Multi-Throat Brane Inflation and Heating

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hep-th/0408084; 0501184; astro-ph/0507053;

hep-th/0602136 with H. Tye

hep-th/0605045 with M. Huang, S. Kachru, G. Shiu.

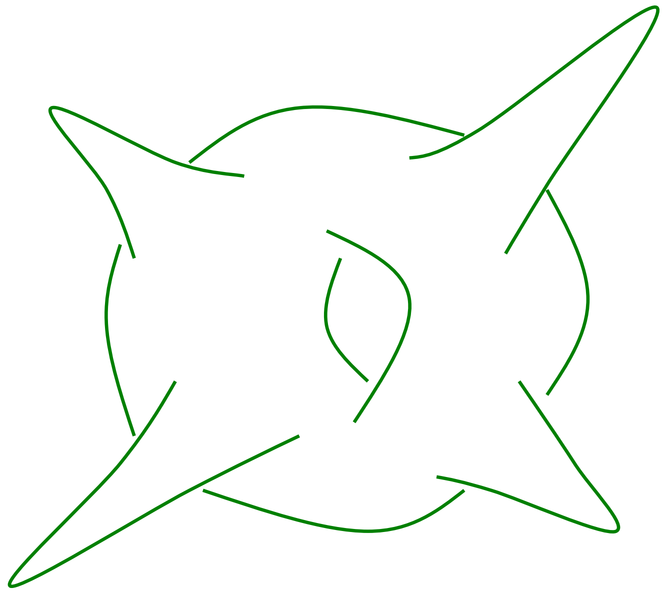
Brane Inflation in Warped Compactification

- Brane inflation (Dvali, Tye, 98; Burgess,Majumdar,Nolte,Quevedo, Rejesh,Zhang;Dvali,Shafi,Solganik,01)

  Brane position as inflaton;

  Brane annihilation or collision as ending.

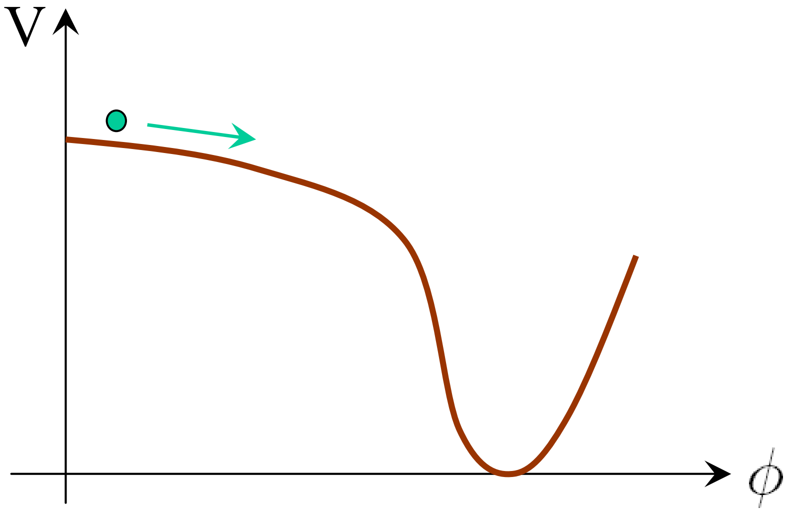
- Warped compactification



Gidding, Kachru, Polchinski, 01

Verlinde, 99

Slow-roll inflation



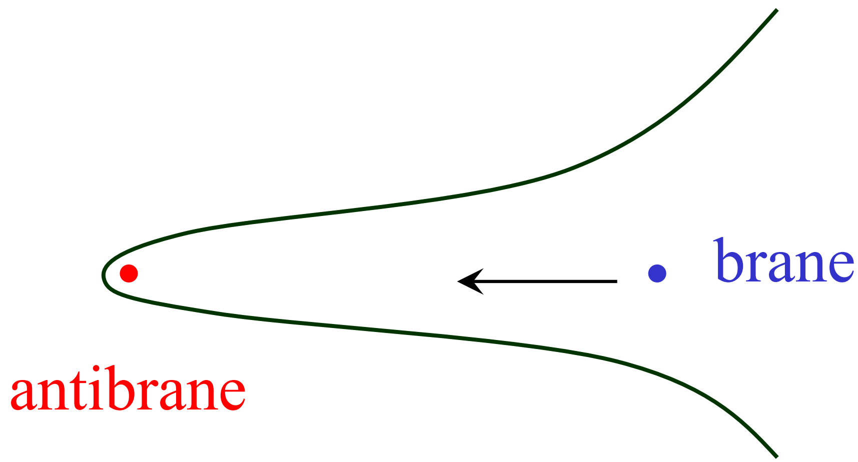
Slow-roll conditions:

$$\epsilon_V = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \quad << 1$$

$$\eta_V = M_{pl}^2 \frac{V''}{V} \quad << 1$$

- η-problem: $V \sim \pm H^2 \phi^2 \Longrightarrow \eta \sim 1$

- Warped space Reduce (red-shift) brane tension:
  → helps slow-roll inflation



KKLMMT, 03

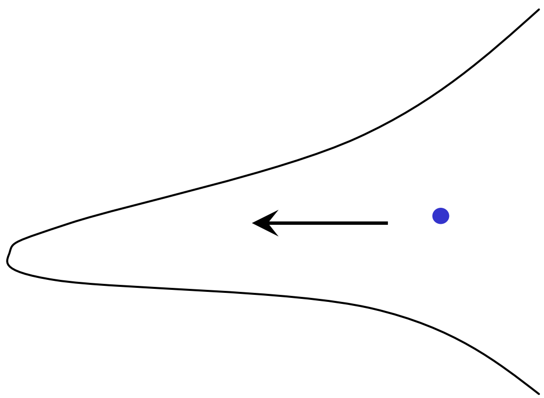
$T_3 \to h^4 T_3$, h is the warp factor.

