Energy Loss of a Quark from AdS/CFT

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Motivation

Applied string theory

- To understand heavy quark physics at RHIC

  string theory →diffusion constant→RHIC observables

- To provide a theoretical laboratory for studying strongly coupled non-abelian plasmas

Herzog, Karch, Kovtun, Kozcaz, Yaffe, JHEP 0607:013, 2006, hep-th/0605158.

Herzog, JHEP 0609:032,2006, hep-th/0605191

Outline

- What is there to understand about heavy quarks at RHIC (and the LHC)?

- What can string theory say about RHIC?

  -- a heavy quark diffusion constant

- What can we learn about strongly interacting non-abelian plasmas?

  -- bounds on friction coefficients

The Relativistic Heavy Ion Collider (RHIC)

- Circular accelerator that collides Au (Pb and other) nuclei at about 200 GeV per nucleon
- 4 detectors (STAR, PHENIX, PHOBOS, BRAHMS)
- Believed to produce a quark-gluon plasma (QGP) at T=250 MeV and $\alpha_s \sim 0.5$

Heavy Quarks at RHIC

- Small quantities of charm and bottom quarks are produced

$$m_c \sim 1.4 \text{ GeV} \quad m_c/T \sim 6$$
$$m_b \sim 4.7 \text{ GeV} \quad m_b/T \sim 20$$

- Quarks are detectable as D and B mesons, charmonium, J/ψ, ...

Some Observables at RHIC

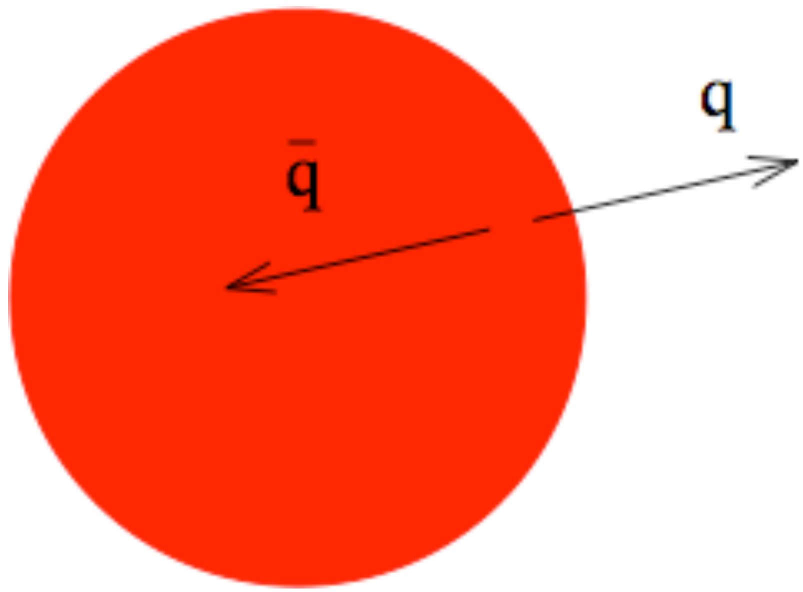
- Elliptic flow -- azimuthal anisotropy of the produced hadrons with respect to the reaction plane

- Jet quenching -- reabsorption of a hard parton as it travels through the fire ball

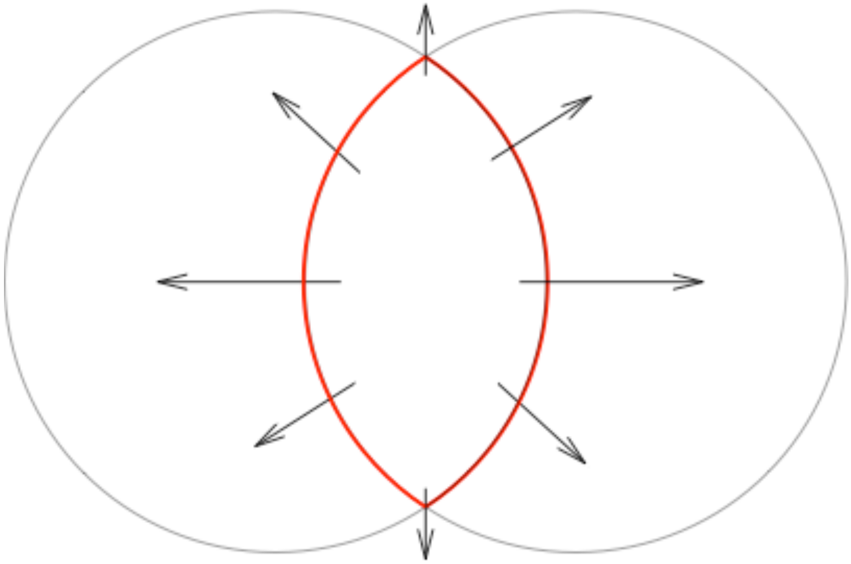
Jet Quenching

Suppression of back-to-back jets

$R_{AA}(p_T)$ suppression factor -- ratio of the meson spectrum in Au-Au collisions to p-p collisions



Elliptic Flow



$$v_2(p_T) = \frac{\int d\phi \frac{dN}{p_T\, dp_T\, d\phi} \cos(2\phi)}{\int d\phi \frac{dN}{p_T\, dp_T\, d\phi}}$$

