

Fermion masses and mixings (Profile of a viable string theory)

G.G.Ross, KITP, Santa Barbara, Sept 2006

The Standard Model

- Gauge structure
 $SU(3) \times SU(2) \times U(1) \subset SU(5), SO(10)?$
 - Multiplet content
 $3(10 + \bar{5} + 1) + H(S?)$
 - Fundamental parameters
 $M_{\text{Higgs}} - \text{SUSY?}$
 $g_i \subset G_5$
quark and lepton masses and mixings
CP violating phase
 - Precision tests - no non-SM ($\pm \nu$) phenomena
- Can obtain gauge and multiplet structure + underlying GUT structure e.g. heterotic string
 - Wilson line breaking at the compactification scale can elegantly solve the doublet triplet splitting problem
- $$G \Rightarrow SU(3) \times SU(2) \times U(1) + 3(10 + \bar{5} + 1) + H_u + H_d \quad (\text{MSSM})$$
- Nucleon stability : Z_2 matter parity? Z_3 baryon parity? $Z_2 \otimes Z_3$ baryon parity
 - Many string possibilities - not necessarily with GUT(?) - possibly with a low scale of unification

String phenomenology

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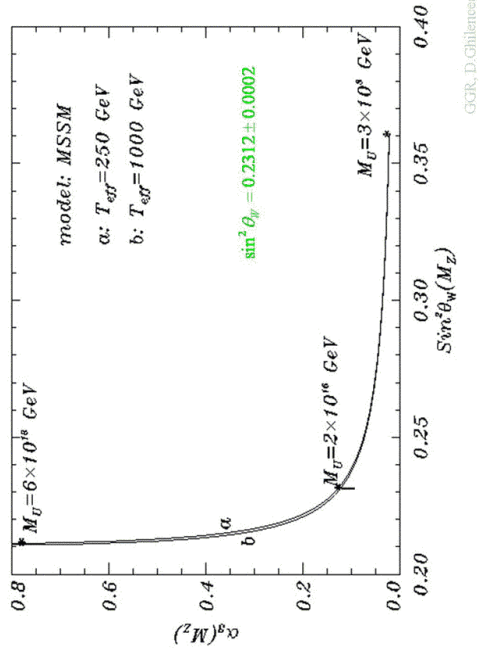
String phenomenology

How can we distinguish them?

Unification Hints - gauge couplings

SUSY UNIFICATION ✓

STRING UNIFICATION ✓



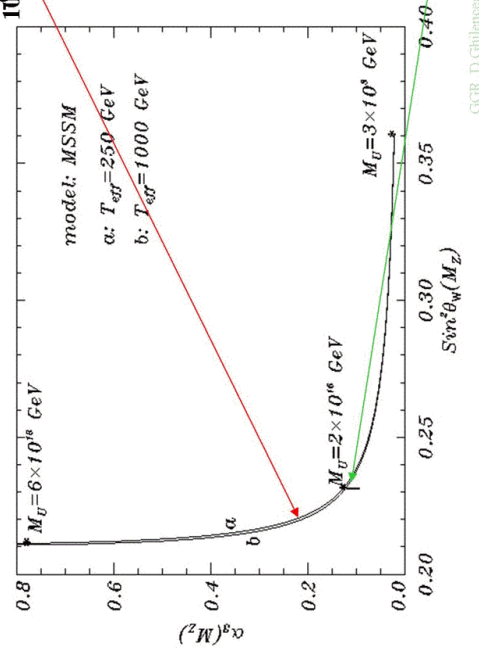
To preserve accuracy :

$$M_s R_c \leq 2$$

$$\sin^2 \theta_W = 0.2334 \pm 0.0025 - 0.25(\alpha_s - 0.119) = 0.2312 \pm 0.0002 \text{ (Expt)}$$

$$\alpha_s = 0.134 \pm 0.01 - 4(\sin^2 \theta_W - 0.2334) = 0.119 \pm 0.003(?) \text{ (Expt)}$$

Unification with gravity?