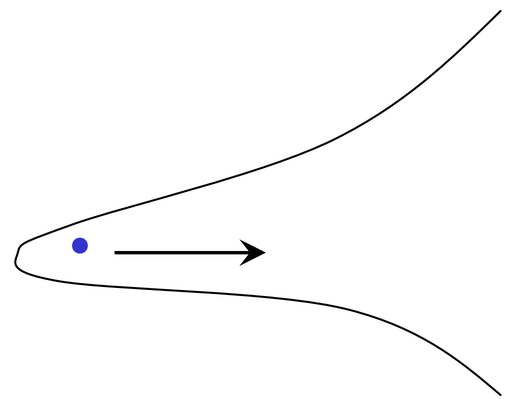
- η-problem:

Volume stabilization;
Volume-scalar coupling; } ⟹ $V \sim \pm H^2 \phi^2 \longrightarrow \eta \sim 1$

- Provide speed-limit: generate DBI inflation.

$$ds^2 \propto \frac{\phi^2}{\lambda} ds_4^2 + \frac{\lambda}{\phi^2} d\phi^2 \quad \longrightarrow \quad |\dot{\phi}| \leq \frac{\phi^2}{\lambda}$$





UV model (Silverstein, Tong, 03)

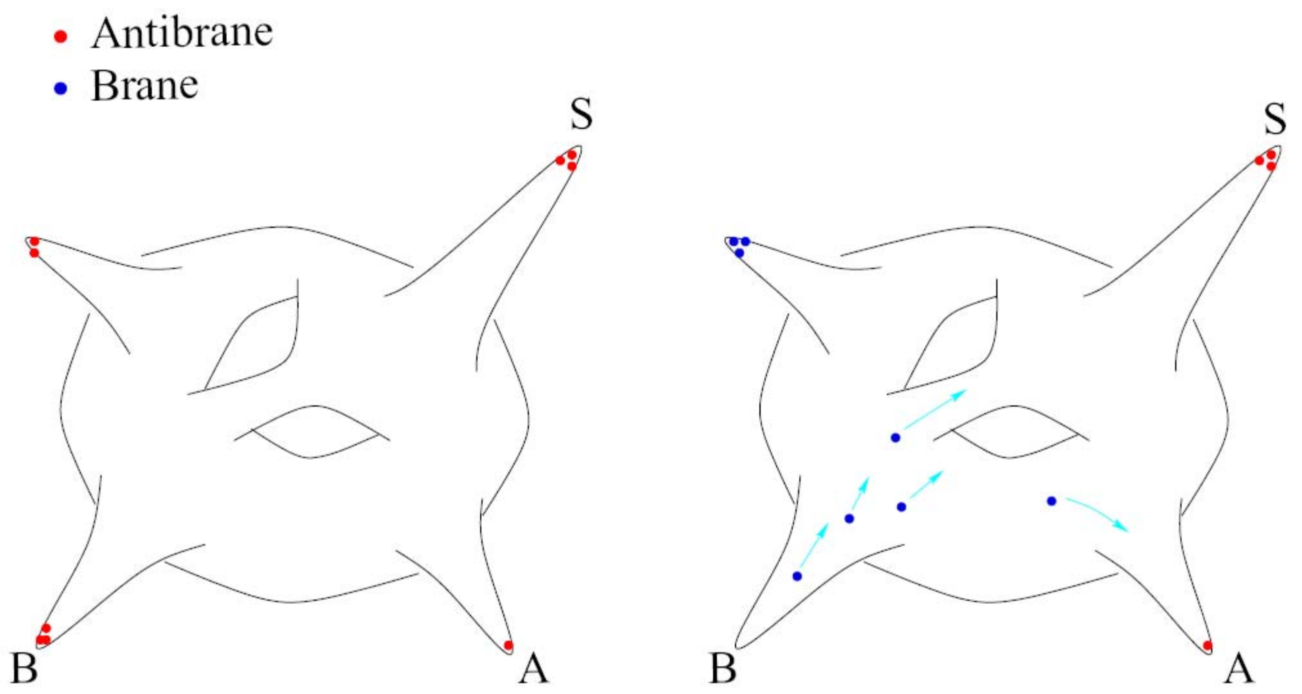
$$V(\phi) \approx \tfrac{1}{2} m^2 \phi^2$$
$$m \gg M_{\rm Pl}/\sqrt{\lambda}$$

IR model (X.C. 04)

$$V(\phi) \approx V_0 - \tfrac{1}{2} m^2 \phi^2$$
$$m \sim H$$

- Source of potential
- Range of scalar

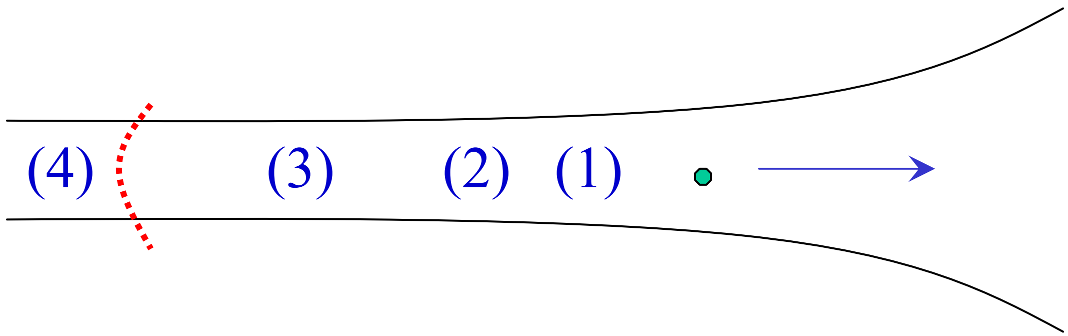
- Multi-throat brane inflation (X.C. 04)



- Antibrane-flux annihilation (Kachru, Pearson, Verlinde, 01)
- Generate branes as candidate inflatons
- Exit B-throat, roll through bulk, settle down in another throat
- Enough warping: DBI inflation;
  Flat potential: slow-roll inflation.

Brane Dynamics (X.C. 04, 05)

$$V = V_0 - \frac{1}{2}\beta H^2 \phi^2 , \qquad \phi = -\frac{\sqrt{\lambda}}{t} + \frac{9\sqrt{\lambda}}{2\beta^2 H^2 t^3} + \cdots , \quad t \ll H^{-1}$$



1) $h > \sqrt{\beta}\frac{RH}{\lambda^{1/8}}$: Field theory DBI action applies;

2) $\frac{RH}{\lambda^{1/4}} < h < \sqrt{\beta}\frac{RH}{\lambda^{1/8}}$: Open string creation (Stringy quantum fluctuations);

3) $\frac{RH}{\lambda^{1/2}} < h < \frac{RH}{\lambda^{1/4}}$: Closed string creation starts;

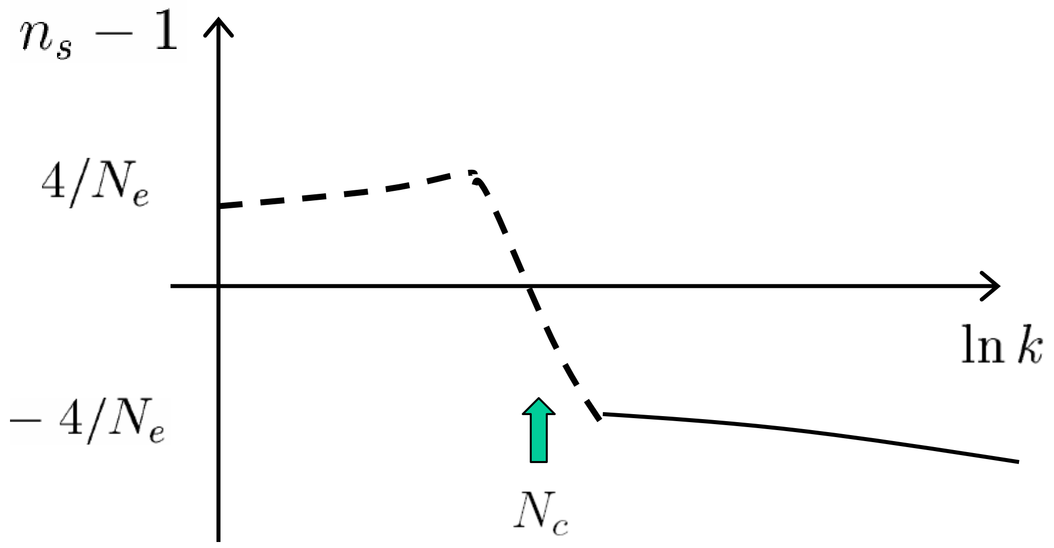
4) $h < \frac{RH}{\lambda^{1/2}}$: Closed strings smooth out background (de Sitter back-reaction cuts off the throat).

Density Perturbations (X.C. 04, 05)

- Density perturbations: $$\delta_H = \frac{2}{5} H \delta t \approx \frac{N_e^2}{5\pi\sqrt{\lambda}}$$
- Spectrum index: $$n_s - 1 \approx -\frac{4}{N_e}$$
- A phase transition:
  - Hubble scale < string scale: $$\gamma H \lesssim T_3^{1/4} h$$
  - Fluctuation speed < speed of light: $$\Delta r \leq \Delta r_{max} \approx \gamma^{-1} H^{-1} h^2$$

➡ Phase transition at: $$N_c \sim \frac{\lambda^{1/8}}{\beta^{1/2}}$$

$$\delta_H = \frac{2}{5} H \delta t \lesssim \frac{18}{5\beta^2 N_e^2}$$ ➡ $$n_s - 1 \sim \frac{4}{N_e} \quad \text{if} \quad N_e > N_c$$



Large running of n_s around the critical e-fold N_c ,
i.e. large and negative $dn_s/d\ln k$;
away from N_c , the running is negligible.

(c.f. WMAP3, $dn_s/d\ln k = -0.055^{+0.029}_{-0.035}$)

Background charge (or Euler number) of string compactification;
Hubble-Hagedorn phase transition.

Large Non-Gaussianities in CMB

(Alishahiha,Silverstein,Tong,04; X.C. 05;Gruzinov,04)

- Non-Gaussianities in general single field inflation are characterized by 5 parameters:

$$c_s\ , \quad \frac{\lambda}{\Sigma}\ , \quad \epsilon\ , \quad \eta\ , \quad s.$$

(X.C., Huang, Kachru, Shiu, 06)

c.f. slow-roll inflation, 2 parameters:

$$\epsilon\ , \quad \eta$$

(Maldacena, 02; Seery, Lidsey, 05)

- Leading Non-Gaussianities:

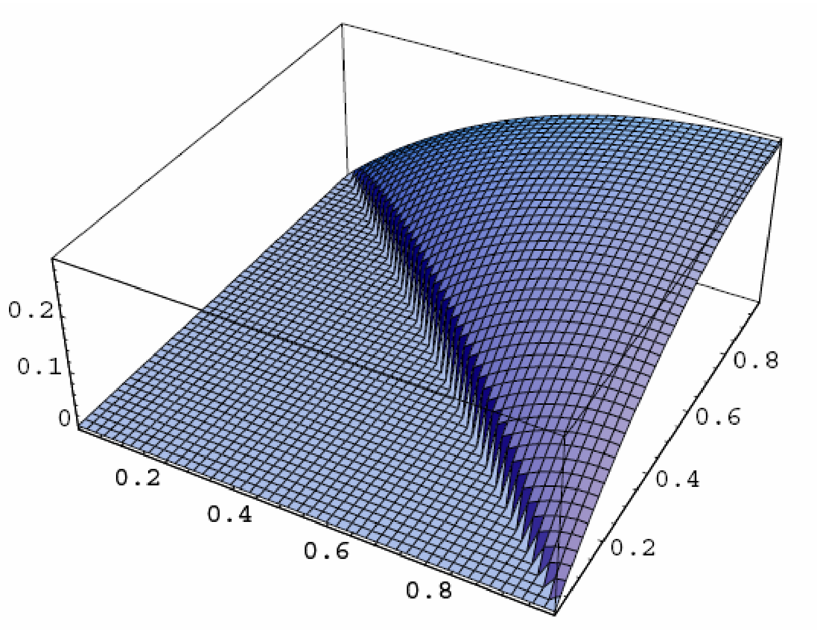
$$f_{NL}^c = \frac{35}{108}\left(\frac{1}{c_s^2}-1\right)\ , \quad f_{NL}^\lambda = -\frac{5}{81}\left(\frac{1}{c_s^2}-1-\frac{2\lambda}{\Sigma}\right)$$

For DBI inflation, $c_s \ll 1$, $\frac{1}{c_s^2}-1=\frac{2\lambda}{\Sigma}$;

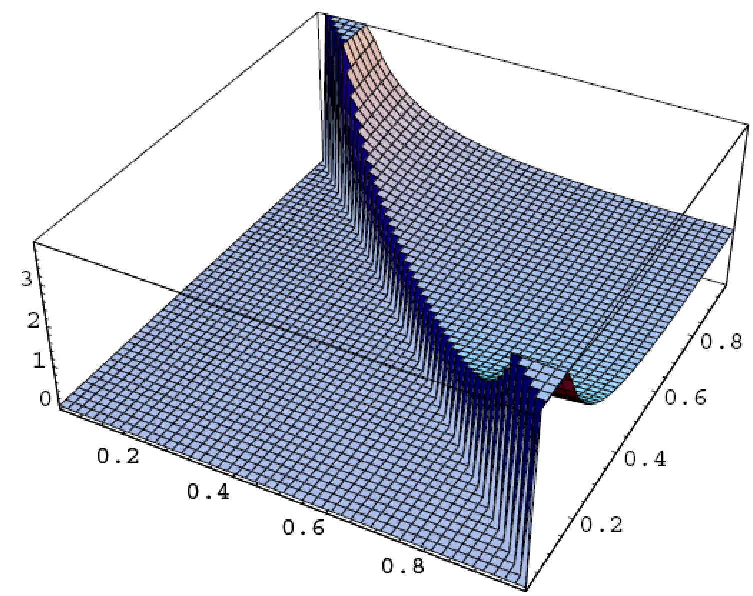
For slow-roll inflation, $c_s=1$, $\lambda=0$.

Shape of Non-Gaussianities

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3) \rangle$$



DBI inflation



Slow-roll inflation

Babich, Creminelli, Zaldarriaga, 04; X.C., Huang, Kachru, Shiu, 06.

Running of Non-Gaussianities (X.C. 05)

- Running index: $$n_{NG} - 1 \equiv \frac{d \ln f_{NL}}{d \ln k}$$

Time dependence of warp factor h

⇨ Lorentz factor $\gamma = 1/c_s$ ⇨ Non-Gaussianity f_{NL}

- For AdS geometry, $f_{NL} \approx 0.036\beta^2 N_e^2$, $n_{NG} - 1 \approx -\frac{2}{N_e}$.

For a constant warp factor, $n_{NG} - 1 \approx \frac{2}{N_{tot} - N_e}$.

Shape and running of non-Gaussianities

⇕

Geometries of string compactification

Other signatures

- UV DBI model, spectrum index, tensor modes, non-Gaussianities:

  Alishahiha, Silverstein, Tong, hep-th/0404084;
  Shandera, Tye, hep-th/0601099;
  Kecskemeti, Maiden, Shiu, Underwood, hep-th/0605189;
  Shiu, Underwood, hep-th/0610151;
  Seery, Lidsey, astro-ph/0610398;
  Huang, Shiu, hep-th/0610235;
  Baumann, McAllister, hep-th/0610285.

- Cosmic strings

  Jones, Stoica, Tye, hep-th/0203163;
  Sarangi, Tye, hep-th/0204074;
  Copeland, Myers, Polchinski, hep-th/0312067.

- Reviews:

  Polchinski, hep-th/0412244; Tye, hep-th/0610221

(Re)Heating Problem

Brane-antibrane annihilation

→ Heavy closed strings

(Sen, 02, Lambert, Liu, Maldacena, 03)
(X.C. 03, Leblond, 05)

→
- Light closed strings
- KK modes
- Gravitons
- A fraction of open strings ??

But, during BBN,

- Mostly photons, neutrinos and electrons (open strings)
- Graviton density < a few %
- Tiny amount of stable massive relics $\sim 10^{-6}$

Questions:

Closed string degree of freedom

Open string degree of freedom with

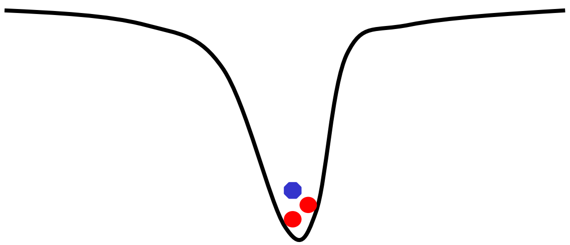
- Negligible gravitons
- Non-lethal amount of relics

(Barnaby, Burgess, Cline, 04; Kofman, Yi, 05,
Chialva, Shiu, Underwood, 05; Frey, Mazumdar, Myers, 05;
X.C., Tye, 06; Langfelder, 06; ……)

Different Cases

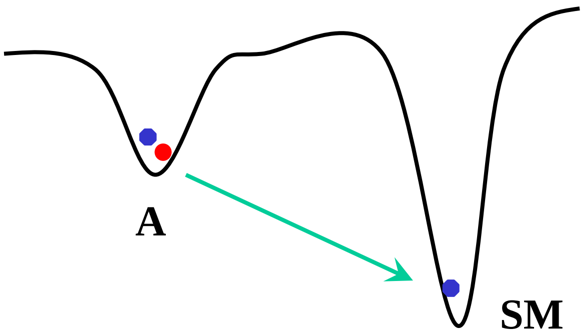
- Single throat case

  Brane annihilation and SM branes in the same throat.



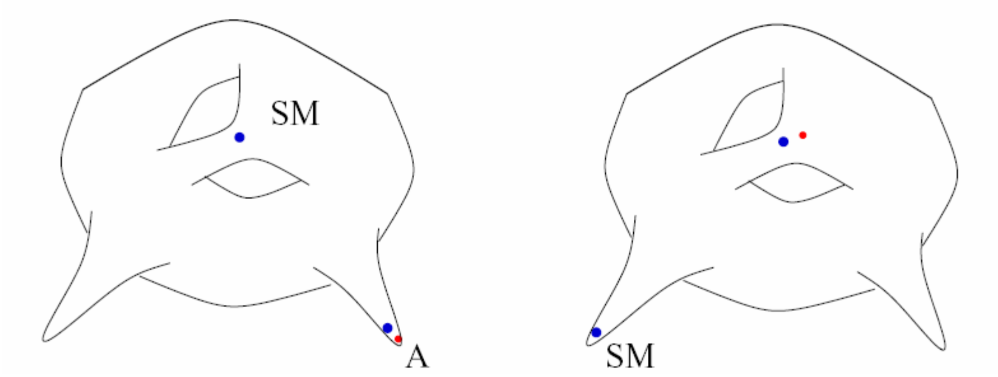
- Double throat case

  In different throats.
  KK modes have to tunnel to heat the SM branes.



- Bulk case

  One of them in the bulk.
  Tunnel is also required.



Previous Studies

- Important observations: (Barnaby, Burgess, Cline, 04)

KK: peak at bottom of throat.
Graviton: uniform in extra dimensions.

⇨ Graviton emission is suppressed by h^2 (h: warp factor).

- Unsolved problems:

➢ Stable KK relics
Previous estimation: $\Omega_{KK} \gg 1 \quad (\sim 10^{22} \text{ for } m \sim 10^{14} \text{ GeV})$
(Kofman, Yi, 05)

➢ Double throat case
Tunneling rate << initial graviton production rate,
over-production of Gravitons from KK annihilation.
(Chialva, Shiu, Underwood, 05)
(Barnaby, Burgess, Cline, 04)

Key Issue:

Thermal history of KK particles
in warped compactification.

Other new ingredient:

Use GKP setup instead RS
---- more relevant to brane inflation.

We will see:

Properties of warped compactification
nicely solve the problems,
with several interesting bonus.

(X.C., Tye, 06)

KK Modes in a Throat

- Local throat metric

$$ds^2 = A^{-1/2}(-dt^2 + d\mathbf{x}^2 + h_{\mu\nu}dx^\mu dx^\nu) + A^{1/2}(dr^2 + r^2 d\Omega_5^2) \ ,$$

$$A(r) = 1 + \frac{R^4}{r^4} \ .$$

- S-wave of metric perturbations

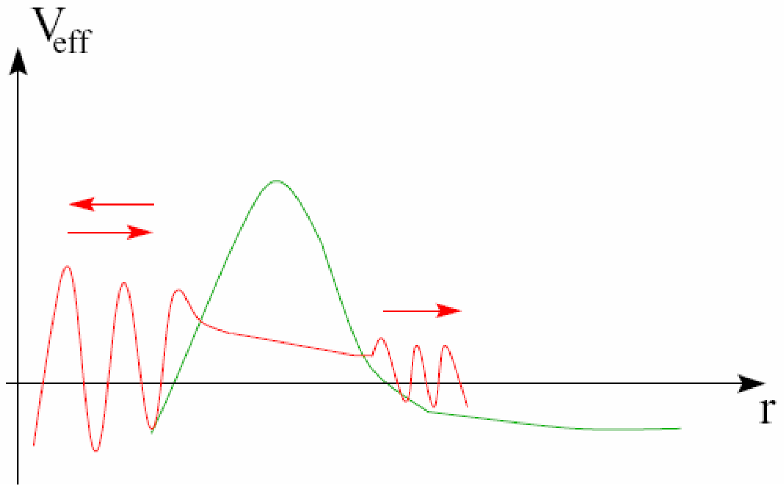
$$h_{\mu\nu} = \tilde{\psi}_{\mu\nu}(x)\phi(r) \ .$$

- E.O.M. for s-wave KK modes

$$\frac{1}{\rho^5}\frac{d}{d\rho}\left(\rho^5\frac{d\phi}{d\rho}\right) + \left(1 + \frac{(mR)^4}{\rho^4}\right)\phi = 0 \ ,$$

$m^2 = -p^2$ is the mass of the KK mode with $\tilde{\psi}_{\mu\nu} \propto e^{ipx}$, $\rho \equiv mr$.

Tunneling-out Behavior



$$\rho \ll m^2R^2: \qquad \phi = -i\frac{32\sqrt{2}}{\pi^{3/2}} m^{-5} R^{-5} \rho^{-3/2} \cos\left(\frac{m^2R^2}{\rho} - \frac{5\pi}{4}\right) ,$$

$$m^2R^2 \ll \rho \ll 1: \qquad \phi = -\frac{4i}{\pi}\rho^{-4} ,$$

$$\rho \gg 1: \qquad \phi = \sqrt{\frac{2}{\pi}}\rho^{-5/2} e^{i(\rho - \frac{5\pi}{4})} .$$

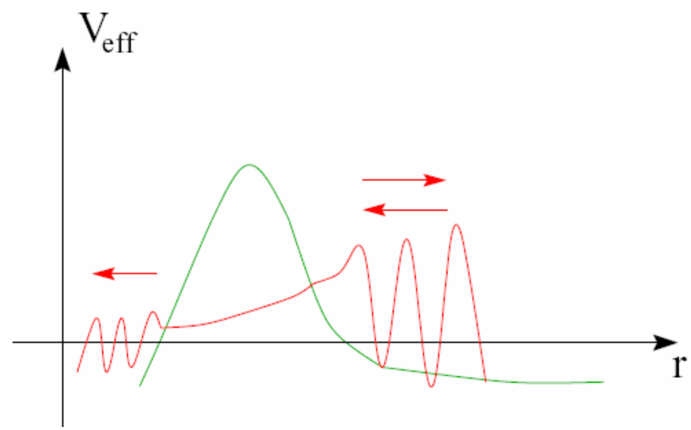
Tunneling probability: $P = \frac{\pi^2}{2^8} m^8 R^8$

Mass quantization: $\Delta m \approx h_0 R^{-1}$

Tunneling between throats

- Absorption cross section

$$\sigma = \frac{\pi^4}{8} m^3 R^8$$



- Tunneling between A and X throat
  - Bulk resonance: $L > h^{-1} R$

$$\Gamma_{A \to X} \approx \text{Min}\ (\Gamma_{A \to bulk}, \Gamma_{bulk \to X}) \lesssim h^9 R^{-1}$$

  - Non-resonance

$$\Gamma_{A \to X} \approx h^{17} R^{-1}$$

Note: in both cases, much smaller than 5d RS case $\Gamma \sim h^5 R^{-1}$
from Dimopoulous, Kachru, Kaloper, Lawrence, Silverstein, 01

Interactions

- KK self-interactions: $KK + KK \to KK$

$$\sigma_{KK} \approx \left(\frac{L}{R}\right)^6 \frac{h_0^{-2}}{M_{\rm Pl}^2} .$$

- KK-SM interactions: $H + H \to KK$

$$\sigma_{HK} \approx \left(\frac{L}{R}\right)^6 \frac{h_0^{-2}}{M_{\rm Pl}^2}$$

- Graviton productions: $KK + KK \to g, \ H + H \to g$

$$\sigma_{Hg} \approx \sigma_{Kg} \approx \frac{1}{M_{\rm Pl}^2}$$

Initial State after Brane Inflation

- Density: $\rho \approx M_{\rm Pl}^2/t^2$

Initially, $\rho \approx h_A^4 T_3 \longrightarrow t_0 \approx g_s^{1/2} h_A^{-2} M_{\rm Pl}/m_s^2.$

- Closed strings $\xrightarrow{\textbf{decay}}$ lighter strings, KK, gravitons

$\Delta t \sim g_s^{-2} h_A^{-1} m_s^{-1} \ll t_0$ (Iengo, Russo, 06)

- KK interaction rate Γ >> Expansion rate H

$\Longrightarrow$ Thermal equilibrium

$\rho \approx \frac{\pi^2}{30} g T^4$ (g: d.o.f.)

$\longrightarrow T_0 \approx (g g_s)^{-1/4} h_A m_s$ < red-shifted string scale
> lowest KK mode

- Initial state: A tower of thermalized relativistic KK with possible brane fields at bottom of mass spectrum.

Single Throat Heating

- Temperature

$$T(t) \approx g^{-1/4} \left(\frac{M_{\rm Pl}}{t} \right)^{1/2}$$

As T drops below $m = n_{KK} h_0 R^{-1}$

n_{KK}th KK $\longrightarrow$ non-relativistic

$$n_{nr} = g \left(\frac{mT}{2\pi} \right)^{3/2} e^{-m/T}$$

Then decouples as $\Gamma = n_{nr} \sigma_{KK} v$ < $H(t_{dec})$

- Relic number density:

$$n_{dec}^{KK} \approx H(t_{dec}) \sigma_{KK}^{-1} v^{-1}(t_{dec}) \approx H(t_{nr}) \sigma_{KK}^{-1}$$

up to some logarithmic factor.

KK Relic density:

$$
\begin{aligned}
\Omega^{KK}_{RDMD} &\equiv \left(\frac{\rho^{KK}}{\rho^{tot}}\right)_{RDMD} \\
&= \sum_{n_{KK}} \left(\frac{\rho^{KK}}{\rho^{tot}}\right)_{t_{dec}} \frac{a(t_{RDMD})}{a(t_{dec})} \\
&\approx \sum_{n_{KK}} g^{-1/4} M_{\rm Pl}^{-1} \rho_{RDMD}^{-1/4} \sigma_{KK}^{-1} \\
&\approx \sum_{n_{KK}} g^{-1/4} \left(\frac{R}{L}\right)^6 \frac{M_{\rm Pl}}{\rho^{1/4}_{RDMD}} h_0^2
\end{aligned}
$$

Eg. GKP: $\left(\frac{R}{L}\right)^6 h_0^2 \lesssim 10^{-27}$

$L \approx 100R$ and $h_0 \approx 10^{-8}$

RS: $h_0 \lesssim 10^{-14}$

Graviton Density

$$\rho^{grav}(t) \approx \int_{t_0}^{t} T(t_1)\ n^2(t_1)\ \sigma_{Kg}\ \frac{a^4(t_1)}{a^4(t)}\ dt_1$$

Dominant contribution comes from initial moment t_0 over a period of order t_0

$$\Omega^{grav}_{BBN\ \ or\ \ RDMD} \approx g_s^{-1/4} g^{1/4} \frac{m_s}{M_{\rm Pl}} h_0$$

Naturally very small to satisfy current bound.

Double Throat Heating

Brane-antibrane annihilate in A-throat (typically short), then KK particles tunnel through barrier to heat S-throat.

Questions:

- Is tunneling too slow?
- Is tunneling efficient? E.g. oscillate back and forth?
- Too much graviton production?
- Shorter A-throat has bigger warp factor,
  but is there enough suppression for KK relics?

Double Throat Heating

- Similar to single throat case, until after $T(t) < h_A R_A^{-1}$

Universe RD $\longrightarrow$ MD tunneling phase.

$$\Delta t_{\rm tun} \sim h_A^{-9} R_A \quad \text{or} \quad h_A^{-17} R_A$$

- KK relics density in A-throat

$$\Omega_{RDMD}^{KK} \approx \sum_{n_{KK}} \left(\frac{\rho^{KK}}{\rho^{tot}}\right)_{t_{Adec}} \frac{M_{\rm Pl}^{1/2} t_{tun}^{-1/2}}{\rho_{RDMD}^{1/4}}$$
$$\approx \sum_{n_{KK}} g^{-1/4} \frac{M_{\rm Pl}}{\rho_{RDMD}^{1/4}} \left(\frac{R}{L}\right)^6 h_A^2 \left(\frac{t_{Anr}}{t_{tun}}\right)^{1/2}$$

- Graviton density due to A-throat

$$\Omega_{RDMD}^{grav} \approx g^{1/4} g_s^{-1/4} \frac{m_s}{M_{\rm Pl}} h_A \left(\frac{t_{Anr}}{t_{tun}}\right)^{2/3}$$

Extra suppression factors of $\frac{t_{Anr}}{t_{tun}}$ are due to the extra MD phase during tunneling.

Tunneling from A to S

- Lowest KK in A $\xrightarrow{\text{tunnel}}$ Excited KK in S
  ↳ Decay to lower KK
  - Universe MD $\longrightarrow$ RD
  - KK in S thermalize to lower T
  - Cannot tunnel back, one-way process.

- KK in A-throat quickly decoupled during tunneling
  $\longrightarrow$ Negligible graviton production,
  plus further dilution by the MD phase.

- Only constraint: at the end of tunneling,
  $\rho_{KK}(t_{tun}) \approx M_{\rm Pl}^2 \Gamma_{tun}^2$ is not over-cooled.

Numerical examples

- No over-cooling:

	Resonance		Non-res
MeV:	$h_A \gtrsim 10^{-4}$	or	$h_A \gtrsim 10^{-2}$
TeV:	$h_A \gtrsim 10^{-2}$	or	$h_A \gtrsim 10^{-1}$

c.f. CMB fitting for slow-roll models $h_A \approx 1/4 \text{ to } 10^{-3}$

- KK relics in A-throat:

Resonance: $\left(\frac{R_A}{L}\right)^6 h_A^{5.5} \lesssim 10^{-31}$

$m_s \approx 10^{10}$ GeV $\quad L \approx 10^3 R_A$ and $h_A \approx 10^{-3}$

Non-resonance: $\left(\frac{R_A}{L}\right)^6 h_A^{9.5} \lesssim 10^{-31}$

$m_s \approx 10^{10}$ GeV $\quad L \approx 10^2 R_A$ and $h_A \approx 10^{-2}$

Multi-Throat Heating

- Selection of long throats

Tunneling requires Mass Level Matching b.t. A and X.

The width of the KK mode in A is Γ_{tun} ,

If $h_X \lesssim \Gamma_{tun} R_X$,

then matching satisfies generically.

$\longrightarrow$ $h_X \lesssim h_A^9$

Long throats are preferentially heated.

Dynamical selection of a long SM throat.

Brief Summary

A short A-throat provides a long period of tunneling phase, but not too long to overcool the universe.

This MD phase in turn reduces the redshift, introducing extra suppression factor to KK relics and graviton production.

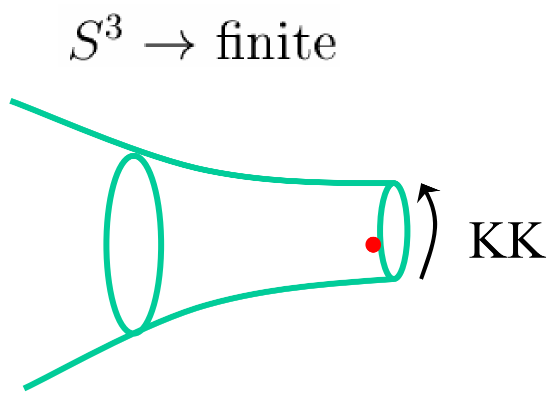
Statistically such a tunneling prefers long throats and is a one-way process.

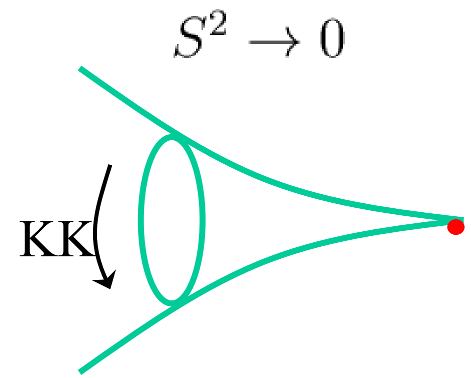
Previous Questions:

- Is tunneling too slow?
- Is tunneling efficient? E.g. oscillate back and forth?
- Too much graviton production?
- Shorter A-throat has bigger warp factor, but is there enough suppression for KK relics?

Warped KK dark matter in S-throat

- Klebanov-Strassler type throat: $\sim AdS^5 \times S^3 \times S^2$





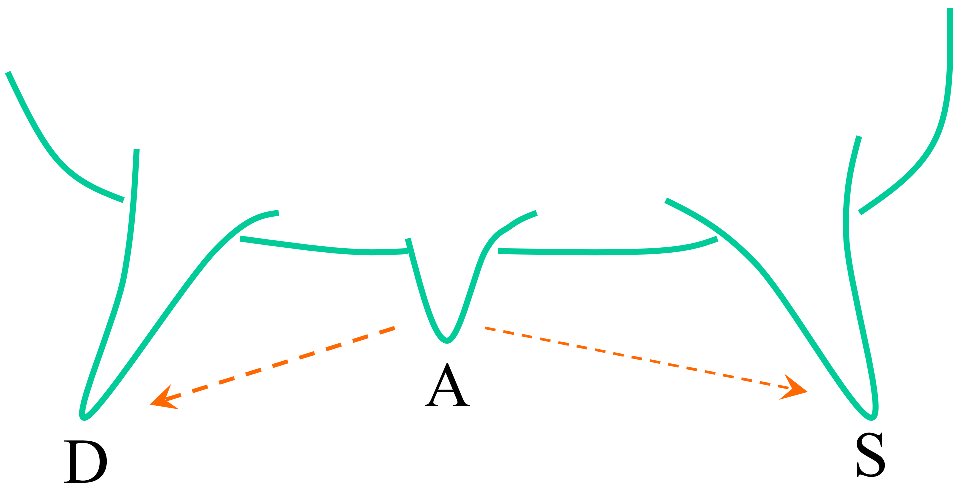
Angular modes associated with S^2 does not interact with SM-branes $\longrightarrow$ Stable Relics.

- In collider,

SM + SM $\longrightarrow$ s-wave KK $\longrightarrow$ l-th KK + l-th KK

c.f. KK of SM particles in (Dienes, Dudas, Gherghetta, 98)
(Cheng, Feng, Matchev, 02)

Hidden Dark Matter



Two possibilities:

- HD in D-throats: through tunneling from A to D;
- HD in A-throat: KK left in A.

Hidden Dark Matter in D-throats

1) Coupling:

LSP: Excited if $E > m_{LSP}$

HD: Does not even if $E > m_{HD}$, only through graviton mediation

2) Thermal history

LSP: Thermal equilibrium $\longrightarrow$ Freeze out

HD: Completely independent thermal history;
D-throats have independent Radiation-Matter transition

3) Relics Density

LSP: Thermal decoupling condition

HD: Initial condition during tunneling

Dark Matter Coincidence Problem

Requirements:

$$\begin{cases} \left.\dfrac{\rho_D}{\rho_S}\right|_{\rm BBN} < \text{a few } \% \\[2ex] \dfrac{\rho_D}{\rho_S} \approx 5\dfrac{a(t_D^{\rm RDMD})}{a(t_S^{\rm RDMD})} \approx 5\left(\dfrac{t_D^{\rm RDMD}}{t_S^{\rm RDMD}}\right)^{1/2} \end{cases}$$

$$\Longrightarrow \quad t_S^{\rm RDMD} > 10^4 t_D^{\rm RDMD}$$

Consequence: when dark matter becomes non-relativistic,

LSP: $\rho_{\rm DM} \ll \rho_{\rm tot}\,,\ t \lesssim 10^{-22}\ t_S^{\rm RDMD}$

HD: $\rho_{\rm DM} \sim \rho_{\rm tot}\,,\ t \lesssim 10^{-4}\ t_S^{\rm RDMD}$

Reducing $10^{-22} \rightarrow 10^{-4}$

HD provides different possibilities to address the coincidence problem.

Hidden Dark Matter in A-throat

- We know cosmic strings in A-throat

$$\frac{\rho_{\rm cs}}{\rho_{\rm tot}} < \frac{\mu}{M_{\rm Pl}^2} < 10^{-6}$$

- Now we see in A-throat, we can also have KK-relics

  - Can have sizable contribution to total matter density
  - Long-living against tunneling

    Lifetime of *l*-th partial wave has extra factor h_A^{-4l}
  - Very heavy

Note: This hidden dark matter share the thermal history, and belongs to thermal relics. (unlike hidden dark matter in D-throats)

(Ultra) High Energy Cosmic Rays from Hidden Dark Matter

1) Evade a famous bound of Griest and Kamionkowski

$$M_{\text{thermal relics}} \lesssim 100 \text{ TeV}$$

Heavier relics → { Smaller cross section / More red-shift ⇐ }

Reason:
Extra MD phase during tunneling greatly reduces the redshift.

2) Can occasionally tunnel to visible sector (S-throat)

Lifetime of *l*-th partial wave has extra factor h_A^{-4l}

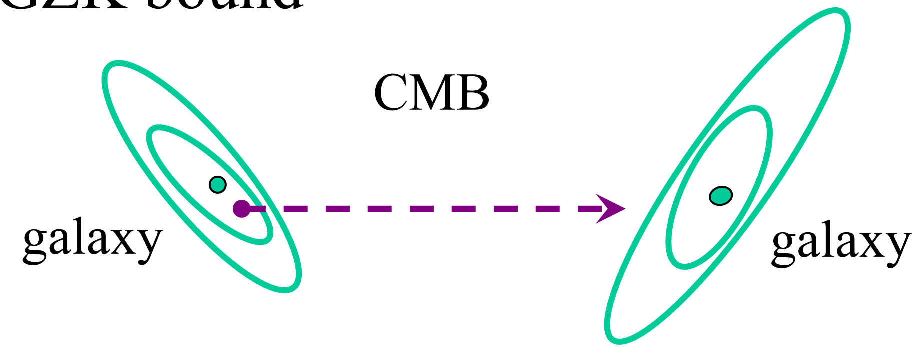
$\tau > 10^{17}\text{s if } l > 2$

- Ultra heavy thermal relics being the source of ultra high energy cosmic rays.

c.f. ultra heavy non-thermal relics being the source, Chung, Kolb, Riotto, 98.

3) Can violate GZK bound

- GZK bound



$E < 10^{20}$ eV

- Here, hidden dark matter is within our galaxy and can have mass $M_{\rm HD} \sim 10^{20}$ eV

- Distribution $\propto \rho_{\rm HD}(x)$

- $M_{\rm HD} \sim \mu_{\rm cosmic\ strings}^{1/2} \sim V_{\rm inflation}^{1/4}$

- Decay process can be very stringy --- compare with string theory.

Conclusions

- Multi-throat brane inflation
  IR DBI inflation
  Spectrum index
  Non-Gaussianities

- Reheating
  Solving KK and graviton problems
  Warped KK dark matter
  Hidden dark matter
  Dark matter coincidence problem
  High energy cosmic rays

String cosmology can solve old problems,
and are making testable predictions

⇕

Future observations (WMAP, Planck, astrophysics …)
can teach us about properties of
stringy theory and string compactifications.