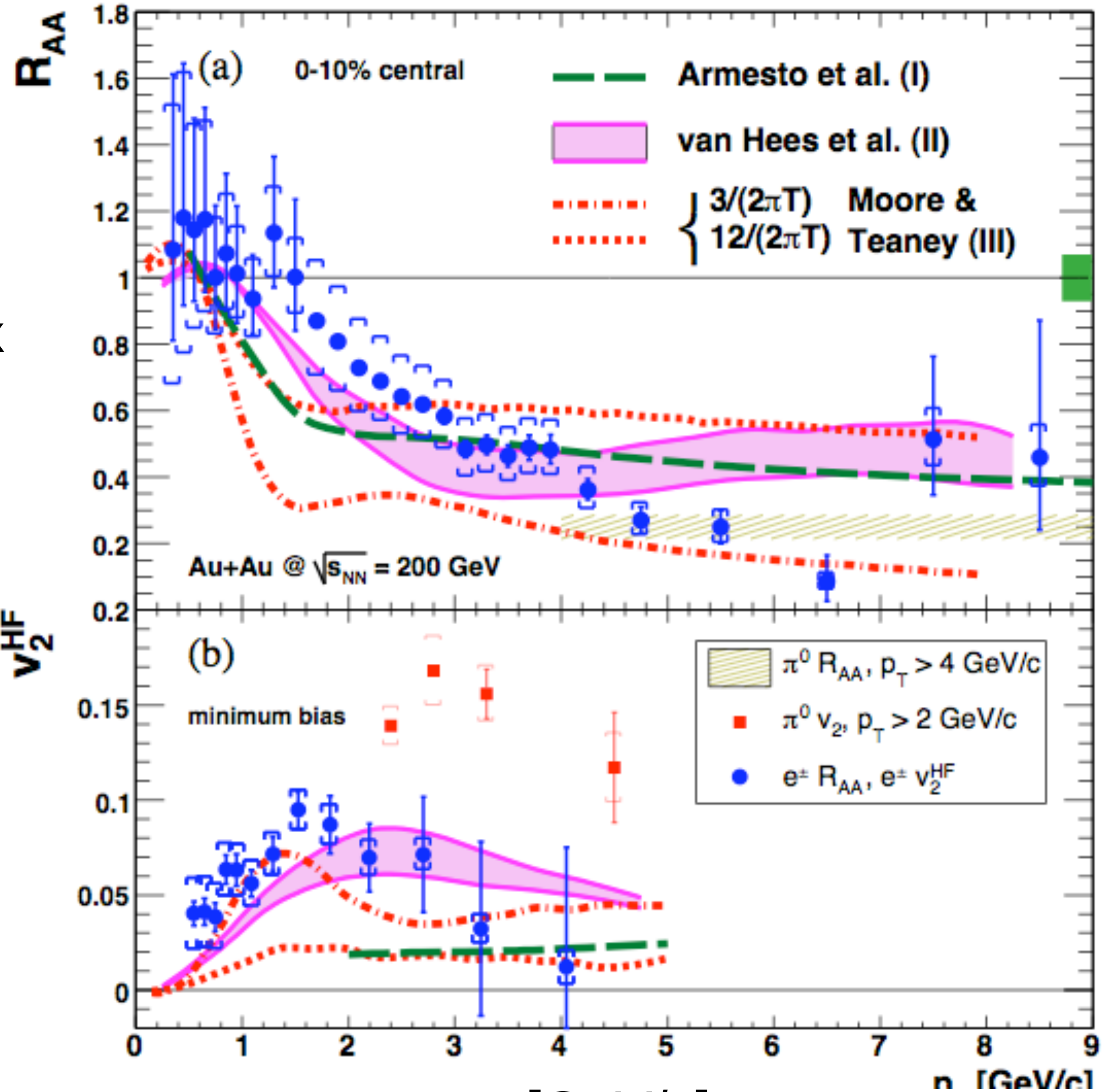
A large elliptic flow supports the claim that QGP behaves like a nearly ideal liquid (small viscosity) with a small mean free path

Heavy Quark Observables

- Measure the elliptic flow with respect to the hadrons containing a bottom or charm quark
- Look at jets formed from charm or bottom quarks
- These observables are sensitive to the rate at which heavy quarks lose energy to the QGP.

data from PHENIX

nucl-ex/0611018



p_t [GeV/c]

A perturbative calculation?

- The diffusion constant $D \sim (3 -- 12) / 2\pi T$ needs to be small to agree with data.
- The perturbative result is large.

$$D = \frac{72\pi}{d_A g^4 T}\left[\left(\ln\frac{2T}{m_D}+\frac{1}{2}-\gamma_E+\frac{\zeta'(2)}{\zeta(2)}\right)+\frac{N_f}{2N_c}\left(\ln\frac{4T}{m_D}+\frac{1}{2}-\gamma_E+\frac{\zeta'(2)}{\zeta(2)}\right)\right]^{-1}$$

$m_D \sim gT$ is the Debye mass, M the quark mass, $d_A = N_c^2 - 1$

To be valid: $g \ll 1\,,\;\; M/T \gg 1\,,\;\; m_D/T \ll 1$

But at RHIC: $m_D/T \sim 1\,,\;\; \alpha_s \sim 1$

Plugging in the numbers Moore and Teaney estimated

$D \sim 6 / 2\pi T$

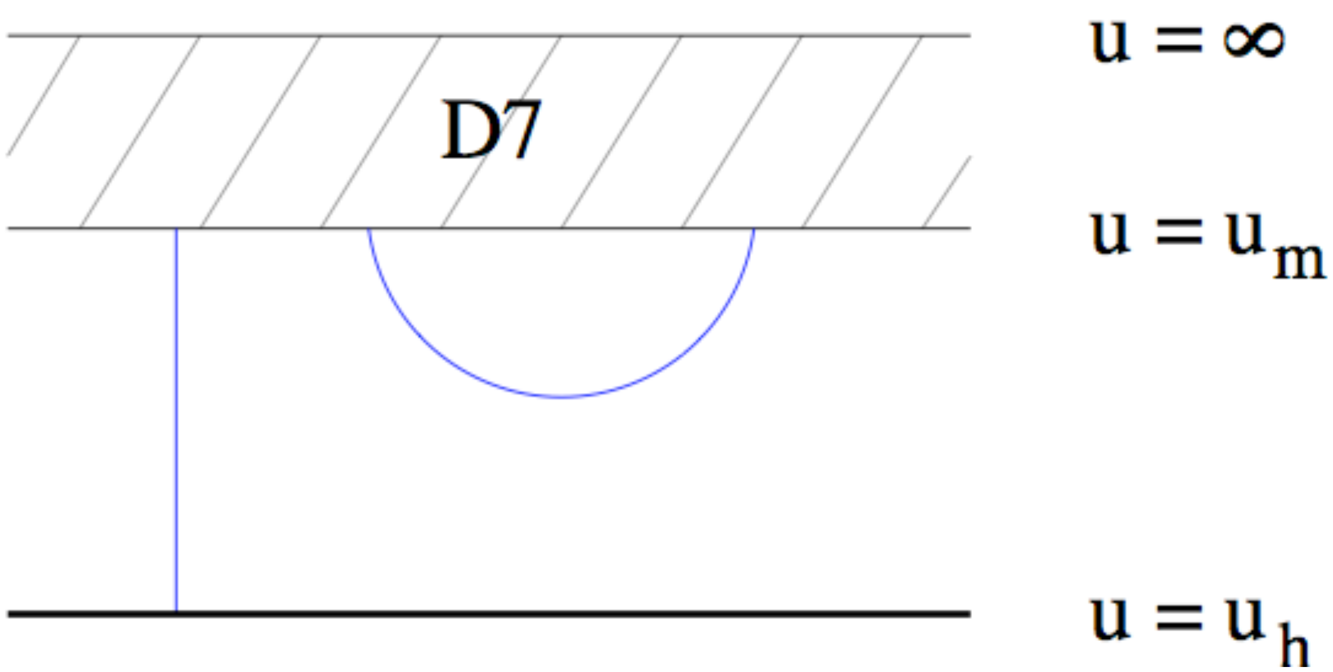
Why should string theory be useful?

