Higher derivative corrections and the violation of the η/s bound

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0812.3572 , 0903.3244

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Window into Strong Coupling

More than a decade of AdS/CFT:

- **Deeper insight into gauge/gravity duality (e.g. microscopic constituents of black holes)**

- **A <u>new way of thinking</u> about strongly coupled gauge theories**

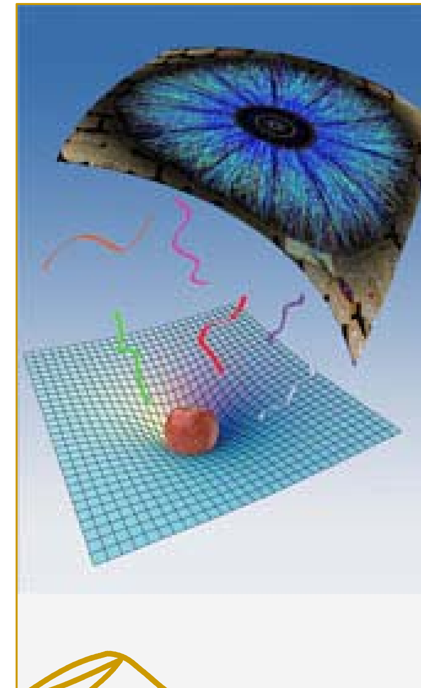
Powerful <u>tool</u> to investigate thermal and hydrodynamic properties of field theories at strong coupling

AdS

CFT

Probing non-equilibrium strongly coupled gauge theories

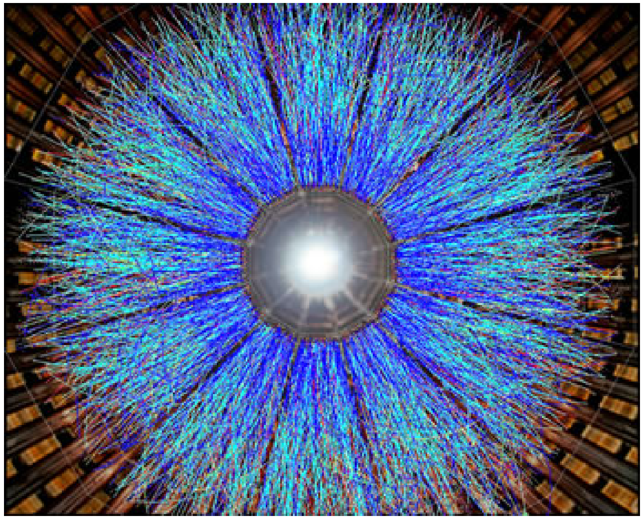
- QFTs at strong coupling hard to study
- Theoretical tools for studying such systems limited :
    - Lattice simulations work well for static (*equilibrium)* processes
    - Dynamics? Lattice methods fail



Why AdS/CFT ?

→ window into *non-equilibrium processes*

Insight into the Quark Gluon Plasma?

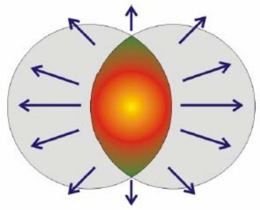


- RHIC → probing behavior of strongly coupled QCD plasma
  (real-time dynamics, transport coefficients)

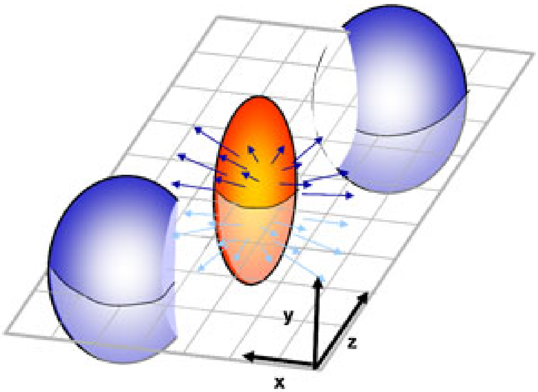
Can we use CFTs to study properties of QCD?