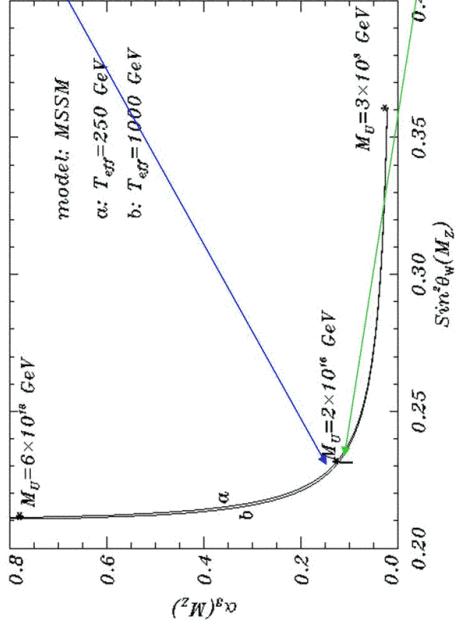
$$M_U^{WCHS} = 3.6 \times 10^{17} \text{ GeV?}$$

$$M_U = (2.5 \pm 2) \cdot 10^{16} \text{ GeV}$$

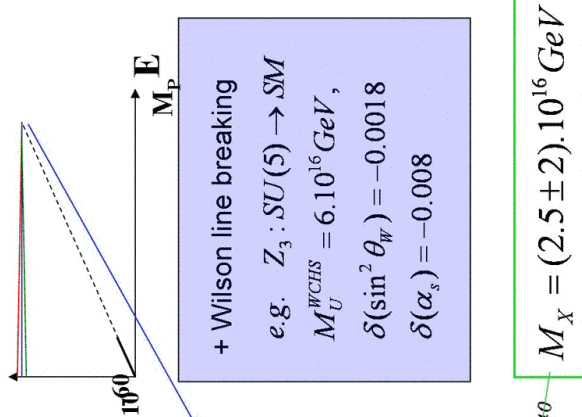
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Unification with gravity?



String unification with gravity



$$\sin^2 \theta_W = 0.2334 \pm 0.0025 - 0.25(\alpha_s - 0.119) = 0.2312 \pm 0.0002 \quad (Expt)$$

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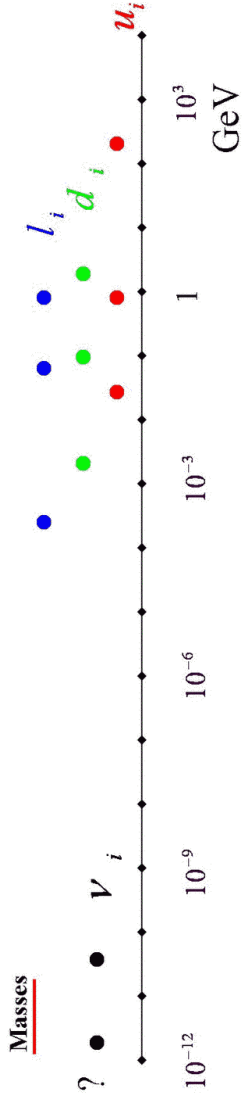
Fermion Masses

The Standard Model

$$L_{Yukawa} = Y_{ij}^u Q^i u^{c,j} H + Y_{ij}^d Q^i d^{c,j} \bar{H}$$

$$M_{ij}^u = Y_{ij}^u < H^0 > \quad M_{ij}^d = Y_{ij}^d < \bar{H}^0 >$$

DATA :



Mixing

Quarks

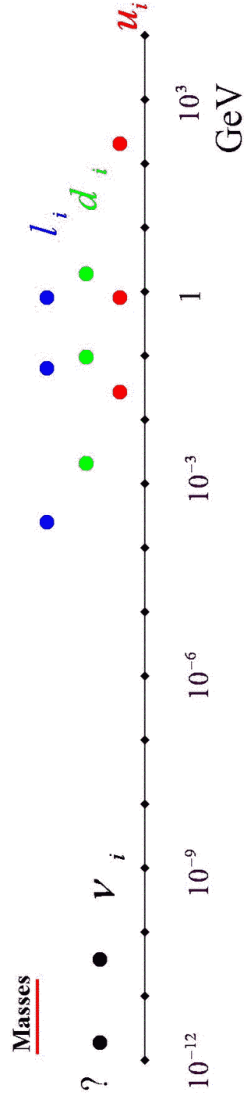
$$V_{CKM} \approx \begin{pmatrix} 1 & 0.218 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 1 & 0.032 - 0.048 \\ 0.004 - 0.015 & 0.03 - 0.048 & 1 \end{pmatrix}$$

$$V_{CKM} = V_L^\dagger U_L$$

$$M^u = V_L^\dagger M_{D^{ia_g}}^u V_R$$

$$M^d = U_L^\dagger M_{D^{ia_g}}^d U_R$$

DATA :