- String theory suggests searching for a duality to understand strong coupling.
- The AdS/CFT correspondence:

  $\mathcal{N}$=4 $SU(N)$ SYM ~ type IIB string theory in $AdS_5 \times S^5$

- Adding a black hole to AdS_5 is dual to raising the temperature, $u_h = \pi T$
- Adding a D7-brane that wraps AdS_5 down to some minimal radius u_m is dual to adding a massive $\mathcal{N}$=2 hypermultiplet

The geometric dual picture



Classical strings model single quarks and mesons.

The mass of the quark is to first approximation

$M \sim u_m - u_h$

What does our AdS/CFT model say?

two gedanken experiments

$$\frac{dp}{dt} = -\mu p + f$$

- Hit the quark and watch it slow down

$$p(t) = p(0)e^{-\mu t}$$

- Drag the quark at constant velocity and figure out how much force is needed.

$$M\mu = f/v$$

Some technical details

- Our line element for the black hole is

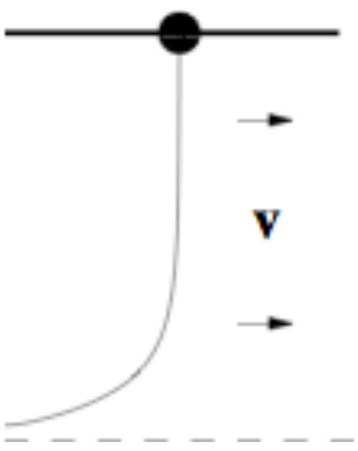
$$ds^2 = L^2\left(\frac{du^2}{h} - h\,dt^2 + u^2 d\vec{x}^2\right) \quad \text{where} \quad h = u^2\left(1 - \left(\frac{u_h}{u}\right)^4\right)$$

  and where *L* is the radius of curvature
- The classical string is governed by the action

$$S = -T_0 \int d\sigma d\tau \sqrt{-\det g_{ab}}$$

  where g_{ab} is the induced metric on the worldsheet and T_0 is the string tension

“Dragging the String”



- There exists an analytic solution corresponding to a single quark moving at constant velocity in response to an electric field.
- This solution has a momentum current

$$\frac{dp}{dt} = -\frac{\pi}{2}\sqrt{\lambda}T^2\frac{v}{\sqrt{1-v^2}}$$

- Assuming a relativistic dispersion relation, one finds

$$\mu = \frac{\pi}{2}\frac{\sqrt{\lambda}T^2}{M_{\text{kin}}}$$

More Results

- Assuming the quark obeys a Langevin type equation, we can extract a jet quenching parameter (may not be valid relativistically)

$$\hat{q} = \frac{d}{dt}\langle(\vec{p}_t)^2\rangle = 4T\mu M_{\text{kin}} = 2\pi\sqrt{\lambda}T^3$$

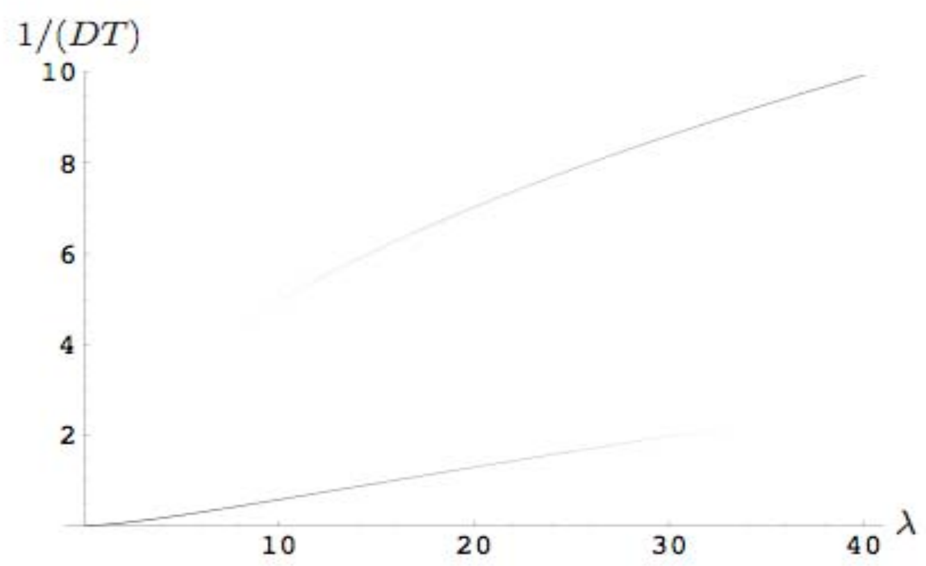
- From μ, we can also extract a diffusion constant

$$D = \frac{T}{\mu M_{\text{kin}}} = \frac{2}{\pi\sqrt{\lambda}T}$$

The perturbative result

- Chesler and Vuorinen (hep-ph/0607148) calculated the *D* in the limit $\lambda \to 0$ for SYM

$$D = \frac{12\pi}{d_A g^4 T}\left[\ln\frac{2T}{m_D} + \frac{13}{12} - \gamma_E + \frac{1}{3}\ln 2 + \frac{\zeta'(2)}{\zeta(2)}\right]^{-1}$$



from hep-ph/ 0607148

Comparing with Experiment

- For SYM, α_s ~ 0.5 corresponds to λ ~ 20.
- One simplistic idea is to approximate the strongly coupled *D* for QCD as

$$D_{\mathrm{QCD},\alpha_s\sim 0.5} \sim D_{\mathrm{QCD},\alpha_s\to 0}\frac{D_{\mathrm{SYM},\lambda\sim 20}}{D_{\mathrm{SYM},\lambda\to 0}}$$

A simple comparison

- We find the ratio

$$\frac{D_{\mathrm{QCD},\alpha_s\to 0}}{D_{\mathrm{SYM},\lambda\to 0}} = \frac{6}{1+N_f/2N_c} \sim 4$$

- Moreover $D_{\mathrm{SYM},\lambda\sim 20} \sim \frac{1}{7T}$

- From which we conclude $D_{\mathrm{QCD},\alpha_s\sim 0.5} \sim \frac{4}{7T}$

- The data suggests $D_{\mathrm{QCD},\alpha_s\sim 0.5} \sim \frac{3 \text{ to } 13}{7T}$

which is in reasonable agreement...