- $\mathcal{N} = 4$ SYM at finite temperature is NOT QCD but:
  - Some features *qualitatively* similar to QCD (for $T \sim T_c$ - $3T_c$)
    - nearly conformal (very small bulk viscosity)
  - Some properties of the plasma may be <u>universal</u>

Elliptic Flow at RHIC



Off-central heavy-ion collisions at RHIC:



Anisotropic Flow

"Elliptic flow" ∼ ability of matter to flow freely locally → shear viscosity η

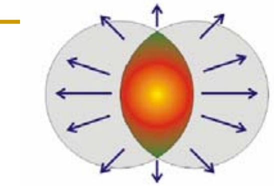
Well described by hydrodynamical calculations with very small shear viscosity/entropy density ratio -- "perfect fluid"

RHIC data favors $0 < \eta/s < 0.3$

- D. Teaney nucl-th/0301099
- Luzum, Romatschke 0804.4015
- H. Song, U.W. Heinz 0712.3715

(different fireball initial conditions)

Nearly ideal, strongly coupled QGP

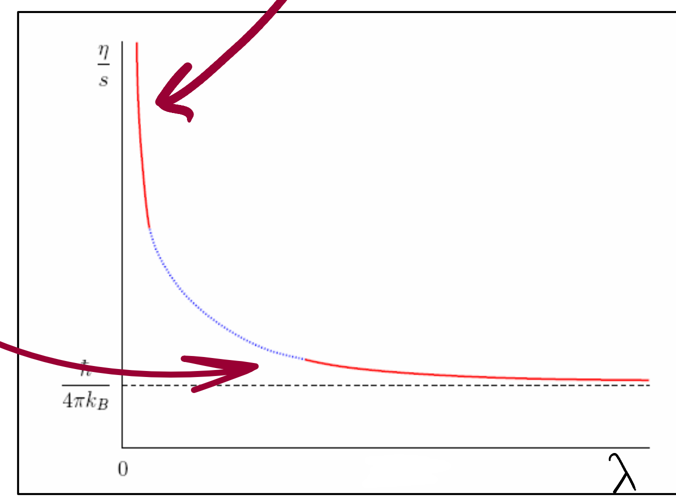


Contrast to weak coupling calculations in thermal gauge theories (Boltzmann eqn)

$$\begin{cases} \eta \sim \dfrac{N_c\, T^3}{\lambda^4 \log 1/\lambda^2} \\ S \sim N_c T^3 V_3 \end{cases} \Longrightarrow \boxed{\frac{\eta}{s} \sim \frac{1}{\lambda^4 \log 1/\lambda^2} \gg 1}$$

Weak Coupling Prediction

$\eta/s << 1 \longrightarrow$ **Strong Coupling Regime**



Small η implies a short λ_{mfp}
for momentum transport in the fluid
(hot matter equilibrates very quickly)

Strong coupling → natural setting for AdS/CFT applications

Shear Viscosity/Entropy Bound

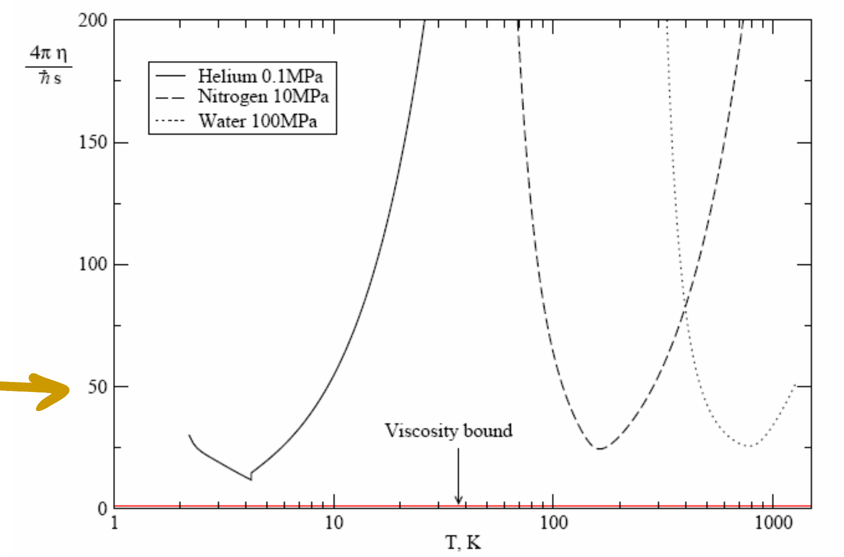
Evidence from AdS/CFT:

- **Conjectured lower bound for field theory at finite T (Kovtun, Son, Starinets 0309213)**

$$\boxed{\frac{\eta}{s} \geq \frac{1}{4\pi}}$$

Fundamental in nature?

lower than any observed fluid



- **Gauge theories with Einstein GR dual saturate the bound (Buchel, Liu th/0311175)**

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi} \sim .08}$$

UNIVERSAL

$\lambda, N \to \infty$

The RHIC value is at most a few times

Corrections to the Bound

Bound saturated in leading SUGRA approximation

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi} \sim .08}$$

String theory corrections ?

