Nothing for Branes

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What can we learn from Tachyons?

- Induces transitions between different backgrounds (including topology change)
- Window into "off-shell" string theory
- New understanding of some black hole and cosmological singularities
- Can lead to new 'phases' of the theory and new processes that could be useful in more experimentally motivated endeavors

Condensing Tachyons

-A negative mass state (closed string tachyon) ⟷ a relevant operator on the worldsheet

-The condensation can be treated by deforming the worldsheet CFT by the (dressed) relevant operator

$$S = S_{cft} - \mu \int d^2\sigma f(X_0)\mathcal{O}_\Delta + \int d^2\sigma \Phi(X_0) R$$

Δ is the scaling dimension of $\mathcal{O}$ and the dressing $f(X_0)$ has dimension $2-\Delta$

-This will generically act like a mass term for the closed string modes.

-Massing up closed strings leads to the **decay/vanishing of spacetime**!

Beyond Perturbative Strings

-Closed strings are lifted, but it says nothing about non-perturbative physics (i.e. Branes).

-This is hard to study when worldsheet and spacetime interpretations break down.

What's at Stake? **Spacetime Unitarity!**

-Branes leaking into regions without spacetime descriptions ('nothing state' / 'no man's land') ⟶ non-unitary spacetime evolution

-In emergent theories, unitarity of the non-perturbative description can still be maintained (as we will see).

So…Is this really a problem?

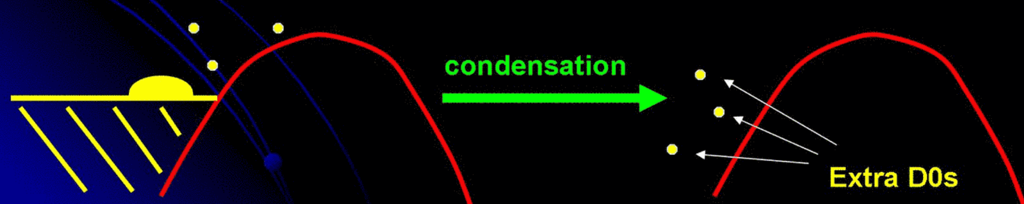
c=1 Matrix Model

-Closed strings in d=2 are dual to open string tachyons on ZZ branes at the boundary (free fermion in matrix model)

-Spacetime physics described by collective excitations on Fermi sea

-Tachyon condensation can be put in by hand, causes draining of the fermi sea Karczmarek and Strominger, hep-th/0403169

-Additional free fermions separated from the draining sea → Branes in the absence of a spacetime description



What about D0s on non-SUSY orbifolds?

-Consider D0 brane probe of $\mathbb{C}/\mathbb{Z}_N$
Adams, Polchinski and Silverstein, hep-th/0108075

-The closed string tachyon in the twisted sector condenses.

-Closed string tachyon *vevs* modify the scalar potential.

$$V_{Higgs} = \frac{1}{2}\sum(|Z_{j+1,j}|^2 - |Z_{j-1,j}|^2 - \lambda_j)^2 - \lambda_j^2$$

-With $\lambda = 0$ the moduli space is $\mathbb{C}/\mathbb{Z}_N$

$\lambda \neq 0$ the moduli space smoothed out.

-If we require that the brane sits at the tip $Z_{i,j} = 0$ → the brane is at a higher energy (i.e. branes are more massive)

What we want to do differently?

c=1 matrix model
- Branes are unstable anyway
- 'tachyon' is massless

$\mathbb{C}/\mathbb{Z}_N$
-tachyon is spatially localized

Plan: Study D-branes in non-localized tachyon background

I. Review worldsheet theory of the 'nothing state'

II. Add boundaries to the worldsheet and discuss boundary states and one-point functions

III. Interpretation via field theory model

IV. General CFT applications

The Tachyon at the End of the Universe

McGreevy and Silverstein, hep-th/0506130

-Milne universe with anti-periodic boundary conditions on fermions.