"Hitting the String"

and universality

- Take a straight string that stretches from the D7-brane to the horizon and consider linearized fluctuations.
- One can extract from this linearized analysis a non-relativistic dispersion relation

$$E = M_{\text{rest}} + \frac{p^2}{2M_{\text{kin}}}$$

- One can extract μ in the large mass limit:

$$\mu \sim 1/M$$

- Or μ in the small mass limit: $\mu = 2\pi T$

Universality

We can repeat the above three calculations with a metric of the form

$$ds^2 = g_{tt}(u)dt^2 + g_{uu}(u)du^2 + g_{xx}(u)\delta_{ij}dx^i dx^j$$

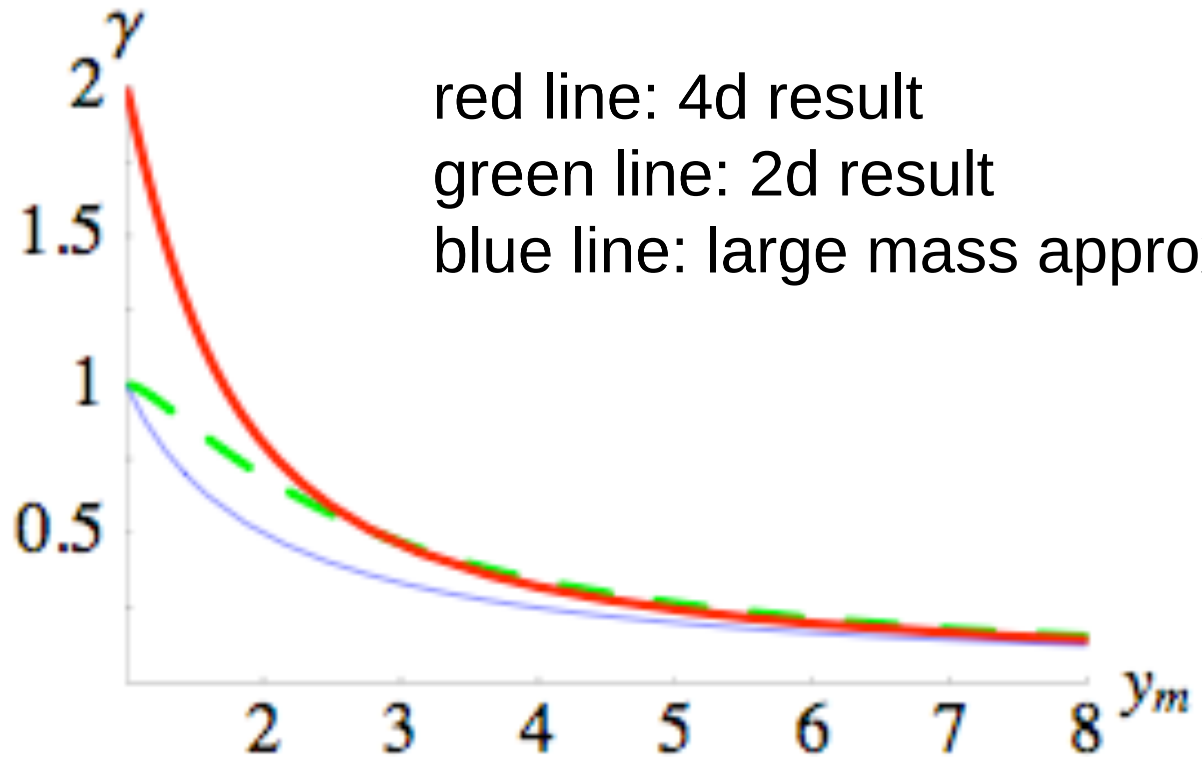
and they remain true!

assumptions:

1) metric is asymptotically AdS, $u \to \infty$
2) regular power series expansion near the horizon, $u \sim u_h$
3) $g'_{xx}(u_h) > 0$ entropy condition

The friction coefficient as a function of u_m

(black hole in AdS)



red line: 4d result
green line: 2d result
blue line: large mass approximation

$\gamma = \mu / u_h$

$u_h d = 4\pi T$

$y_m = u_m / u_h$

bound?

Issues with the friction coefficient bound

- Assumed we can model the string with a 5D Nambu-Goto action, c.f. Caceres and Guijosa
- It is possible to set $(g_{xx})'(u_h) < 0$ in some backgrounds, N=2* of Buchel and Liu, Antonyan
- Light quark limit of a heavy quark system

  --can make λ large, phase transition

  --ongoing project with Jensen and Karch

Concluding Remarks

- Calculation of a diffusion constant for a strongly coupled theory.
- Conjectured friction coefficient bound.
- There are few techniques available for computing time dependent quantities in the QGP: perturbation theory, string theory.
- Perturbation theory does not appear to be valid in the regime relevant for RHIC.
- String theory is valid in the right regime but describes the wrong theory.
- Asking the right questions! It took some time to think to look at η/s.

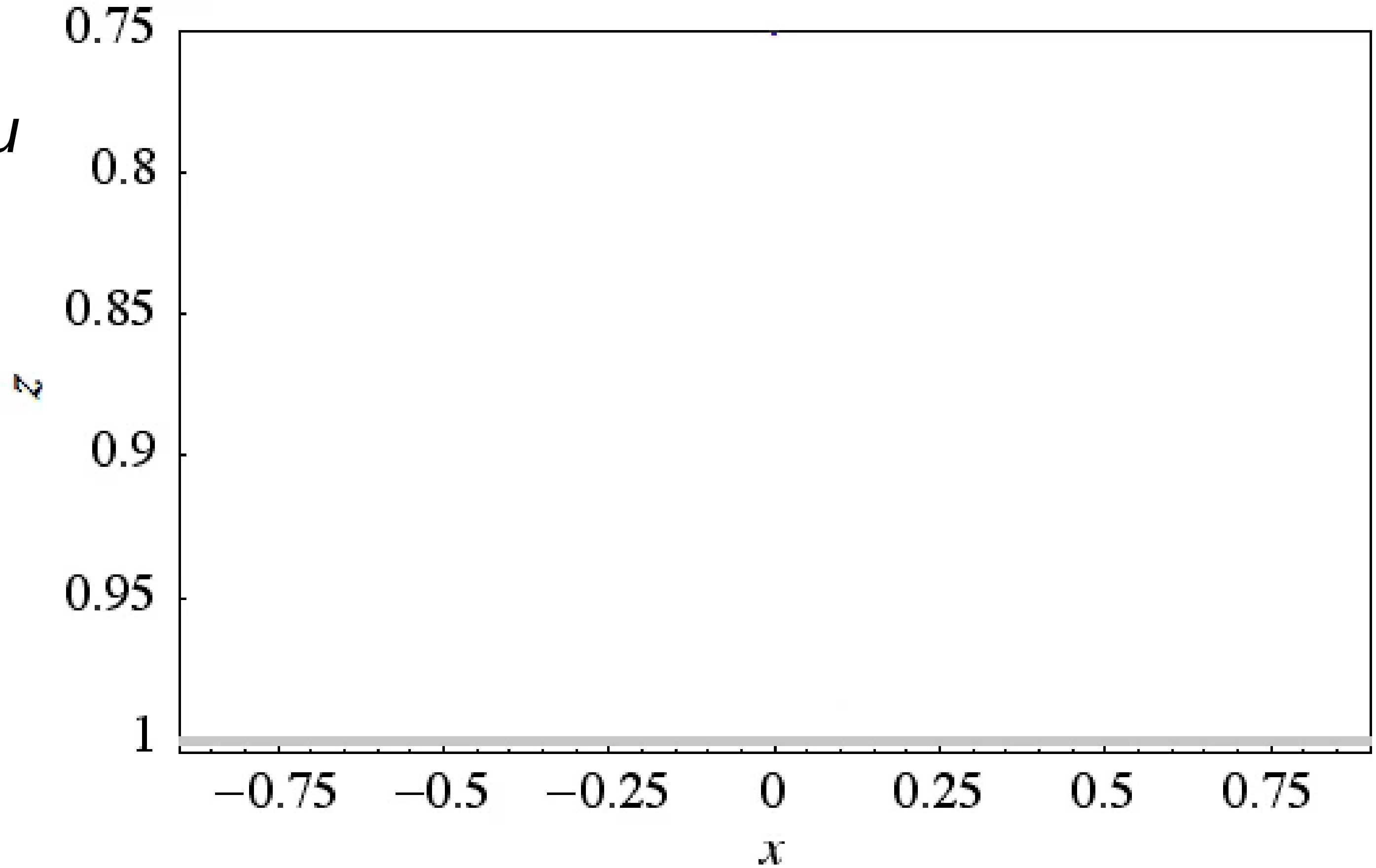
“Hitting the String” again

Unsatisfied with the linear, small velocity analysis, we performed a full numeric simulation of the slowing string.

- Create a quark-antiquark pair flying apart from each other at high energy.
- This configuration corresponds to an expanding semi-circular string.
- Use the Polyakov action and a PDE solver to calculate the motion of the string.
- Measure how quickly the quarks (endpoints of the string) slow down.

quark-antiquark pair slowing down

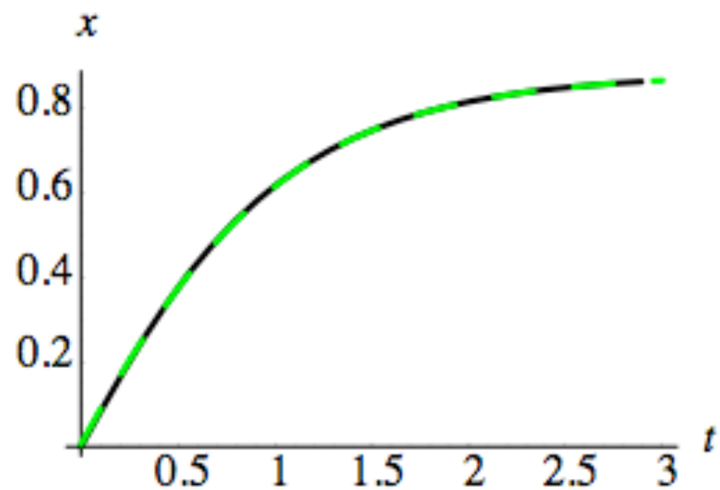
$z = u_h/u$



Fitting to find μ

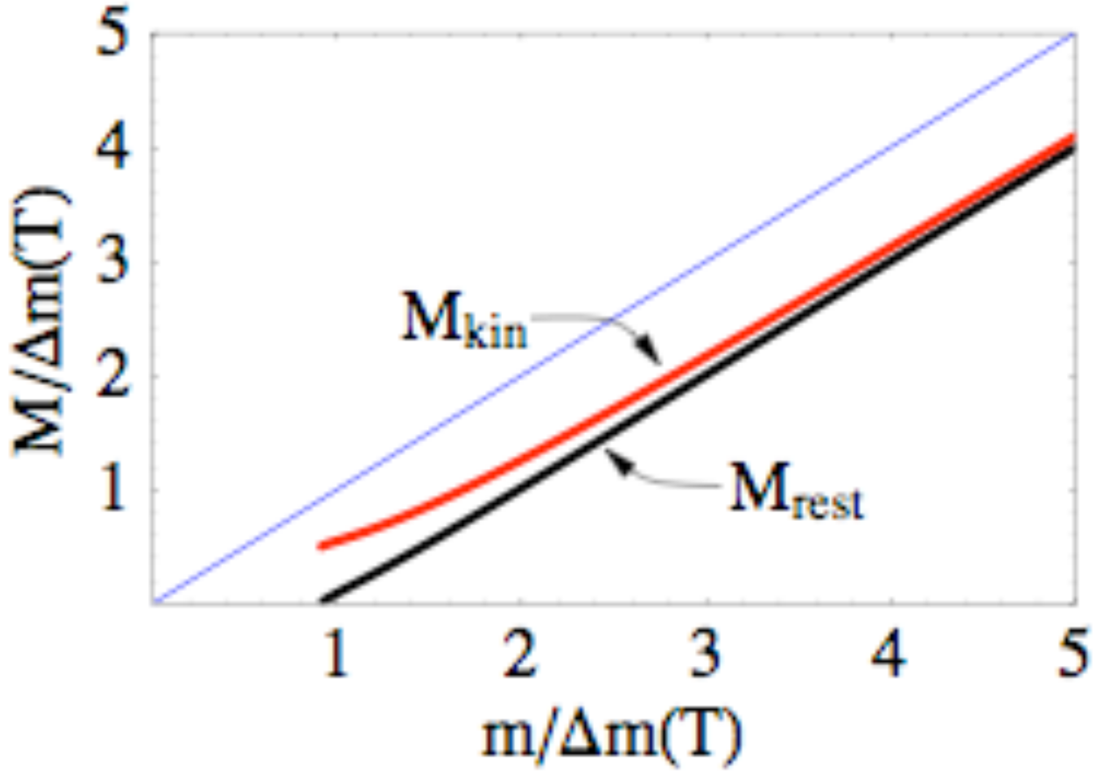
- We fit the motion of the endpoint assuming a relativistic dispersion relation.

$$E = M_{\text{rest}} - M_{\text{kin}} + \sqrt{p^2 + M_{\text{kin}}^2}$$



- In this particular example, the result is $\mu/\pi T = 1.40$. All numeric values for μ agree well with the linear analysis just presented.
- The solid black line is the endpoint while the dashed green line is the best fit.
- Small deviation at early times from the quark potential.