- **Leading α' correction on $AdS_5 \times S^5$ ($\mathcal{N}=4$ SYM) increased the ratio (Buchel, Liu, Starinets th/0406264)**

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + 15\zeta(3)\lambda^{-3/2} + \ldots\right]$$

$\alpha'^3 R^4$ in IIB

- **Possible bound violations ? YES**
  (Brigante et al, Buchel et al, Kats & Petrov arXiv:0712.0743)

$$S = \int d^D x \sqrt{-g}\left(\frac{R}{2\kappa} - \Lambda + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)$$

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - 4(D-4)(D-1)\frac{c_3}{L^2/\kappa}\right]$$

violation if $c_3 > 0$

Outline for rest of talk

S.C., K. Hanaki, J. Liu, P. Szepietowski
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- Explore string theory corrections (D=5 N = 2 gauged SUGRA) with finite chemical potential (SUSY corrected terms)

  $R^2 + ARR + F^4 + \ldots$

- Effects on thermodynamics and hydrodynamics (shear viscosity)

- At two-derivative level, chemical potential does not affect η/s
  With higher derivatives?

- Bound is violated AND R-charge makes violation worse $1/N$ effect

- Any connection with fundamental GR constraints?

Why explore higher derivative corrections?

$$\mathcal{L} = R - \frac{1}{2n!} F_n^2 + \ldots + \alpha' R^2 + \alpha'^2 R^3 + \alpha'^3 R^4 + \ldots$$

- **Supergravity is an *effective* low-energy description of string theory**

  → Higher derivative corrections <u>natural</u> from an EFT point of view

  - **Interesting applications to black hole physics (smoothing out singularity of small b.h.)**

- **From more "phenomenological" viewpoint:**

  Hope that corrections will bring observable quantities closer to observed values

Pathologies of higher derivative gravity?

$$\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R_{\mu\nu}^2 + \alpha_3 R_{\mu\nu\rho\sigma}^2 + \dots$$

Higher derivative corrections can lead to undesirable features:

- **Modify graviton propagator**
- **ill-poised Cauchy problem (no generalization of Gibbons-Hawking term)**

Both issues related to presence of <u>four-derivative terms</u>.

However:

- **pathologies show up only at the Planck scale**
- **Perturbative parameters** $\alpha_1, \alpha_2, \alpha_3$

D=5 $\mathcal{N}$=2 gauged SUGRA

Motivation:

- Natural setting for AdS/CFT (with 4D SUSY dual)
- D=5 black holes: rich playground (nice N=2 attractor story)

Interested in "full theory" with R-charge (chemical potential)

To leading order:

$$\mathcal{L}_0 = -R - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\lambda\sigma}F_{\mu\nu}F_{\rho\lambda}A_\sigma + 12g^2$$

gauged SUGRA coupling constant

Higher derivative corrections start at R^2: sensitive to susy

- Include graviphoton $\rightarrow$ $A \wedge Tr(R \wedge R)$
- We would like to use supersymmetric correction terms

SUSY R^2 terms in 5D

- R^2 terms in principle can be derived directly from ST
- However this would require specific choice of string compactification to 5D (Sasaki-Einstein)

- Instead make use of SUSY:
  - Susy completion of mixed gauge-gravitational CS term $A \wedge Tr(R \wedge R)$ coupled to arbitrary # of vector multiplets

hep-th/0611329
Hanaki, Ohashi, Tachikawa

<u>Superconformal Formalism for SUGRA</u>

- *Off-shell formulation of N=2, D=5 SUGRA*
- *SUSY invariants using susy tensor calculus*