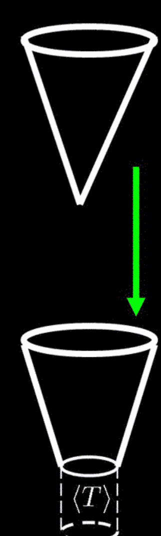
-String scale circle ⇒ winding tachyon

-Worldsheet theory deformed by the operator

$$\int d^2\theta(2i\mu e^{-\kappa\tilde{X}_0}\cos(w\tilde{\tilde{\Omega}}))$$

- After wick rotation, the X_0 zero mode can be integrated out, and one finds

$$Z = \frac{-1}{\kappa}(\ln(\frac{\mu}{\mu^*}) + i\frac{\pi}{2})Z(\mu = 0)$$



- This is the theory without the tachyon (and a thermal two point function) BUT **there is no contribution at early times.**

Aside on Wick Rotation

-Subtle issue in string theory, but is the choice of state of spacetime theory

-Solutions to time dependant problems (to date) rely crucially on Wick rotations / analytic continuations

-Known to fail in some cases Nakayama, Rey & Sugawara, hep-th\0605013

-We will take the more standard rotation (ignoring the subtleties)

$$\tau \equiv e^{-i\gamma}\tau_\gamma, X_0 \equiv e^{i\gamma}X_{0,\gamma}, \kappa \equiv e^{-i\gamma}\kappa_\gamma, v \equiv e^{-i\gamma}v_\gamma$$

-Choice of state will be made via Wick rotation but results are not crucially dependant on details of the rotation.

-The background charge will also get shifted under this rotation to maintain semiclassical conformality

Adding Boundaries to the Worldsheet

-Boundaries generically break SUSY

-We will maintain some SUSY by adding terms to the boundary action. Given an action of the form

$$\int d^2z d^2\theta \mathcal{L}(\theta, \bar{\theta})$$

-Then we should add a term to the boundary of the form

$$i\eta \oint dx \mathcal{L}(\theta = \bar{\theta} = 0) \qquad (\eta = \pm 1)$$

-In our case, this boundary action is

$$S_{bdy} = -\eta \oint dx (i\psi_\mu \bar{\psi}^\mu - 2\mu \cos(w\bar{\Omega}) e^{-\kappa X_0})$$

-With added terms, we preserve $Q + \eta\bar{Q}$

Boundary Conditions

-Use the variation of the action to get natural BCs

e.g. the boundary terms arising from X_0 variation are

$$\oint dx 2\delta X_0 \left(\frac{\partial X_0}{\partial y} - \eta\kappa\mu \cos(w\bar{\Omega}) e^{-\kappa X_0} \right)$$

-Dirichlet conditions give spacelike branes, these are basically just like the D0s in $\mathbb{C}/\mathbb{Z}_N$

-The modified Neumann condition is what we will need but it is not particularly nice.

-The cosine is a function of the T-dual coordinate,

Take Dirichlet conditions on the T-dual coordinate to make life easier.

T-Duality

-Type II on a Scherk-Schwarz circle ⟷ Type 0 on a circle with a $\mathbb{Z}_2$ orbifold. This orbifold acts by $S \cdot (-1)^{F_R}$

-Two types of branes (for a given dimensionality) in type 0, but are not globally defined under the orbifold action

-The type of brane $(\eta = \pm 1)$ in type 0 ⟷ Sign of the Wilson line on the Neumann brane in type II.

-In type II and type 0, the D-brane that is invariant under GSO and orbifold is a combination of the 'electric' and 'magnetic' branes in type 0.

Boundary States

-Boundary state is the state annihilated by the boundary condition (operator), i.e. $\widehat{b.c.}|B\rangle = 0$

-For a wrapped brane, $|B\rangle \propto \delta(\bar{\Omega} - \bar{\Omega}_0)$

-The extra spatial directions and fermion conditions give the usual contributions to the boundary state.

-The remaining complication is the boundary condition on X_0

$$\left(\frac{\partial X_0}{\partial y} - \eta\kappa\mu\cos(w\bar{\Omega}_0)e^{-\kappa X_0}\right)|B,\eta,\mu\rangle = 0$$

This looks just like the FZZT brane B.C.

Minisuperspace Wavefunctions

- Minisuperspace truncation (of Liouville theory) gives the same answer (for the one-point function) as the $b \rightarrow 0$ limit of the exact solutions

But, it gives better intuition than exact answers

- The Boundary state 'wavefunction' is given by

$$\Psi_b(X_0) = \exp(-\eta\mu e^{-\kappa X_0}\cos(w\bar{\Omega}_0))$$

-To construct a GSO invariant boundary state, we will need combinations of states with $\eta = \pm 1$

-Wrapped brane will have exponentially growing **AND** decaying contributions to the boundary state (from the zero modes)

Approximation Scheme

-Taking leading order in everything, including the tachyon *vev*.
This will require that the tachyon *vev*, κ and μ are small.