The Kinetic and Rest Mass as a Function of the Lagrangian Mass



$$\Delta m(T) = \frac{1}{2}\sqrt{\lambda}T$$

Related Papers

1) S.D. Avramis, K. Sfetsos, D. Zoakos, ON THE VELOCITY AND CHEMICAL-POTENTIAL DEPENDENCE OF THE HEAVY-QUARK INTERACTION IN N=4 SYM PLASMAS. [HEP-TH 0609079]

2) E. Shuryak, THE CONICAL FLOW FROM QUENCHED JETS IN SQGP. [NUCL-TH 0609013]

3) E. Nakano, S. Teraguchi, W.-Y. Wen, DRAG FORCE, JET QUENCHING, AND ADS/QCD. [HEP-PH 0608274]

4) M. Asakawa, S.A. Bass, B. Muller, ANOMALOUS TRANSPORT PROCESSES IN ANISOTROPICALLY EXPANDING QUARK-GLUON PLASMAS. [HEP-PH 0608270]

5) P.C. Argyres, M. Edalati, J. F. Vazquez-Poritz, NO-DRAG STRING CONFIGURATIONS FOR STEADILY MOVING QUARK-ANTIQUARK PAIRS IN A THERMAL BATH. [HEP-TH 0608118]

6) E.V. Shuryak, STRONGLY COUPLED QUARK-GLUON PLASMA: THE STATUS REPORT. [HEP-PH 0608177]

7) K.-Y. Kim, S.-J. Sin, I. Zahed, DENSE HADRONIC MATTER IN HOLOGRAPHIC QCD. [HEP-TH 0608046]

8) S.C. Huot, S. Jeon, G.D. Moore, SHEAR VISCOSITY IN WEAKLY COUPLED N = 4 SUPER YANG-MILLS THEORY COMPARED TO QCD. [HEP-PH 0608062]

9) S. Caron-Huot, P. Kovtun, G.D. Moore, A. Starinets, L.G. Yaffe, PHOTON AND DILEPTON PRODUCTION IN SUPERSYMMETRIC YANG-MILLS PLASMA. [HEP-TH 0607237]

10) E. Caceres, M. Natsuume, T. Okamura, SCREENING LENGTH IN PLASMA WINDS. [HEP-TH 0607233]

11) T. Matsuo, D. Tomino, W.-Y. Wen, DRAG FORCE IN SYM PLASMA WITH B FIELD FROM ADS/CFT. [HEP-TH 0607178]

12) M. Chernicoff, J.A. Garcia, A. Guijosa, THE ENERGY OF A MOVING QUARK-ANTIQUARK PAIR IN AN N=4 SYM PLASMA. [HEP-TH 0607089]

13) P.M. Chesler, A. Vuorinen, HEAVY FLAVOR DIFFUSION IN WEAKLY COUPLED N=4 SUPER YANG-MILLS THEORY. [HEP-PH 0607148]

14) H. Liu, K. Rajagopal, U. Achim Wiedemann, AN ADS/CFT CALCULATION OF SCREENING IN A HOT WIND. [HEP-PH 0607062]

15) J.J. Friess, S.S. Gubser, G. Michalogiorgakis, S.S. Pufu, THE STRESS TENSOR OF A QUARK MOVING THROUGH N=4 THERMAL PLASMA. [HEP-TH 0607022]

15) Y.-H. Gao, W.-S. Xu, D.-F. Zeng, WAKE OF COLOR FIELDS IN CHARGED N=4 SYM PLASMAS. [HEP-TH 0606266]

16) N. Armesto, J.D. Edelstein, J. Mas, JET QUENCHING AT FINITE `T HOOFT COUPLING AND CHEMICAL POTENTIAL FROM ADS/CFT. [HEP-PH 0606245]

17) K. Peeters, J. Sonnenschein, M. Zamaklar, HOLOGRAPHIC MELTING AND RELATED PROPERTIES OF MESONS IN A QUARK GLUON PLASMA. [HEP-TH 0606195]

18) S.D. Avramis, K. Sfetsos, SUPERGRAVITY AND THE JET QUENCHING PARAMETER IN THE PRESENCE OF R-CHARGE DENSITIES. [HEP-TH 0606190]

19) F.-L. Lin, T. Matsuo, JET QUENCHING PARAMETER IN MEDIUM WITH CHEMICAL POTENTIAL FROM ADS/CFT. [HEP-TH 0606136]

20) E. Caceres, A. Guijosa, ON DRAG FORCES AND JET QUENCHING IN STRONGLY COUPLED PLASMAS. [HEP-TH 0606134]

21) S.-J. Sin, I. Zahed, AMPERE'S LAW AND ENERGY LOSS IN ADS/CFT DUALITY. [HEP-PH 0606049]

22) J.F. Vazquez-Poritz, ENHANCING THE JET QUENCHING PARAMETER FROM MARGINAL DEFORMATIONS. [HEP-TH 0605296]

23) J.J. Friess, S.S. Gubser, G. Michalogiorgakis, DISSIPATION FROM A HEAVY QUARK MOVING THROUGH N=4 SUPER-YANG-MILLS PLASMA. [HEP-TH 0605292]

24) J. Erdmenger, N. Evans, J. Grosse, HEAVY-LIGHT MESONS FROM THE ADS/CFT CORRESPONDENCE. [HEP-TH 0605241]

25) E. Caceres, A. Guijosa, DRAG FORCE IN CHARGED N=4 SYM PLASMA. [HEP-TH 0605235]

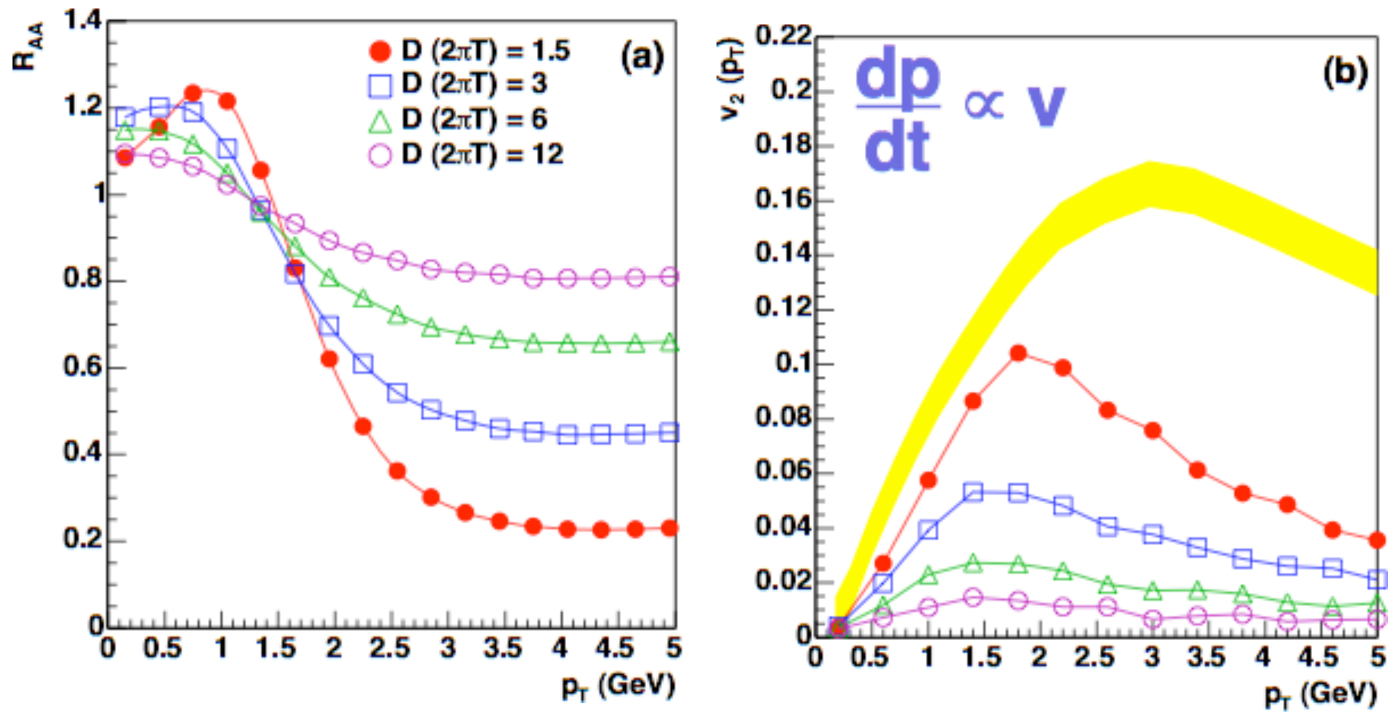
26) C.P. Herzog, ENERGY LOSS OF HEAVY QUARKS FROM ASYMPTOTICALLY ADS GEOMETRIES. [HEP-TH 0605191]

27) S.S. Gubser, DRAG FORCE IN ADS/CFT. [HEP-TH 0605182]

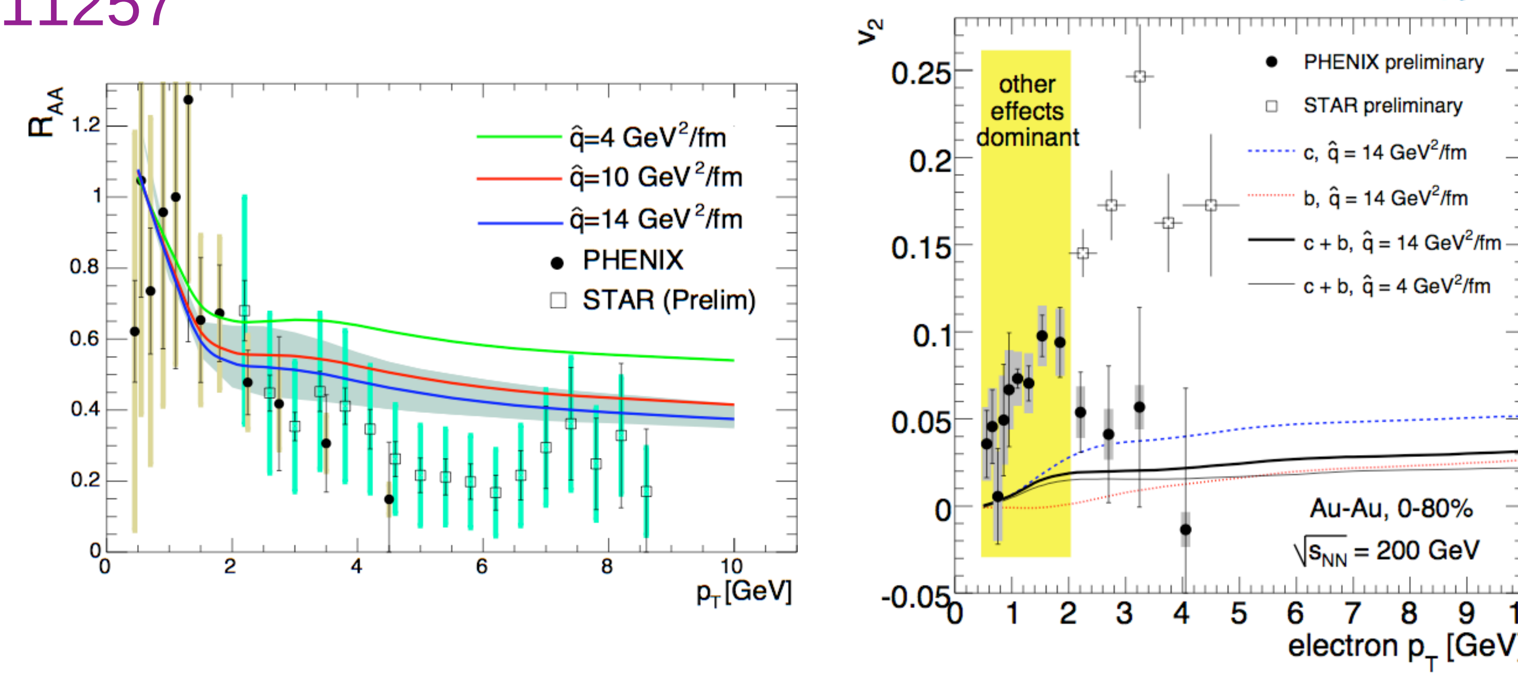
28) J. Casalderrey-Solana, D. Teaney, HEAVY QUARK DIFFUSION IN STRONGLY COUPLED N=4 YANG-MILLS. [HEP-PH 0605199]

29) H. Liu, K. Rajagopal, U. Achim Wiedemann, CALCULATION OF THE JET QUENCHING PARAMETER FROM ADS/CFT. [HEP-PH 0605178]

hep-ph/0412346



hep-ph/0511257



But we want QCD, not SYM!
Maybe the two are not so different.

Hot non-abelian plasma with Debye screening and finite spatial correlation lengths.