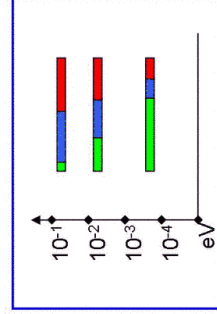


Mixing

Quarks

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$$V_{MNS} = \begin{pmatrix} 0.79 - 0.88 & 0.48 - 0.61 & < 0.2 \\ 0.27 - 0.49 & 0.45 - 0.71 & 0.52 - 0.82 \\ 0.28 - 0.5 & 0.51 - 0.65 & 0.57 - 0.81 \end{pmatrix}$$



$$\approx \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-Tri Maximal
Mixing ...
Non Abelian
Structure?

- Data ↔ Theory (Y_{ij})

...underconstrained - need mixing angles but only $V_{CKM} = V_L^\dagger U_L$ measured

- Hierarchy ↔ Democracy

$$\begin{pmatrix} a \epsilon^2 & \epsilon \\ \epsilon & 1 \end{pmatrix} \leftrightarrow \begin{pmatrix} 1 + 2\epsilon + a\epsilon^2 & 1 - a\epsilon^2 \\ 1 - a\epsilon^2 & 1 - 2\epsilon + a\epsilon^2 \end{pmatrix} / 2$$

$$\theta_C = \theta^u - \theta^d$$

Quark Textures

Hierarchical : $M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}$

$$\begin{aligned} V_{23} &\cong \frac{m_{23}}{m_{33}} + \frac{m_{32}}{m_{33}} \frac{m_s}{m_b} \\ V_{12} &\cong \frac{m_{12}}{m_{22}} + \frac{m_{21}}{m_{22}} \frac{m_d}{m_s} \\ V_{13} &\cong \frac{m_{13}}{m_{33}} + \frac{m_{31}}{m_{33}} \frac{m_d}{m_b} \end{aligned}$$

Expansion parameter $\epsilon = 0.15$

$$\frac{M^d}{m_b} = \begin{pmatrix} \leq \epsilon^4 & ? & ? \\ m_d & \leq \epsilon^2 & ? \\ V_{us} & \leq \epsilon^3 & \epsilon^2 \\ V_{ub} & V_{cb} & 1 \end{pmatrix}$$

Quark Textures

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Expansion parameter $\epsilon = 0.15$

$$\frac{M^d}{m_b} = \begin{pmatrix} \leq \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \leq \epsilon^3 & \leq \epsilon^2 & \epsilon^2 \\ \leq \epsilon & \leq 1 & 1 \end{pmatrix}$$

$\text{Det}[M] = m_s$

Data ↔ Theory (Y_{ij} ∝ M_{ij})

Symmetric fit

$$\frac{M^d}{m_b} = \begin{pmatrix} 0 & 1.5\epsilon^3 & 0.4e^{i20}\epsilon^3 \\ 1.5\epsilon^3 & \epsilon^2 & 1.3\epsilon^2 \\ 0.4e^{i20}\epsilon^3 & 1.3\epsilon^2 & 1 \end{pmatrix} \quad \epsilon = 0.15$$

$$\frac{M^u}{m_t} = \begin{pmatrix} 0 & \epsilon'^3 & ?\epsilon'^3 \\ \epsilon'^3 & \epsilon'^2 & ?\epsilon'^2 \\ ?\epsilon'^3 & ?\epsilon'^2 & 1 \end{pmatrix} \quad \epsilon' = 0.05$$

Asymmetric fit

$$\frac{M^d}{m_b} = \begin{pmatrix} 0 & 1.7\epsilon^3 & 0 \\ 1.7\epsilon^3 & 0 & 5\epsilon^2 \\ 0 & 0.3 & 1 \end{pmatrix} \quad \frac{M^u}{m_t} = \begin{pmatrix} 0 & 2\epsilon'^3 & 0 \\ \epsilon'^3 & 0 & 2\epsilon' \\ 0 & 0.6\epsilon' & 1 \end{pmatrix}$$

Roberts
Romanino
GGR
Velasco Sevilla

Texture zero

$$\frac{M^d}{m_b} = \begin{pmatrix} 0 & \epsilon^3 & \leq \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \leq \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ ? & ? & 1 \end{pmatrix}$$

GATTO, SARTORI, TONIN

symmetric

CP SM phase

$$|V_{us}| = \sqrt{\frac{m_d}{m_s} - e^{i\delta} \frac{m_u}{m_c}}$$

Fritzsch,
Weinberg
Roberts,
Romanino,
GGR, Velasco

$$0.217 - 0.222 \text{ c.f. } |(0.216 - 0.214) - (0.07 - 0.076)e^{i\delta}| = 0.213 - 0.223, \delta = 90^\circ$$

Origin of M_{quark} structure?