-This will allow us to work in conformal perturbation theory where the semiclassical action is a good approximation

How should one use the minisuperspace description (which is not a consistent approximation)?

It gives the same answers as the exact solutions in this limit.

Calculate one point function using minisuperspace but keep only the leading order in everything

What do we gain? An integral representation of the answer

Example 1

$\cos(w\bar{\Omega}_0) = 1$ -The boundary state factorizes, and we can deal with X_0 and $\bar{\Omega}_0$ separately.

Ω part is easy to deal with. Use method of images

X_0 part is hard. Non-trivial boundary conditions

Want to use minisuperspace wavefunctions to get the X_0 part.

Solve SUSY QM, picking modes that decay behind 'Liouville wall'

→ 1-point function is inner product with boundary state

$$\langle e^{ipX_0} \rangle \simeq \int dX_0 K_{ip}(\mu e^{-\kappa X_0}) e^{-\eta \mu e^{-\kappa X_0}}$$

Example 2

$\cos(w\bar{\Omega}_0) = 0$ - The boundary conditions are simple

$$\partial X_0 |B\rangle = 0$$

-This is just the usual Neumann conditions, so we see

$$|B, w\bar{\Omega} = \tfrac{\pi}{2}, \mu\rangle \equiv |B, w\bar{\Omega} = \tfrac{\pi}{2}, \mu = 0\rangle$$

This brane has no modification to the boundary state

-Up to v corrections, this boundary state can be explicitly constructed

-One point function is still heavily modified by closed string mass.

-Consistent with minisuperspace wavefunction since $\Psi_B \propto \mathbb{I}$

Field Theory Analogue

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - \mu^2 \kappa^2 e^{2\kappa t} \phi^2 + j(t,x)\phi$$

-We can solve for $\langle\phi\rangle$ directly using the Green's function

$$G(z,y) \propto \Theta(z-y) K_{\vec{p}}(z) I_{\vec{p}}(y) + \Theta(y-z) K_{\vec{p}}(y) I_{\vec{p}}(z)$$

$$(z = \mu \exp(\kappa t))$$

-Using $t \ll \tilde{t}$ and $\vec{p} \simeq 0$ this gives the one-point function

$$\langle \phi(t) \rangle \simeq \int d\tilde{t}\, j(\tilde{t}) \exp(-\mu |e^{\kappa t} - e^{\kappa \tilde{t}}|)$$

-This is the same as the worldsheet example in the same limit.

-Thus, in this analogy we should have $\Psi_B \sim j$

Interpretation

-Hard to separate effects of closed string mass from D-brane behaviour in exact answers

-From the field theory model, we can see how we can separate the effects

-If we measure the mass as the coupling to gravitons (closed strings) then

$$\tau \propto \Psi_B \propto 1 + \eta\mu\kappa \cos(w\bar{\Omega}_0) e^{-\kappa X_0} + \ldots$$

-Since the tension comes from boundary state normalization might guess to group zero mode dependence into the mass anyway

For some branes, this tension is insensitive to the tachyon

-**This is only the 'BARE' mass**. The Branes source fields that are effected. Should gain large 'dressed' masses anyway.

Choice of State

-Closed string masses grow exponentially with time →
A natural choice of (spacetime) state is the vacuum (but not necessary)

-In the D-brane context, there is less control of the choice of state (D-brane scattering is less natural from worldsheet perspective)

-A non-perturbative description would likely allow the same choice as for the D-branes (and a calculation of the 'dressed mass')

-The choice of state is important for spacetime unitarity.

Comparison with Previous Results

c=1 Matrix Model

-All the work here is done by worldsheet SUSY, not present in c=1

-ZZ branes in c=1 have an open string tachyon (and $\hat{c} = 1$)

-Open string tachyon can be tuned so that decay occurs after closed string tachyon condenses

But, this does not make bosonic theories 'more stable'

$\mathbb{C}/\mathbb{Z}_N$

-Branes appear to get massive as expected.