End result: off shell action, lots of auxiliary fields, susy-complete R^2 term

Off-shell Lagrangian, N=2, D=5 gauged SUGRA

Physical fields $g_{\mu\nu},\ A^I_\mu,\ M^I$

$$\mathcal{N} = \tfrac{1}{6} c_{IJK} M^I M^J M^K$$

Auxiliary fields $D,\ v_{\mu\nu},\ V^{ij}_\mu,\ Y^I_{ij},\ \mathcal{A}^\alpha_i$

$$\begin{aligned}\mathcal{L}_0 = & \boxed{\tfrac{1}{4} D(2\mathcal{N} + \mathcal{A}^2)} + \boxed{R\left(\tfrac{3}{8}\mathcal{A}^2 - \tfrac{1}{4}\mathcal{N}\right)} + v^2(3\mathcal{N} - \tfrac{1}{2}\mathcal{A}^2) \\ & + 2\mathcal{N}_I v^{\mu\nu} F^I_{\mu\nu} + \mathcal{N}_{IJ}\left(\tfrac{1}{4} F^I_{\mu\nu} F^{J\,\mu\nu} - \tfrac{1}{2}\mathcal{D}^\mu M^I \mathcal{D}_\mu M^J\right) + \tfrac{1}{24} c_{IJK} \epsilon^{\mu\nu\rho\lambda\sigma} A^I_\mu F^J_{\nu\rho} F^K_{\lambda\sigma} \\ & - \mathcal{N}_{IJ} Y^I_{ij} Y^{J\,ij} + 2\left[\mathcal{D}^\mu \mathcal{A}^{\bar\alpha}_i \mathcal{D}_\mu \mathcal{A}^i_\alpha + \mathcal{A}^{\bar\alpha}_i (g\,M)^2 \mathcal{A}^i_\alpha + 2g Y^{ij}_{\alpha\beta} \mathcal{A}^{\bar\alpha}_i \mathcal{A}^\beta_j\right].\end{aligned}$$

Canonical EH term $\rightarrow \mathcal{A}^2 = -2$

D equation of motion $\rightarrow \mathcal{N} = 1$

Scalars parametrize a *very special* manifold (Kahler moduli not indep.)

Integrating out auxiliary fields

$$\mathcal{L} = -R - \tfrac{3}{4} F^2_{\mu\nu} + \tfrac{1}{4}\epsilon^{\mu\nu\rho\lambda\sigma} A_\mu F_{\nu\rho} F_{\lambda\sigma} + 12 g^2$$

Off-shell Lagrangian, Higher Derivative Terms

$$\mathcal{L}_1^{\text{ungauged}} = \frac{1}{24}c_{2I}\Big[\frac{1}{16}\epsilon_{\mu\nu\rho\lambda\sigma}A^{I\,\mu}R^{\nu\rho\alpha\beta}R^{\lambda\sigma}{}_{\alpha\beta} + \frac{1}{8}M^I C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \boxed{\frac{1}{12}M^I D^2 + \frac{1}{6}F^I_{\mu\nu}v^{\mu\nu}D}$$
$$-\frac{1}{3}M^I C_{\mu\nu\rho\sigma}v^{\mu\nu}v^{\rho\sigma} - \frac{1}{2}F^{I\,\mu\nu}C_{\mu\nu\rho\sigma}v^{\rho\sigma} + \frac{8}{3}M^I v_{\mu\nu}\nabla^\nu\nabla_\rho v^{\mu\rho}$$
$$-\frac{16}{9}M^I v^{\mu\rho}v_{\rho\nu}R^\nu_\mu - \frac{2}{9}M^I v^2 R + \frac{4}{3}M^I\nabla^\mu v^{\nu\rho}\nabla_\mu v_{\nu\rho} + \frac{4}{3}M^I\nabla^\mu v^{\nu\rho}\nabla_\nu v_{\rho\mu}$$
$$-\frac{2}{3}M^I\epsilon_{\mu\nu\rho\lambda\sigma}v^{\mu\nu}v^{\rho\lambda}\nabla_\delta v^{\sigma\delta} + \frac{2}{3}F^{I\,\mu\nu}\epsilon_{\mu\nu\rho\lambda\sigma}v^{\rho\delta}\nabla_\delta v^{\lambda\sigma} + F^{I\,\mu\nu}\epsilon_{\mu\nu\rho\lambda\sigma}v^\rho{}_\delta\nabla^\lambda v^{\sigma\delta}$$
$$-\frac{4}{3}F^{I\,\mu\nu}v_{\mu\rho}v^{\rho\lambda}v_{\lambda\nu} - \frac{1}{3}F^{I\,\mu\nu}v_{\mu\nu}v^2 + 4M^I v_{\mu\nu}v^{\nu\rho}v_{\rho\lambda}v^{\lambda\mu} - M^I(v^2)^2\Big],$$