Symmetric fit

$$\frac{M^d}{m_b} = \begin{pmatrix} 0 & 1.5 \epsilon^3 & 0.4 e^{i20} \epsilon^3 \\ 1.5 \epsilon^3 & \epsilon^2 & 1.3 \epsilon^2 \\ 0.4 e^{i20} \epsilon^3 & 1.3 \epsilon^2 & 1 \end{pmatrix} \quad \epsilon = 0.15$$

$$\frac{M^u}{m_t} = \begin{pmatrix} 0 & \epsilon'^3 & ? \epsilon'^3 \\ \epsilon'^3 & \epsilon'^2 & ? \epsilon'^2 \\ ? \epsilon'^3 & ? \epsilon'^2 & 1 \end{pmatrix} \quad \epsilon' = 0.05$$

Roberts
Romanino
GGR
Velasco Sevilla

Asymmetric fit

$$\frac{M^d}{m_b} = \begin{pmatrix} 0 & 1.7 \epsilon^3 & 0 \\ 1.7 \epsilon^3 & 0 & 5 \epsilon^2 \\ 0 & 0.3 & 1 \end{pmatrix} \quad \frac{M^u}{m_t} = \begin{pmatrix} 0 & 2 \epsilon'^3 & 0 \\ \epsilon'^3 & 0 & 2 \epsilon' \\ 0 & 0.6 \epsilon' & 1 \end{pmatrix}$$

Origin of M_{quark} structure

- Third generation heavy $h_{t,b} \propto g$ OK (specific string calculations + IRFP) (GUT?)

$$h_b = h_\tau = (?)h_t$$

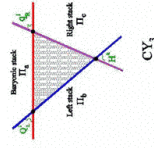
- Hierarchy :
 \Rightarrow spontaneously broken family symmetry - mass matrix elements ordered by :

... dimension of operator $\bar{\psi}_L \psi_R H \left(\frac{\theta}{M} \right)^n$ 0.2 Froggatt-Nielsen

... order of radiative corrections $\propto (h^2 / 16\pi^2)^n$

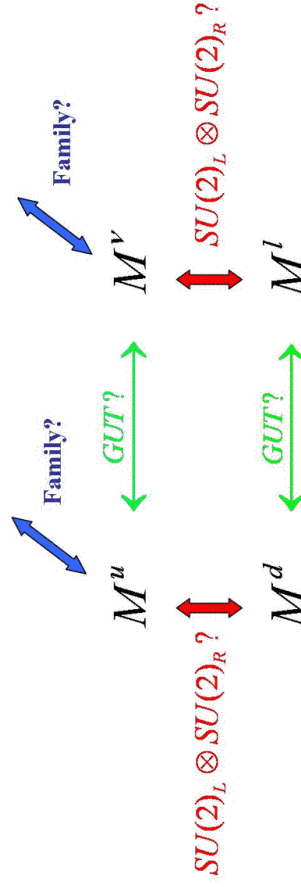
\Rightarrow spatial separation

$$Y_{ijk} \propto h_{qu} \otimes \left(\frac{\theta}{M} \right)^n d_{ijk} e^{-\frac{d_{ijk}(\theta)}{2\gamma} \frac{\theta}{M}} e^{i\alpha} \gamma_{ij}^k \otimes \mathbf{1}_0$$



Textures and flavour models

Symmetries Coherent picture of quark and lepton masses and mixing?



Symmetries and textures

Hierarchical structure strongly suggests a broken symmetry

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \xrightarrow{\langle \theta \rangle \neq 0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & a\varepsilon^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$a = O(1), \quad \varepsilon^2 = \frac{\langle \theta \rangle}{M}$$



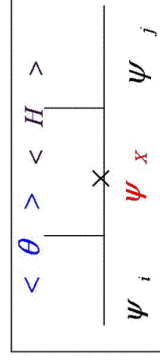
MESSENGER SECTOR?

$$m_{ij} = \frac{\langle \theta \rangle \langle H \rangle}{M_X}$$



FAMILY SYMMETRY?