-The exact time dependence of growth is unknown (we know leading order at best) → No contradiction

CFTs and Bulk Deformations

-Bulk perturbations can induce boundary flows

Fredenhagen, Gaberdiel & Keller, hep-th\0609034

-Consider perturbations of the form

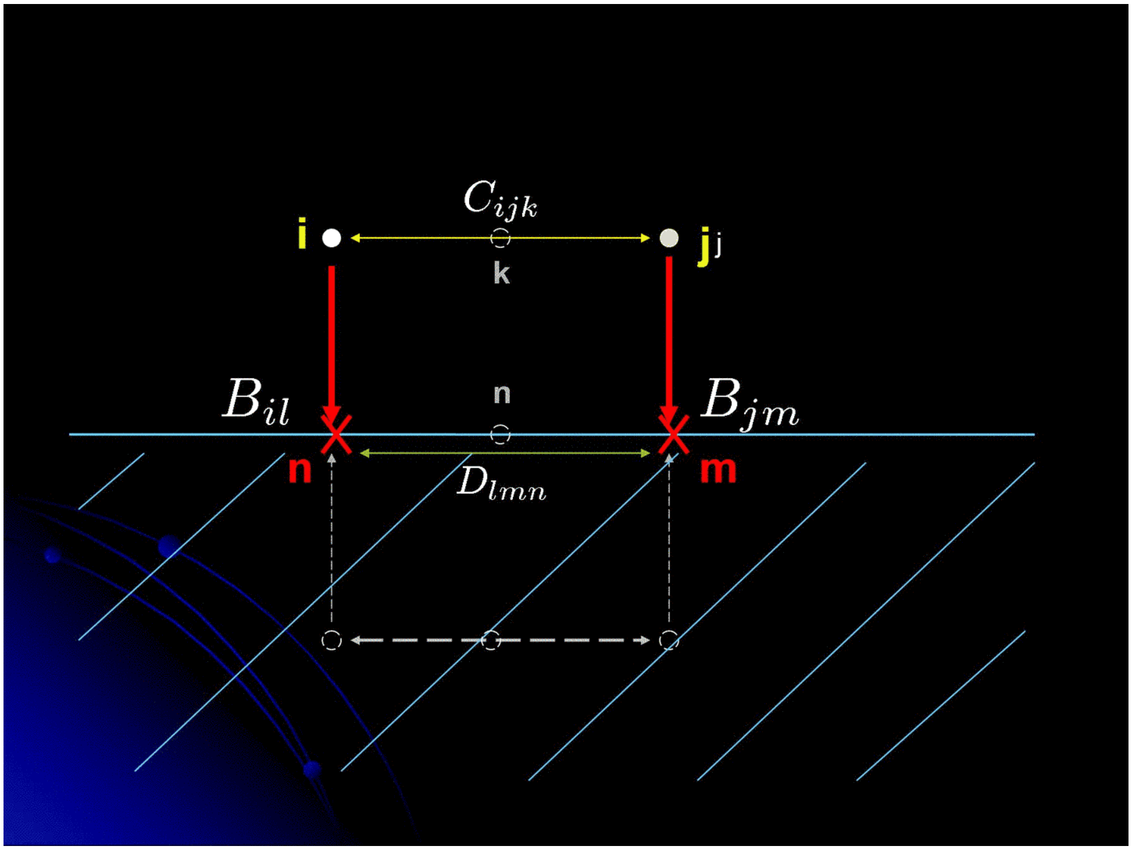
$$S = S_{CFT} + \sum_i \lambda_i l^{2-h_\phi} \int d^2z \phi_i(z) + \sum_j \mu_j l^{1-h_\psi} \oint dx \psi_j(x)$$

-The bulk and boundary RG flows are

$$\dot{\lambda}_k = (2 - h_{\phi_k})\lambda + \pi C_{ijk}\lambda_i\lambda_j + \ldots$$
$$\dot{\mu}_k = (1 - h_\psi)\mu_k + \tfrac{1}{2}B_{ik}\lambda_k + D_{ijk}\mu_i\mu_j + \ldots.$$

-Even if the bulk perturbation is exactly marginal (without a boundary) the perturbation can induce a boundary flow.

-At the one-loop level, the bulk-boundary OPE tells us what happens to the branes in closed string backgrounds.



CFT applications

-Deform a free theory by a momentum operator of the form

$$\phi(z) = \sin(pX(z))$$

-Then, if have Dirichlet conditions such that $X(z=\bar{z})=0$ then

$$B_{\phi,j} = 0$$

-But, if we have other boundary conditions, it induces a flow!

-We can do the same with winding operators and Wilson lines

So, if we sit at the zero of the tachyon and nothing happens

This is NOT what happened earlier. There is no zero in the *vev*
SUSY imposes additional structure that lets us do something similar

More CFT applications

Consider a general CFT with a bulk perturbation.

-If there are conformal boundary conditions such that

$$\langle \phi \rangle \propto \sum B_{\phi j} \langle \psi_j \rangle = 0$$

then

$$B_{\phi j} = 0 \text{ or } \langle \psi_j \rangle = 0$$

and

$$B_{\phi \mathbb{I}} = 0$$

-This does not forbid boundary flows, but restricts the types of possible flows significantly.

e.g. think of operators of the form $\phi(z) = \prod \phi_i(z)$

Open Questions

-In GLSM model descriptions ‘dust vacua’ arise on Coulomb branch

Adams et al, hep-th/0502021

-Speculated that this could be related to sign of (Lorentzian) F terms

McGreevy and Silverstein, hep-th/0506130

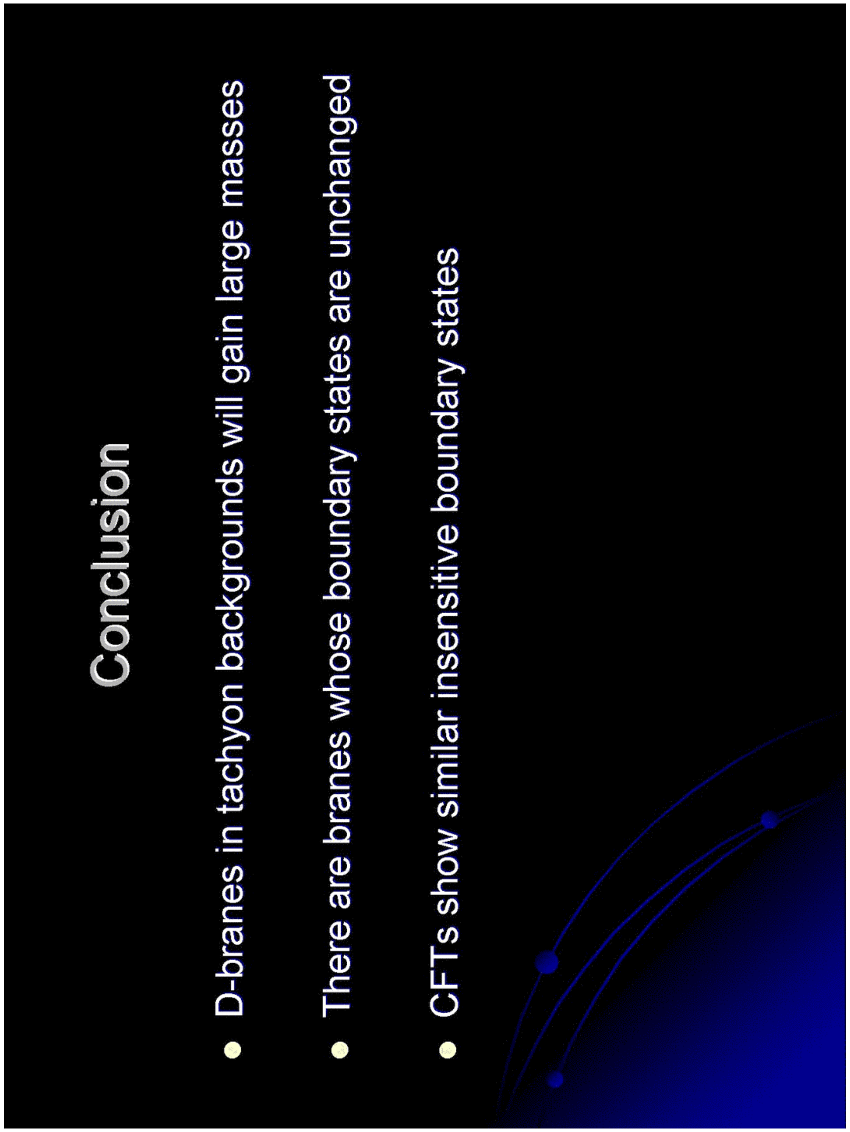
-Unmodified branes sit at specific values of the potential

Are these things related? Do D-branes survive in dust vacua?

-Higher order ⟷ late/early time behaviour

What happens with the dilaton?

- OPEN STRING SPECTRUM unknown


Conclusion
D-branes in tachyon backgrounds will gain large masses
There are branes whose boundary states are unchanged
CFTs show similar insensitive boundary states