$$\mathcal{L}_1^{\text{gauged}} = \frac{1}{24}c_{2I}\Big[-\frac{1}{12}\epsilon_{\mu\nu\rho\lambda\sigma}A^{I\,\mu}R^{\nu\rho\,ij}(U)R^{\lambda\sigma}_{ij}(U)$$
$$-\frac{1}{3}M^I R^{\mu\nu\,ij}(U)R_{\mu\nu\,ij}(U) - \frac{4}{3}Y^I_{ij}v_{\mu\nu}R^{\mu\nu\,ij}(U)\Big],$$

***c*$_{2I}$**	**effective expansion parameters (control strengh of corrections)**

***D* variation** $\longrightarrow$ $\boxed{\mathcal{N} = 1 - \frac{c_{2I}}{72}(DM^I + F^{I\,\mu\nu}v_{\mu\nu})}$

***modified* very special geometry constraint**

On-shell Lagrangian (minimal SUGRA)

arXiv:0812.3572

S.C., K. Hanaki, J.Liu, P. Szepietowski

Truncation to minimal SUGRA

$$M^I = \bar{M}^I + c_2 \hat{M}^I, \qquad A^I_\mu = \bar{M}^I A_\mu, \qquad c_2 \equiv c_{2I}\bar{M}^I$$

Graviphoton

$$\begin{aligned}\mathcal{L} &= -R - \frac{1}{4}F^2 + \frac{1}{12\sqrt{3}}\left(1 - \frac{1}{6}c_2 g^2\right)\epsilon^{\mu\nu\rho\lambda\sigma}A_\mu F_{\nu\rho}F_{\lambda\sigma} + 12g^2 \\ &\quad + \frac{c_2}{24}\left[\frac{1}{48}RF^2 + \frac{1}{576}(F^2)^2\right] + \mathcal{L}_1^{\text{ungauged}},\end{aligned}$$

$$\begin{aligned}\mathcal{L}_1^{\text{ungauged}} = \frac{c_2}{24}\Big[&\frac{1}{16\sqrt{3}}\epsilon_{\mu\nu\rho\lambda\sigma}A^\mu R^{\nu\rho\delta\gamma}R^{\lambda\sigma}{}_{\delta\gamma} + \frac{1}{8}C^2_{\mu\nu\rho\sigma} + \frac{1}{16}C_{\mu\nu\rho\lambda}F^{\mu\nu}F^{\rho\lambda} - \frac{1}{3}F^{\mu\rho}F_{\rho\nu}R^\nu_\mu \\ &- \frac{1}{24}RF^2 + \frac{1}{2}F_{\mu\nu}\nabla^\nu\nabla_\rho F^{\mu\rho} + \frac{1}{4}\nabla^\mu F^{\nu\rho}\nabla_\mu F_{\nu\rho} + \frac{1}{4}\nabla^\mu F^{\nu\rho}\nabla_\nu F_{\rho\mu} \\ &+ \frac{1}{32\sqrt{3}}\epsilon_{\mu\nu\rho\lambda\sigma}F^{\mu\nu}(3F^{\rho\lambda}\nabla_\delta F^{\sigma\delta} + 4F^{\rho\delta}\nabla_\delta F^{\lambda\sigma} + 6F^\rho{}_\delta\nabla^\lambda F^{\sigma\delta}) \\ &+ \frac{5}{64}F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} - \frac{5}{256}(F^2)^2\Big].\end{aligned}$$

Physical Meaning of c_2 ?

- **Parameters of 5D SUGRA action contain info about 10D string theory description**

$$\mathcal{L} = \frac{1}{16\pi G_5}\left[-R - \frac{1}{4}F^2 + \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\lambda\sigma}A_\mu F_{\nu\rho}F_{\lambda\sigma} + 12g^2 + \frac{c_2}{192}C^2_{\mu\nu\rho\sigma} + \cdots\right]$$

- **Ungauged case (e.g. D=11 SUGRA on CY_3) c_2 related to topological data**

  c_{2I} is 2nd Chern class

$$\int_{M_{11}} C_3 \wedge \left[(\mathrm{Tr}R^2)^2 - 4\,\mathrm{Tr}R^4\right] \longrightarrow \int_{M_5} c_{2I}\, A^I \wedge R \wedge R$$

- **Gauged case:**

  $c_2 = 0$ for IIB on S^5 (no R^2 terms with maximal sugra)

  For us: IIB on Sasaki-Einstein → meaning of c_2 less clear

We can use AdS/CFT to relate c_2 to central charges of dual CFT via:

- **Holographic trace anomaly**
- **R-current anomaly** ← see paper

Using the dual CFT ($\mathcal{N}$=1)

- **4D CFT central charges $\boxed{a\,,\,c}$ defined in terms of trace anomaly:**
  (CFT coupled to external metric)

$\text{Weyl}^2 = R^2_{\mu\nu\rho\sigma} - 2R^2_{\mu\nu} + \frac{1}{3}R^2$

$$\langle T^\mu_\mu \rangle = \frac{c}{16\pi^2} C - \frac{a}{16\pi^2} E$$

$R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$

$$\langle T^\mu_\mu \rangle = \frac{1}{16\pi^2}\left[\left(\frac{c}{3} - a\right) R^2 + (4a - 2c) R^2_{\mu\nu} + (c - a) R^2_{\mu\nu\rho\sigma}\right]$$

$$\Updownarrow$$

$$\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R^2_{\mu\nu} + \alpha_3 R^2_{\mu\nu\rho\sigma} + \ldots$$

sensitive to higher derivative corrections

Extracting c_2 : the holographic trace anomaly

- **Prescription for obtaining trace anomaly for higher derivative gravity**

Blau, Narain, Gava (th/9904179), Nojiri, Odintsov (th/9903033)

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(-R + 12g^2 + \alpha R^2 + \beta R_{\mu\nu}^2 + \gamma R_{\mu\nu\rho\sigma}^2 + \cdots\right)$$

$$\langle T^\mu_\mu \rangle = \frac{2L}{16\pi G_5}\left[\left(-\frac{L}{24} + \frac{5\alpha}{3} + \frac{\beta}{3} + \frac{\gamma}{3}\right)R^2 + \left(\frac{L}{8} - 5\alpha - \beta - \frac{3\gamma}{2}\right)R_{\mu\nu}^2 + \frac{\gamma}{2}R_{\mu\nu\rho\sigma}^2\right]$$

$$g = \frac{1}{L}\left[1 - \frac{1}{6L^2}(20\alpha + 4\beta + 2\gamma)\right]$$

$$a = \frac{\pi L^3}{8G_5}, \qquad c = \frac{\pi L^3}{8G_5}\left(1 + \frac{c_2}{24L^2}\right), \qquad g = \frac{1}{L} \quad \Rightarrow \quad c_2 = \frac{24}{g^2}\,\frac{c-a}{a}$$

R-charged black holes

- **Lowest order theory admits a two-parameter family of solutions** [Behrndt, Cvetic, Sabra]

$$ds^2 = H^{-2} f dt^2 - H\left(f^{-1}dr^2 + r^2 d\Omega_{3,k}^2\right) \qquad H(r) = 1 + \frac{Q}{r^2},$$

$$A = \sqrt{\frac{3(kQ+\mu)}{Q}}\left(1 - \frac{1}{H}\right)dt, \qquad f(r) = k - \frac{\mu}{r^2} + g^2 r^2 H^3$$

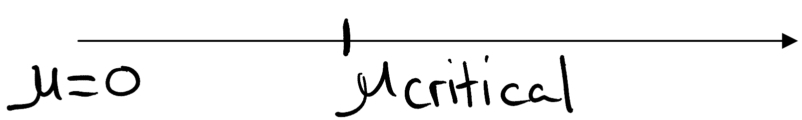
Q R-charge

μ non-extremality parameter

- **Special cases:**
  - **k=1, μ=0 : BPS solution, naked singularity (superstar)**

    Horizon formation with higher derivatives?

    Possible, BUT cannot trust perturbative analysis



  - **k=0, finite μ : near-extremal D3 brane → use for thermo + hydro analysis**

Thermodynamics

- **Einstein GR: entropy → area of event horizon**
- **Higher derivative terms → area law is modified** ⟶ $\boxed{S = 2\pi \int_\Sigma d^{D-2}x\sqrt{-h}\,\frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}}\epsilon_{\mu\nu}\epsilon_{\rho\sigma}}$

$$S = \frac{A}{4G_5}\left[1 + c_2 g^2 \frac{Q + 3r_0^2}{48\, r_0^2}\right] \qquad T \sim T_0 + c_2 \delta T$$

corrected horizon area

- **Entropy in terms of dual CFT central charges**

$$\boxed{s = \frac{2a(r_0^2 + Q)^{3/2}}{\pi L^6}\left(1 + \frac{c-a}{a}\,\frac{3Q^2 - 14Qr_0^2 - 21r_0^4}{8r_0^2(Q - 2r_0^2)}\right)}$$

$$s = \frac{2a(r_0^2+Q)^{3/2}}{\pi L^6}\left(1+\frac{c-a}{a}\frac{3Q^2-14Qr_0^2-21r_0^4}{8r_0^2(Q-2r_0^2)}\right)$$

non-trivial T dependence

- **No R-charge** → $S \sim T^3$
- **R charge introduces a new scale** $\longrightarrow S = S(T, Q)$

- **Zero R-charge:**

$$s = 2\pi^2 a T^3 \left(1+\frac{9}{4}\frac{c-a}{a}\right)$$

$$\Downarrow$$

- ***N* = 4 SYM:**

Free field result

$$\boxed{a = c \quad \Rightarrow \quad s = 2\pi^2 a T^3 = \frac{3}{4} s_0}$$

Hydrodynamics

0903.3244 and 0903.2834

- Want to go beyond description of equilibrium (thermodynamic) quantities

- Long-distance, low-frequency behavior of any interacting theory at finite temperature is described by hydrodynamics

effective description of dynamics of the system
at large wavelengths and long time scales

→ Use AdS/CFT to describe transport phenomena

Relativistic Hydrodynamics:

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - \sigma^{\mu\nu}$$

SHEAR

BULK

$$\sigma_{ij} = \eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u^k\right) + \zeta\delta_{ij}\partial_k u^k$$

Shear Viscosity

- η **can be extracted from certain correlators of the boundary $T_{\mu\nu}$:**
  (Kubo's formula: retarded Green's fn of stress tensor)

$$G^R_{xy,xy}(\omega,\mathbf{0}) = \int dt\, d\mathbf{x}\, e^{i\omega t}\theta(t)\langle[T_{xy}(t,\mathbf{x}),\, T_{xy}(0,\mathbf{0})]\rangle = -i\eta\omega + O(\omega^2)$$

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\,\mathrm{Im}\, G^R_{xy,xy}(\omega,\mathbf{0})$$

- **Use (Minkowski) modification of standard AdS/CFT recipe (Son & Starinets):**

$$\left\langle \exp\left(\int d^{D-1}x\, \phi_0(x) T_{12}(x)\right)\right\rangle_{\mathrm{CFT}} = e^{-S_{\mathrm{SG}}[\phi]}\Big|_{\mathrm{AdS}}$$

- **AdS/CFT dictionary: source for $T_{\mu\nu}$ is the metric**

→ **Set up appropriate metric perturbations**

$$g_{xy} \to g_{xy} + h_{xy}$$

Bound Violation

COMBINE

$$s = \frac{2a(r_0^2+Q)^{3/2}}{\pi L^6}\left(1+\frac{c-a}{a}\,\frac{3Q^2-14Qr_0^2-21r_0^4}{8r_0^2(Q-2r_0^2)}\right)$$

$$\eta \sim \frac{(1+Q)^{3/2}}{16\pi}\left[1-\frac{c-a}{a}\,\frac{5Q^2+6Q+5}{8(Q-2)}\right]$$

$$\Longrightarrow \boxed{\frac{\eta}{s} = \frac{1}{4\pi}\left[1-\frac{c-a}{a}(1+Q)\right]}$$

Suprisingly simple dependence on R-charge: some form of universality?

- Bound violated for c - a > 0
- R-charge makes violation worse

$$0 \le Q \le 2 \quad \Longrightarrow \quad \boxed{\frac{1}{4\pi}\left(1-3\frac{c-a}{a}\right) \le \frac{\eta}{s} \le \frac{1}{4\pi}\left(1-\frac{c-a}{a}\right)}$$

Violation is SMALL !

