0_0}}_{\frac{dy}{-\kappa}}\, \frac{C}{\mu_E}\, e^{-S_E^{(T=0)}}\, e^{-C e^{-\kappa X^0_0}}$$

where $C = \int \mu_E\, e^{-\kappa \hat{X}^0}\, \hat{T}$. The 0-mode integral is of the form $\left(y \equiv e^{-\kappa X^0_0}\right)$

$$\int_{\simeq 0}^{\infty} dy\, e^{-Cy} \simeq \frac{1}{C}$$

$\leftarrow$ ($v \ll 1 \Rightarrow$ will be self-consistent)

This yields (cf Gupta, Trivedi, Wise ; Bershadsky, Klebanov '90 in LFT)

$$Z_h = \Big(\underbrace{-\frac{\ln \mu/\mu_*}{k}} \; \underbrace{- i \frac{\pi}{2k}} \Big) \hat{Z}_h (T=0)$$

* Range of X^0 restricted to bulk region where $T \to 0$

($\Delta X^0_* \sim \frac{l_s}{v}$ = time to would-be singularity)

continuation back to $\mu = e^{-\frac{i\pi}{2}} \mu_E$

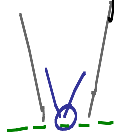
↳ thermal state: $X^0 \to X^0 - i\beta$ with $\beta = \frac{\pi}{k}$ will be corroborated below.

nonzero mode contribution to $T=0$ bulk theory

⇒ • (sum over) closed string states lifted in $\langle T \rangle$ phase

• Back reaction from quantum stress-energy controllably small

The 2-point function $\langle \int V_\omega \int V_\omega \rangle = \frac{\beta}{\alpha}$



gives the Bogoliubov coefficients:

- The X_0^0 integral $\Rightarrow \langle \int V_\omega \int V_\omega \rangle \propto \mu_E^{-\sum_n \frac{\omega_n}{\kappa}}$
- The magnitude of the result is 1 in the Euclidean continuation (total reflection off a Liouville wall)

$$\left| \frac{\beta_{\vec{k},n}}{\alpha_{\vec{k},n}} \right| = e^{-\omega(\vec{k},n)\frac{\pi}{\kappa}} \quad \underset{|\beta|^2 \mp |\alpha|^2 = 1}{\Rightarrow} \quad \boxed{\ast \; N_{\vec{k},n} = \frac{1}{e^{\frac{\pi}{\kappa}(2\omega)} \mp 1}}$$

$$N_{\vec{k},n} = |\beta_{\vec{k},n}|^2$$

Thermal distribution of created pairs at temp $\frac{\kappa}{\pi}$ as above

Finally, one can assess the singularity structure of other perturbative amplitudes similarly. Again as in Liouville, find divergences at $\sum \omega_n (\pm)_n = 0$ (where X_0^0 integral unsuppressed in bulk: these are expected divergences from physical states).

In earlier work, some issues with continuation of 3-pt ftns — cf Schomerus ...

Remarks: • $\langle T \rangle$ and the "Nothing Phase": our result that $\langle T \rangle$ shuts off support of amplitudes lines up with many heuristic arguments, including:

- In matter sector, tachyon vertex operator is relevant $\to$ lose degrees of freedom: cf spatially localized cases

- Worldline QFT analogue (a.k.a. minisuperspace) is an exponentially increasing mass (Strominger ...)

$$S_{\text{worldline}} = \int d\tau \left(-(\partial_\tau X^0)^2 + (\partial_\tau \vec{X})^2 - (m_0^2 + \mu^2 e^{-2\kappa X^0}) \right)$$

- There is some large $\leftrightarrow$ small radius correlation between tachyonic systems and those with Witten "bubble of nothing" decays

$*$ Note that this phase is not necessarily static, but is adiabatic

• These results indicate a perturbative, ~~stringy~~ mechanism for "starting time from nothing"

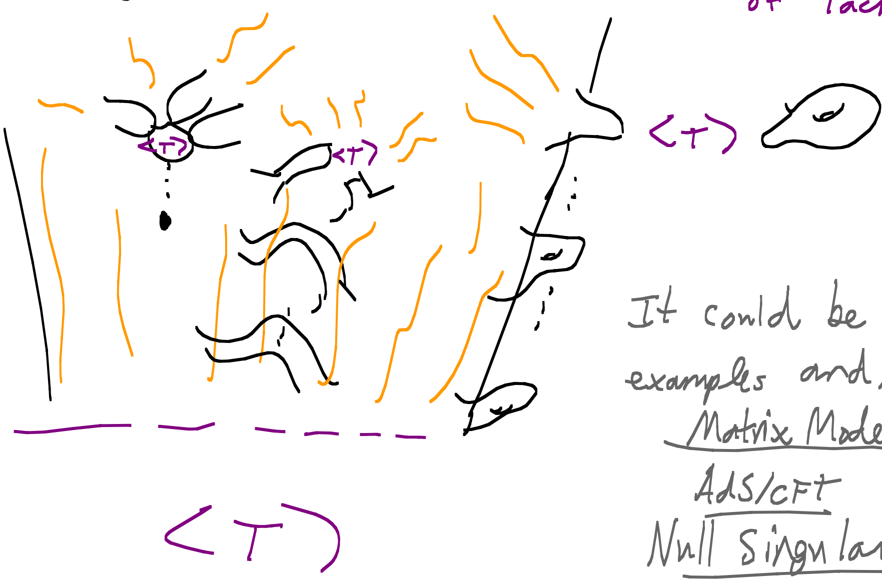
cf. • perturbative, stringy topology change & Baby universe production by winding tachyons (ALMSS 0502021) • note finite time

• winding-string catalyzed perturbative bubble of nothing (Horowitz)

These are subjects previously studied via Euclidean quantum gravity instantons (cf Hartle, Hawking, Linde, Vilenkin, Giddings, Strominger & more recently Tye..., Dijkgraaf, Gopakumar, Gukov, Ooguri, Vafa, Verlinde ...

A lesson I take from this is that although (because!) they are instabilities, Tachyonic modes play a useful role in addressing problems of gravity.

Big Picture that is suggested in this perturbative regime: Regions of bulk spacetime smoothly end at regions of Tachyon condensate $\langle T \rangle$



It could be interesting to relate to other examples and/or approaches e.g.

- Matrix Model $\langle T \rangle$: Polchinski, Karczmarek, Strominger, ...
- AdS/CFT Shenker et al, Hertog/Horowitz ...
- Null Singularities Liu Moore Seiberg ... Berkooz et al
  - → matrix theory Craps Sethi Verlinde
- Euclidean Q.G.   String FT Zweibach

Ongoing Work

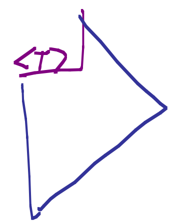
• D-branes in $\langle T \rangle$ phase (Green, Starr)

w/ G. Horowitz

• Status of other putative vacua: cf BRST anomaly

• the states with unitary "reflection off $X^0 = X^0_*$"

may allow us to microphysically check for some cases the "Black Hole Final State" proposal of Horowitz & Maldacena (cf Gottesman, Preskill...)



• In any case, these results may apply to the spacelike singularities in black hole physics. cf Horowitz

• Large ν regime: