Bubbles Unbound:

Bubbles of Nothing Without Kaluza-Klein

Keith Copsey

hep-th/0610058

Outline

I. Background and Motivation

II. Geometric Properties

III. Initial Dynamics

IV. High Curvatures and Quantum Corrections

V. Discussion

Witten's Bubble

Nucl. Phys. B 195 (1982) 481

- Begin with a 5d Schwarzschild metric

$$ds^2 = -(1 - \frac{{r_s}^2}{r^2})dt^2 + \frac{dr^2}{(1-\frac{{r_s}^2}{r^2})} + r^2(d\theta^2 + \sin^2(\theta)d\Omega_2)$$

- Then analytically continue

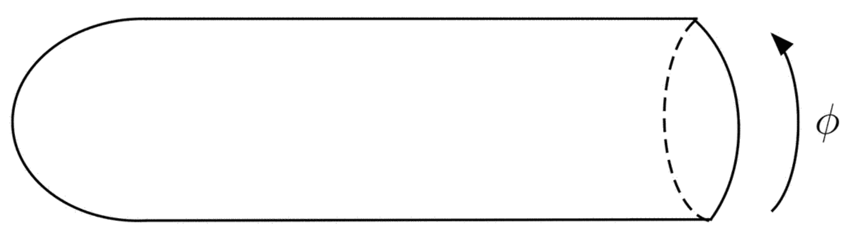
$$t \longrightarrow i\phi \quad \text{and} \quad \theta \longrightarrow \frac{\pi}{2} + i\tau$$

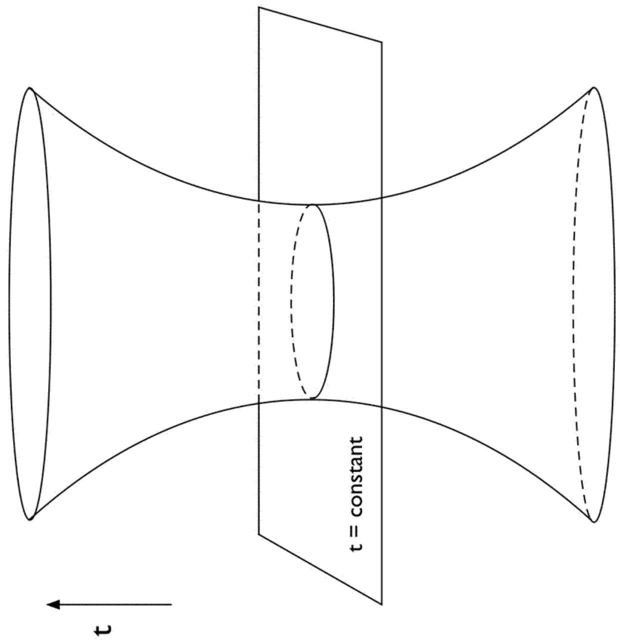
$$ds^2 = -r^2 d\tau^2 + (1 - \frac{{r_s}^2}{r^2})d\phi^2 + \frac{dr^2}{(1-\frac{{r_s}^2}{r^2})} + r^2\cosh^2(\tau)d\Omega_2$$

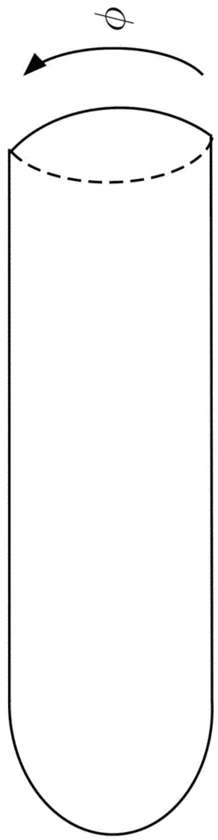
- $r = r_s + \frac{u^2}{2r_s}$

$$ds^2 \approx du^2 + \frac{u^2}{{r_s}^2}d\phi^2 + {r_s}^2[-d\tau^2 + \cosh^2(\tau)d\Omega_2]$$

metric regular iff ϕ has period $2\pi r_s$




t = constant
t


ϕ

Are there bubbles in asymptotically flat space?

- Simplest such solutions would have two angles with closed orbits; one to pinch off and one for minimal sphere

  $\longrightarrow$ 5d

- Initial skepticism: asymptotic S^3 has no circles
- Interior symmetries of a spacetime need not be same as asymptotics
- $S^1 \times S^2$ and S^3 are cobordant
- Pure gravity solution?

  $\longrightarrow$ Black Rings!

- Just need find solution where instead of S^1 going to finite value (size of ring) goes to zero smoothly

---

- Would have no inherent scale (unlike KK) so could find ones of arbitrary size or mass
- Witten's SUSY bdy condition argument doesn't rule out
- Analytic continuations from known solutions do not seem to produce desired bubbles
- Consider instead looking for initial states that are momentarily static that could produce quantum mechanically
- Looking for time symmetric initial data which satisfies Hamiltonian constraints

  $\longleftrightarrow \quad {}^{(4)}R = 0$

A New Bubble

- Motivated by form of BR metrics and C metric solutions consider

$$ds^2 = \frac{R^2}{(x-y)^2}\Big[A(x)\Big[-G(y)d\psi^2 - \frac{F(y)}{G(y)}dy^2\Big] + B(y)\Big[H(x)d\phi^2 + \frac{J(x)}{H(x)}dx^2\Big]\Big]$$

  - U(1) X U(1) symmetry
  - Factorizable aside from overall $\frac{1}{(x-y)^2}$

- Look for bubble solution where ψ pinches off, leaving a minimal S^2 parametrized by x and ϕ

$\longrightarrow$ $g_{\psi\psi}$ vanishes at two values of y and $g_{\phi\phi}$ vanishes at two values of x

Requiring that where $g_{\psi\psi}$ and $g_{\phi\phi}$ vanish metric is smooth

$$ds^2 = \frac{R^2}{(x-y)^2}\Big[A(x)\Big[-\frac{P(y)}{B(y)}d\psi^2 - \frac{B(y)}{P(y)}dy^2\Big] + B(y)\Big[\frac{P(x)}{A(x)}d\phi^2 + \frac{A(x)}{P(x)}dx^2\Big]\Big]$$

where

$$P(\xi) = Q(\xi_4 - \xi)(\xi - \xi_3)(1 - \xi_2\xi)(1 - \xi_5\xi)$$

$$A(x) = k_1(1 - k_2 x)^2$$

and

$$B(y) = k_3(1 - k_4 y)^2$$

- Note if $k_2 = k_4 = 0$ would just be Euclidean charged C-metric
- $k_2,\ k_4$ allow elimination of conical singularities
- 4 zeroes of P(ξ_i), 4 scaling constants (R, Q, k_1, k_3) and k_2 and k_4

- Regularity does not allow both x and y to have arbit. large ranges $\longrightarrow$ take x finite
- If y also has a finite range may take

$$1)\ \frac{1}{\xi_2} \le y \le \xi_3 \le x \le \xi_4$$

Remaining zero must be outside of relev. ranges, i.e.

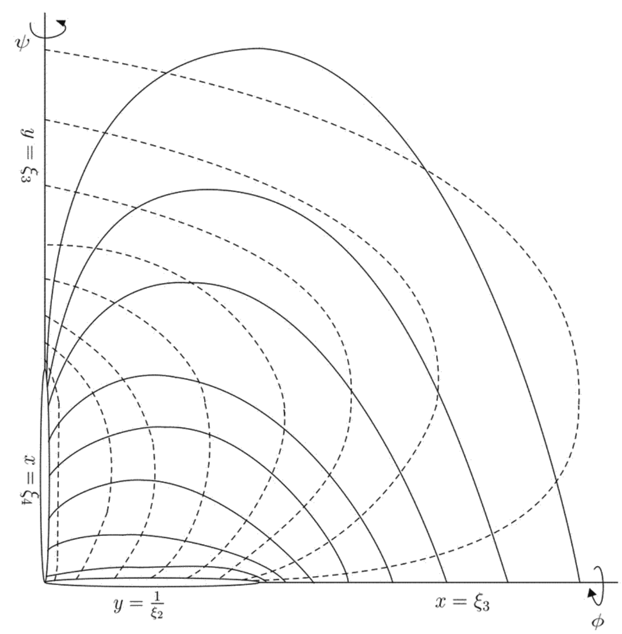
$$a)\xi_5 = 0$$

$$b)\frac{1}{\xi_5} < \frac{1}{\xi_2}$$

$$c)\xi_4 < \frac{1}{\xi_5}$$

- Alternatively, allowing range of y to be semi-infinite

$$2) -\infty < y \le \xi_3 \le x \le \xi_4 < \frac{1}{\xi_5}$$



- Demanding the absence of a conical singularity as $x \longrightarrow \xi_4$ sets period of ϕ

$$\left|\frac{P'(\xi_4)}{2A(\xi_4)}\right|\Delta\phi = 2\pi$$

- Similarly period of ψ set by absence of conical sing. as $y \longrightarrow \frac{1}{\xi_2}$

$$\left|\frac{P'(\frac{1}{\xi_2})}{2B(\frac{1}{\xi_2})}\right|\Delta\psi = 2\pi$$

Simplifying the Solution

- By writing parameters in terms physical quantities can simplify metric greatly
- Consider bubble at constant y (y-bubble) and define its size r_0 via its area: $A = 4\pi r_0^2$
- Area of bubble at constant x (x-bubble): $A' = 4\pi {r'_0}^2$
  - Introduce a parameter $\omega\,(0 < \omega < 1):\ \ A' = 4\pi {r_0}^2\left(\frac{1}{\omega} - 1\right)$

$$\omega \ll 1 \longleftrightarrow {r'}_0 \gg r_0$$
$$\omega \sim 1 \longleftrightarrow {r'}_0 \ll r_0$$
$$\omega = \tfrac{1}{2} \longleftrightarrow {r'}_0 = r_0$$

Note: $\omega = \dfrac{{r_0}^2}{{r'_0}^2 + {r_0}^2}$ and $1 - \omega = \dfrac{{r'_0}^2}{{r'_0}^2 + {r_0}^2}$ so $\omega \leftrightarrow 1 - \omega$ if $r_0 \leftrightarrow r'_0$

- Now define

$$\bar{P}(\xi) = (\xi_4 - \xi)(\xi - \xi_3)(1 - \xi_2\xi)(1 - \xi_5\xi)$$

and rescaled angles $\bar{\psi}$ and $\bar{\phi}$ such that the new angles have period 2π, i.e.

$$\bar{\psi} = \frac{2\pi}{\Delta\psi}\psi \quad \text{and} \quad \bar{\phi} = \frac{2\pi}{\Delta\phi}\phi$$

$$ds^2 = r_0^2 \frac{(1-\xi_2\xi_3)(1-\xi_5\xi_4)(1-\xi_2\xi_4)^2}{(1-k_2\xi_4)^2(k_4-\xi_2)^2(x-y)^2}\Bigg\{(1-k_2x)^2(1-k_4y)^2\left[\frac{dx^2}{\bar{P}(x)} - \frac{dy^2}{\bar{P}(y)}\right]$$

$$-\frac{4\left(1-\frac{k_4}{\xi_2}\right)^4}{\left(\bar{P}'(\frac{1}{\xi_2})\right)^2}\frac{(1-k_2x)^2}{(1-k_4y)^2}\bar{P}(y)d\bar{\psi}^2 + \frac{4(1-k_2\xi_4)^4}{\left(\bar{P}'(\xi_4)\right)^2}\frac{(1-k_4y)^2}{(1-k_2x)^2}\bar{P}(x)d\bar{\phi}^2\Bigg\}$$

and the scaling parameters (R, Q, k_1, k_3) have vanished in favor of r_0

- Can further shift and rescale the coordinates x and y:

$$\xi = \mathcal{A}\tilde{\xi} + \mathcal{B}$$

Zeroes of $P(\xi)$ automatically shift and scale appropriately (e.g. $\xi_4 = \mathcal{A}\tilde{\xi}_4 + \mathcal{B}$) but if form of metric is to be unaltered must require if $k_2 \neq 0$

$$\frac{1}{k_2} = \mathcal{A}\frac{1}{\tilde{k}_2} + \mathcal{B}$$

and if $k_4 \neq 0$

$$\frac{1}{k_4} = \mathcal{A}\frac{1}{\tilde{k}_4} + \mathcal{B}$$

- Automatic if no conical singularities and can choose $\mathcal{A}$ and $\mathcal{B}$ to eliminate two parameters. In particular, may take $\xi_3 = 0$ and $\xi_4 = 1$.

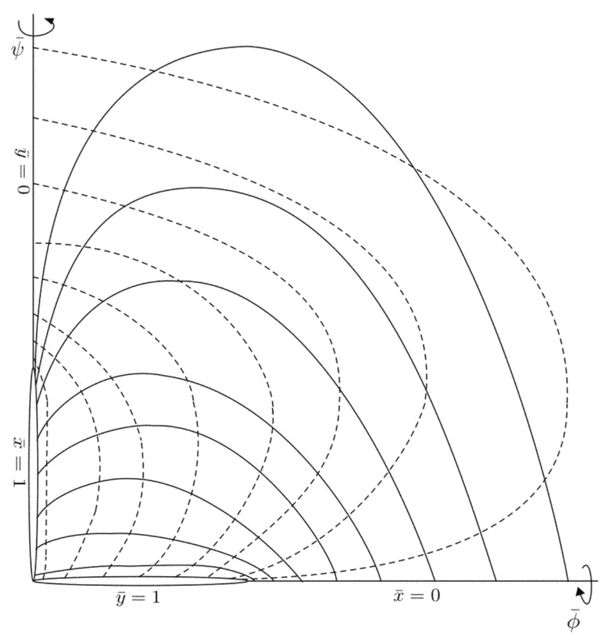
- Still gauge freedom left. Define phys. coord. $\bar{x}$ by fraction of area of y-bubble covered in a disk from pole at x = 0 to a given x:

$$\bar{x} = \frac{2\pi}{4\pi {r_0}^2} \int_0^x dx' \sqrt{g_{x'x'} g_{\bar{\phi}\bar{\phi}}} = \frac{(1-\xi_2)x}{1-\xi_2 x}$$

- Similarly defining a physical coord. $\bar{y}$ by fraction of area of x-bubble

$$1 - \bar{y} = \frac{2\pi}{4\pi {r_0'}^2} \int_{\frac{1}{\xi_2}}^y dy' \sqrt{g_{y'y'} g_{\bar{\psi}\bar{\psi}}} = \frac{1-\xi_2 y}{1-y}$$

- By construction $0 \leq \bar{x} \leq 1$ and $0 \leq \bar{y} \leq 1$



$$ds^2 = \frac{{r_0}^2}{\omega(\bar{x}+\bar{y}-\bar{x}\bar{y})^2}\Big[A(\bar{x})\Big[\frac{4P(\bar{y})}{B(\bar{y})}d\bar{\psi}^2 + \frac{B(\bar{y})}{P(\bar{y})}d\bar{y}^2\Big]$$
$$+B(\bar{y})\Big[\frac{A(\bar{x})}{R(\bar{x})}d\bar{x}^2 + \frac{4R(\bar{x})}{A(\bar{x})}d\bar{\phi}^2\Big]\Big]$$

where

$$A(\bar{x}) = \Big[1-\Big(1-\sqrt{1-\omega}\Big)\bar{x}\Big]^2$$
$$B(\bar{y}) = \Big[1-\Big(1-\sqrt{\omega}\Big)\bar{y}\Big]^2$$
$$P(\bar{y}) = (1-\bar{y})\bar{y}(1-(1-\omega)\bar{y})$$

and

$$R(\bar{x}) = (1-\bar{x})\bar{x}(1-\omega\bar{x})$$

Note $\frac{{r_0}^2}{\omega} = {r'_0}^2 + {r_0}^2$ and so solution symmetric under $r_0 \leftrightarrow r'_0$

Geometric Properties

$$\bar{x} = \frac{4({r_0}^2+{r'_0}^2)\cos^2(\theta)}{r^2+4{r'_0}^2\cos^2(\theta)+4{r_0}^2} \qquad \bar{y} = \frac{4({r_0}^2+{r'_0}^2)\sin^2(\theta)}{r^2+4{r_0}^2\sin^2(\theta)+4{r'_0}^2}$$

$$ds^2 = \Big[1+\frac{\delta_{rr}}{r^2}+\mathcal{O}\Big(\frac{1}{r^4}\Big)\Big]dr^2 + r^2\Big[1+\frac{\delta_{\theta\theta}}{r^2}+\mathcal{O}\Big(\frac{1}{r^4}\Big)\Big]d\theta^2 + \mathcal{O}\Big(\frac{1}{r^3}\Big)dr\,d\theta$$
$$+r^2\sin^2(\theta)\Big[1+\frac{\delta_{\bar{\psi}\bar{\psi}}}{r^2}+\mathcal{O}\Big(\frac{1}{r^4}\Big)\Big]d\bar{\psi}^2 + r^2\cos^2(\theta)\Big[1+\frac{\delta_{\bar{\phi}\bar{\phi}}}{r^2}+\mathcal{O}\Big(\frac{1}{r^4}\Big)\Big]d\bar{\phi}^2$$

where

$$\delta_{rr} = 4(r_0+r'_0)\bar{r} - 6\bar{r}^2 + 2(r_0-r'_0)(r_0+r'_0-2\bar{r})\cos(2\theta)$$
$$\delta_{\theta\theta} = 4(r_0+r'_0)\bar{r} - 2\bar{r}^2 + 2(r_0-r'_0)(r_0+r'_0-2\bar{r})\cos(2\theta)$$
$$\delta_{\bar{\psi}\bar{\psi}} = 4\Big({r_0}^2 + (r'_0-r_0)\bar{r} + \bar{r}(r_0+r'_0-\bar{r})\cos(2\theta)\Big)$$
$$\delta_{\bar{\phi}\bar{\phi}} = 4\Big({r'_0}^2 + (r_0-r'_0)\bar{r} + \bar{r}(\bar{r}-r_0-r'_0)\cos(2\theta)\Big)$$

and $\bar{r} = \sqrt{{r_0}^2+{r'_0}^2}$

- The ADM mass of these solutions is then

$$E = \frac{2\pi}{G_5}\sqrt{{r_0'}^2 + {r_0}^2}\left[r_0 + r_0' - \sqrt{{r_0'}^2 + {r_0}^2}\right]$$

Via the triangle ineq. E is positive definite, as required by the positive energy thms

R. Schoen and S.T. Yau: Comm. Math. Phys. 65 (1979) 45
Ibid 79 (1981) 47
Phys. Rev. Lett. 42 (1979) 547
E. Witten: Comm. Math. Phys. 80 (1981) 381

- Note if one bubble much larger than other

$$E \approx \frac{2\pi r_0 r_0'}{G_5}$$

- More generically, can show $E(r_>, \frac{r_<}{r_>})$ monotonically incr. ftn of $\frac{r_<}{r_>}$
- Often easier regard $E(r_0, \omega)$ (monotonic. incr. ftn of ω)

In particular consider geometry of bubbles with r_0 fixed, ${r'}_0 \ll r_0 \ (\omega \sim 1)$

- Curvature of y-bubble becomes vanishingly small away from x-bubble.

Defining

$$\rho = 2r_0\sqrt{\bar{x}}$$

metric on the y-bubble becomes

$$ds^2 = \frac{\left(1 + \sqrt{1-\omega}\frac{\bar{x}}{1-\bar{x}}\right)^2}{\left(1 + (1-\omega)\frac{\bar{x}}{1-\bar{x}}\right)}d\rho^2 + \frac{\left(1 + (1-\omega)\frac{\bar{x}}{1-\bar{x}}\right)}{\left(1 + \sqrt{1-\omega}\frac{\bar{x}}{1-\bar{x}}\right)^2}\rho^2 d\bar{\phi}^2$$

and so provided $\sqrt{1-\omega} \ll 1$ until $\bar{x}$ gets close to one

$$1 - \bar{x} \sim \sqrt{1-\omega}$$

the metric is approximately flat

- In same limit (r_0 fixed, $\omega \sim 1$) x-bubble becomes small undistorted S^2

  To see this, define

$$\sin(\theta) = 2\sqrt{\bar{y}(1-\bar{y})} \qquad \theta(0) = 0$$

  Then the metric on the x-bubble becomes

$$ds^2 = {r_0'}^2 \left[\frac{\left(1-(1-\sqrt{\omega})\sin^2(\frac{\theta}{2})\right)^2}{1-(1-\omega)\sin^2(\frac{\theta}{2})} d\theta^2 + \frac{1-(1-\omega)\sin^2(\frac{\theta}{2})}{\left(1-(1-\sqrt{\omega})\sin^2(\frac{\theta}{2})\right)^2} \sin^2(\theta) d\bar{\psi}^2 \right]$$

  where recall

$${r'}_0 = r_0\sqrt{\tfrac{1}{\omega} - 1}$$

Horizons

- Eqn for apparent horizon yields complicated nonlinear PDE
- Any such horizon will be symmetric under $\frac{\partial}{\partial\bar{\psi}}$ and $\frac{\partial}{\partial\bar{\phi}}$
- Consider null geodesics with $\dot{\bar{\psi}} = \dot{\bar{\phi}} = 0$ in Gaussian normal coords (i.e. $N = 1, N^a = 0$)

$$0 = \dot{l}^\alpha \dot{l}_\alpha = g_{\bar{x}\bar{x}}\dot{\bar{x}}^2 + g_{\bar{y}\bar{y}}\dot{\bar{y}}^2 + g_{tt}\dot{t}^2$$

$$= \frac{({r_0}^2 + {r'_0}^2)A(\bar{x})B(\bar{y})}{(\bar{x}+\bar{y}-\bar{x}\bar{y})^2}\left[\frac{\dot{\bar{x}}^2}{R(\bar{x})} + \frac{\dot{\bar{y}}^2}{P(\bar{y})}\right] - E^2$$

  where $E = -g_{tt}\dot{t}$ is the conserved energy of a geodesic

- Defining $V = \frac{-E^2}{2({r_0}^2 + {r'_0}^2)} \frac{(\bar{x}+\bar{y}-\bar{x}\bar{y})^2}{A(\bar{x})B(\bar{y})}$

and coordinate transformations

$$X = \int_0^{\bar{x}} \frac{d\bar{x}'}{\sqrt{R(\bar{x}')}} = \int_0^{\bar{x}} \frac{d\bar{x}'}{\sqrt{(1-\bar{x}')\bar{x}'(1-\omega\bar{x}')}} = 2\,F(\arcsin\sqrt{x}, \sqrt{\omega})$$

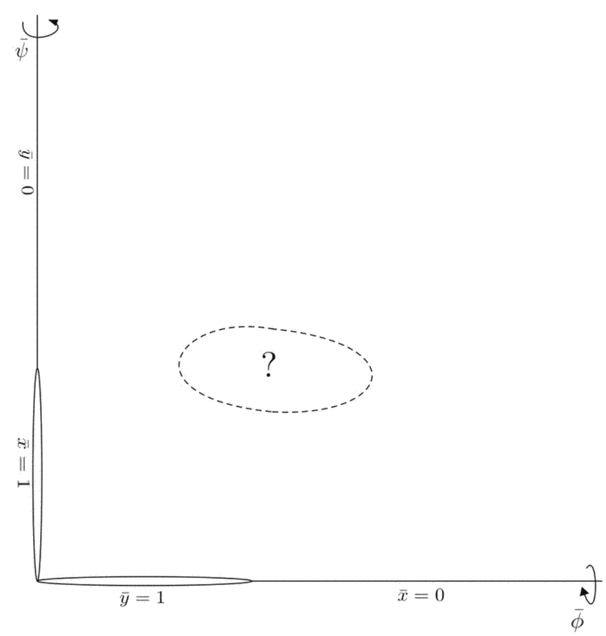
and

$$Y = \int_0^{\bar{y}} \frac{d\bar{y}'}{\sqrt{P(\bar{y}')}} = \int_0^{\bar{y}} \frac{d\bar{y}'}{\sqrt{(1-\bar{y}')\bar{y}'(1-(1-\omega)\bar{y}')}} = 2\,F(\arcsin\sqrt{y}, \sqrt{1-\omega})$$

Equivalent to problem of a particle of unit mass moving in effective potential V with zero energy

$$\frac{1}{2}\left(\dot{X}^2 + \dot{Y}^2\right) + V = 0$$

- If further consider motion along bubble or axis becomes 1-d problem of particle with higher energy than any potential barrier
  $\longrightarrow$ no trapped surfaces intersecting bubbles or axis



Initial Dynamics

- None of solutions are static.
- Analytically can study initial time dependence.
- Go to Gaussian normal coordinates
- At moment of time symmetry (t = 0) Ham. evolution eqns give (vacuum solns)

$$\ddot{h}_{ab}(0) = \frac{N}{8\pi G\sqrt{h}}\Big(\dot{\pi}^G_{ab} + \frac{\dot{\pi}^G}{2-d}h_{ab}\Big)$$
$$= \tfrac{N}{8\pi G\sqrt{h}}\Big[-16\pi GN\sqrt{h}\Big({}^{(d-1)}R_{ab} - \tfrac{{}^{(d-1)}R}{2}h_{ab}\Big) + \tfrac{h_{ab}}{2-d}\Big(-16\pi GN\sqrt{h}\Big)\Big(1-\tfrac{d}{2}\Big){}^{(d-1)}R\Big] = -2\ {}^{(d-1)}R_{ab}$$

- Easy to check metrics retain $U(1)\times U(1)$ symmetry and are diagonal with exception of development of an xy cross-term
- Note further any zeroes of an angle, say $\bar{\psi}$, remain at same coordinates for any regular initial data since

$$\ddot{h}_{\bar{\psi}\bar{\psi}} = -2^{(d-1)}R_{\bar{\psi}\bar{\psi}} = -2^{(d-1)}R_{ab}\left(\frac{\partial}{\partial\bar{\psi}}\right)^a\left(\frac{\partial}{\partial\bar{\psi}}\right)^b = 0$$

- Then since the area of the y-bubble is given (at least through order t^2)

$$A = 2\pi\int_0^1 d\bar{x}\sqrt{g_{\bar{x}\bar{x}}\,g_{\bar{\phi}\bar{\phi}}}$$

evaluated at $\bar{y} = 1$

$$\ddot{A} = 2\pi\int_0^1 d\bar{x}\frac{\ddot{g}_{\bar{x}\bar{x}}\,g_{\bar{\phi}\bar{\phi}} + g_{\bar{x}\bar{x}}\,\ddot{g}_{\bar{\phi}\bar{\phi}}}{2\sqrt{g_{\bar{x}\bar{x}}\,g_{\bar{\phi}\bar{\phi}}}} = -2\pi\int_0^1 d\bar{x}\frac{{}^{(d-1)}R_{\bar{x}\bar{x}}\,g_{\bar{\phi}\bar{\phi}} + g_{\bar{x}\bar{x}}\,{}^{(d-1)}R_{\bar{\phi}\bar{\phi}}}{\sqrt{g_{\bar{x}\bar{x}}\,g_{\bar{\phi}\bar{\phi}}}}$$

- For present bubbles gives

$$\ddot{A} = \frac{8\pi}{3\sqrt{1-\omega}}\left[3\sqrt{\omega} - 2\sqrt{1-\omega} - (1+\omega)\right]$$

Accel. of area dimensionless and hence depends only on ${r'}_0/r_0$ or equiv. ω.

In fact, monotonically increasing ftn of ω.

- For $\omega \ll 1\,({r'}_0 \gg r_0)$

$$\ddot{A} \sim -8\pi$$

while for $\omega \sim 1\,({r'}_0 \ll r_0)$

$$\ddot{A} \sim \frac{8\pi}{3\sqrt{1-\omega}}$$

- Transition between expansion and contraction occurs at

$\omega \approx 0.803815$ or equivalently at $\frac{r'_0}{r_0} \approx 0.494032$

- Accel of x-bubble can be obtained via $\omega \leftrightarrow 1-\omega$ symmetry
- If bubbles are of comparable size, both collapse
- If one significantly larger than other

$$r_> \gtrapprox 2.02416\ r_<$$

larger bubble expands while smaller collapses

- Bubbles which collapse (semiclassically) should produce black holes; may be relatively easy to produce even in models without large extra dim.
- In case where one expands, other contracts; result presumably is small BH in expanding bubble. KK version previously described by Emparan and Reall hep-th/0110258
- If collapsing bubble not heavy enough could get quantum state with no semiclassical description

- If bubble expand for any sig. period of time spacetime far away from initial disturbance is radically altered

$\longrightarrow$ Instability!

- Appears unlikely that once bubble starts expanding will halt
  - Numerical studies KK bubbles

    O. Sarbach and L. Lehner hep-th/008116
  - KK bubbles from Kerr with circle which grows asymptotically expand

    O. Aharony, M. Fabinger, G.T. Horowitz and E. Silverstein hep-th/0204158
  - Solutions include bubbles of arbit. size which expand so any process stoping would have to be entirely dynamical
  - Self-gravity strongest at beginning; energy relaxes away

Large Curvatures and Quantum Corrections

- Would like to write spacetime d-dimensional (present case d = 5) quantities in terms of (d - 1)-dim. quantities from initial data
- Using Gauss-Codacci relation

$$^{(d-1)}R_{abc}{}^{d} = h_a^f h_b^g h_c^k h_j^d \, {}^{(d)}R_{fgk}{}^{j} - K_{ac}K_b^d + K_{bc}K_a^d$$

where h_{ab} is the metric induced on the spatial slice of the initial data and K_{ab} is the extrinsic curvature of that slice

- Going to Gaussian normal coordinates for time symmetric initial data $(K_{ab} = 0)$

$$^{(d)}R_{abcd}\,{}^{(d)}R^{abcd} = {}^{(d-1)}R_{abcd}\,{}^{(d-1)}R^{abcd} + 4\,{}^{(d-1)}R_{ab}\,{}^{(d-1)}R^{ab}$$

Recalling that $^{(d-1)}R_{ab} = -\ddot{h}_{ab}/2$ for time symmetric initial data $^{(d)}R^2$ is sum of curv. due to spatial gradients and curv. due to time dep.