1) Pressure

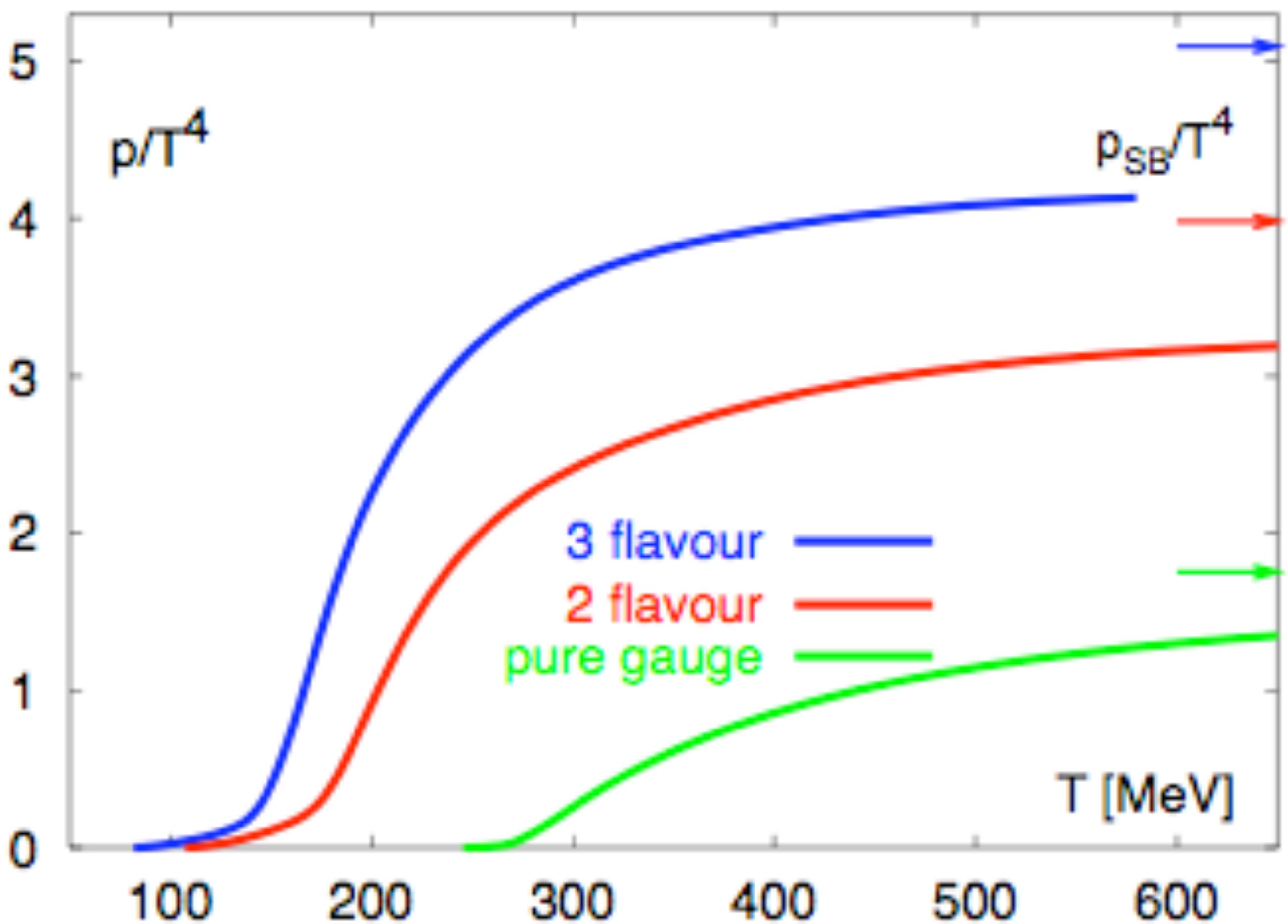
For SYM, Gubser, Klebanov, and Peet (hep-th/9602135) calculated

$$\frac{P_{SYM}(\lambda \to \infty)}{P_{SYM}(\lambda \to 0)} = \frac{3}{4}$$

While for QCD, the ratio of the lattice and perturbative results are

$$\frac{P_{QCD}(T \sim 2T_c)}{P_{QCD}(\alpha_s \to 0)} = 0.8$$

QCD Pressure vs. Temperature

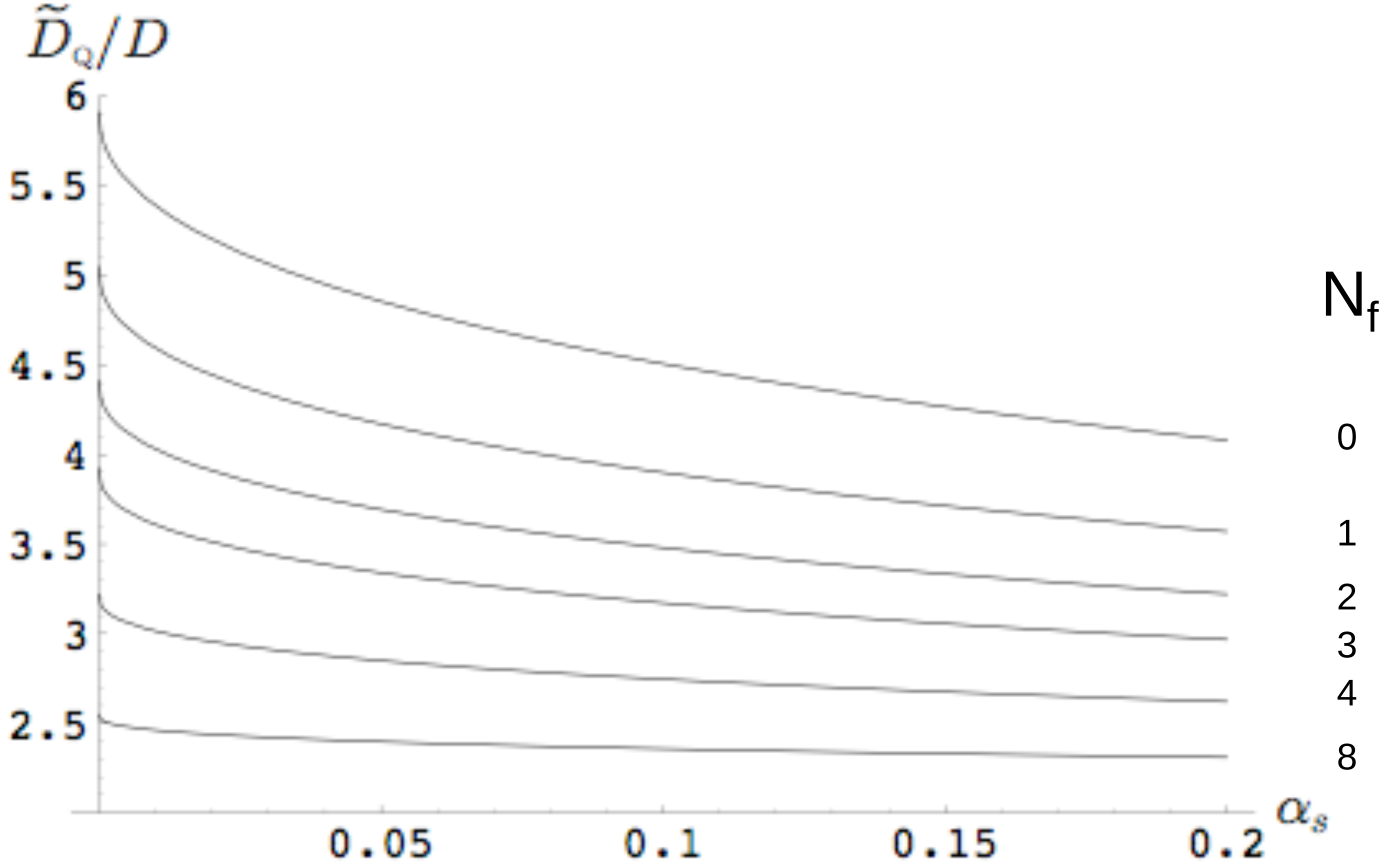


2) Viscosity to entropy ratio η/s (Kovtun, Son, Starinets, hep-th/0405231)

This ratio is $1/4\pi$ in the $\lambda \rightarrow \infty$ limit for all finite *T* field theories with gravity duals.
A viscosity bound?

This value is consistent with measurements of $v_2(p_t)$ at RHIC.

Lesson: We need to be careful about the questions we ask.



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