Abelian, Non-Abelian $\subset (U(3))^6$



Froggatt, Nielsen

Abelian Family symmetry

$$\frac{\bar{\psi}_R M^d \psi_L}{m_b} = \begin{pmatrix} -3 & 2 & 1 \\ \bar{d}_L & \bar{s}_L & \bar{b}_L \end{pmatrix} \theta_{CKM} \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^4 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon \\ \varepsilon^4 & \varepsilon & 1 \end{pmatrix} \begin{pmatrix} d_R & -3 \\ s_R & 2 \\ b_R & 1 \end{pmatrix}$$

$O(\varepsilon^8)$ ✓

$$V_{cb} = (40.4 \pm 1.8) 10^{-3} = O\left(\frac{m_d}{m_s}\right)$$

$$Q_{Higgs} = -2,$$

$$\varepsilon = \frac{\langle \theta \rangle}{M_X} = \frac{\langle \bar{\theta} \rangle}{M_X}$$

Ibanez GGR

Non-Abelian Family symmetry?

$$\frac{\bar{\psi}_R M^d \psi_L}{m_b} = \begin{pmatrix} -3 & 2 & 1 \\ \bar{d}_L & \bar{s}_L & \bar{b}_L \end{pmatrix}$$

$$\begin{pmatrix} 0 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & \epsilon^2 \end{pmatrix} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \begin{matrix} -3 \\ 2 \\ 1 \end{matrix}$$

$$V_{cb} = (40.4 \pm 1.8) 10^{-3} = O\left(\frac{m_d}{m_s}\right)$$

$$Q_{Higgs} = -2, \quad \epsilon = \frac{1}{M_X} \frac{\langle \theta \rangle}{M_X} < \theta \rangle < \bar{\theta} \rangle$$

Ibanez GGR

● Extension to charged leptons – GUT?

$$Det(M^l) = Det(M^d) |_{M^x}$$

$$\frac{M^l}{m_\tau} = \begin{pmatrix} 0 & \epsilon^3 & ?\epsilon^3 \\ \epsilon^3 & \epsilon^2 & ?\epsilon^2 \\ ?\epsilon^2 & ?\epsilon^2 & 1 \end{pmatrix} \begin{matrix} ? \\ ? \\ ? \end{matrix}$$

$$\frac{m_b}{m_\tau} (M^x) = 1$$

● Extension to charged leptons – GUT?

$$Det(M^l) = Det(M^d) |_{M^x}$$

$$\frac{M^l}{m_\tau} = \begin{pmatrix} 0 & \epsilon^3 & ?\epsilon^3 \\ \epsilon^3 & 3\epsilon^2 & ?\epsilon^2 \\ ?\epsilon^2 & ?\epsilon^2 & 1 \end{pmatrix}$$

$\frac{m_s(M_x)}{m_\mu} = \frac{1}{3}$
 $\frac{m_d(M_x)}{m_e} = 3$
 Georgi Jarlskog

$\frac{m_b(M_x)}{m_\tau} = 1$

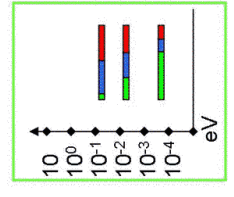
● Extension to neutrinos???

$$\mathcal{L}^{eff} = a \psi_i \theta_{23}^i \psi_j \theta_{23}^j + b \psi_i \theta_{123}^i \psi_j \theta_{123}^j$$

$$\theta_{23} = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$$

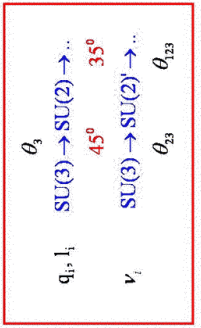
$$\theta_{123} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Tri-Bi-Maximal mixing



$$\mathcal{L}^{eff} = a \psi_i \theta_3^i \psi_3^c \theta_3^j + \dots$$

$$\theta_3 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$



???

Vacuum alignment+decoupling

Strings and fermion masses

● Majorana masses ✓
 Occur in some string compactifications
 e.g. 3 generation Calabi Yau

● Family symmetries?
Abelian symmetries $\Pi_i U(1)_i$ ✓
Non-Abelian symmetries $SU(3)_f$ ✗
 $\Delta(27)...$ ✓

e.g. Calabi Yau manifold $Z_3 \boxtimes Z_2 \boxtimes \mathcal{O}_3^{\diamond} \leftarrow Z_3^* \leftarrow \dots$ Tian Yau
 $+n(f + \bar{f})$
 (but 3 generation model $R \boxtimes R_0 / \mathcal{O}_3 \boxtimes Z_3^* \dots$)

also in small $\frac{R_c}{R_e}$ limit Yukawa coupling calculable : $SU(2)^n$ symmetry
 (a possible origin of the non Abelian structure?)