Violation is 1/N correction

- **Correction is 1/N**

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + \frac{c-a}{a}(1+Q)\right]}$$

- **Contrast to IIB on AdS$_5$ x S^5** $\longrightarrow \alpha'^3 R^4 \longrightarrow \lambda^{-3/2}$

- **For N = 4 SYM** $a = c$ → **no R^2 corrections** $(AdS_5 \times S^5)$
- **In general** $a = c = \mathcal{O}(N^2)$ **and** $\frac{c-a}{a} \sim \frac{1}{N}$

Which higher derivative terms matter?

$$\mathcal{L} = -R - \frac{1}{4}F^2 + \cdots + \frac{c_2}{g^2}\Big[\alpha_1 C^2_{\mu\nu\rho\sigma} + \alpha_2 C_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

$$+\left(\frac{\alpha_4}{2} - \alpha_3\right) R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \quad + \cancel{\alpha_3 \nabla^\mu F^{\nu\rho}\nabla_\mu F_{\nu\rho} + \alpha_4 \nabla^\mu F^{\mu\rho}\nabla_\nu F_{\rho\mu}} + \cdots\Big]$$

$$\Rightarrow \quad \boxed{\frac{\eta}{s} = \frac{1}{4\pi}\Big[1 - 4\bar{c}_2\big(2\alpha_1 - q(\alpha_1 + 6\tilde{\alpha}_2)\big)\Big]} \quad \text{Riem } F^2$$

Only terms with explicit dependence on Riemann tensor

- **Having SUSY completion of higher derivative terms did not play much of a role**
- **Shear viscosity seems to depend only on horizon data** $\rightarrow \partial_r \Pi = 0 + \mathcal{O}(\omega^2)$
- **Analog of Wald's entropy for shear viscosity?**

Brustein's proposal → ***effective gravitational coupling***

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}\frac{k^2_{rt}}{k^2_{xy}}}$$

$$\boxed{\frac{1}{k^2_{\mu\nu}} \sim \frac{\delta \mathcal{L}}{\delta R_{ab}{}^{cd}}\,\epsilon_{ab}\epsilon^{cd}}$$

Sign of c-a ?

- Bound is <u>always</u> violated if c-a > 0

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{c-a}{a}(1+Q)\right]}$$

- CFTs give <u>both</u> c-a < 0 and c-a > 0

- So far no CFT examples with stringy/SUGRA dual with c-a < 0
  Not enough examples studied?

- Is gravity somehow <u>constraining</u> the sign of c-a to be positive ?
  Constraining CFTs?
  Landscape/swampland?

$c_2 \, \text{Riemann}^2$

Final Remarks - I

- AdS/CFT : playground to explore strongly coupled field theories (new set of tools)

- GR higher derivative corrections associated with finite N and λ corrections
  → phenomenological interest (e.g. QGP)

- Shear viscosity bound is violated (1/N correction from R^2 terms)

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{\#}{N}(1+Q) + \frac{\#}{\lambda^{3/2}} + \cdots\right]$$

"Phenomenological approach" :

- (R-charged) Chemical potential: one more parameter we can "tune"
  - Caveat: small baryon number chemical potential at RHIC $\frac{\mu_B}{T} \lesssim 0.15$

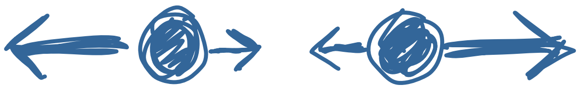
- "Scan" CFT landscape by considering various corrections and "tune" parameters to better agree with data? $\lambda,\ N,\ Q$

Final Remarks - II

Interesting *fundamental* questions:

- **Can we relate bound violation to constraints on GR side ?**

  Gravity is the weakest force (Vafa et al, AH et al.)

  - **Unnatural to have infinite number of exactly stable particles**
  - **There must be particles with smaller M/Q than extremal b.h.**
  - **Higher derivative corrections to M/Q ? Kats et al. th/0606100**

$$\frac{M}{|Q|} = 1 - \frac{2}{5q^2}\left(2c_2 + 8c_3 + \frac{2c_5}{\kappa^2} + \frac{2c_6}{\kappa^2} + \frac{8c_7}{\kappa^4} - \frac{2c_8}{\kappa^2} - \frac{c_9}{\kappa^2}\right)$$

  - **Our solutions don't have a nice extremal b.h. limit (superstar)**

- **Analog of Wald's entropy for shear viscosity? (Brustein's proposal)**
- **SUGRA/stringy constraints on sign of c-a and allowed CFTs ?**

The End