- For bubbles R^2 is complicated but in limit r_0 fixed, $\omega \sim 1 \ (r'_0 \ll r_0)$

$$\frac{12(\bar{x}+\bar{y}-\bar{x}\bar{y})^2}{{r_0}^4(1-\bar{x}+\bar{x}\sqrt{1-\omega})^{12}}\Big[(1-\bar{x})^6(1-\omega)\Big(\bar{x}^2(3\bar{x}^2-2\bar{x}+3)$$

$$+2\bar{x}\bar{y}(3\bar{x}^2+1)(1-\bar{x})+(1-\bar{x})^2(3\bar{x}^2+2\bar{x}+3)\bar{y}^2\Big)+\mathcal{O}\Big((1-\omega)^2(1-\bar{x})^5\Big)\Big]$$

$\longrightarrow$ Away from $\bar{x} \sim 1$, space is approximately flat

- Near x-bubble $(\bar{x} \sim 1)$

$$\frac{1}{\Big(1-\bar{x}+\bar{x}\sqrt{1-\omega}\Big)^{12}{r_0}^4}\Big[24(1-\omega)^4+96(1-\omega)^{\frac{7}{2}}(1-\bar{x})+192(1-\omega)^3(1-\bar{x})^2$$

$$+288(1-\omega)^{\frac{5}{2}}(1-x)^3+312(1-\omega)^2(1-x)^4+192(1-\omega)^{\frac{3}{2}}(1-\bar{x})^5+48(1-\omega)(1-\bar{x})^6+\dots\Big]$$

becomes large when $1-\bar{x} \lesssim \sqrt{1-\omega}$

- R^2 largest at $\bar{x}=1$ with value

$$\frac{24}{(1-\omega)^2{r_0}^4}+\mathcal{O}\left(\frac{1}{(1-\omega)^{\frac{3}{2}}{r_0}^4}\right) = \frac{24}{{r'_0}^4}+\mathcal{O}\left(\frac{\sqrt{1-\omega}}{{r'_0}^4}\right)$$

- Region of high curvature confined to small area of larger bubble and bounded by a 3 surface of constant $\bar{x}$ ($1-\bar{x} \gg \sqrt{1-\omega}$) of area

$$A_x \sim 16\pi^2{r_0}^3(1-\bar{x})^2$$

- In order to be able to describe entire solution (and in particular dynamics) semiclassically need size of smaller bubble much greater than string length

- Even with smaller bubble much smaller than string or planck length string thy should have a valid description with same mass

Discussion

- Have presented a two parameter family of asymptotically flat solutions describing pairs of bubbles where can either choose the sizes of the bubbles or the size of one bubble and the total mass
- If one bubble much larger (2x) than other larger one expands while smaller one contracts $\longrightarrow$ stimulated instability of asym. flat 5d space
- Most important open question is time evolution of this initial data:
  Numerical Relativists please apply!
- Numerically would also like to investigate horizons
- Should be able to embed these solutions in any locally flat system provided much smaller than any scale in background
  - -AdS, compactified dimensions
- Many more bubbles even in 5d--stay tuned

- Solutions in other dimensions very likely
- Possible to emerse solutions in flux or other matter so solution with bubble has less energy than without?
- Production rate (speculative)

  First guess based on analogy with Witten's bubble for ${r'}_0 \ll r_0$ is

$$P \sim \underbrace{\left(\frac{L}{r_0}\right)^4}_{\sim 10^{120}} e^{-C\left(\frac{r_0}{l_p}\right)^{n_0}}$$

  - If ${r'}_0 \gg l_s$ P may or may not be strongly suppressed, depending on C

- For nucleating pairs of BHs of mass m in background magnetic field B

$$\frac{P}{V} \sim e^{-\frac{\pi m}{B}}$$

trading magnetic energy for BH mass

- Starting with some mass m_i which is absorbed in transition process most likely form of probability for $r'_0 \ll r_0$ would seem

$$P \sim \left(\frac{L}{r_0}\right)^4 e^{-D\left(\frac{m_b}{m_i}\right)^n}$$

- Instantons or other solid estimation
- Understanding quantum corrections

The End