● CP violation SUSY CP problem - $\phi_A = Arg(Am_{1/2}^*)$, $\phi_B = Arg(Bm_{1/2}^*) < 10^{-2}$?

In string theory CP is a discrete gauge **symmetry** :

Dine, Leigh, MacIntyre

↑ CP conserved by compactification
 - spontaneously broken on family symmetry breaking

↑ CP violation from Yukawa sector ✓?

$$P = \left(\frac{\rho}{M}\right)^{\alpha(i,j)} Q_i q_j^c H_a + ..$$

$$A_{ij} Y_{ij} \tilde{Q}_i \tilde{q}_j^c H_a = (3 + \alpha(i,j)) < P^* > Y_{ij} \tilde{Q}_i \tilde{q}_j^c H_a$$

▶ Cannot align Yukawa and A terms

● OK with suitable family symmetry

$\mu \rightarrow e\gamma$, EDM close to experimental limits

$$\frac{M^d}{m_b} = \begin{pmatrix} \leq \epsilon^4 & \epsilon^3 & \leq \epsilon^3 \\ \leq \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \leq \epsilon^2 & \leq \epsilon^2 & 1 \end{pmatrix}$$

$(m_i, \Delta S \neq 0, \Delta Q = 0)$ CP (A_{ij}) Vives, GGR
 $\Delta S \neq 0, \Delta Q = 0$ CP (A_{ij})

String Profile

- Z_3 baryon parity or above
- $M_s R_c \leq 2$
- Wilson line breaking
- Non-Abelian (discrete) family symmetry
- Spontaneously broken CP symmetry

Phenomenological implications

- [P-decay dimension 6 and above](#)
- ? (High scale compactification)
- ? (Wilson line breaking)
- [A new solution to the SUSY flavour problem](#) - SUSY scalar spectrum
 n edm, $\mu \rightarrow e\gamma$ close to present limits

Light Pseudo Goldstone familon

- Axion

Nucleon decay

$$P_{D=4} = \lambda L L e^c + \lambda' L Q d^c + \lambda'' u^c d^c d^c$$

R – parity violating
(forbidden in MSSM)

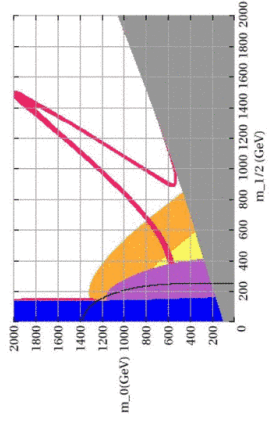
$$\Delta L = 1 \quad \Delta L = 1 \quad \Delta B = -1$$

	$D = 4 \propto \frac{1}{M_d^2}$	×
	$D = 5 \propto \frac{1}{M_{H_T}}$	$M_{H_T} > 10^{25} \text{ GeV}$
	$D = 6 \propto \frac{1}{M_X}$	$M_X > 10^{16} \text{ GeV}$

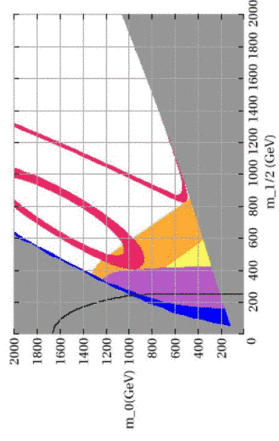
Baryon Parity : $\lambda'' = 0$
(Anomaly free)

$Z_3 : g(Q, u^c, d^c, L, e^c, H_1, H_2) = (1, \alpha^2, \alpha, \alpha^2, \alpha, \alpha^2)$

Ibanez, GGR ;
Allanach, Dedes, Dreiner



$$m_{Q_1} = m_{u_i^c} = m_{d_i^c} = m_H = m_0$$



$$m_{Q_{i \neq 3}} = m_{u_i^c} = m_{d_i^c} = m_H = m_0$$

$$m_{Q_3} = m_{Q_{i \neq 3}} (1 \pm 0.2)$$

ϕ_3 family symmetry breaking

Ramage, GGR